Title: QFT1 Lecture - 103123

Speakers: Gang Xu

Collection: Quantum Field Theory 1 2023/24

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- charge conjugation

$$\mathcal{L} = \bar{\psi}\left((i\not\partial - m) - e\gamma^\mu A_\mu\right)\psi$$

e.o.m $\quad ((i\not\partial - m))\psi - e\gamma^\mu A_\mu \psi = 0$

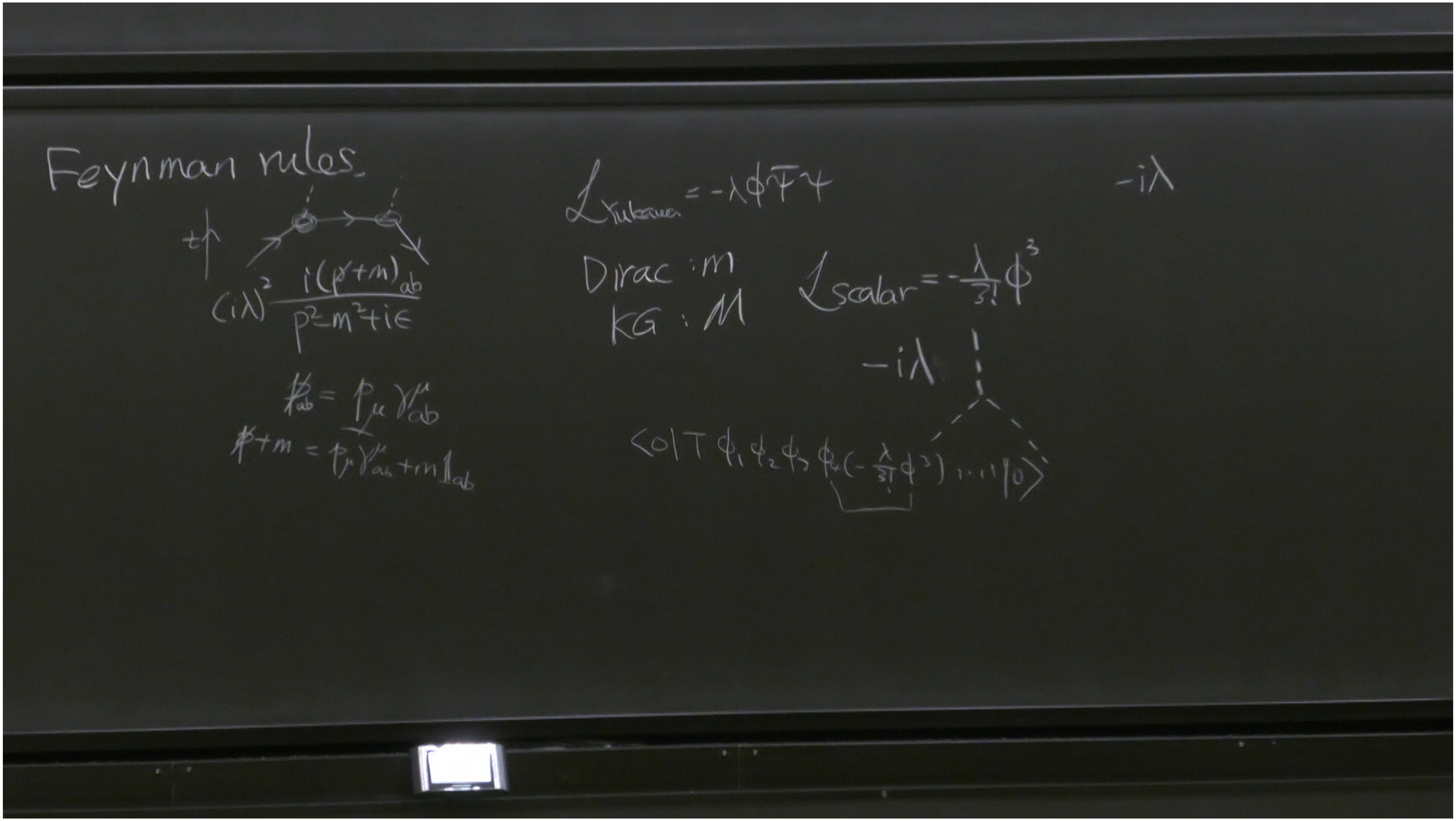
Step 1: take c.c.

