Title: QFT2 Lecture - 111723

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Collection: Quantum Field Theory 2 2023/24

Date: November 17, 2023 - 9:00 AM

URL: https://pirsa.org/23110020

Euclidean $(++++)$

$$S_E[\phi] = \int d^d x \; \frac{1}{2}(\partial_\mu \phi)^2 + V(\phi)$$

$$Z_E[j] = \int D[\phi] \; \exp\left(-\frac{1}{\hbar}\left(S_E[\phi] - j\cdot\phi\right)\right)$$

$$W_E[j] = \hbar \operatorname{Log} Z_E[j]$$

Connected correlation (Green) Functions

j classical ϕ quantum

Minkowski $(-+++)$ East Coast

$$S[\phi] = \int d^d x \; \frac{1}{2}(\partial_t \phi)^2 - \frac{1}{2}(\partial_{\vec{x}} \phi)^2 - V(\phi)$$

$$Z[j] = \int D[\phi] \; \exp\left(\frac{i}{\hbar}\left(S[\phi] + j\cdot\phi\right)\right)$$

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Classical limit $\hbar \to 0$

E.O.M $\quad \dfrac{\delta S[\phi]}{\delta \phi(x)} = 0 \Rightarrow \phi(x)$ solution of e.o.m.

$\phi(x) \Rightarrow \phi(x) + \delta\phi(x)$

$\cdot\phi))$

Quantum : E.O.M ? Action ?

Schwinger-Dyson (DS) Equations

Euclidean $(++++)$

$$S_E[\phi] = \int d^d x \ \frac{1}{2}(\partial_\mu \phi)^2 + V(\phi)$$

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$$S[\phi] = \int d^d x \ \frac{1}{2}(\partial_t \phi)^2 - \frac{1}{2}(\partial_{\vec{x}} \phi)^2 - V(\phi)$$

$$Z[j] = \int D[\phi] \ \exp\left(\frac{i}{\hbar}\left(S[\phi] + \underbrace{j\cdot\phi}_{\int d^d x \ j(x)\phi(x)}\right)\right)$$

② $\phi(x) \longrightarrow \underset{\substack{\uparrow \\ \text{integrated} \\ \text{over}}}{\phi(x)} + \underbrace{\delta\phi(x)}_{\text{classical variation}} = \phi'(x)$

$\delta\phi(x)$ is choosen before performing the integration over ϕ

$$Z[j] = \int D[\phi'] \exp\left(-\frac{1}{\hbar}\left(S[\phi'] - j\cdot\phi'\right)\right)$$

(a). $S[\phi'] = S[\phi] + \underbrace{\underline{\delta\phi} \cdot \frac{\delta S[\phi]}{\delta\phi}}_{\int d^d x\, \delta\phi(x)\cdot\frac{\delta S[\phi]}{\delta\phi(x)}} + \dots$, (b). $j\cdot\phi' = j\phi + j\cdot\underline{\delta\phi}$

(c) $D[\phi'] = \prod_x d\phi'(x) = \prod_x d\phi(x) = D[\phi]$

$\uparrow$ measure Lebesgue on $\mathbb{R}$

local observable

of course $\delta Z[j] = 0$ for all $\delta\phi(x)$

$$\delta Z[j] = \left(-\frac{1}{\hbar}\right)\int d^dx\, \delta\phi(x) \underbrace{\int D[\phi]\, e^{-\frac{1}{\hbar}(S[\phi]-j\cdot\phi)} \left(\frac{\delta S[\phi]}{\delta\phi(x)} - j(x)\right)}_{\text{this is } 0 \text{ for all } x} = 0$$

$$\left\langle \frac{\delta S[\phi]}{\delta\phi(x)} - j(x) \right\rangle_{j\neq 0} = 0$$

$j \neq 0$ not yet fixed

v.e.v for a QFT with action $S[\phi] - j\cdot\phi$ which has a vaccum state $|\Omega[j]\rangle$

$D[\phi]$

J classical $\quad \phi$ quantum

$$\frac{\delta S[\phi]}{\delta \phi(x)} = -\Delta_x \phi(x) + V'(\phi(x)) \quad \overset{SD.}{\Rightarrow} \quad -\Delta_{x_0} \langle \phi(x_0) \rangle_j + \langle V'(\ldots$$

