Title: QFT2 Lecture - 111423

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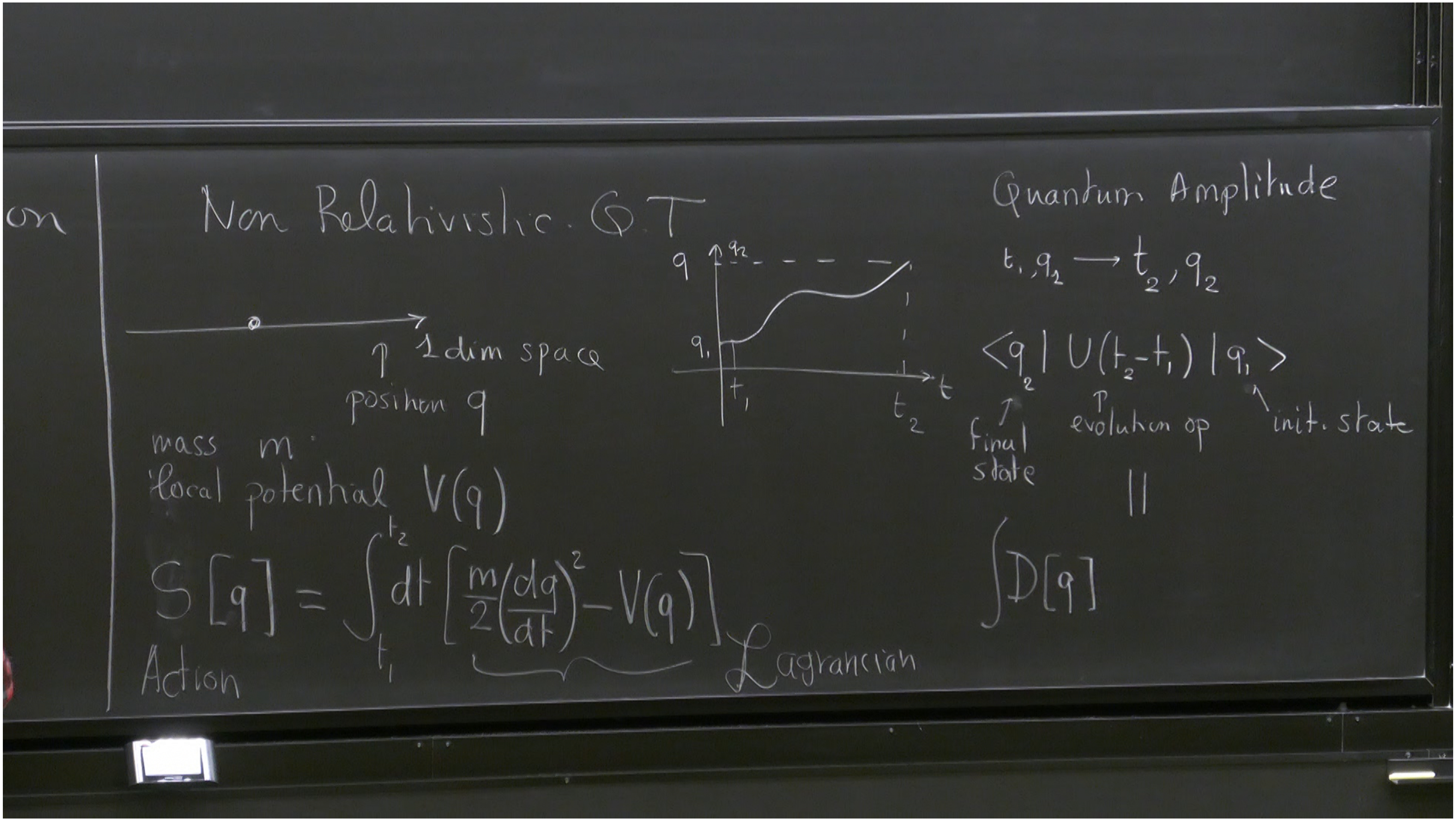
Collection: Quantum Field Theory 2 2023/24