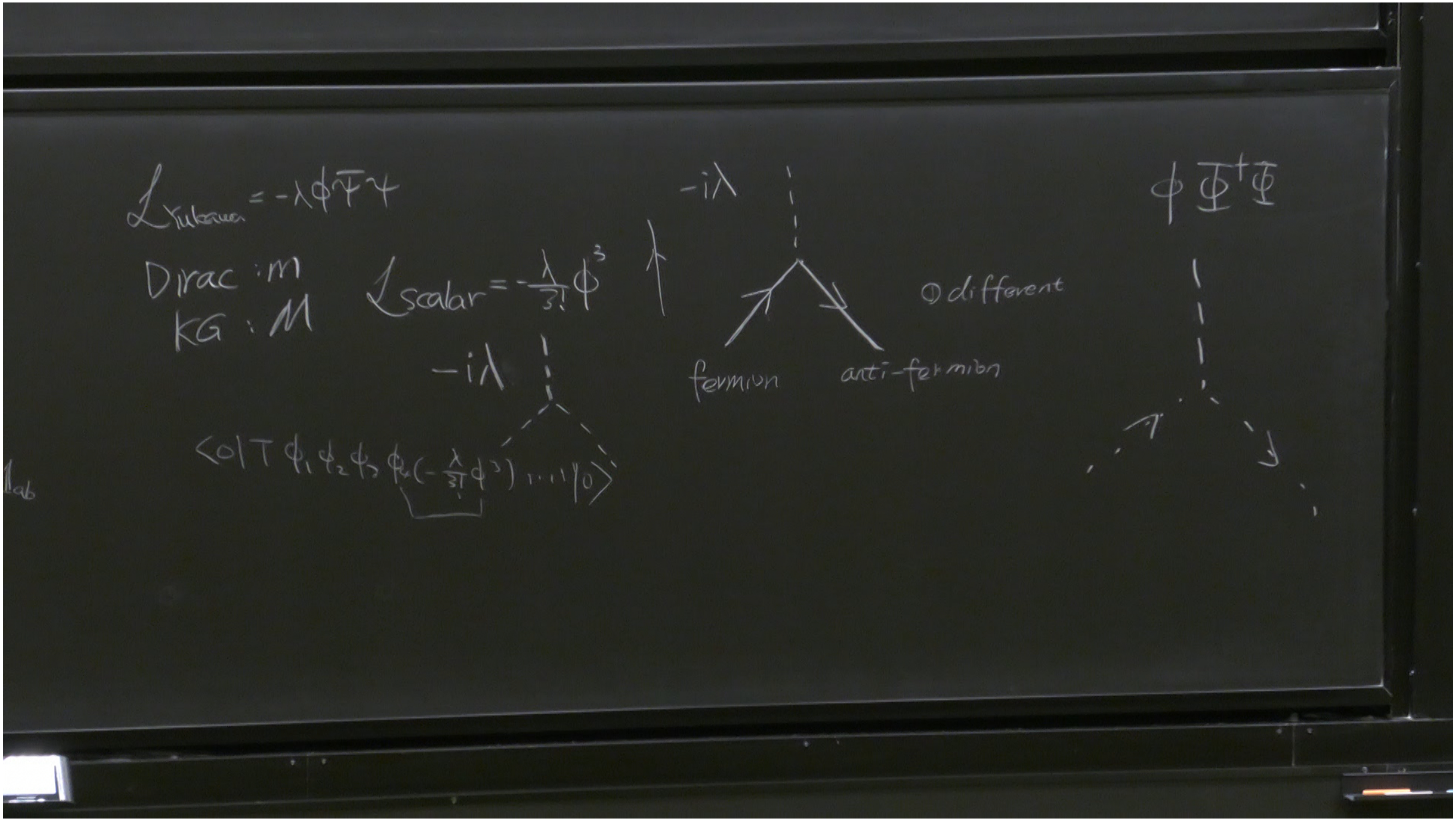
$C^{-1}C$

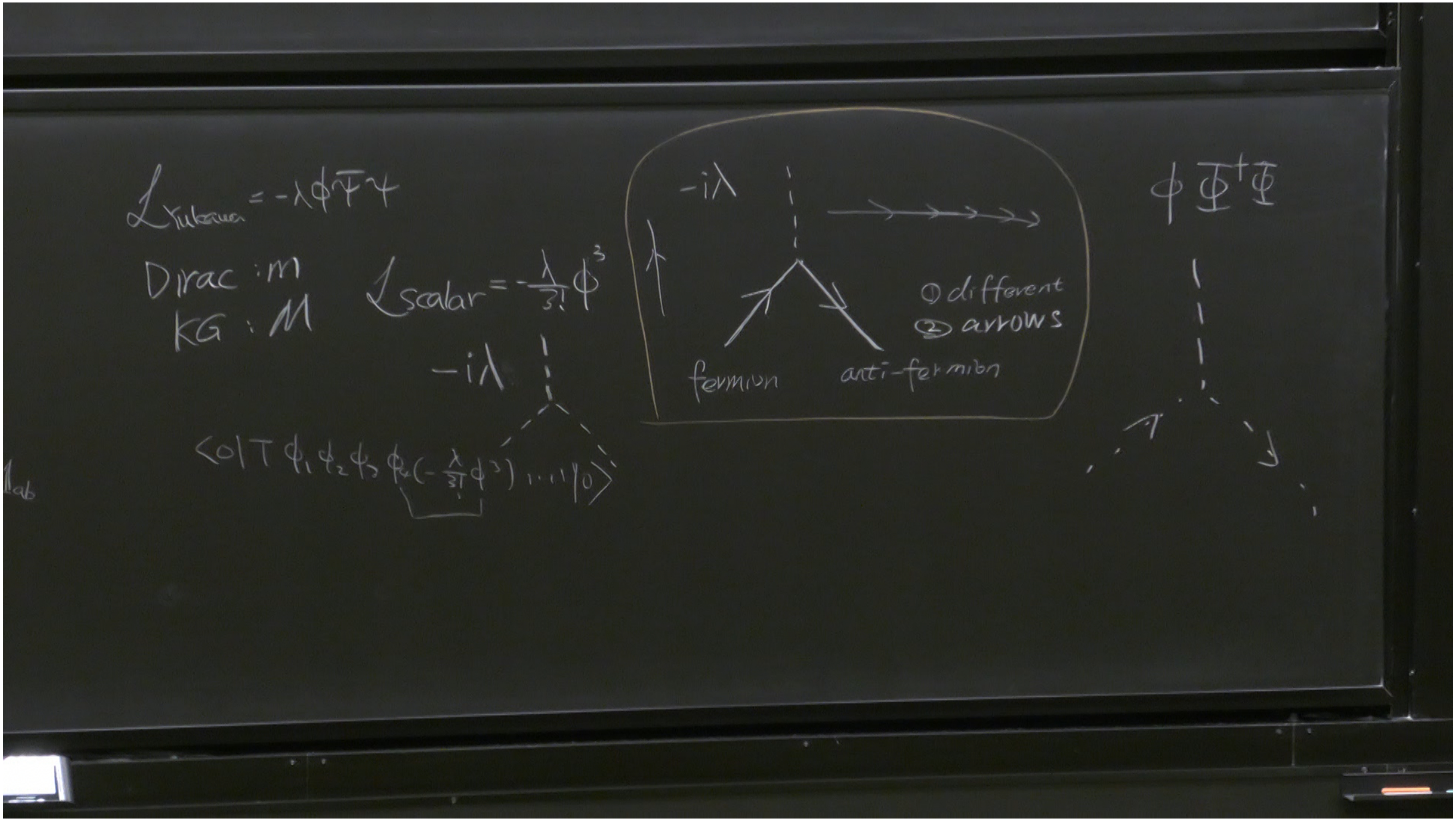
$C\gamma^{\mu *}C^{-1} = -\gamma^\mu$

$$C(-i\partial_\mu(\gamma^\mu)^* - m)\psi^* - Ce(\gamma^\mu)^* A_\mu \psi^* = 0$$

$$(i\not\partial - m)\psi^{(c)} + e\gamma^\mu A_\mu \psi^{(c)} = 0$$


Feynman rules
$(i\lambda)^2 \frac{i(\not p+m)_{ab}}{p^2-m^2+i\epsilon}$
$\not p_{ab} = p_\mu \gamma^\mu_{ab}$
$\not p + m = p_\mu \gamma^\mu_{ab} + m \mathbb{1}_{ab}$
$\mathcal{L}_{Yukawa} = -\lambda \phi \bar\Psi \Psi$
Dirac : m
KG : M
$\mathcal{L}_{scalar} = -\frac{\lambda}{3!}\phi^3$
$-i\lambda$
$\langle 0|T\phi_1\phi_2\phi_3\phi_4(-\frac{\lambda}{3!}\phi^3)\cdots|0\rangle$
$-i\lambda$