$\uparrow$ Laplace operator

$$N=1 \quad \frac{\delta}{\delta j(x_1)} SD(x_0)\Big|_{j=0} = -\Delta_{x_0} \langle \phi(x_0)\phi(x_1) \rangle + \langle V'(\phi(x_0)) \cdot \phi(x_1) \rangle - \delta^d(\ldots$$

classical contribution

$$\frac{\delta}{\delta j(x_1)} \cdots \frac{\delta}{\delta j(x_N)} \; " \; = \boxed{-\Delta_{x_0} \langle \phi(x_0)\, \phi(x_1) \cdots \phi(x_N) \rangle + \langle V'(\phi(x_0))\, \phi(x_1) \cdots \phi(x_N) \rangle \ldots}$$

qua…

$$x_0 \langle \phi(x_0) \rangle_j + \langle V'(\phi(x_0)) \rangle_j - \hbar\, j(x_0) = 0$$

equation between v.e.v of operators

Dirac distribution $\Leftarrow \dfrac{\delta j(x_0)}{\delta j(x_1)} = \delta(x_0 - x_1) \leftarrow \dfrac{\partial x_i}{\partial x_j} = \delta_{ij}$

$$\phi(x_0) \cdot \phi(x_1) \rangle - \hbar\, \delta^d(x_1 - x_0) = 0$$

quantum

$$(\phi(x_0))\, \phi(x_1) \cdots \phi(x_N) \rangle - \hbar \boxed{\sum_{i=1}^{N} \delta^d(x_i - x_N) \langle \phi(x_1)\, \cancel{\phi(x_i)}\, \phi(x_N) \rangle}$$

j classical ϕ quantum

$$\frac{\delta S[\phi]}{\delta\phi(x)} = -\Delta_x \phi(x) + V'(\phi(x)) \quad \overset{SD.}{\Rightarrow} \quad -\Delta_{x_0} \langle \phi(x_0) \rangle_j + \langle V$$

↑ Laplace operator

$$N=1 \quad \frac{\delta}{\delta j(x_1)} SD(x_0) \Big|_{j=0} = -\Delta_{x_0} \langle \phi(x_0)\phi(x_1) \rangle + \langle V'(\phi(x_0))\,\phi(x_1) \rangle - \hbar\,\delta($$

classical contribution

$$\frac{\delta}{\delta j(x_1)} \cdots \frac{\delta}{\delta j(x_N)} \;\; " \;\; = \boxed{-\Delta_{x_0} \langle \phi(x_0)\,\phi(x_1)\cdots\phi(x_N) \rangle + \langle V'(\phi(x_0))\,\phi(x_1)\cdots\phi(x_N)}$$

qu

③

$$\phi(x_0)\rangle_j + \langle V'(\phi(x_0))\rangle_j - \hbar\, j(x_0) = 0$$

equation between v.e.v of operators

Dirac distribution $\Leftarrow \dfrac{\delta j(x_0)}{\delta j(x_1)} = \delta(x_0 - x_1) \leftarrow \dfrac{\partial x_i}{\partial x_j} = \delta_{ij}$

$$\phi(x_1)\rangle - \hbar\,\delta^d(x_1 - x_0) = 0$$

quantum

$$x_0))\phi(x_1)\cdots\phi(x_N)\rangle - \hbar \boxed{\sum_{i=1}^{N}\delta^d(x_0 - x_i)\langle\phi(x_1)\,\cancel{\phi(x_i)}\,\phi(x_N)\rangle}$$

Free Theory $V(\phi) = \frac{m^2}{2}\phi^2 \Rightarrow V'(\phi(x)) = m^2\phi(x)$

$N=1$

$$\boxed{\left(-\Delta_{x_0} + m^2\right)\langle\phi(x_0)\phi(x_1)\rangle_0 = \hbar\,\delta(x_0 - x_1)}$$

