Title: QFT1 Lecture - 110623

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Collection: Quantum Field Theory 1 2023/24

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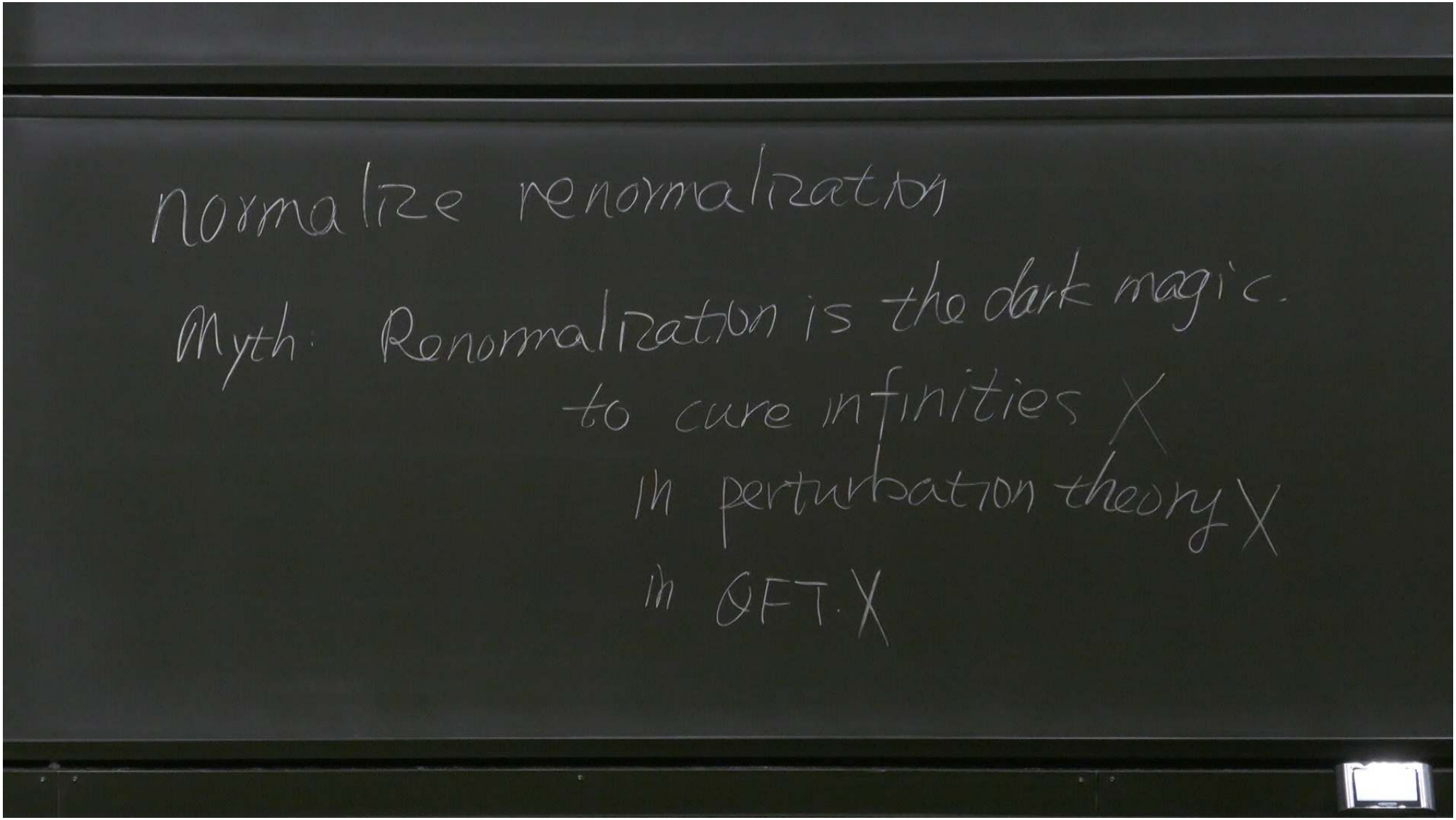
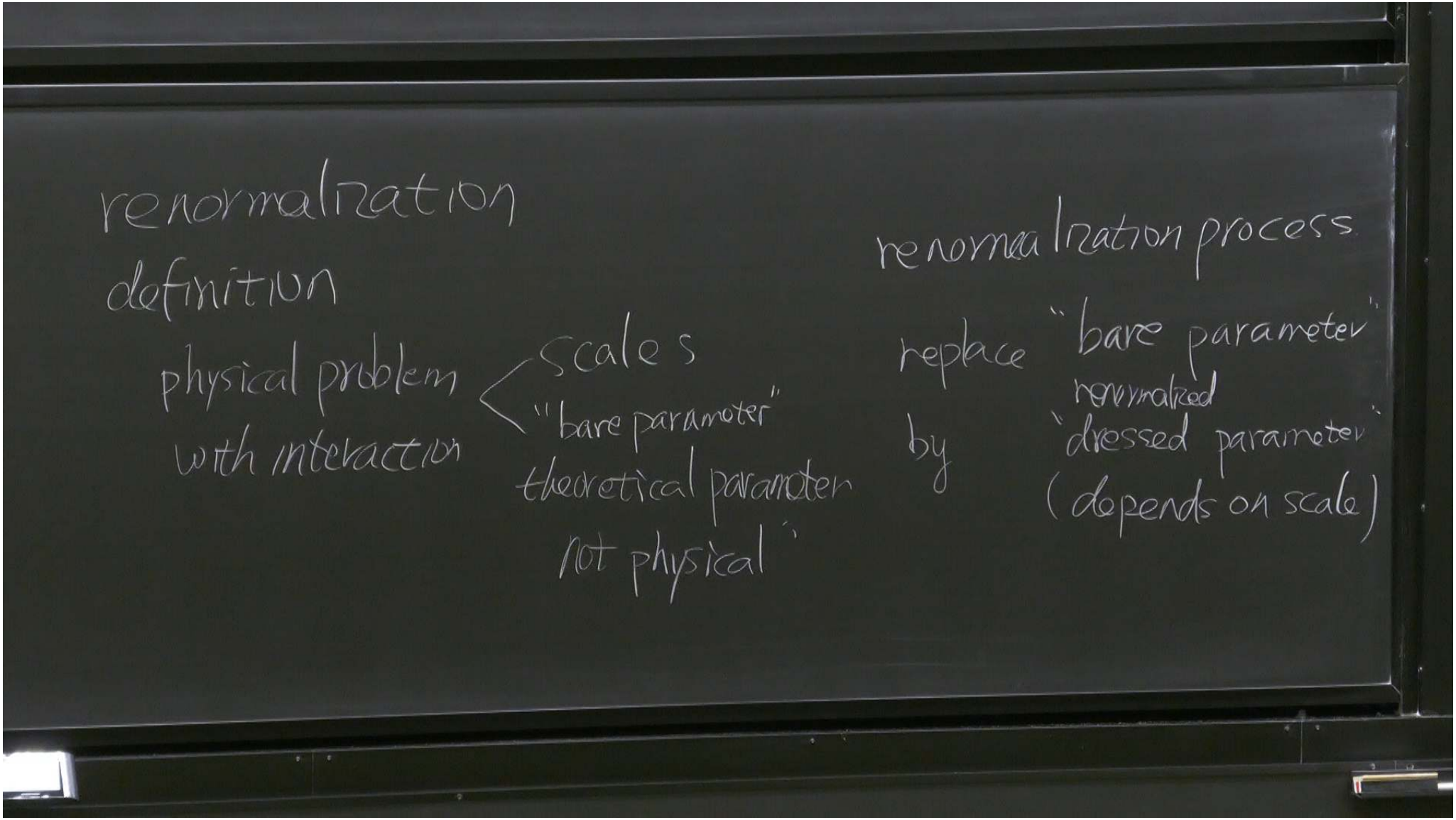