$\mathcal{L}_{\text{Yukawa}} = -\lambda \phi \bar{\psi} \psi$
Dirac : m
KG : M
$\mathcal{L}_{\text{scalar}} = -\frac{\lambda}{3!} \phi^3$
$-i\lambda$
$\langle 0 | T \phi_1 \phi_2 \phi_3 \phi_4 (-\frac{\lambda}{3!} \phi^3) \cdots | 0 \rangle$
$-i\lambda$
fermion
anti-fermion
① different
$\phi \Phi^\dagger \Phi$


$\mathcal{L}_{Yukawa} = -\lambda\phi\bar{\Psi}\Psi$
Dirac : m
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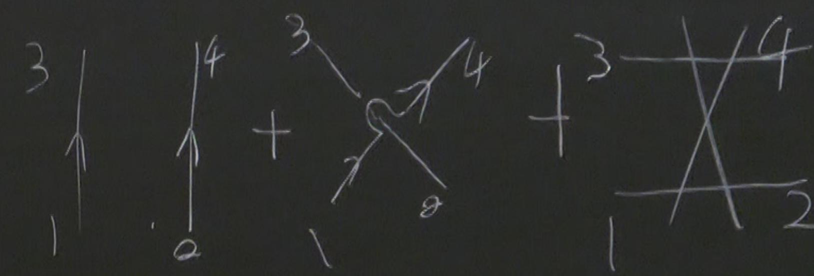
$-i\lambda$
$\langle 0|T\phi_1\phi_2\phi_3\phi_4(-\frac{\lambda}{3!}\phi^3)\cdots|0\rangle$
$-i\lambda$
① different
② arrows
fermion
anti-fermion
$\phi\,\Phi^\dagger\Phi$

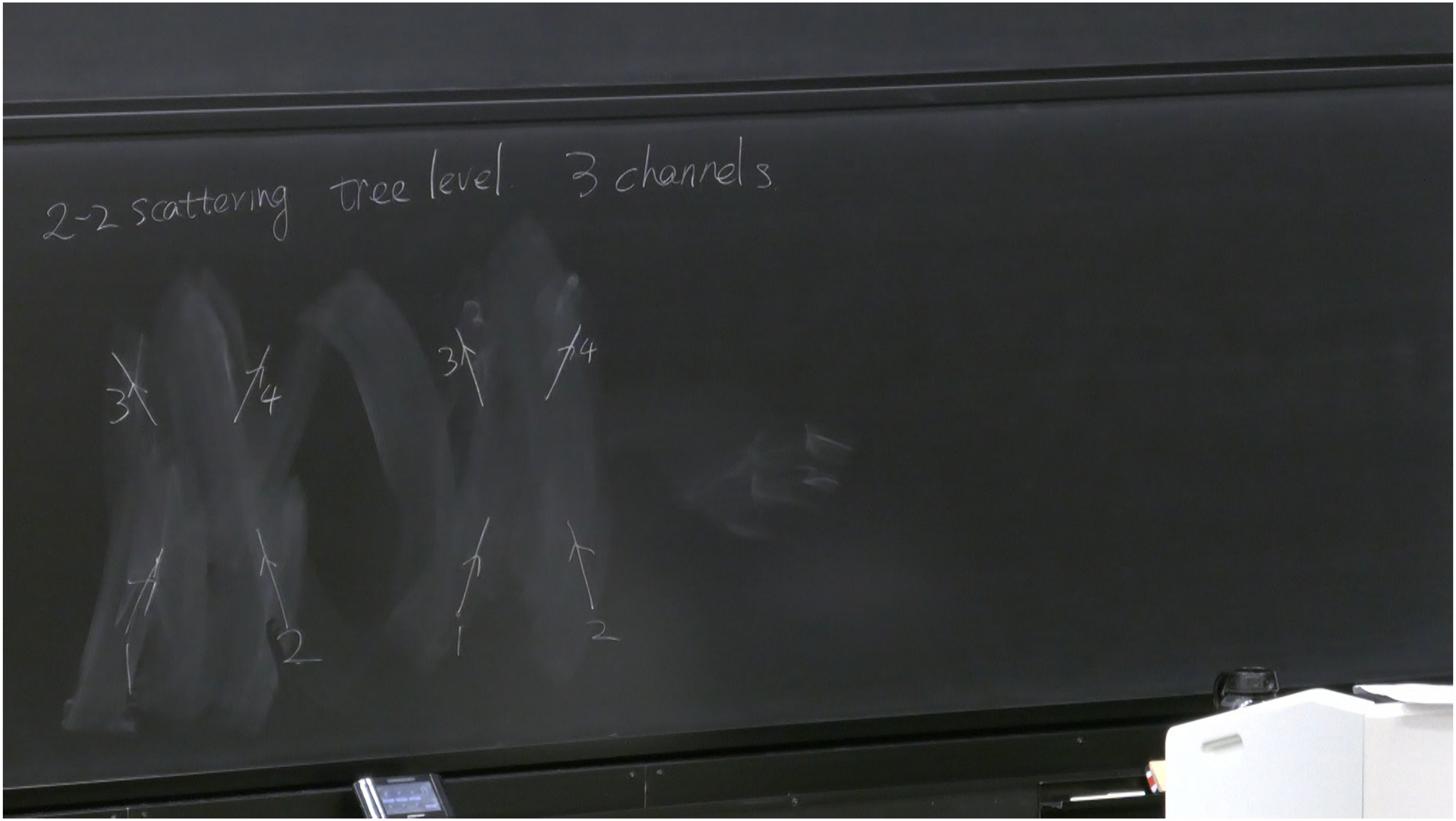
$\langle 0|T\psi_4\psi_3\bar{\psi}_1\bar{\psi}_2|0\rangle$ 0th order