$\uparrow g=0$

Free propagator in E-space

$$\langle\phi(x_0)\,\phi(x_1)\rangle = \int\frac{d^dk}{(2\pi)^d}\frac{e^{ik\cdot x}}{k^2+m^2}\,\hbar$$

$$\prod_x d\phi'(x) = \prod_x d\phi(x) = D[\phi]$$

measure Lebesgue on $\mathbb{R}$

$\boxed{\text{S.D equations} \quad \text{not yet fixed}}$

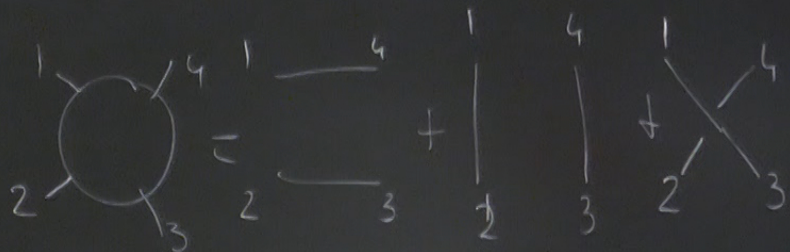
state $|\Omega[j]\rangle$

$N=3$ 4pt 2pt functions

$$(-\Delta_{x_0}+m^2)\langle\phi(x_0)\phi(x_1)\phi(x_2)\phi(x_3)\rangle_0 = \hbar\Big(\delta(x_0-x_1)\langle\phi(x_2)\phi(x_3)\rangle + \delta(x_0-x_3)\langle\phi(x_1)\phi(x_3)\rangle$$

$$+\ \delta(x_0-x_3)\langle\phi(x_1)\phi(x_2)\rangle$$

$$= \langle\phi(x_0)\phi(x_1)\rangle\langle\phi(x_2)\phi(x_3)\rangle + \langle\phi(x_0)\phi(x_3)\rangle\langle\phi(x_1)\phi(x_2)\rangle + \langle\phi(x_0)\phi(x_2)\rangle \times \langle\phi(x_1)\phi(x_3)\rangle$$



$\boxed{\text{Wick Theorem} \Leftarrow \text{DS equations}}$

$$\int D[\phi]\, \exp\left(-\frac{1}{\hbar}\left(S[\phi]-j\cdot\phi\right)\right)\left(-\Delta_{x_0}\phi(x_0)+V'(\phi(x_0))-j(x_0)\right)=0$$

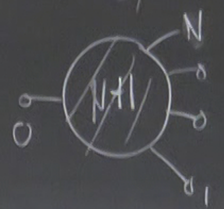
$$\hbar\frac{\delta}{\delta j(x_1)}\left[\quad '' \quad\right]$$

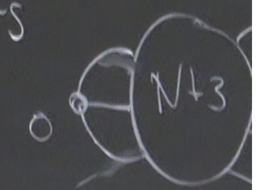
$$\phi(x_1) \qquad\qquad \hbar\,\delta(x_1-x_0)$$

