Title: QFT1 Lecture - 110723

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Collection: Quantum Field Theory 1 2023/24

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QED Renormalization

Superficial divergence

D = power of P = numerator − denominator

$= 4\times \underbrace{L}_{\text{loop}} - 2\underbrace{P_\gamma}_{\text{photon propagator}} - 1\underbrace{P_e}_{\text{fermion propagator}}$

relationship

$1\cdot V = \underbrace{N_\gamma}_{\text{external photon}} + 2P_\gamma$

$2V = \underbrace{N_e}_{\text{external fermion}} + 2P_e$

internal

$V - \underset{\uparrow}{I} +$

$P_e + P_\gamma$

$D = 4 - \frac{3}{2}$

internal thing

$$V - I + L = 1$$

$P_e + P_\gamma$

$$D = 4 - \frac{3}{2} N_e - N_\gamma$$

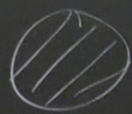
$-2P_\gamma$

al

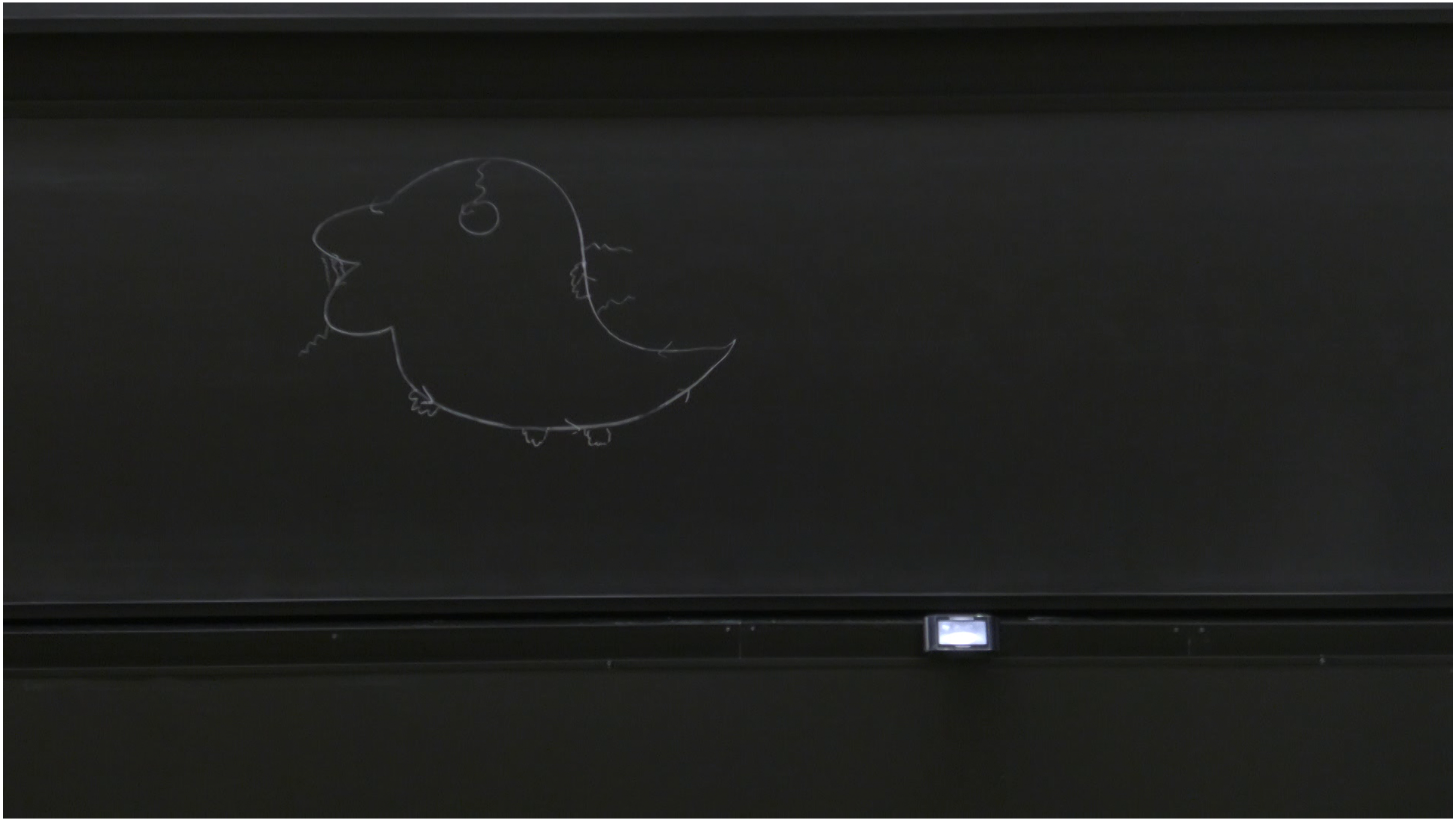
$+2P_e$

nal

rom

$D=4$  don't care

$D=3$



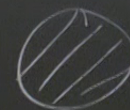
internal thing

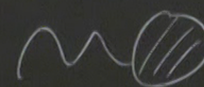
$$V - I + L = 1$$

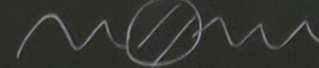
$$\uparrow$$

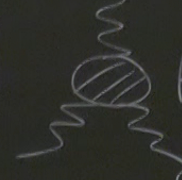
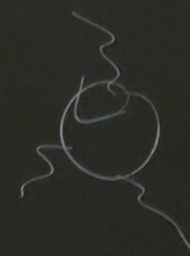
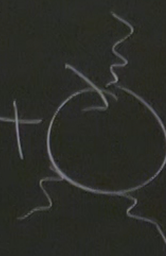
$$P_e + P_\gamma$$