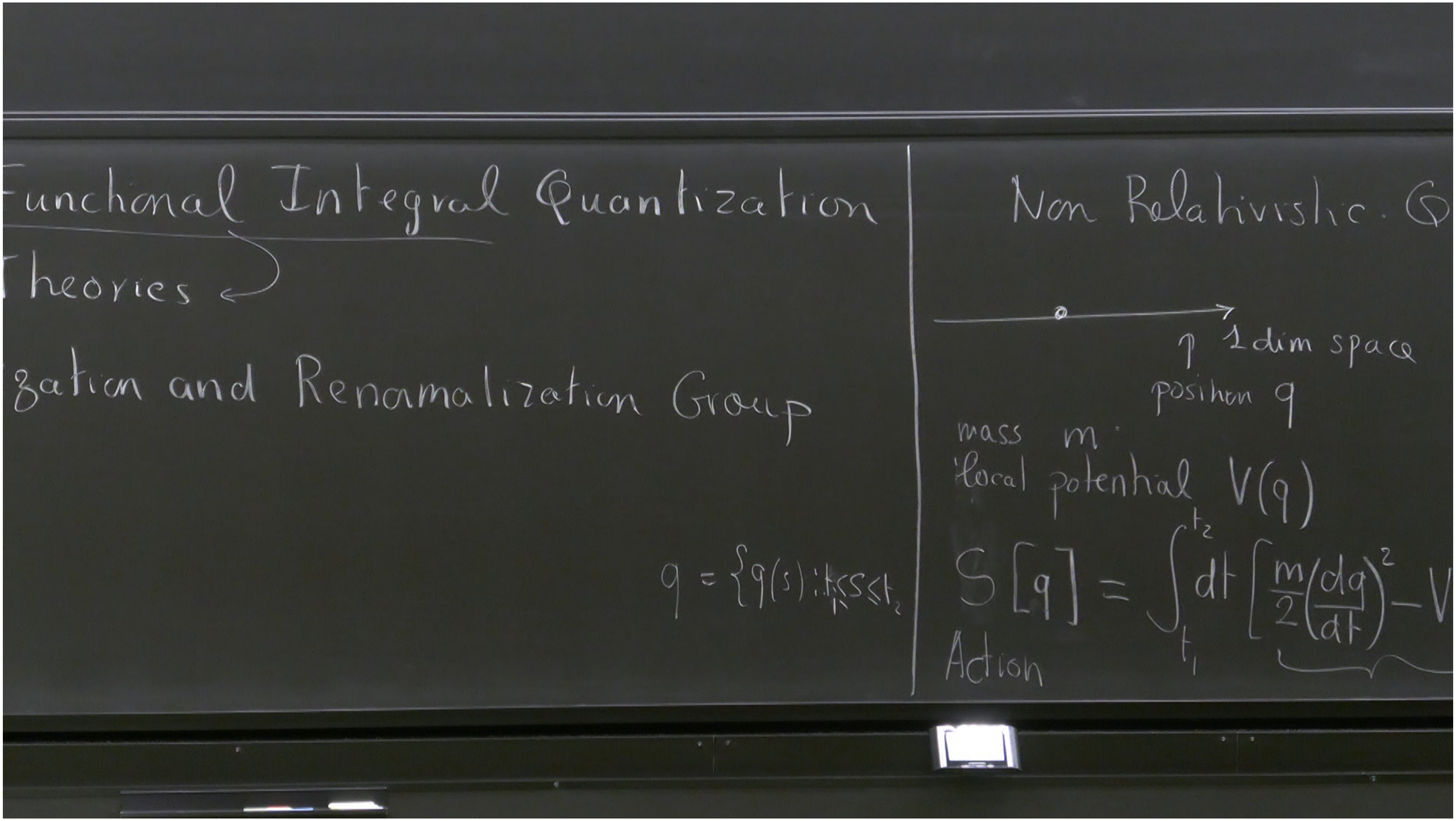
Date: November 14, 2023 - 9:00 AM

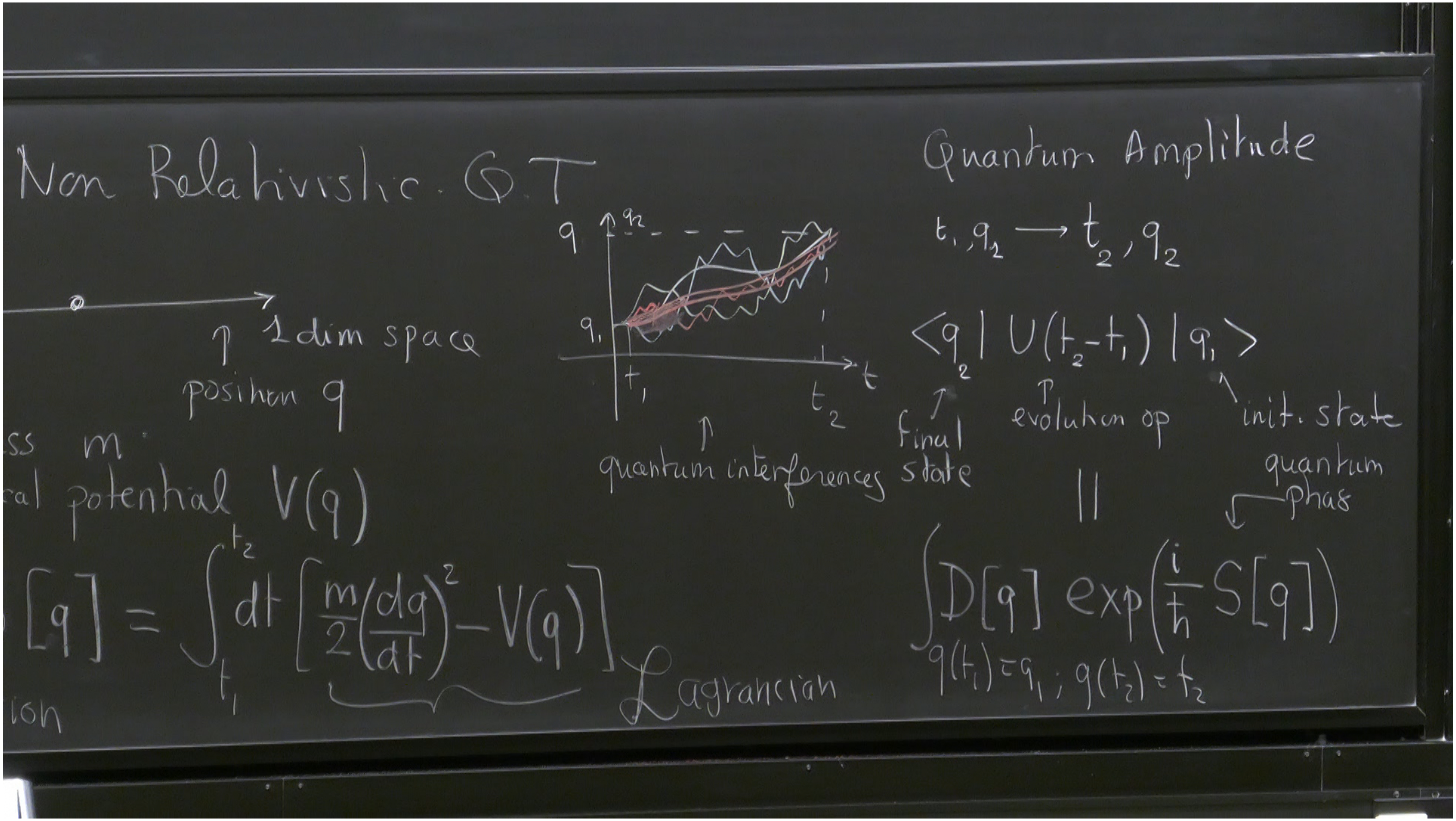
URL: https://pirsa.org/23110018

- Path & Functional Integral Quantization
- Gauge Theories
- Renormalization and Renormalization Group

Today


on
Non Relativistic Q.T
1 dim space
position q
mass m
local potential V(q)
$S[q] = \int_{t_1}^{t_2} dt \left[\frac{m}{2}\left(\frac{dq}{dt}\right)^2 - V(q) \right]$
Action
Lagrangian
q
q_2
q_1
t_1
t_2
t
Quantum Amplitude
$t_1, q_2 \longrightarrow t_2, q_2$
$\langle q_2 | U(t_2 - t_1) | q_1 \rangle$
final state
evolution op
init. state
$\int D[q]$


Functional Integral Quantization
Theories
zation and Renomalization Group
$q = \{q(s) : t_1 \leq s \leq t_2\}$
Non Relativistic Q
1 dim space
position q
mass m
local potential $V(q)$
$S[q] = \int_{t_1}^{t_2} dt \left[\frac{m}{2} \left(\frac{dq}{dt} \right)^2 - V \right.$
Action


