Title: QFT1 Lecture - 110623

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Collection: Quantum Field Theory 1 2023/24

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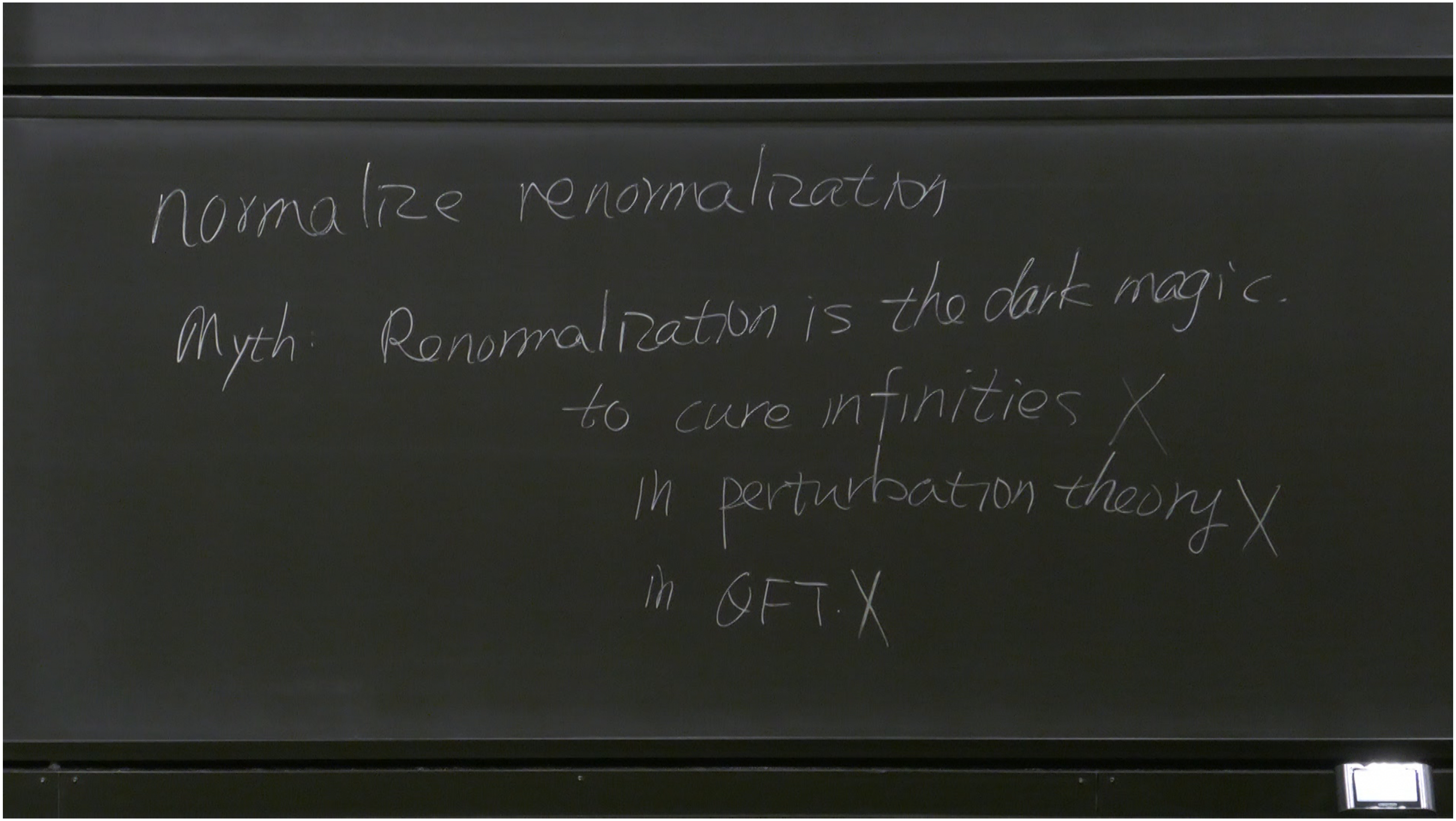

normalize renormalization
Myth: Renormalization is the dark magic.
to cure infinities X
in perturbation theory X
in QFT. X

renormalization

definition

physical problem with interaction

- scales
- "bare parameter" theoretical parameter not physical"

renormalization process

replace "bare parameter"

by renormalized "dressed parameter" (depends on scale)

PSI Award

Feedback.