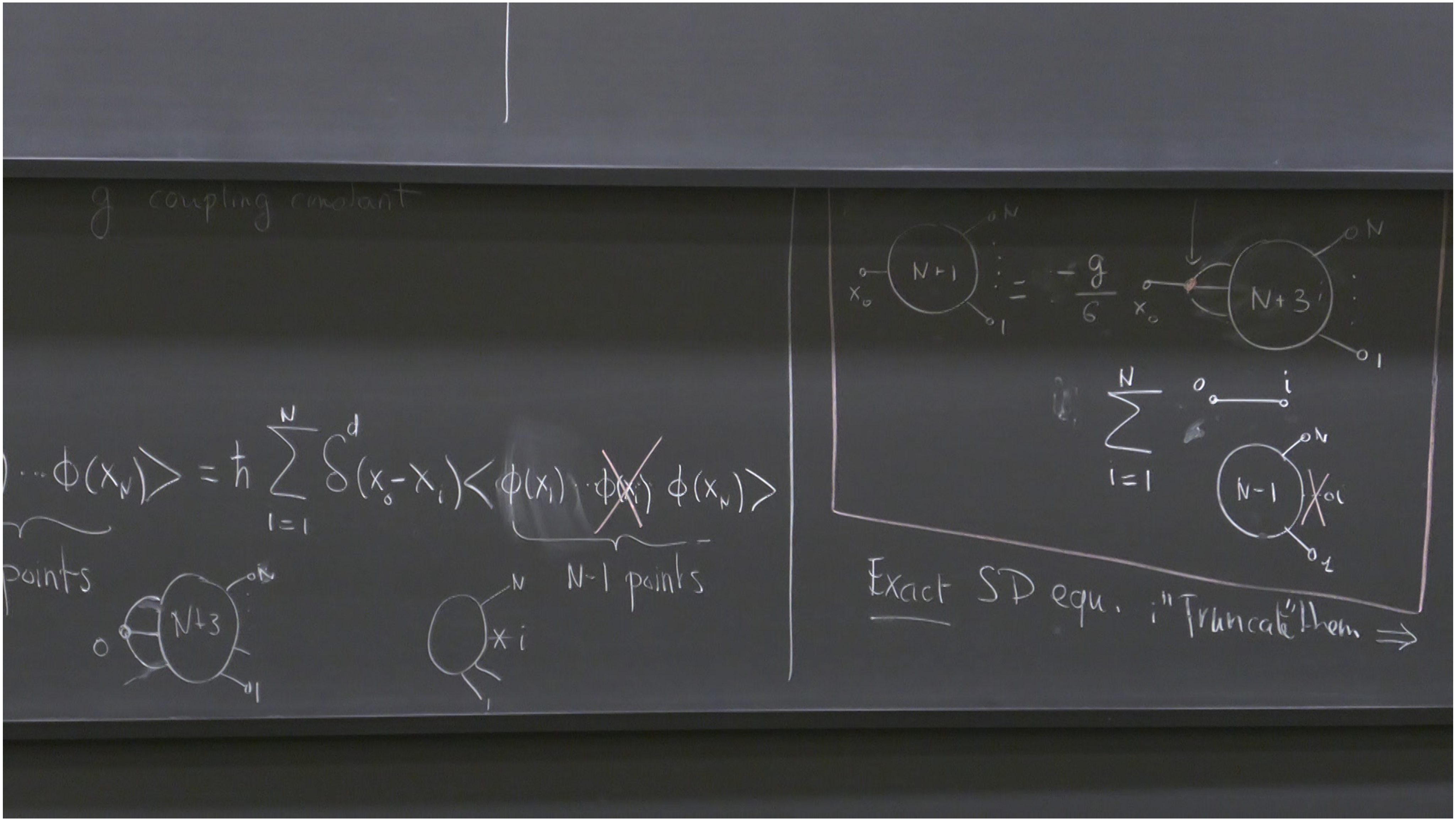
(5) Interactions ϕ $\quad V(\phi) = \frac{m}{2}\phi + \frac{g}{4!}\phi$ $\quad g$ coupl