Non Relativistic Q.T
1 dim space
position q
ss m
cal potential V(q)
[q] = ∫ dt [m/2 (dq/dt)² − V(q)]
Lagrangian
ion
q
q₂
q₁
t₁
t₂
t
quantum interferences
Quantum Amplitude
t₁, q₂ → t₂, q₂
<q₂| U(t₂−t₁) |q₁>
final state
evolution op
init. state
quantum phas
∫ D[q] exp(i/ħ S[q])
q(t₁) = q₁ ; q(t₂) = t₂

classical physics $\hbar \longrightarrow 0$

$S[q]$ is extremum of S

$$\int D[\phi(t,\vec{x})]\ \exp\left(\frac{i}{\hbar} S\right.$$

Quantum Fields Scalar Field. Klein-Gordon

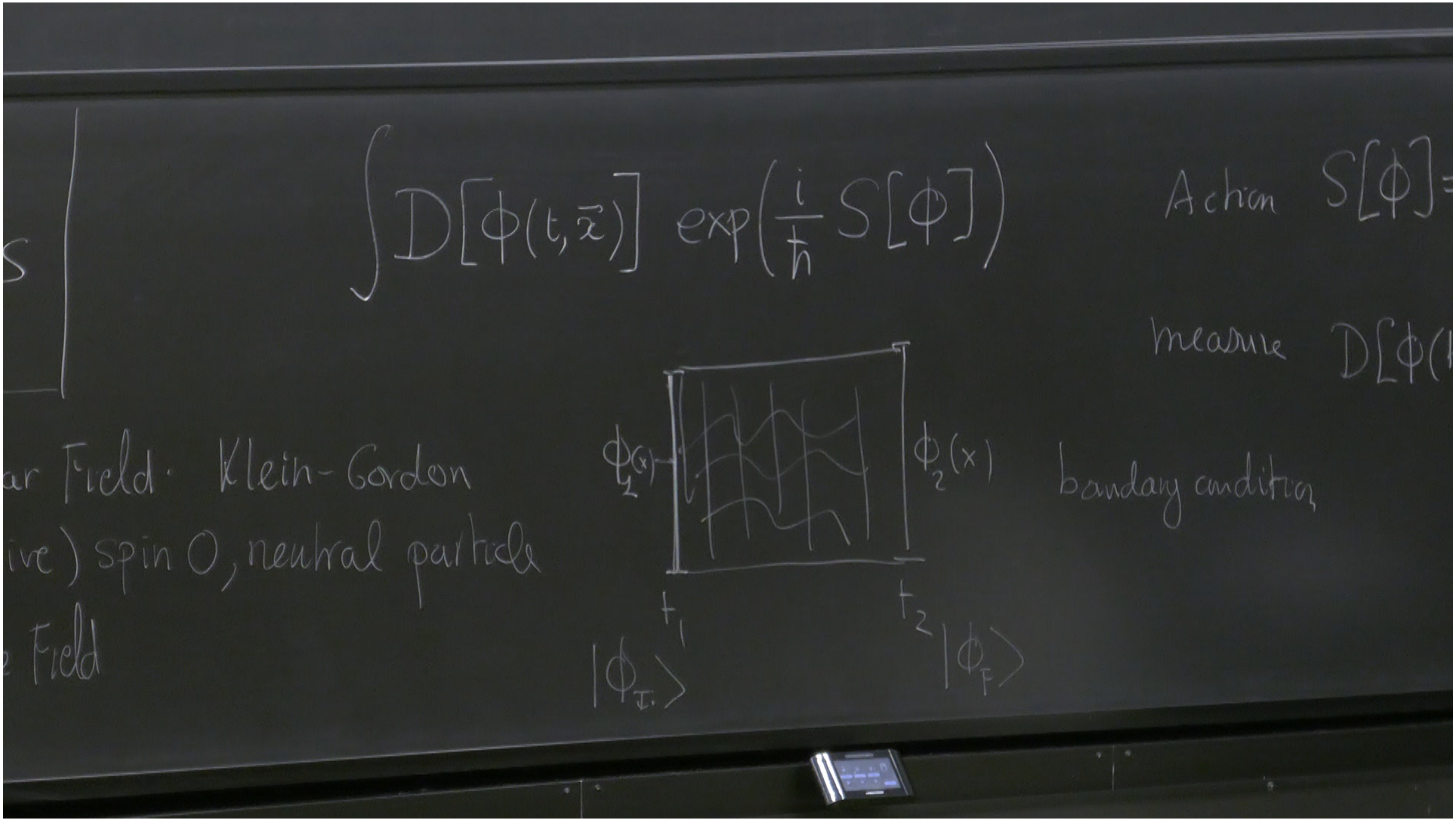
(massive) spin 0, neutral particle

$\phi(t,\vec{x})$ real Field Free Field

$\phi])$

Action $S[\phi] = \int dt \int_{\text{space}} d\vec{x} \left(\underbrace{\frac{1}{2}\left(\frac{\partial \phi}{\partial t}\right)^2}_{\text{"Kinetic en."}} - \underbrace{\left(\frac{1}{2}\left(\frac{\partial \phi}{\partial \vec{x}}\right)^2 + \frac{m^2}{2}\phi^2\right)}_{\text{"pot" energy}} \right)$

measure $D[\phi(t,\vec{x})] \text{ "}=\prod_{t} \prod_{\vec{x}}\text{" } d\phi(t,\vec{x})$


$\int D[\phi(t,\vec{x})] \exp\left(\frac{i}{\hbar} S[\phi]\right)$
Action $S[\phi]=$
measure $D[\phi($
ar Field. Klein-Gordon
ive) spin 0, neutral particle
e Field
$\phi_1(x)$
$\phi_2(x)$
boundary condition
t_1
t_2
$|\phi_I\rangle$
$|\phi_F\rangle$

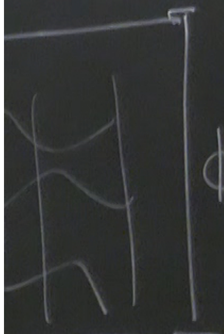
$\frac{i}{\hbar} S[\phi])$

Free Field

Action $S[\phi] = \int dt \int_{\text{space}} d^D\vec{x} \left(\frac{1}{2}\left(\frac{\partial \phi}{\partial t}\right)^2 - \left(\frac{1}{2}\left(\frac{\partial \phi}{\partial \vec{x}}\right)^2 + \frac{m^2}{2}\phi^2 \right)\right)$