Nature forbids

a naked

single larng tea.

ess

meter"

meter"

scale)

in 1877 Boussinesq. ad. hoc

many scales

a "bare" quantity

replaced by

renormalized quantity

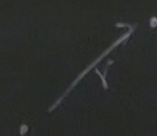
that (depends on scale)

1922. Debye-Hückel e

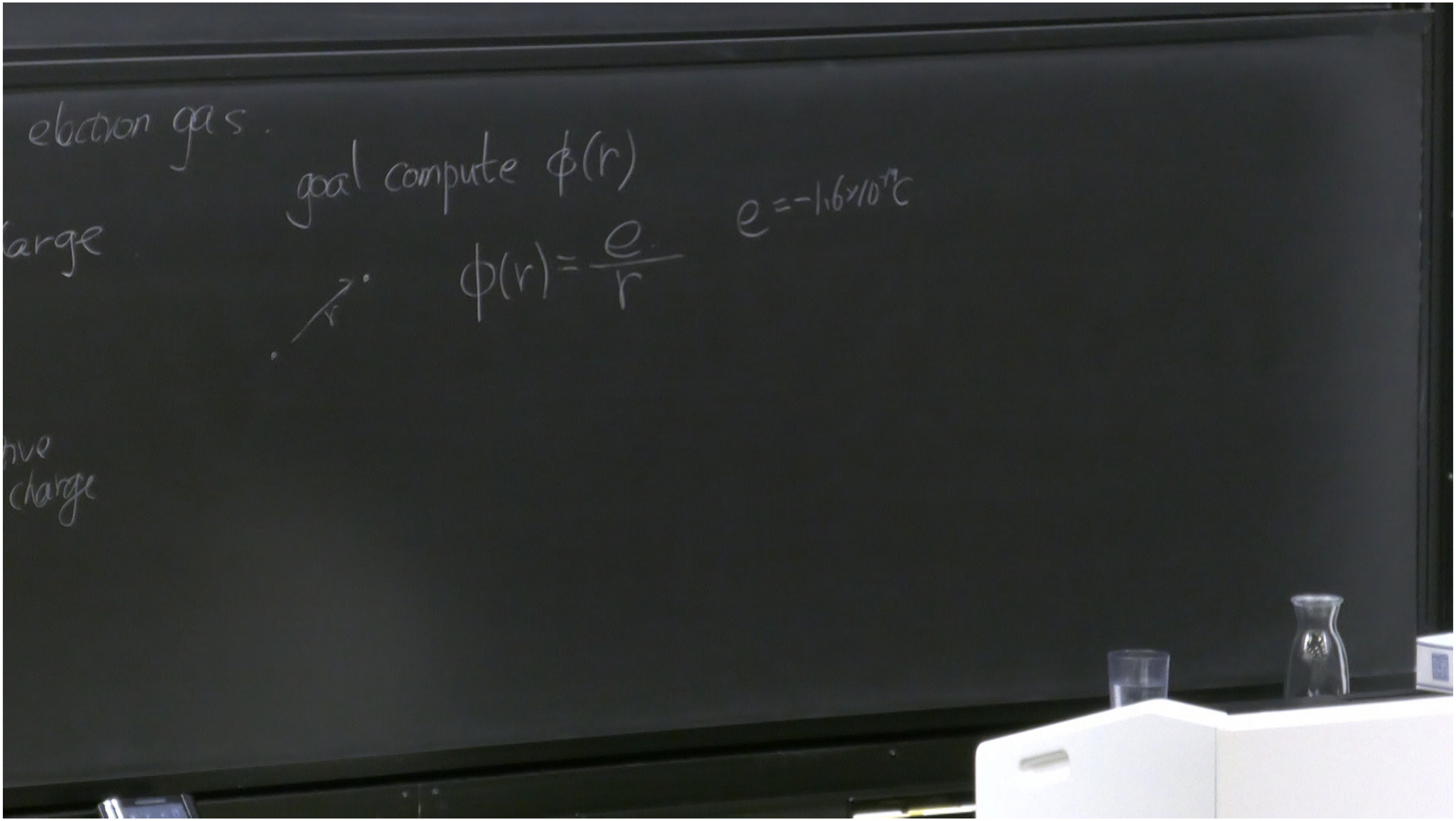
1922. Debye-Hückel electrolyte electron gas.