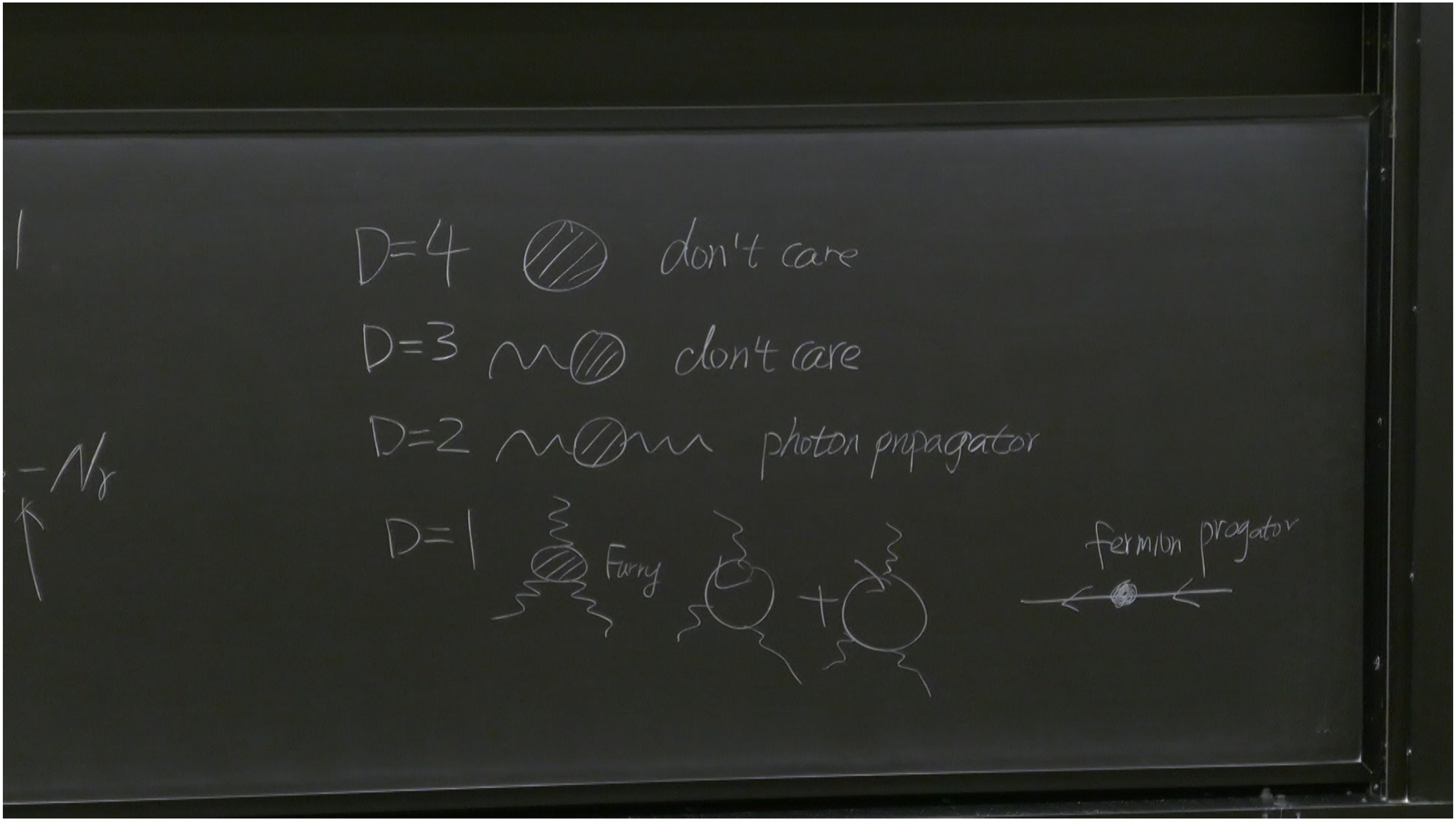
$$D = 4 - \frac{3}{2} N_e - N_\gamma$$

$D=4$  don't care

$D=3$  don't care

$D=2$  photon propagator

$D=1$  Furry  + 


$D=4$ don't care
$D=3$ don't care
$D=2$ photon propagator
$D=1$ Furry
fermion progator
$-N_f$

QED

Maxwell theory

photon propagator

$\partial_\mu\phi\partial^\mu\phi$

$\mathcal{L}_{KG} \propto \phi(\partial^2+m^2)\phi$

propagator $\underline{\dfrac{i}{p^2-m^2}}$

$i\partial_\mu \to p_\mu$

momentum space

KG operator $\underline{-p^2+m^2}$

inverse

Maxwell propagator

$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}$

$= -\frac{1}{4}(\partial_\mu$

$= -\frac{1}{2}(\partial_\mu$

up to $\dfrac{\text{total}}{\text{derivative}} + \frac{1}{2}$

Maxwell propagator should be inverse of the Maxwell operator.

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$= -\frac{1}{4} (\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu)$$

$$= -\frac{1}{2} (\partial_\mu A_\nu \partial^\mu A^\nu - \partial_\mu A_\nu \partial^\nu A^\mu)$$

up to total derivative $+\frac{1}{2} A_\nu (\partial^2 \eta^{\mu\nu} - \partial^\mu \partial^\nu) A_\mu$

$$\parallel$$

$$\partial^2 \eta^{\mu\nu} - \partial^\mu \partial^\nu$$