"Kinetic en." "pot" energy

measure $D[\phi(t,\vec{x})] \text{"="} \prod_t \prod_{\vec{x}} \text{"} d\phi(t,\vec{x})$

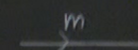


$\phi_2(x)$

t_2

$|\phi_F\rangle$

boundary condition $\longrightarrow$ initial and final states



KG particles

2 particle local interactions

Interactions $\frac{m^2}{2}\phi^2 \longrightarrow \frac{m^2}{2}\phi^2 + \frac{g}{4!}\phi^4$

particle not in a pure state

but in a mixed state ⟵ density matrix : ρ

$|i\rangle \to$ energy E_i $\quad E_0 < E_1 < E_2 < \ldots$

$$\rho = \sum_{\substack{\text{states} \\ i}} p(i) \, |i\rangle\langle i|$$

↖ probability

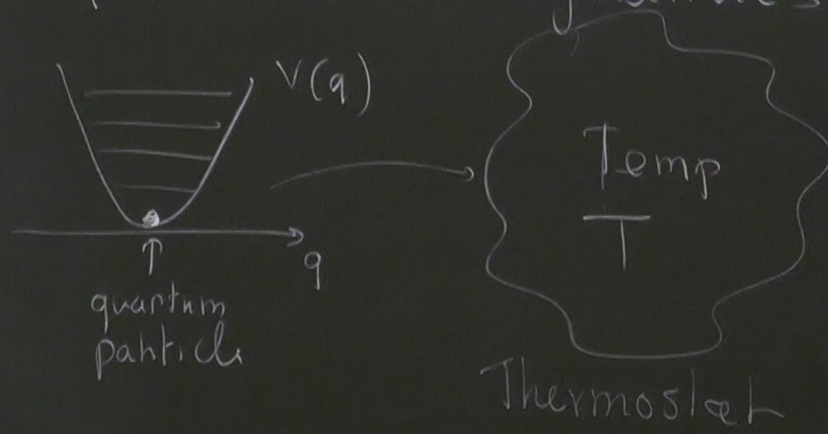
$$p(i) = \frac{1}{Z} e^{-\beta E_i} \qquad \beta = \frac{1}{k_B T}$$

Gibbs probability

$$Z = \sum_i p(i) = \mathrm{Tr}(\rho)$$

partition function

Q.T + Thermodynamics



particle not in a pure state

but in a mixed state ← dens

$|i\rangle \to$ energy E_i $\qquad E_0 < E_1 < E_2$

$$\rho = \sum_{\substack{\text{states} \\ i}} p(i)\, |i\rangle\langle i|$$

density matrix
=
Operator

← probability

$p(i) = \frac{1}{Z}$

Gibbs p

$Z = \sum_i p$

...trix : ρ

E_i

$\beta = \frac{1}{k_B T}$

...lity

partition function

$$\rho = \frac{1}{Z} e^{-\beta H}$$

$H = \sum_i E_i |i\rangle\langle i|$

hamilt op

$$Z = \mathrm{Tr}\left(e^{-\beta H}\right)$$

$\to$ Hermitean $\quad H = H^\dagger$

$$1 = \mathrm{Tr}\,\rho$$

Evolution operator

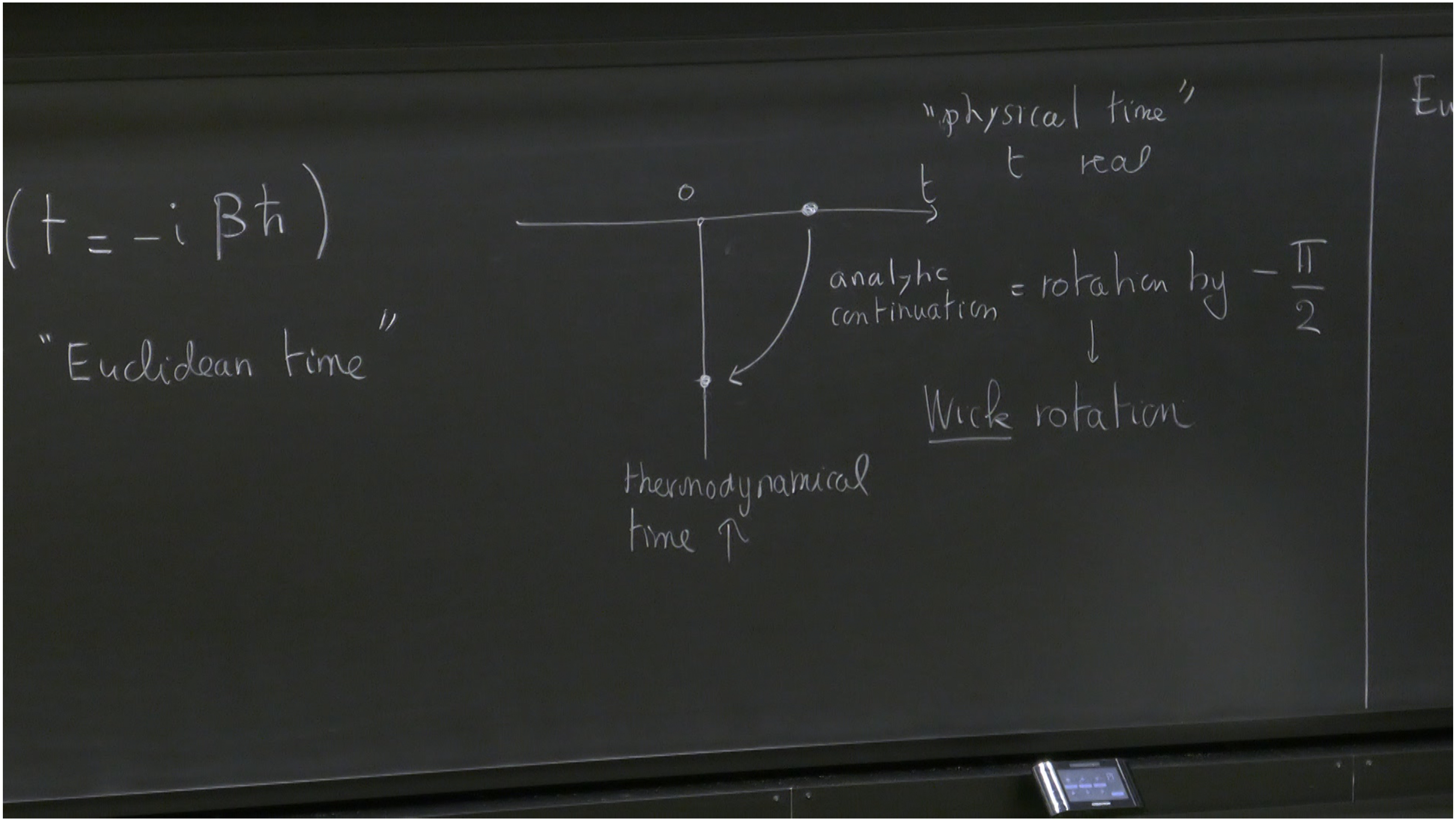
$$U(t) = \exp\left(\frac{t}{i} H\right)$$