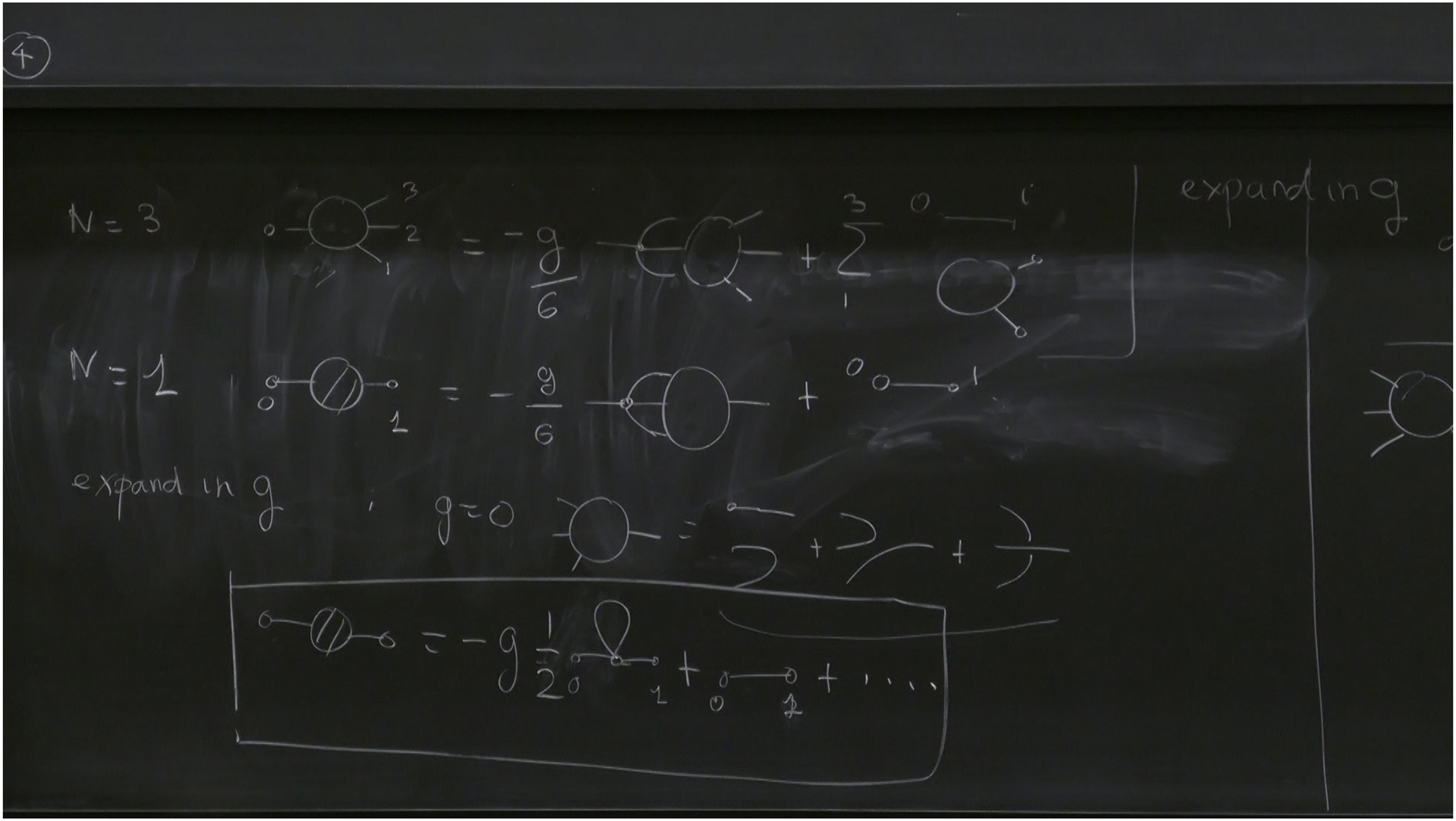
$$V'(\phi) = m^2\phi + \frac{g}{6}\phi^3$$