normalize renormalization
Myth: Renormalization is the dark magic.
to cure infinities X
in perturbation theory X
in QFT. X


renormalization
definition
physical problem
with interaction
scales
"bare parameter"
theoretical parameter
not physical"
renormalization process
replace "bare parameter"
by renormalized "dressed parameter"
(depends on scale)

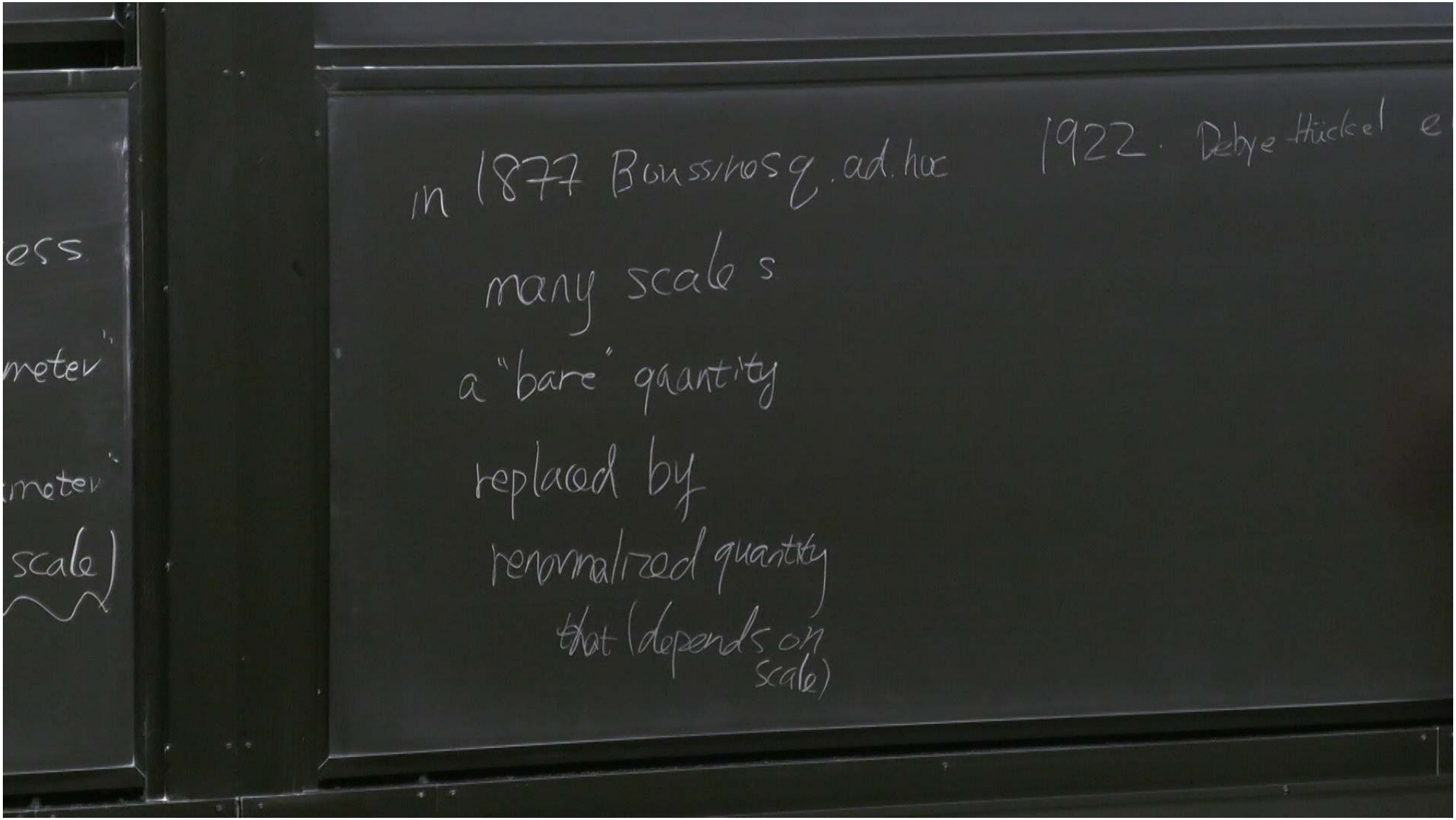
PSI Award

Feedback

Nature forbids

a naked

single lang tea.