Unitary

$$e^{-\beta H} = U(t = -i\beta\hbar)$$

$t = -i\tau \qquad \tau$ "Euclidean time"

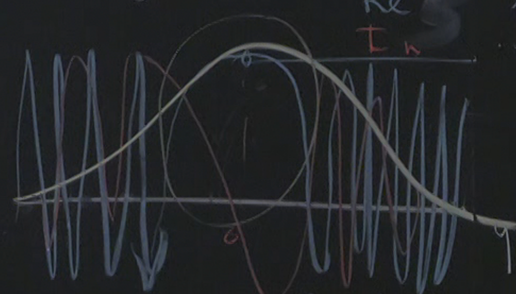
$$\tau = \beta\hbar = \frac{\hbar}{k_B T}$$


$(t = -i\beta\hbar)$
"Euclidean time"
"physical time"
t real
0
t
analytic continuation = rotation by $-\frac{\pi}{2}$
Wick rotation
thermodynamical time

Gaussian integration

$$\int_{-\infty}^{+\infty} dq\, \Psi(q)\, e^{\frac{i}{\hbar}\frac{q^2}{2}}$$

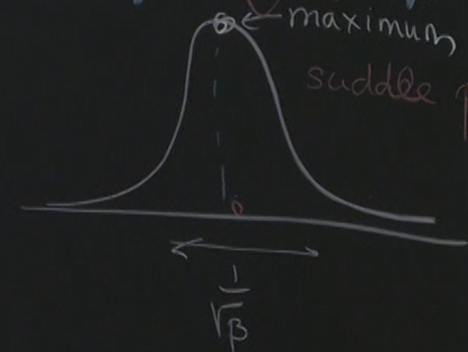
smooth test function



phase is stationary

$$\int_{-\infty}^{+\infty} dq\, e^{-\beta\frac{q^2}{2}}$$

real gaussian



$\beta \to \infty$

$\frac{\pi}{2}$

Euclidean QFT $\longleftrightarrow$ Real time QFT

$(-+++)$ Minkowski spacetime at finite temperature

$$e^{\frac{i}{\hbar}\int dt \int d^d\vec{x}\left(\frac{1}{2}\left(\frac{d\phi}{dt}\right)^2 - \frac{1}{2}\left(\frac{\partial\phi}{\partial\vec{x}}\right)^2\right)}$$

$dt \to -i\,d\tau$, $\quad \frac{1}{2}\left(\frac{d\phi}{dt}\right)^2 \to -\frac{1}{2}\left(\frac{d\phi}{d\tau}\right)^2$

$t \to -i\tau$

$$e^{-\frac{1}{\hbar}\int d\tau \int d\vec{x}\left[\frac{1}{2}\left(\frac{d\phi}{d\tau}\right)^2 + \frac{1}{2}\left(\frac{d\phi}{d\vec{x}}\right)^2\right]}$$