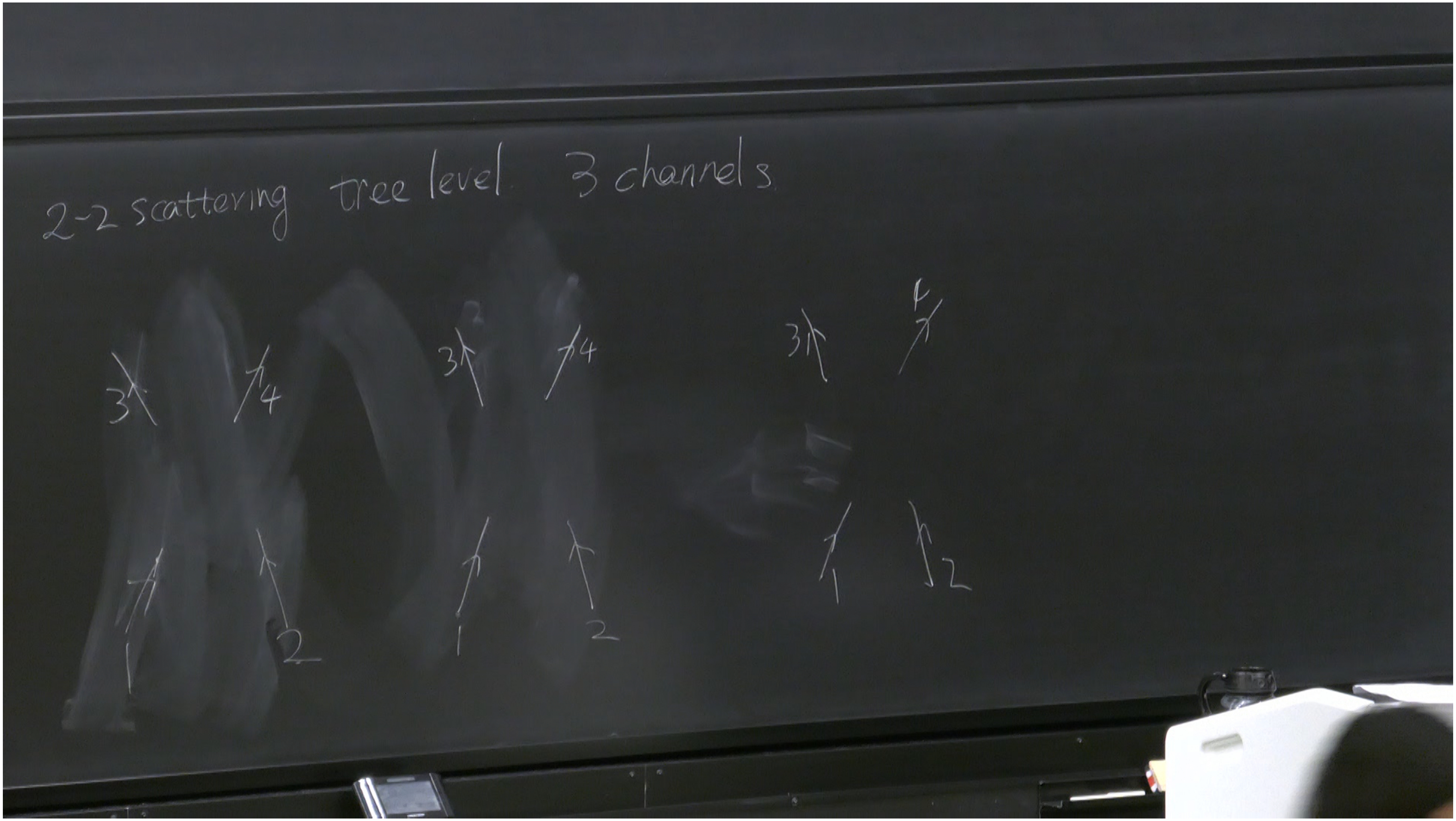
2-2 s