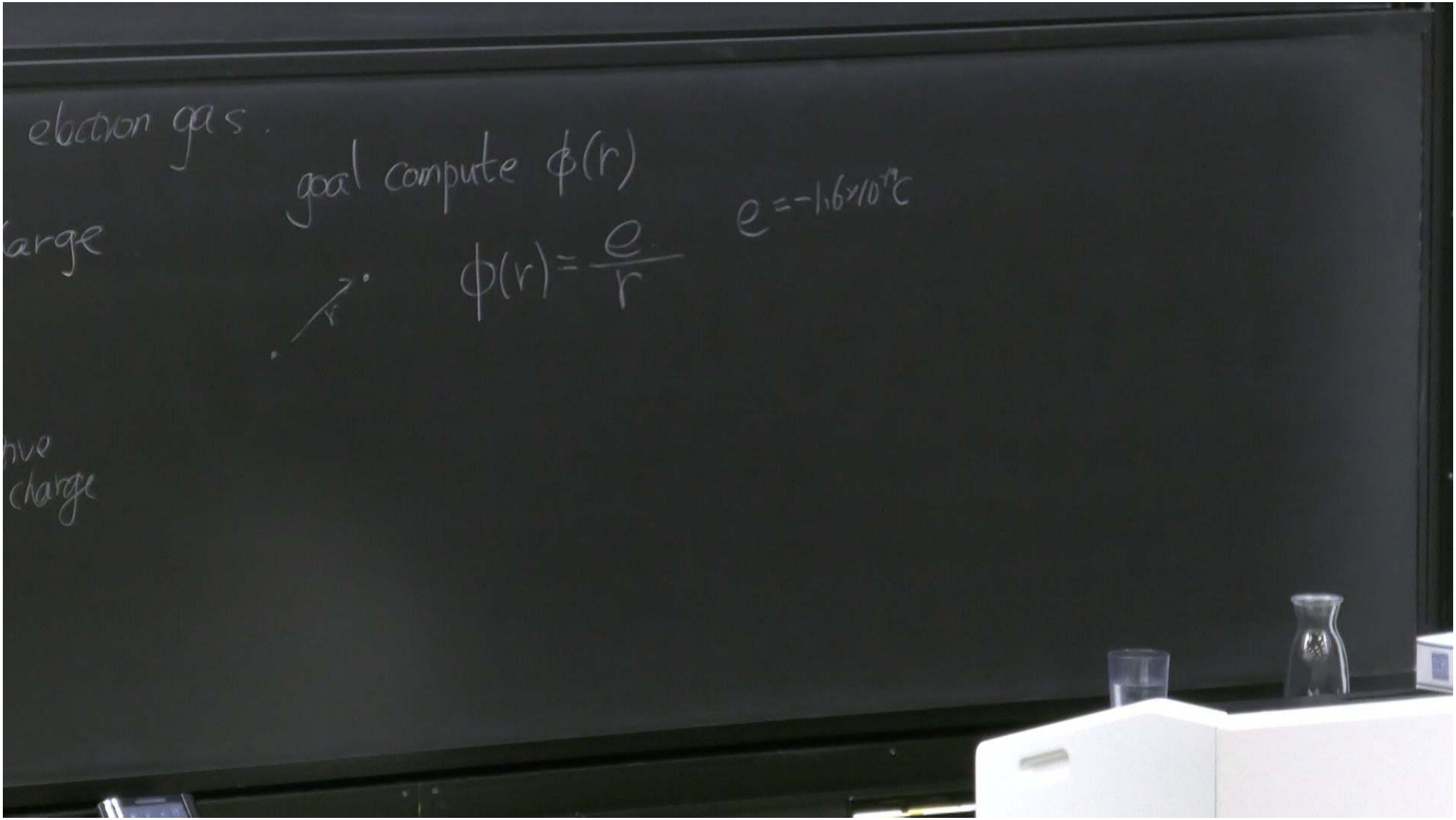

ess
meter
meter
scale)
in 1877 Boussinesq ad hoc
many scales
a "bare" quantity
replaced by
renormalized quantity
that (depends on
scale)
1922. Debye-Hückel e