$\frac{1}{2}\left(\frac{\partial\phi}{\partial\vec{X}}\right)^2$

$(++++)$

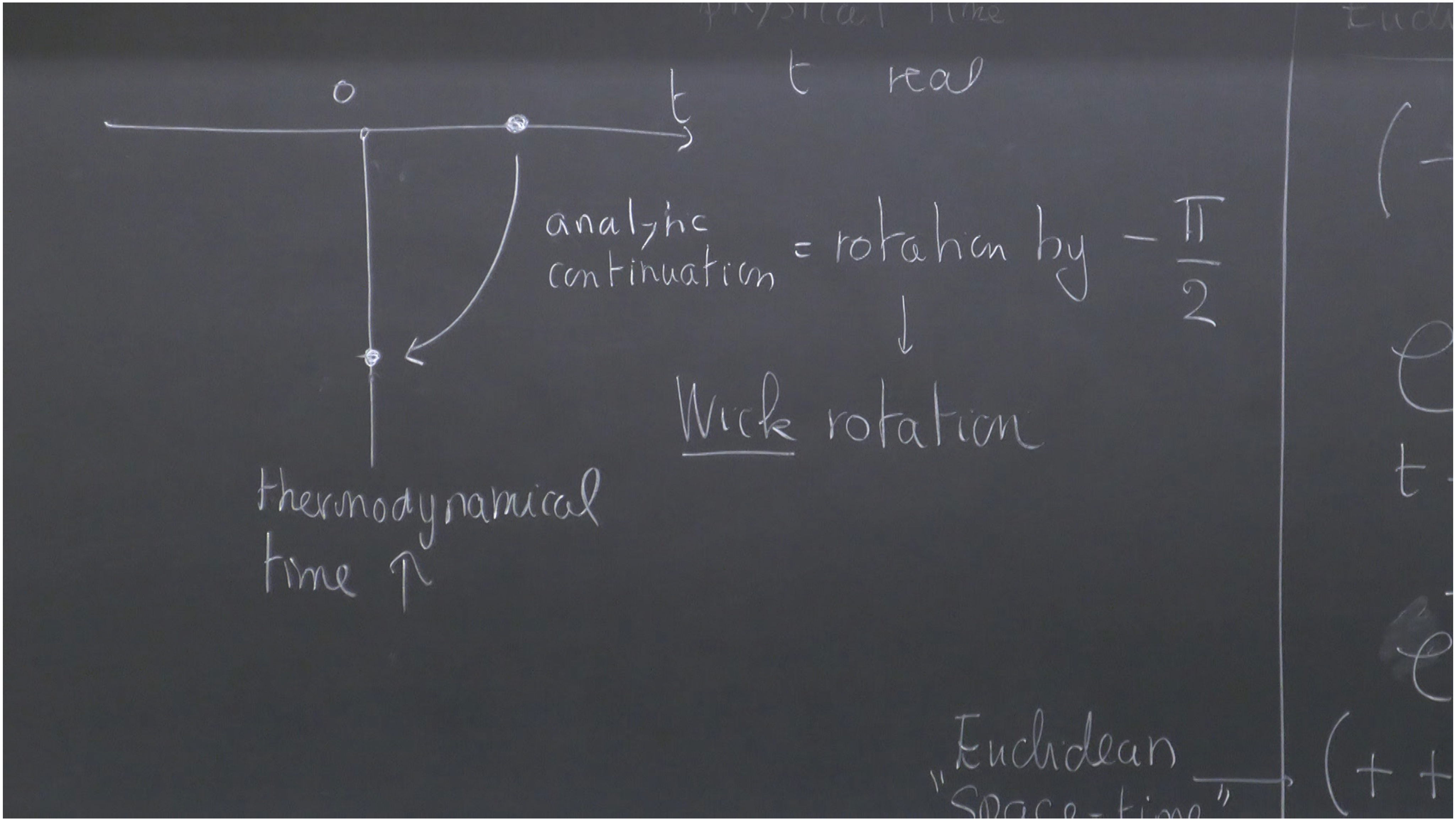
real variables

$\phi(\tau, \vec{x})$

$(\tau, \vec{x}) = \vec{X}$

$1 \quad d \qquad 1+d$ dim vector

ne"


t real
0
t
analytic continuation = rotation by - π/2
Wick rotation
thermodynamical time τ
Euclidean

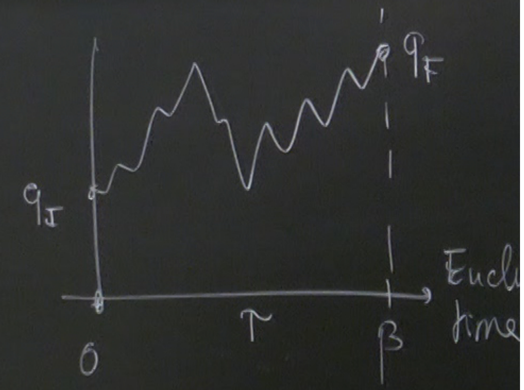
partition function $Z = \mathrm{Tr}\left(e^{-\beta H}\right) = \sum_i \langle i | e^{-\beta H} | i \rangle$

$$= \int_{-\infty}^{+\infty} dq \, \langle q | e^{-\beta H} | q \rangle$$

$$\langle q_F | e^{-\beta H} | q_I \rangle = \int D[q(\tau)] \; e^{-\frac{1}{\hbar}\int_0^\beta d\tau \left[\frac{m}{2}\left(\frac{dq}{d\tau}\right)^2 + V(q)\right]}$$

path integral representation

probability weigh factor



"Euclidean "Space-time" → | $(+ + + +)$ | $\frac{1}{2}\left(\frac{\partial}{\partial x}\right)$ | action

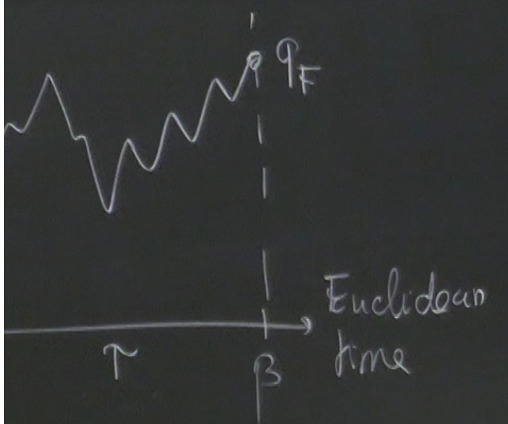
period

$q_I = q_F = q$ and integrated over q

$$Z = \int D[q(\tau)]\, e^{-\frac{1}{\hbar} S_E[q]}$$

periodic trajectory

$$q(0) = q(\tau)$$



Finite Temperature QT $\Longleftrightarrow$ Path / Functional integral in periodic Euclidean Time

"Euclidean "Space-time" $\longrightarrow$ $(+ + + +)$ $\frac{\ }{2}\left(\frac{\partial}{\partial x}\right)$ Euclidean action