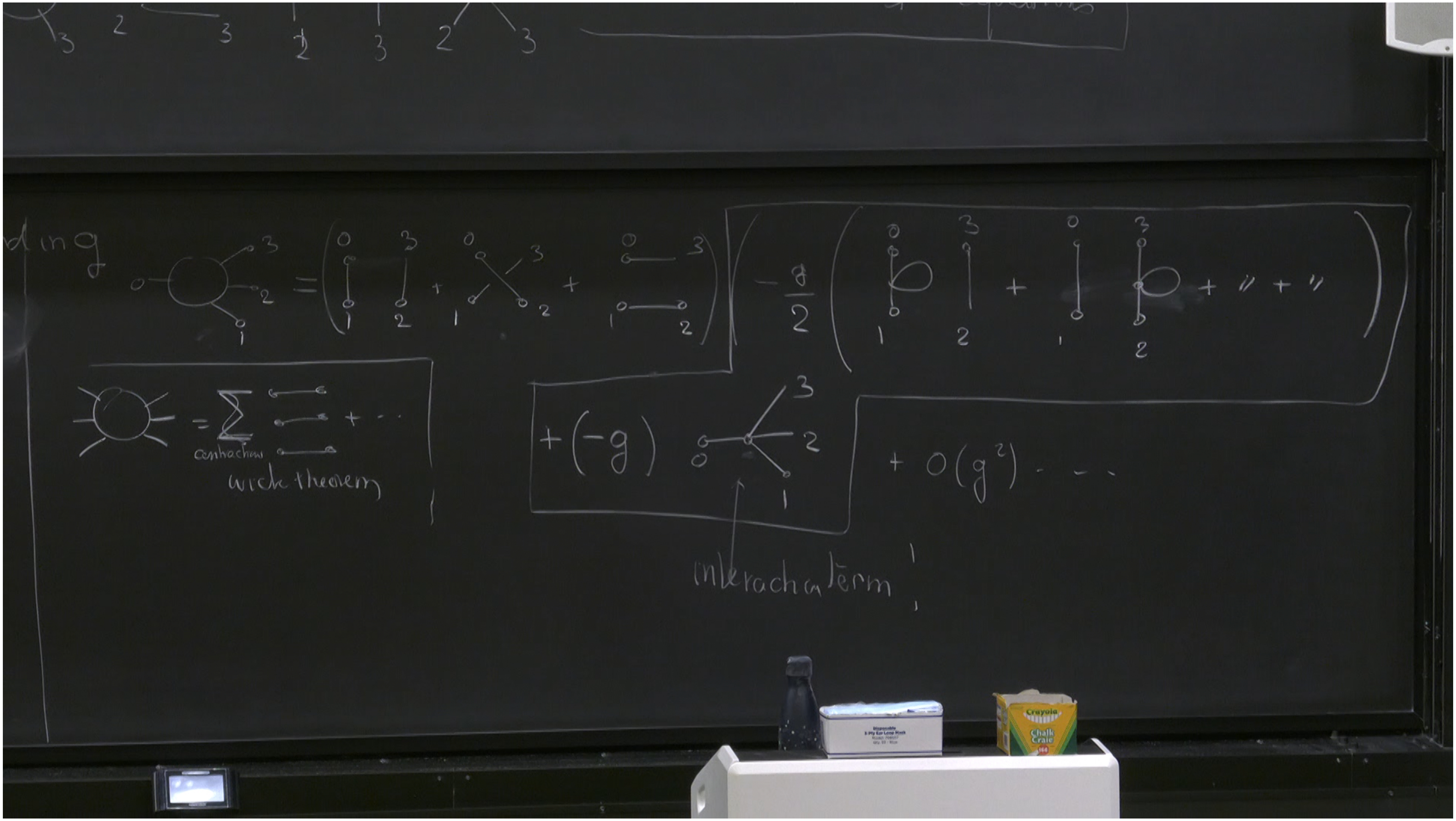
$$\left(-\Delta_{x_0} + m^2\right)\langle \underbrace{\phi(x_0)\,\phi(x_1)\cdots\phi(x_N)}_{N+1 \text{ points}} \rangle + \frac{g}{6}\langle \underbrace{\phi(x_0)^3\,\phi(x_1)\cdots\phi(x_N)}_{N+3 \text{ points}} \rangle =$$