$$\Downarrow \quad i\partial_\mu \to p_\mu$$

$$-p^2 \eta^{\mu\nu} + p^\mu p^\nu$$

- invert!

d be inverse of the Maxwell operator.

$$\| $$

$$\partial^2 \eta^{\mu\nu} - \partial^\mu \partial^\nu$$

$$\Downarrow \; i\partial_\mu \to p_\mu$$

$$-p^2 \eta^{\mu\nu} + p^\mu p^\nu$$

• invert!

$$(-p^2 \eta^{\mu\nu} + p^\mu p^\nu)\, p_\nu$$

$$= -p^2 p^\mu + p^\mu p^2 = 0$$

$$\ldots)(\partial^\mu A^\nu - \partial^\nu A^\mu)$$

$$- \partial_\mu A_\nu \partial^\nu A^\mu)$$

$$\ldots^{\mu\nu} - \partial^\mu \partial^\nu) A_\mu$$

gauge symmetry.

gauge condition $\partial_\mu A^\mu = 0$

can be chosen

gauge fixed

$$\mathcal{L}_{new} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\xi}(\partial_\mu A^\mu)^2$$

↑gauge choice

$$-\frac{1}{2\xi}\partial_\mu A^\mu \partial_\nu A^\nu$$

$$\overset{\text{IBP}}{=} \frac{1}{2\xi} A_\mu \partial^\mu \partial^\nu A_\nu$$

operator

$$-p^2\eta^{\mu\nu} + p^\mu p^\nu\left(1-\frac{1}{\xi}\right)$$

invertible

$$\frac{-i}{p^2+i\epsilon}\left(\eta^{\mu\nu} - (1-\right.$$

ξ

invertible

$$\frac{-i}{p^2+i\epsilon}\left(\eta^{\mu\nu}-(1-\xi)\frac{p^\mu p^\nu}{p^2}\right)$$ — photon propagator