$q_I = q_F = q$ and integrated over q

$$Z = \int \mathcal{D}[q(\tau)]\, e^{-\frac{1}{\hbar} S_E[q]}$$

periodic trajectory

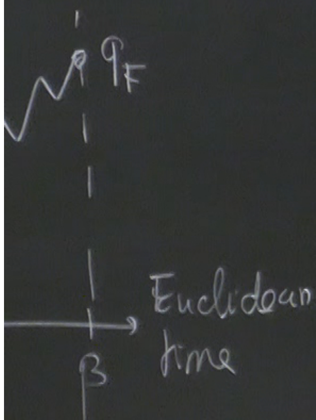
$q(0) = q(\tau)$

period in Euclidean time

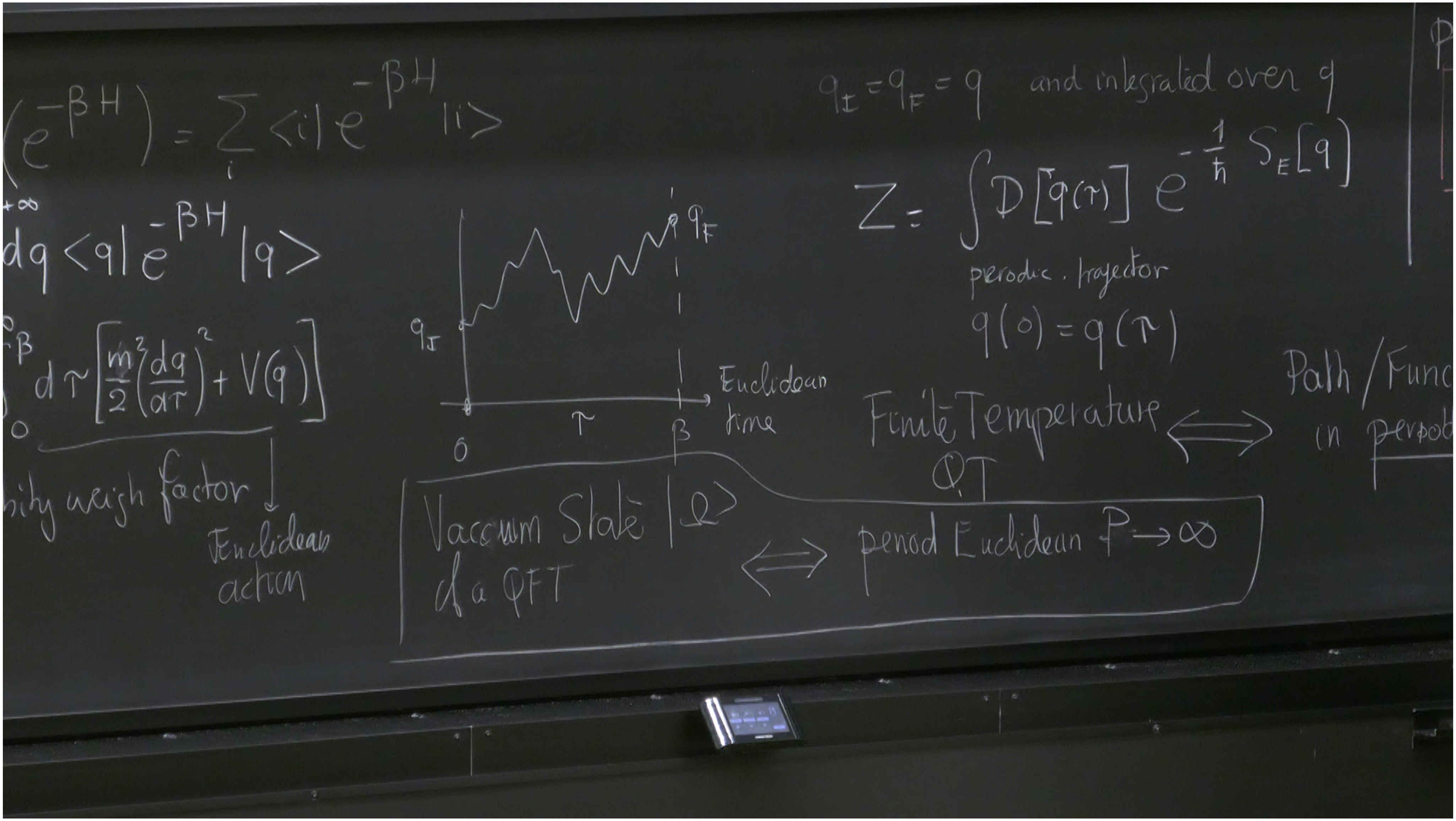
$$\boxed{P_E = \frac{\hbar}{k_B \cdot T}}$$

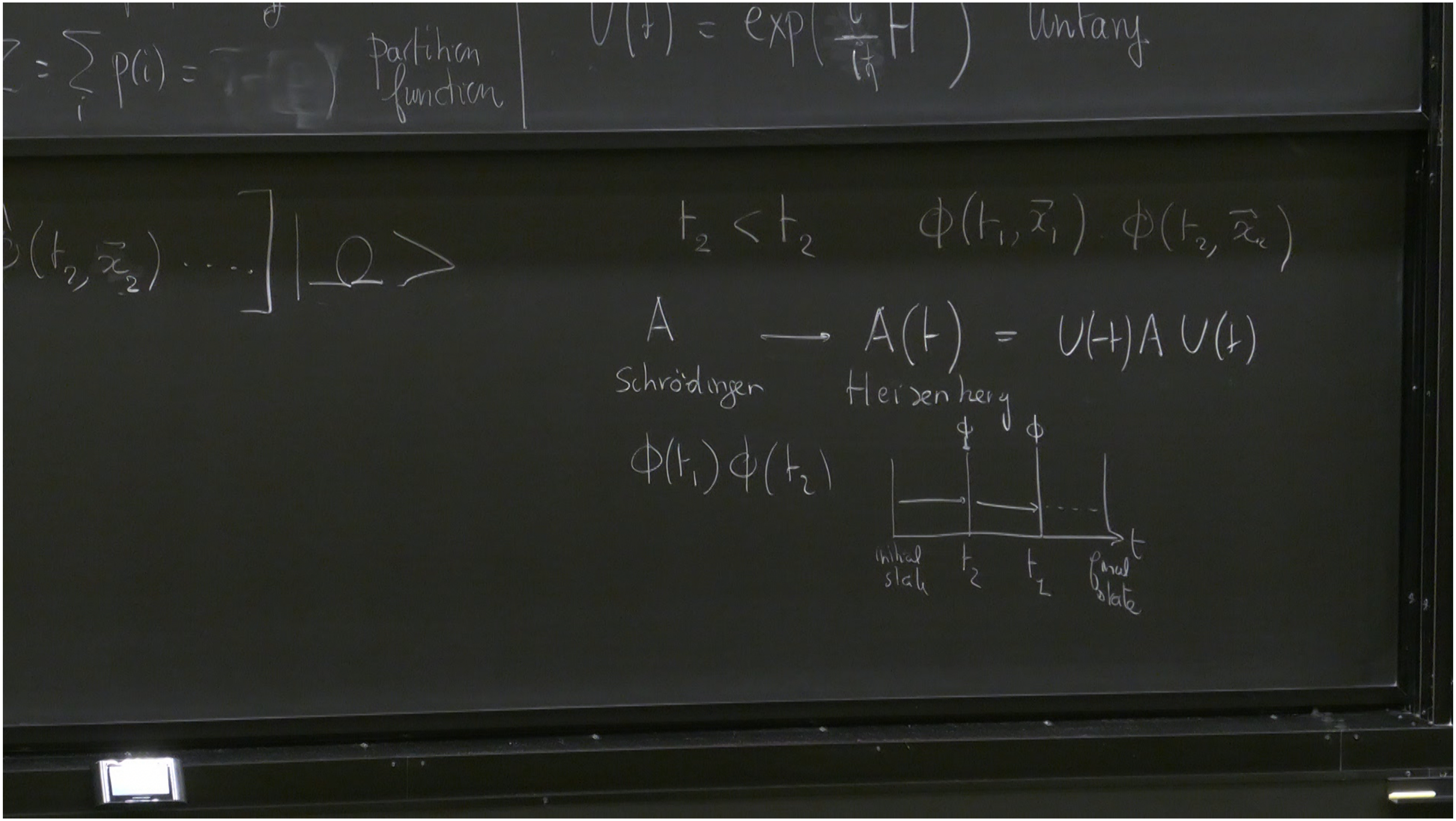
← temp of the quantum system

Boltzman constant



Finite Temperature QT $\Longleftrightarrow$ Path/Functional integral in periodic Euclidean Time