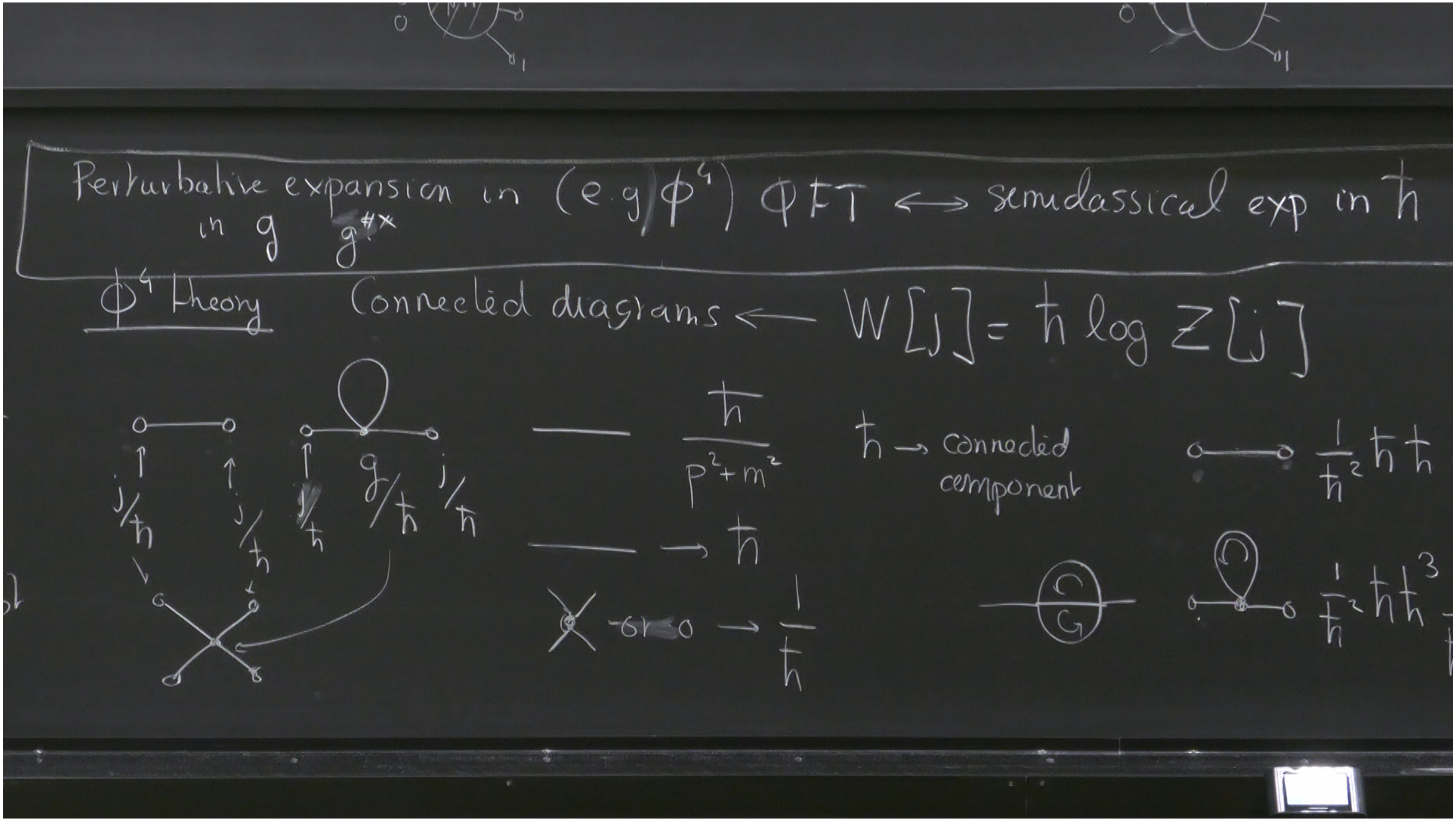



g coupling constant
$\cdots\phi(x_N)\rangle = \hbar \sum_{i=1}^{N} \delta^d(x_0 - x_i)\langle \phi(x_1)\cdots\cancel{\phi(x_i)}\cdots\phi(x_N)\rangle$
points
N-1 points
N+3
N+1
$= -\frac{g}{6}$
x_0
$\sum_{i=1}^{N}$
N-1
Exact SD equ. ; "Truncate" them ⇒