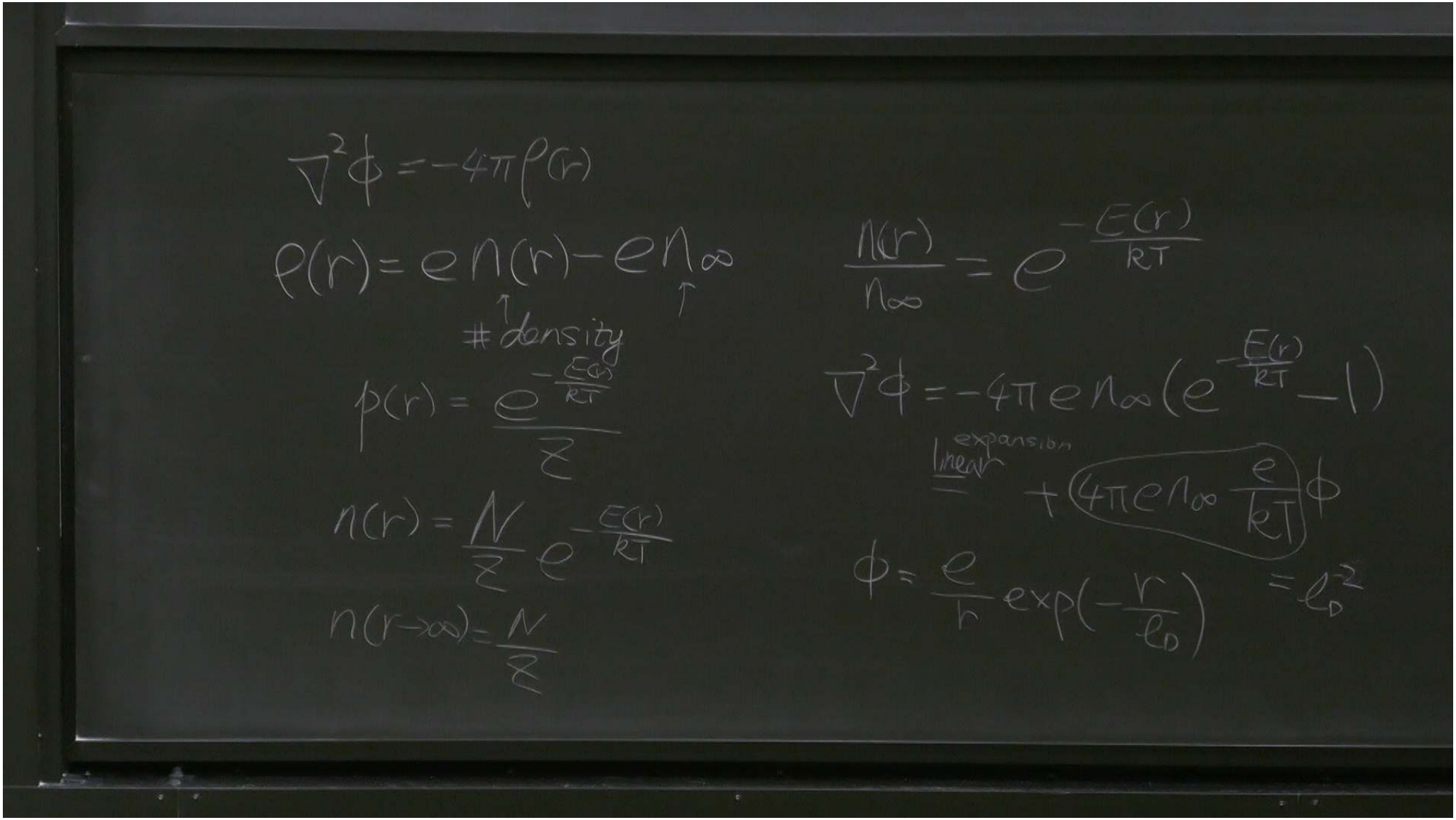
1922. Debye-Hückel electrolyte electron gas.

- charge → renormalized effective charge

goal compute $\phi(r)$

{ system is neutral
  N free electrons
  spatially uniform positive charge


electron gas.
goal compute $\phi(r)$
$\phi(r) = \frac{e}{r}$
$e = -1.6 \times 10^{-19} C$
r
harge
ave
charge


$\nabla^2\phi = -4\pi\rho(r)$
$\rho(r) = en(r) - en_\infty$
density
$p(r) = \frac{e^{-\frac{E(r)}{RT}}}{Z}$
$n(r) = \frac{N}{Z} e^{-\frac{E(r)}{RT}}$
$n(r\to\infty) = \frac{N}{Z}$
$\frac{n(r)}{n_\infty} = e^{-\frac{E(r)}{RT}}$
$\nabla^2\phi = -4\pi e n_\infty \left(e^{-\frac{E(r)}{RT}} - 1\right)$
linear expansion
$+ \left(4\pi e n_\infty \frac{e}{kT}\right)\phi$
$\phi = \frac{e}{r}\exp\left(-\frac{r}{\ell_D}\right)$
$= \ell_D^{-2}$

$e \to e \exp\left(\frac{-r}{r_0}\right)$

Quantum Mechanics

two-state system (solvable, no perturbation theory)