(e^{-βH}) = Σ_i <i| e^{-βH} |i>
dq <q| e^{-βH} |q>
dτ [m/2 (dq/dτ)^2 + V(q)]
ity weigh factor
Euclidean action
q_I
q_F
0
τ
β
Euclidean time
q_I = q_F = q and integrated over q
Z = ∫ D[q(τ)] e^{-1/ħ S_E[q]}
periodic trajectory
q(0) = q(τ)
Finite Temperature QT
Path / Func
in period
Vacuum State |Ω> of a QFT
period Euclidean P → ∞


$Z = \sum_i p(i) =$
partition function
$U(t) = \exp(\frac{t}{i\hbar} H)$
unitary
$\phi(t_2, \bar{x}_2) \ldots] | \Omega >$
$t_2 < t_2$
$\phi(t_1, \bar{x}_1)\ \phi(t_2, \bar{x}_2)$
$A \longrightarrow A(t) = U(-t) A U(t)$
Schrödinger
Heisenberg
$\phi(t_1)\phi(t_2)$
ϕ
ϕ
t
initial state
t_2
t_1
final state

Operator

$$Z = \sum_i p(i) =$$

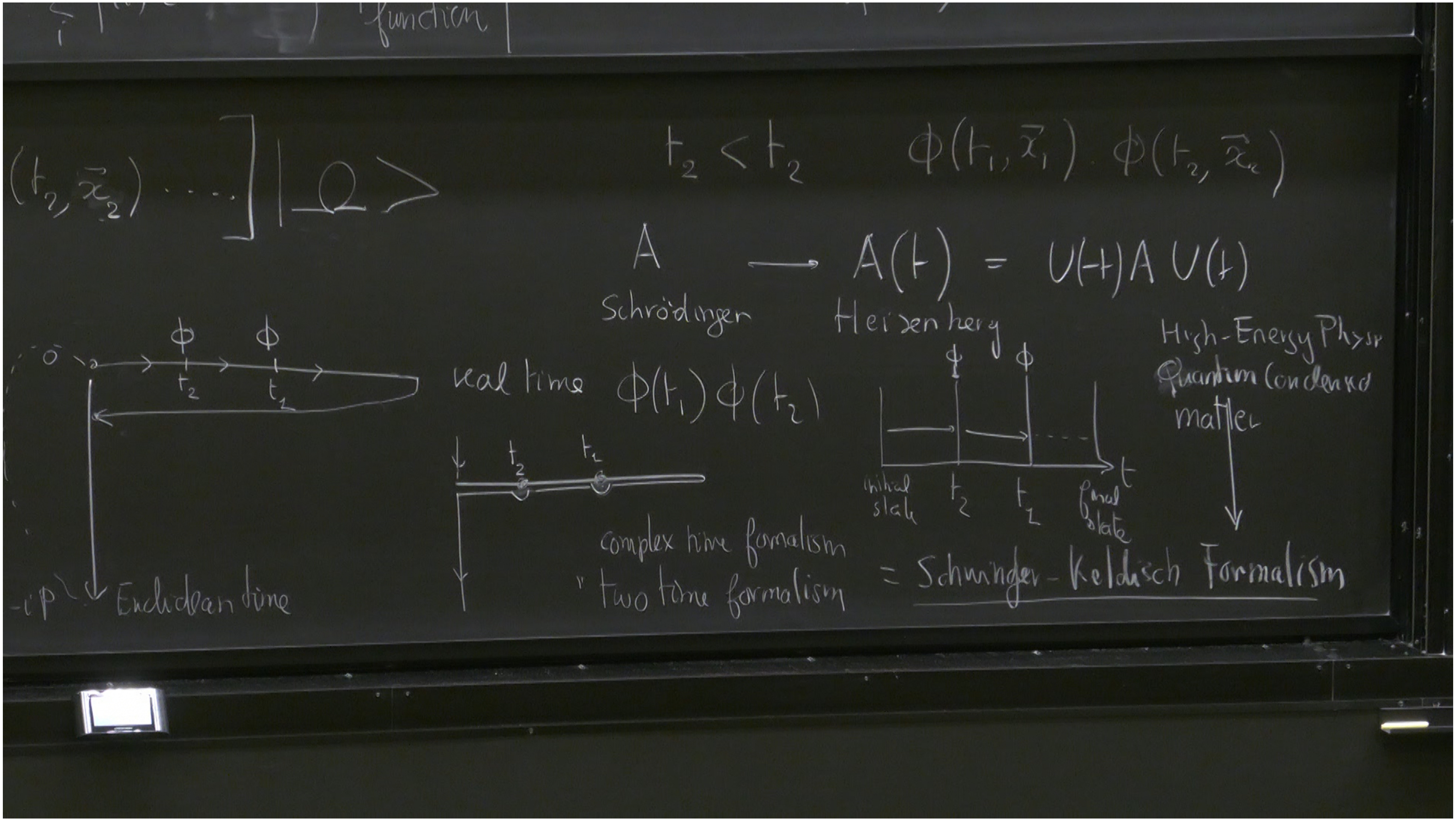
QFT with a scalar Field ϕ

$\hat{\phi}(t, \vec{x})$ local Field operator

$$\langle \Omega | T\left[\hat{\phi}(t_1, \vec{x}_1)\, \hat{\phi}(t_2, \vec{x}_2) \ldots \right]$$

time ordering

$$\int_{\substack{\text{I \& Final} \\ \text{condition}}} D[\phi]\, e^{\frac{i}{\hbar} S[\phi]}\; \phi(t_2, \vec{x}_2)\, \phi(t_1, \vec{x}_1) \ldots$$


function
$(t_2, \vec{x}_2) \cdots] | \Omega >$
$t_2 < t_2$
$\phi(t_1, \vec{x}_1) \phi(t_2, \vec{x}_2)$
$A \longrightarrow A(t) = U(-t) A U(t)$
Schrödinger
Heisenberg
real time $\phi(t_1) \phi(t_2)$
High-Energy Phys
Quantum Condensed matter
initial state
final state
complex time formalism
" two time formalism
= Schwinger-Keldysch Formalism
Euclidean time