$\xi=1$ Feynman gauge

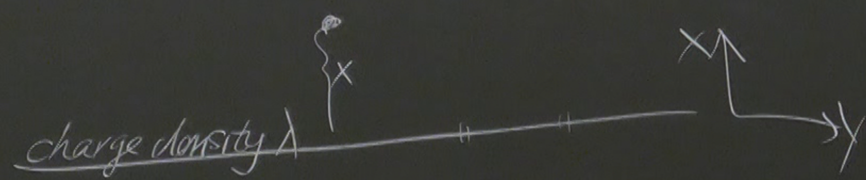
$$\frac{-i\,\eta^{\mu\nu}}{p^2+i\epsilon}$$

1st step introduce regulator

dimension regularization

$$V(x) = \int_{-\infty}^{\infty} \frac{\lambda\, dy}{\sqrt{x^2+y^2}} \to \infty$$

toy model



dim rez

cutoff

$$V(x) = \int_{-\infty}^{\infty} \frac{\lambda\, dy}{\sqrt{x^2+y^2}} \to \infty$$

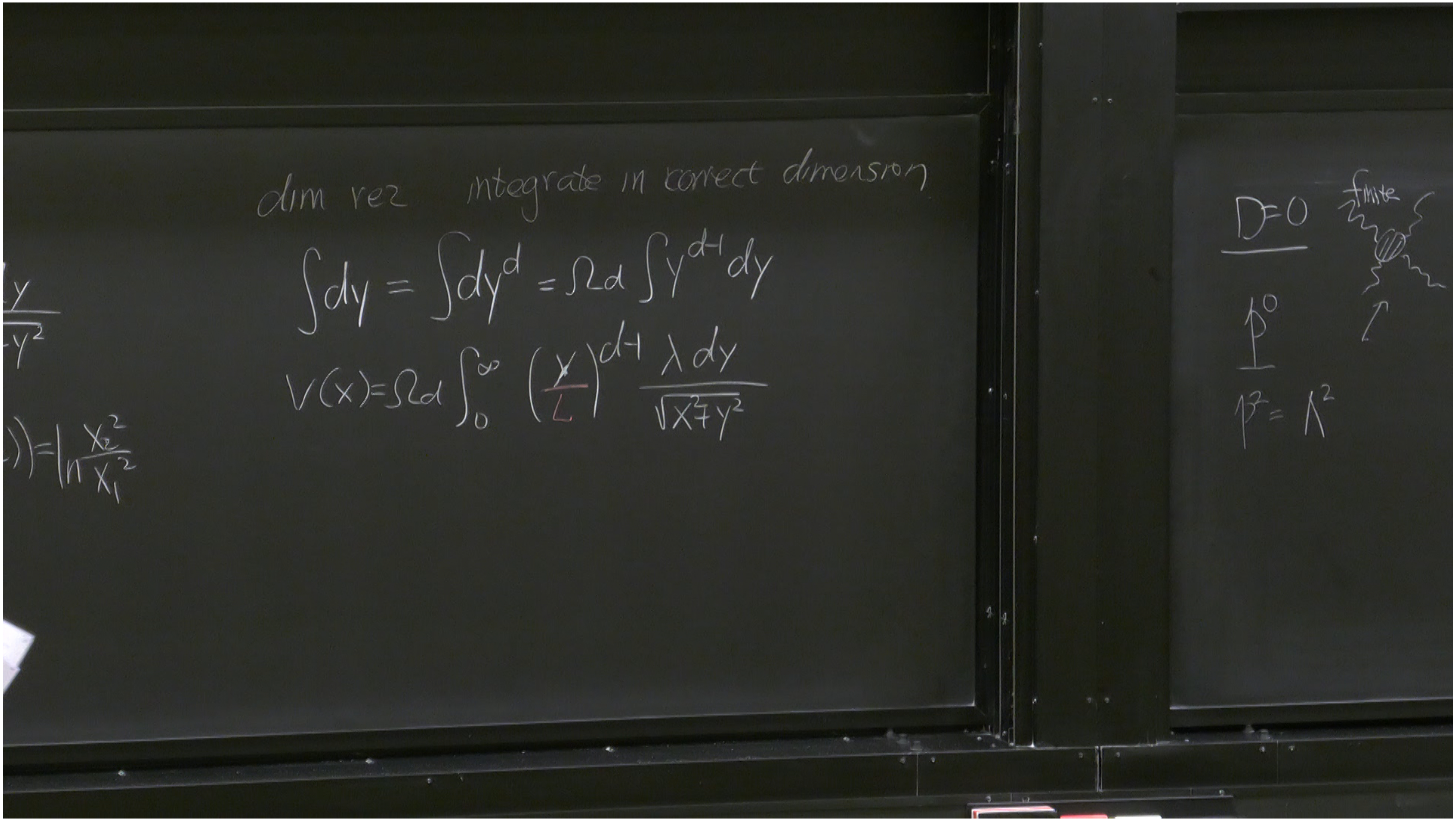
$$V(x) = \int_{-L}^{L} \frac{\lambda\, dy}{\sqrt{x^2+y^2}}$$