$$H = H_0 + H_I$$

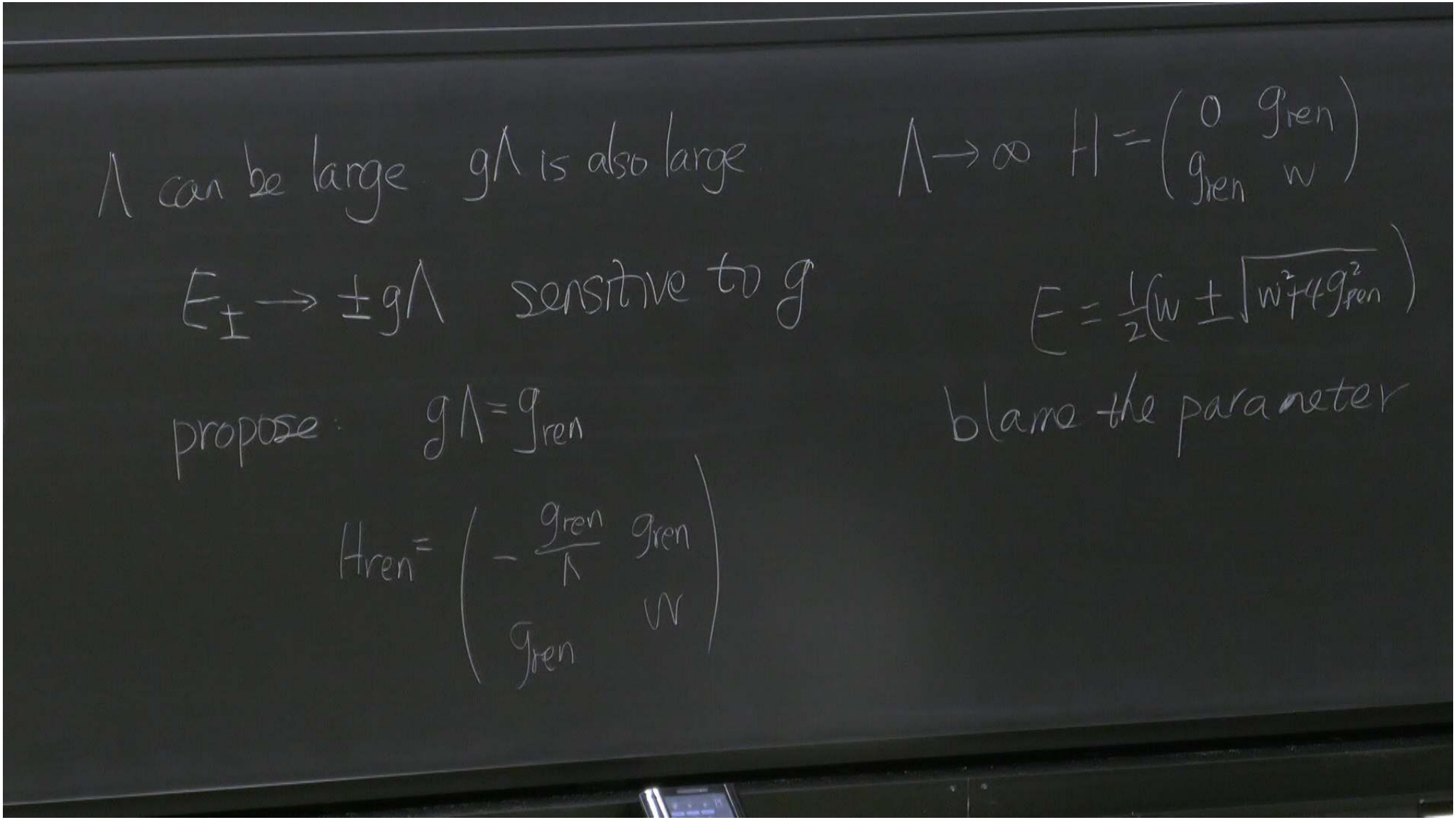
$$H_0 = \begin{pmatrix} 0 & 0 \\ 0 & \omega \end{pmatrix} \qquad H_I = \underset{\text{Mass}}{g} \begin{pmatrix} -1 & \Lambda \\ \Lambda & 0 \end{pmatrix}$$

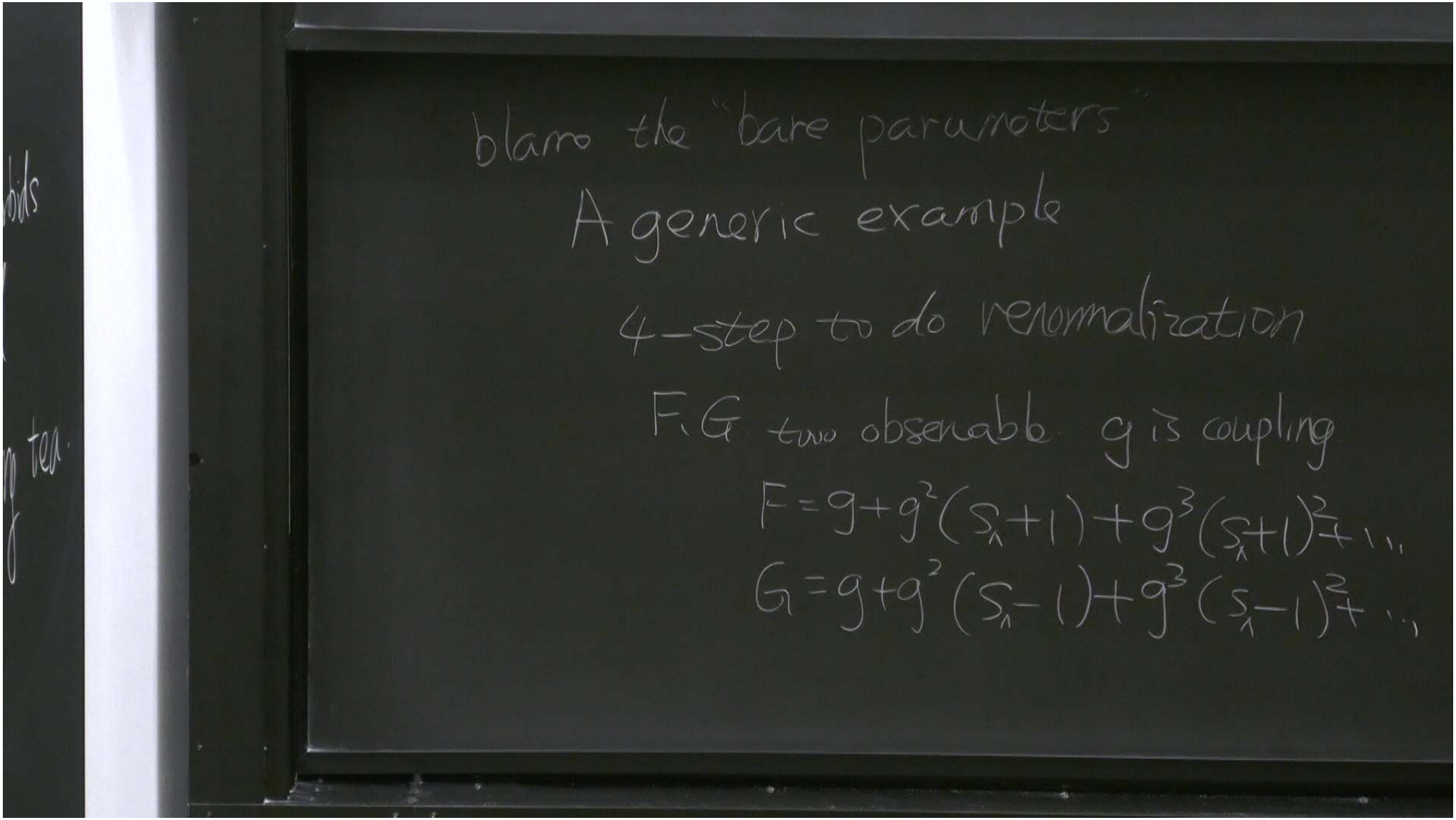
$$H = \begin{pmatrix} -g & g\Lambda \\ g\Lambda & \omega \end{pmatrix}$$