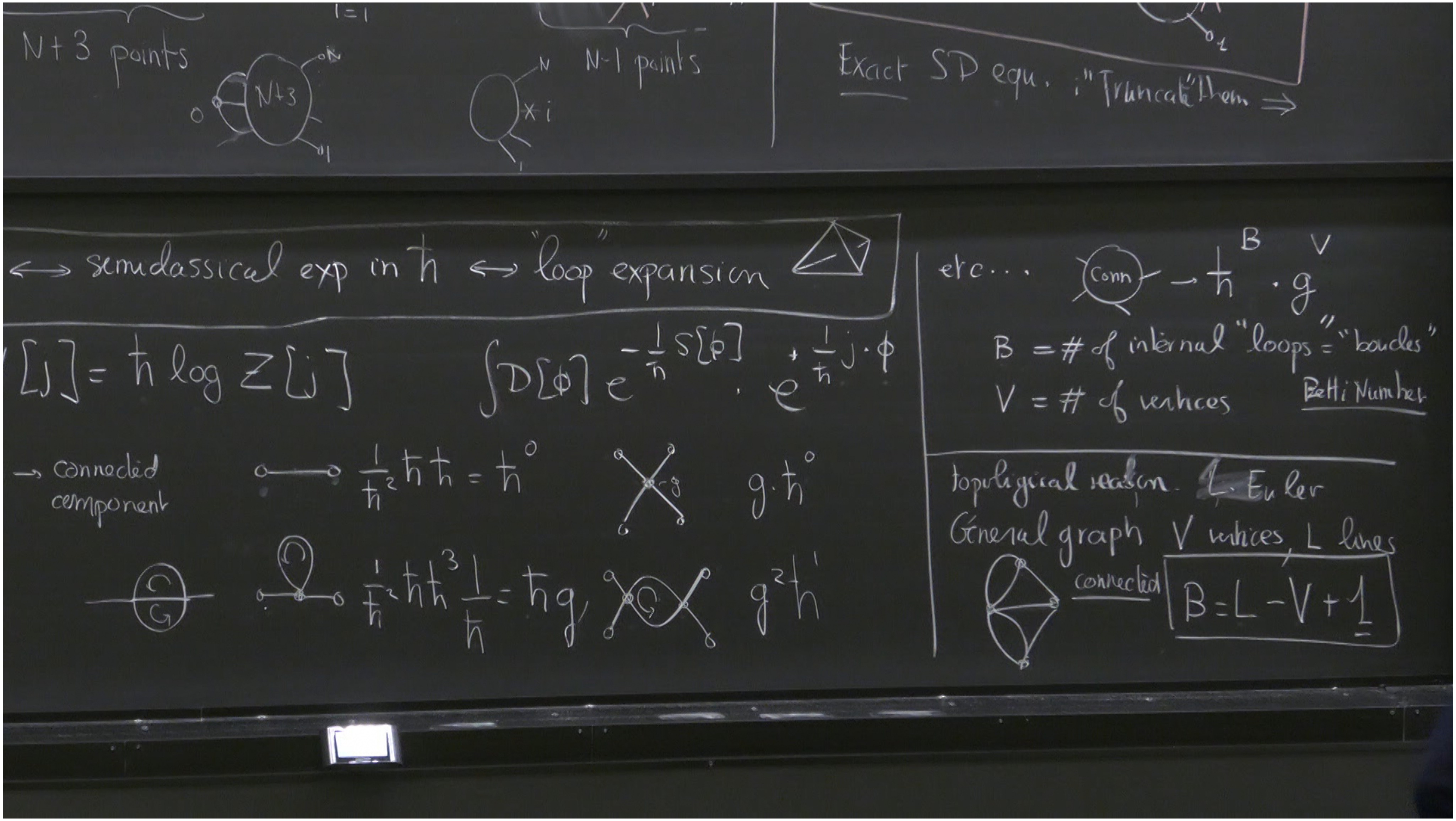

N = 3
= − g/6
N = 1
= − g/6
expand in g
g = 0
= − g 1/2
expanding



ding
contraction
wick theorem
$-\frac{g}{2}$
$+(-g)$
$+ O(g^2) \ldots$
interaction term



Perturbative expansion in (e.g. φ⁴) QFT ⟷ semiclassical exp in ħ
in g   g^{#x}
φ⁴ theory
Connected diagrams ⟵ W[j] = ħ log Z[j]
j/ħ   j/ħ   j/ħ   g/ħ   j/ħ
ħ / (p² + m²)
→ ħ
-or- o → 1/ħ
ħ → connected component
1/ħ² ħ ħ
1/ħ² ħ ħ³


N+3 points
N+3
N-1 points
Exact SD equ. ; "Truncate" them ⇒
⟷ semiclassical exp in ℏ ⟷ "loop" expansion
[j] = ℏ log Z[j]
∫D[φ] e^{-1/ℏ S[φ]} · e^{1/ℏ j·φ}
→ connected component
1/ℏ² ℏ ℏ = ℏ⁰
g·ℏ⁰
1/ℏ² ℏ ℏ³ 1/ℏ = ℏ g
g² ℏ¹
etc... Conn → ℏ^B · g^V
B = # of internal "loops" = "boucles"
V = # of vertices
Betti Number
topological reason. Euler
General graph V vertices, L lines
connected
B = L - V + 1