goal compute $\phi(r)$

- charge → renormalized effective charge



system is neutral
N free electrons
spatially uniform positive charge


electron gas.
goal compute φ(r)
e = -1.6×10^-19 C
arge
φ(r) = e/r
r
tive
charge

$$\nabla^2 \phi = -4\pi \rho(r)$$

$$\rho(r) = e\,n(r) - e\,n_\infty$$

\# density

$$p(r) = \frac{e^{-\frac{E(r)}{kT}}}{Z}$$

$$n(r) = \frac{N}{Z} e^{-\frac{E(r)}{kT}}$$

$$n(r \to \infty) = \frac{N}{Z}$$

$$\frac{n(r)}{n_\infty} = e^{-\frac{E(r)}{kT}}$$

$$\nabla^2 \phi = -4\pi e n_\infty \left(e^{-\frac{E(r)}{kT}} - 1\right)$$

linear expansion

$$+ \left(4\pi e n_\infty \frac{e}{kT}\right)\phi$$

$$= \ell_D^{-2}$$

$$\phi = \frac{e}{r} \exp\left(-\frac{r}{\ell_D}\right)$$

$e \to e \exp\left(\frac{-r}{r_0}\right)$

Quantum Mechanics

two-state system (solvable, no perturbation theory)

$H = H_0 + H_I$

$H_0 = \begin{pmatrix} 0 & 0 \\ 0 & \omega \end{pmatrix}$ $H_I = \underset{\text{mass}}{g} \begin{pmatrix} -1 & \Lambda \\ \Lambda & 0 \end{pmatrix}$

$H = \begin{pmatrix} -g & g\Lambda \\ g\Lambda & \omega \end{pmatrix}$

eigenvalue equation

$$(E+g)(E-\omega)-(g\Lambda)^2=0$$

$$E_\pm=\frac{1}{2}\left(\omega-g\pm\sqrt{(\omega+g)^2+4g^2\Lambda^2}\right.$$

Λ can be large. $g\Lambda$ is also large.

$E_{\pm} \to \pm g\Lambda$ sensitive to g

propose: $g\Lambda = g_{ren}$

$$H_{ren} = \begin{pmatrix} -\frac{g_{ren}}{\Lambda} & g_{ren} \\ g_{ren} & w \end{pmatrix}$$

$\Lambda \to \infty$ $H = \begin{pmatrix} 0 & g_{ren} \\ g_{ren} & w \end{pmatrix}$

$$E = \frac{1}{2}\left(w \pm \sqrt{w^2 + 4 g_{ren}^2}\right)$$

blame the parameter

bids

tea.

blame the "bare parameters"

A generic example

4-step to do renormalization

F, G two observable g is coupling

$$F = g + g^2(S_\Lambda + 1) + g^3(S_\Lambda + 1)^2 + \dots$$

$$G = g + g^2(S_\Lambda - 1) + g^3(S_\Lambda - 1)^2 + \dots$$

g : some divergent stuff.