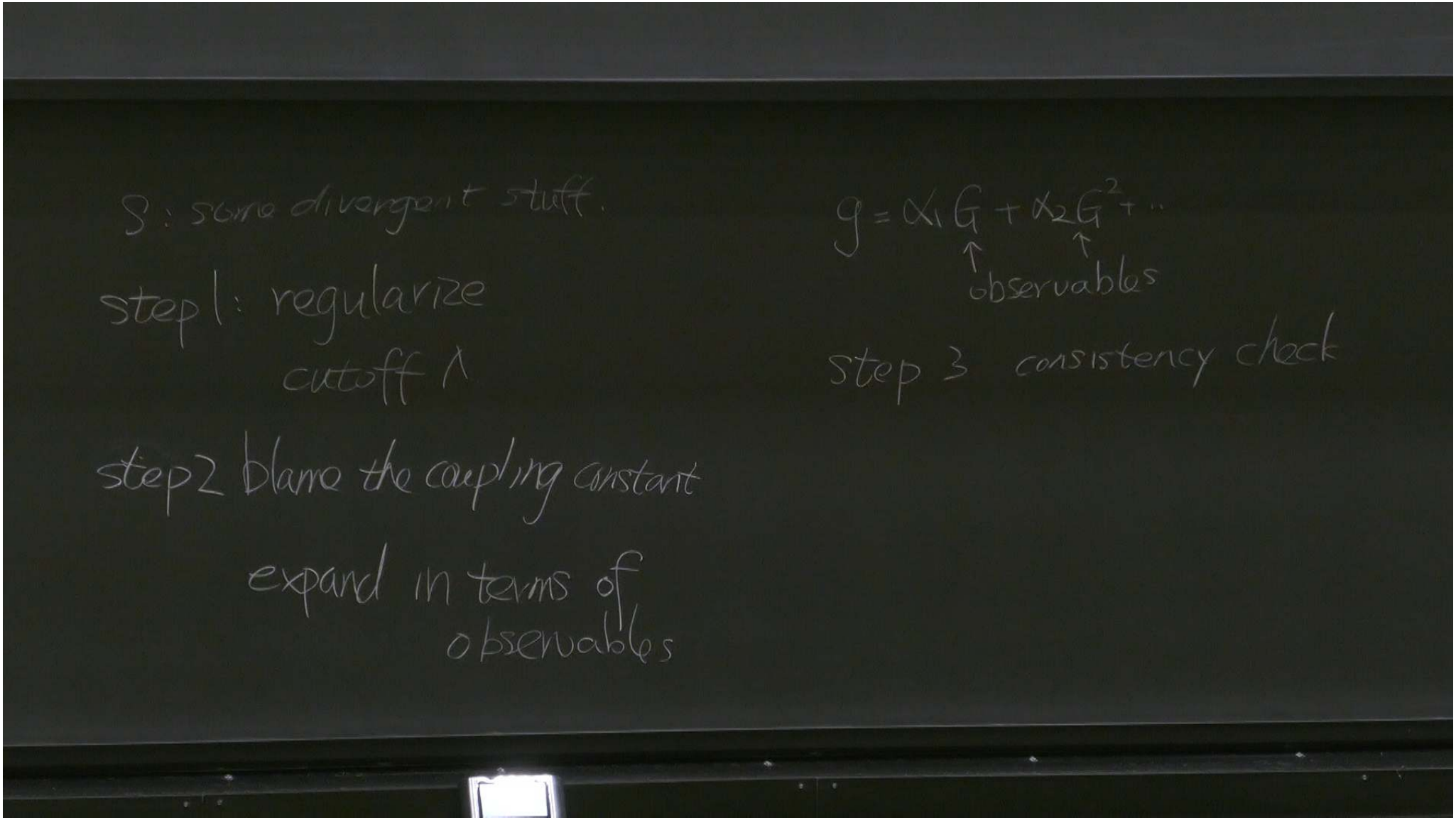
eigenvalue equation

$$(E+g)(E-w)-(g\Lambda)^2=0$$

$$E_\pm=\frac{1}{2}\left(w-g\pm\sqrt{(w+g)^2+4g^2\Lambda^2}\right.$$


Λ can be large gΛ is also large
$\Lambda \to \infty$ $H = \begin{pmatrix} 0 & g_{ren} \\ g_{ren} & w \end{pmatrix}$
$E_I \to \pm g\Lambda$ sensitive to g
$E = \frac{1}{2}(w \pm \sqrt{w^2 + 4g_{ren}^2})$
propose: $g\Lambda = g_{ren}$
blame the parameter
$H_{ren} = \begin{pmatrix} -\frac{g_{ren}}{\Lambda} & g_{ren} \\ g_{ren} & w \end{pmatrix}$


blare the "bare parameters"
A generic example
4-step to do renormalization
F, G two observable g is coupling
$F = g + g^2(s_\Lambda + 1) + g^3(s_\Lambda + 1)^2 + \ldots$
$G = g + g^2(s_\Lambda - 1) + g^3(s_\Lambda - 1)^2 + \ldots$


S : some divergent stuff.
step 1 : regularize
cutoff Λ
step 2 blame the coupling constant
expand in terms of
observables
$g = \alpha_1 G + \alpha_2 G^2 + \cdots$
observables
step 3 consistency check