$$V(x_1) = V(x_2)$$

$$\lim_{L\to\infty} \left(V(x_1) - V(x_2)\right) = \ln \frac{x_2^2}{x_1^2}$$

no force

$$\frac{\partial V}{\partial x} = 0 \leftarrow \text{wrong}$$


dim rez   integrate in correct dimension
$\int dy = \int dy^d = \Omega_d \int y^{d-1} dy$
$V(x) = \Omega_d \int_0^\infty \left(\frac{y}{L}\right)^{d-1} \frac{\lambda\, dy}{\sqrt{x^2+y^2}}$
$= \ln \frac{X_2^2}{X_1^2}$
D=0
finite
p^0
$p^2 = \Lambda^2$

$\to \infty$

cutoff

$$V(x) = \int_{-L}^{L} \frac{\lambda\, dy}{\sqrt{x^2+y^2}}$$

$$\lim_{L\to\infty} \left(V(x_1) - V(x_2)\right) = \ln \frac{x_2^2}{x_1^2}$$

dim reg   integrate in correct dimension

$$\int dy \Rightarrow \int dy^d = \Omega_d \int y^{d-1}\, dy \quad (d \to 1)$$

$$V(x) = \Omega_d \int_0^\infty \left(\frac{y}{L}\right)^{d-1} \frac{\lambda\, dy}{\sqrt{x^2+y^2}}$$

$$d = 1 - \epsilon$$

$$\text{Mathematica} = \left(\frac{L}{X}\right) \frac{\Gamma\left(\frac{\varepsilon}{2}\right)}{\pi^{\frac{\varepsilon}{2}}}$$

$$\int \frac{d^4 l}{(2\pi)^d} \frac{1}{(l^2 - \Delta)^n}$$

$$\lim_{\epsilon \to 0} \left(V(x_1) - V(x_2)\right) = \ln \frac{x_1^2}{x_2^2}$$

$$V(x) = \cancel{\frac{2}{\epsilon}} + (\cancel{-\gamma_E - \log \pi}) + \log \frac{L^2}{x^2} + O(\epsilon)$$

$$\Delta^{\rho}{}_{\nu} = \delta^{\rho}{}_{\nu} - \frac{p^{\rho} p^{\nu}}{p^2}$$

$$= -\frac{i\eta_{\mu\nu}}{p^2} + \frac{-i\eta_{\mu\rho}}{p^2}\Delta^{\rho}{}_{\nu}\left(\Pi(p^2)+\Pi^2(p^2)+\ldots\right)$$

$$= \frac{-i}{p^2(1-\Pi(p^2))}\left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) + \frac{-i}{p^2}\frac{p_\mu p_\nu}{p^2}$$

$p^2 \to 0$ pole is not shifted

the residue stay

$\Pi(p^2)=0$ when $p^2 \to 0$

...e the coupling constant

$g_R = g_0 + \alpha_2 g_0^2$

...ysics

$\Pi^{\mu\nu} = \Pi(p^2)(p^2\eta^{\mu\nu} - p^\mu p^\nu)$

$+$ 1PI

$\boxed{\mathcal{M}_\mu p^\mu = 0}$

$\Delta \rho_v = \delta \rho_v - \ldots$

$= -\frac{i\eta_{\mu\nu}}{p^2} + \frac{-i\eta_{\mu\ldots}}{p\ldots}$

$\frac{-i\eta_{\mu\nu}}{p^2} + \frac{-i\eta_{\mu\rho}}{p^2}\, i\Pi^{\rho\sigma}\, \frac{-i\eta_{\sigma\nu}}{p^2} + \ldots$

$i\mathcal{M}$

$= \epsilon_\mu \epsilon_\nu (\ldots \mathcal{M}^{\mu\nu})$

$\cancel{\mathcal{M}^\mu}$

$p_\mu \epsilon_\rho \epsilon_\sigma \epsilon_\nu \mathcal{M}^{\mu\nu\rho\sigma} = 0$

$= \frac{-i}{p^2(1-\Pi(p^2))}$

$\frac{\quad}{p^2 - m_{phy}^2}$

$\Pi(p^2) = \ldots$