step 1: regularize

cutoff Λ

step 2 blame the coupling constant

expand in terms of observables

$$g = \alpha_1 G + \alpha_2 G^2 + \cdots$$

observables

step 3 consistency check

$$g = \alpha_1 G + \alpha_2 G^2 + \cdots$$

observables

step 3 consistency check

$$\underline{G} = (\alpha_1 \underline{G} + \alpha_2 G^2 \cdots)$$
$$+ (\alpha_1 G + \alpha_2 G^2 \cdots)^2 (S_\Lambda - 1)$$
$$+ \cdots$$

$$\alpha_1 = 1$$

$$\alpha_2 + \alpha_1^2 (S_\Lambda - 1) = 0$$

$$\alpha_2 = -(S_\Lambda - 1)$$

$= 0$

step 4 cutoff can be removed

$$F = \left(G - (s_\Lambda - 1)G^2\right) + \left(G - (s_\Lambda - 1)G^2\right)^2 (s_\Lambda + 1) \cdots$$

$$= G - (s_\Lambda - 1)G^2 + G^2(s_\Lambda + 1)$$

$$= G + 2G^2 + \cdots$$

A toy model

g_0 : "bare" coupling

$F(x)$ FD computation of the coupling.

$$F(x) = g_0 + g_0^2 I_1(x) + g_0^3 I_2(x) + \cdots$$

FD ill defined

experiment $x = \mu$

$F(x=\mu)$ is physical finite

$F(x)$ should be finite function of $F(x=\mu) \equiv g_R$

$$F(x) = \left(F(x=\mu) \right)$$

should be function

of $F(x=\mu) \equiv g_R$

$F(x) = \left(F(x=\mu)\right)$

$g_0 = g_R + \delta_2 g_R + \delta_3 g_R$

step3 consistency check $F(x=\mu) = g_R$

find δ_2, δ_3

step4 $F(x) =$ in terms of $F(\mu)$

$$\lim_{\Lambda \to \infty} F(x, \Lambda, \mu, g_R)$$

concrete example

$$F(x) = g_0 + \underline{g_0^2 I_1(x)} + g_0^3 I_2(x) + \cdots$$

$$I_1(x) = \alpha \int_0^\infty \frac{dt}{t+x}$$

step 1 . $I_{1,\Lambda}(x) = \alpha \int_0^\Lambda \frac{dt}{t+x}$

step 2 2nd order

$$g_0 = g_R + \delta_2 g_R^2 + \cdots$$

step 3 consistency check

$$F_\Lambda(x) = (g_R + \delta_2 g_R^2) + g_R^2 I_{1,\Lambda}(x) + \cdots$$

$$= g_R + (\delta_2 + I_1(x))\, g_R^2 + \cdots$$

$\ldots_2(x)+\cdots$

$\ldots\frac{t}{x}$

$\ldots^{\Lambda}_{0}\frac{dt}{t+x}$

step2 2nd order

$$g_0 = g_R + \delta_2 g_R^2 + \cdots$$

step3 consistency check

$$F_\Lambda(x) = (g_R + \delta_2 g_R^2) + g_R^2 I_{1,\Lambda}(x) + \cdots$$

$$= g_R + (\delta_2 + I_1(x))\, g_R^2 + \cdots$$

$$g_R = F_\Lambda(\mu)$$

$x=\mu$ $\quad \delta_2 + I_1(\mu) = 0$

$$\delta_2 = -I_{1,\Lambda}(\mu)$$

$$= -\alpha \int_0^\Lambda \frac{dt}{t+\mu}$$

$$= -\alpha \ln \frac{\mu+\Lambda}{\mu}$$

$$g_R = F_\Lambda(\mu)$$

$x=\mu$ $\quad \delta_2 + I_1(\mu) = 0$

$$\delta_2 = -I_{1,\Lambda}(\mu)$$

$$= -\alpha \int_0^\Lambda \frac{dt}{t+\mu}$$

$$= -\alpha \ln \frac{\mu+\Lambda}{\mu}$$

step 4 $\quad F_\Lambda(x) = g_R + g_R^2 \left(I_{1,\Lambda}(x) - I_{1,\Lambda}(\mu) \right)$

$$= g_R + g_R^2 \left(\alpha \ln \frac{x+\Lambda}{x} - \alpha \ln \frac{\mu+\Lambda}{\mu} \right)$$

$$= g_R + g_R^2 \alpha \ln \frac{x+\Lambda}{x} \frac{\mu}{\mu+\Lambda}$$

$$= g_R + g_R^2 \alpha \left(\ln \frac{\mu}{x} + \cancel{\ln \frac{x+\Lambda}{\mu+\Lambda}} \right)$$