$g = \alpha_1 G + \alpha_2 G^2 + \cdots$

observables

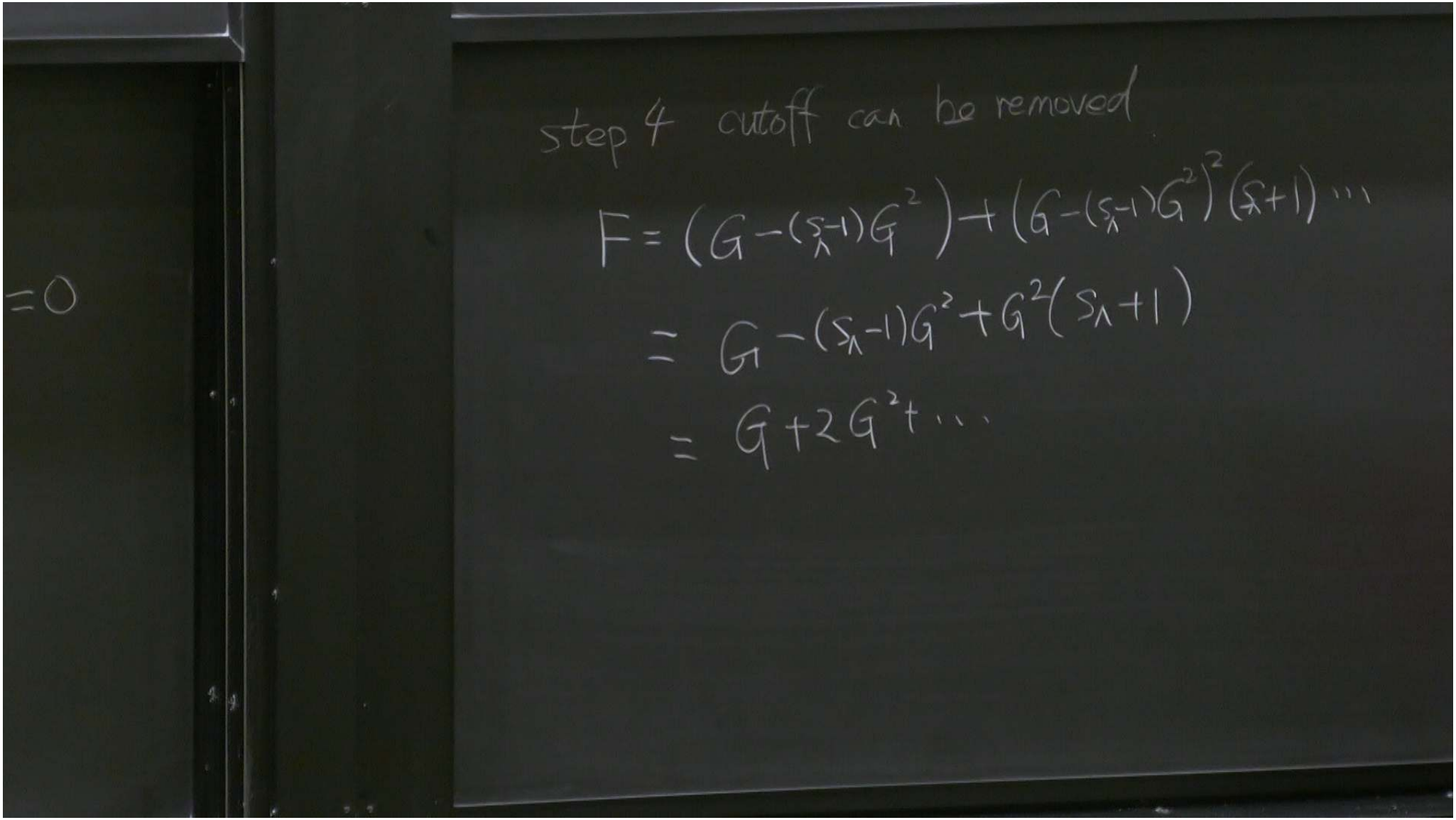
$\alpha_1 = 1$

$\alpha_2 + \alpha_1^2 (S_\Lambda - 1) = 0$

$\alpha_2 = -(S_\Lambda - 1)$

step 3 consistency check

$$G = (\alpha_1 G + \alpha_2 G^2 \cdots) + (\alpha_1 G + \alpha_2 G^2 \cdots)^2 (S_\Lambda - 1) + \cdots$$


step 4 cutoff can be removed
$F = (G - (S_\Lambda - 1)G^2) + (G - (S_\Lambda - 1)G^2)^2 (S_\Lambda + 1) \cdots$
$= G - (S_\Lambda - 1)G^2 + G^2(S_\Lambda + 1)$
$= G + 2G^2 t \cdots$
$= 0$

A toy model

g_0 : "bare" coupling

$F(x)$ FD computation of the coupling.

$$F(x) = g_0 + g_0^2 I_1(x) + g_0^3 I_2(x) + \cdots$$

↑ FD ill defined ↑

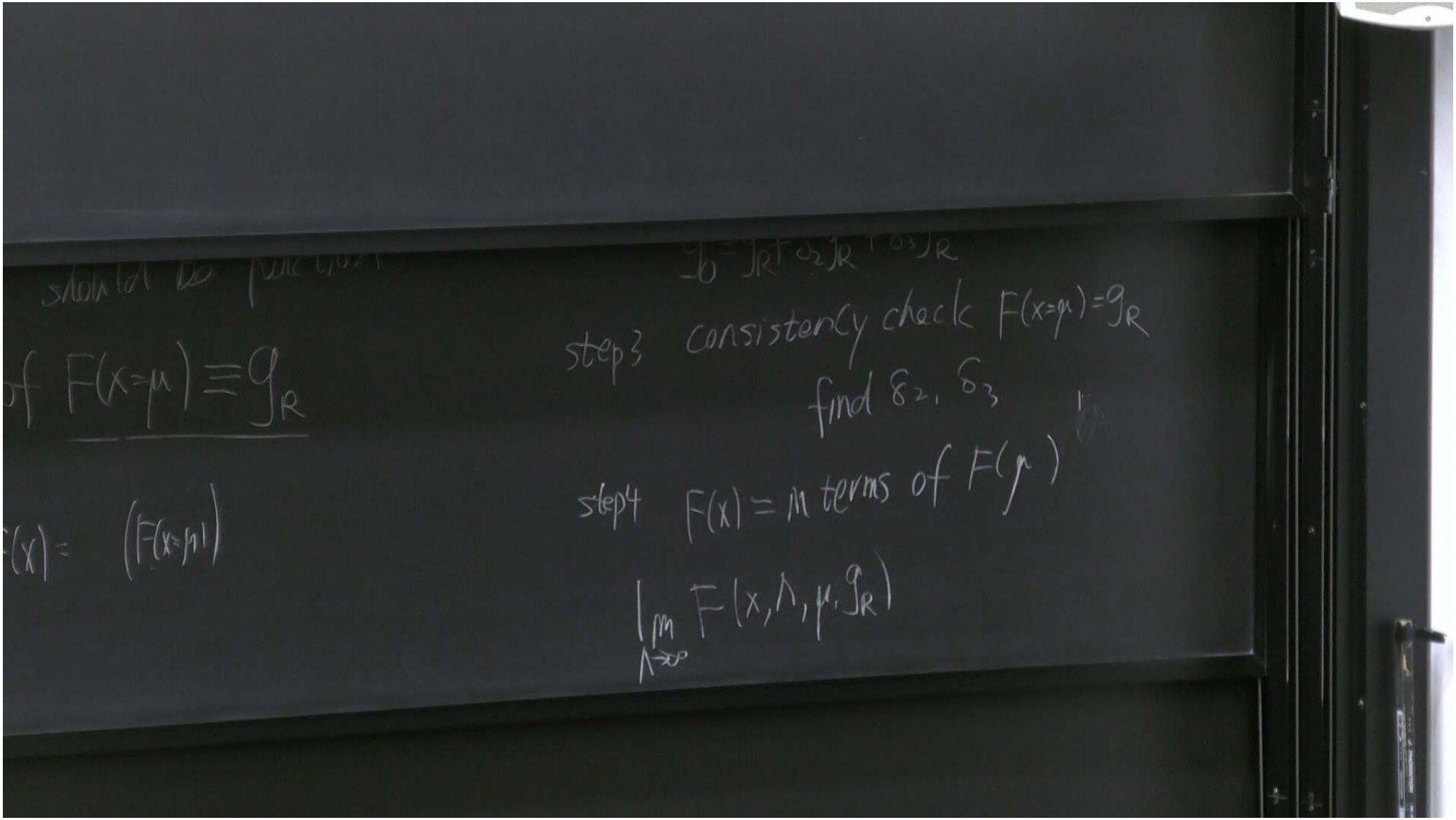
experiment $x = \mu$

$F(x=\mu)$ is physical finite

$F(x)$ should be finite function

of $F(x=\mu) \equiv g_R$

$F(x) = \left(F(x=\mu)\right)$


of $F(x=\mu) \equiv g_R$
$(x) = (F(x=\mu))$
step3 consistency check $F(x=\mu) = g_R$
find δ_2, δ_3
step4 $F(x) =$ in terms of $F(\mu)$
$\lim_{\Lambda \to \infty} F(x, \Lambda, \mu, g_R)$

concrete example

$$F(x) = g_0 + g_0^2 I_1(x) + g_0^3 I_2(x) + \cdots$$

$$I_1(x) = \alpha \int_0^\infty \frac{dt}{t+x}$$

step 1 . $I_{1,\Lambda}(x) = \alpha \int_0^\Lambda \frac{dt}{t+x}$

step 2 2nd order

$$g_0 = g_R + \delta_2 g_R^2 + \cdots$$

step 3 consistency check

$$F_\Lambda(x) = (g_R + \delta_2 g_R^2) + g_R^2 I_{1,\Lambda}(x) + \cdots$$

$$= g_R + (\delta_2 + I_1(x)) g_R^2 + \cdots$$

$_2(x)+\cdots$

$\frac{t}{x}$

$\frac{\Lambda dt}{t+x}$

step 2 2nd order

$$g_0 = g_R + \delta_2 g_R^2 + \cdots$$

step 3 consistency check

$$F_\Lambda(x) = (g_R + \delta_2 g_R^2) + g_R^2 I_{1,\Lambda}(x) + \cdots$$

$$= g_R + (\delta_2 + I_1(x))\, g_R^2 + \cdots$$

$$g_R = F_\Lambda(\mu)$$

$x=\mu$ $\quad \delta_2 + I_1(\mu) = 0$

$$\delta_2 = -I_{1,\Lambda}(\mu)$$

$$= -\alpha \int_0^\Lambda \frac{dt}{t+\mu}$$

$$= -\alpha \ln \frac{\mu+\Lambda}{\mu}$$

$g_R = F_\Lambda(\mu)$

$x=\mu \quad \delta_2 + I_{1,\Lambda}(\mu) = 0$

$\delta_2 = -I_{1,\Lambda}(\mu)$

$= -\alpha \int_0^\Lambda \frac{dt}{t+\mu}$

$= -\alpha \ln \frac{\mu+\Lambda}{\mu}$

step 4 $F_\Lambda(x) = g_R + g_R^2 \left(I_{1,\Lambda}(x) - I_{1,\Lambda}(\mu) \right)$

$= g_R + g_R^2 \left(\alpha \ln \frac{x+\Lambda}{x} - \alpha \ln \frac{\mu+\Lambda}{\mu} \right)$

$= g_R + g_R^2 \alpha \ln \frac{x+\Lambda}{x} \frac{\mu}{\mu+\Lambda}$

$= g_R + g_R^2 \alpha \left(\ln \frac{\mu}{x} + \ln \frac{x+\Lambda}{\mu+\Lambda} \right)$