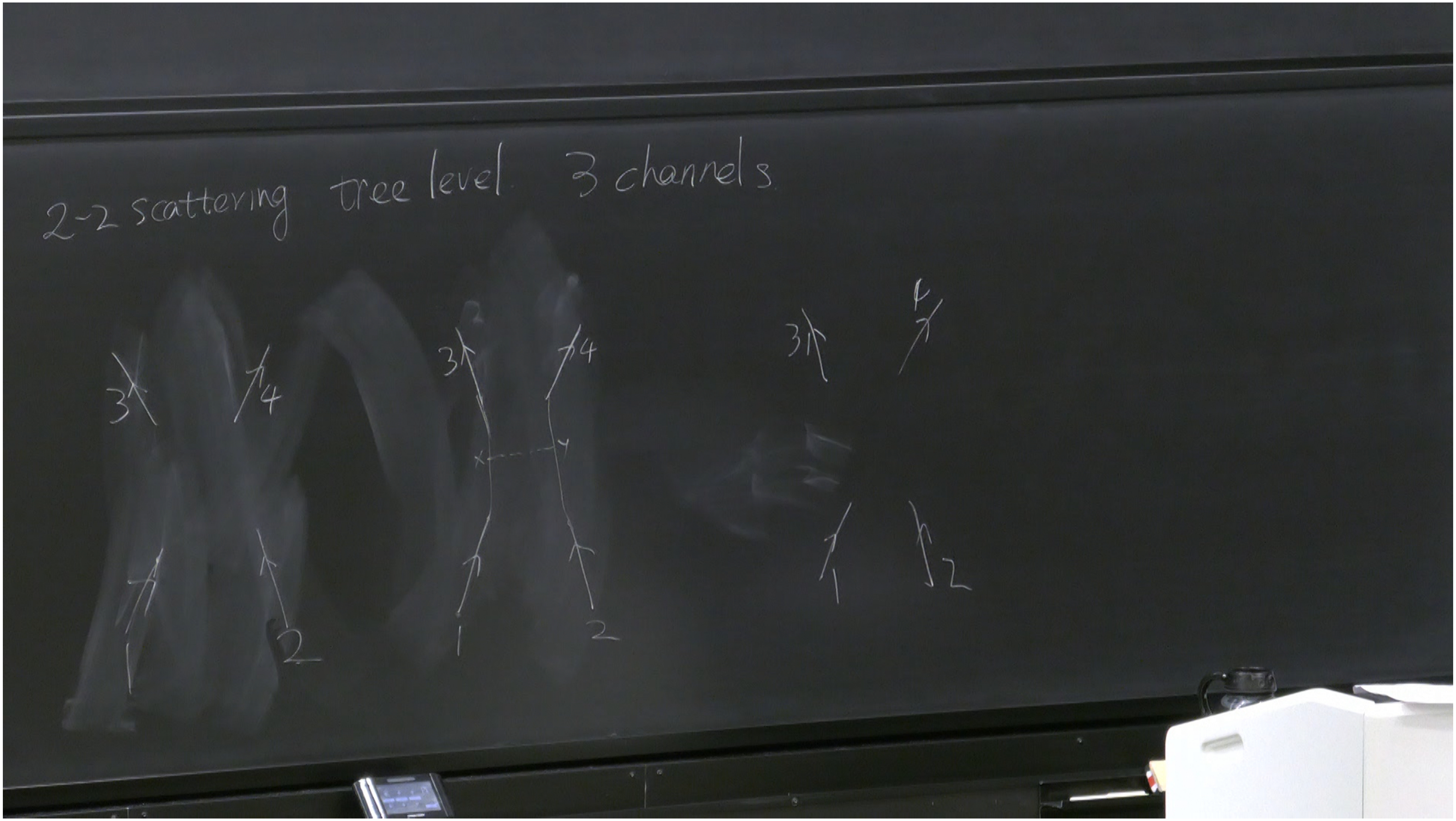


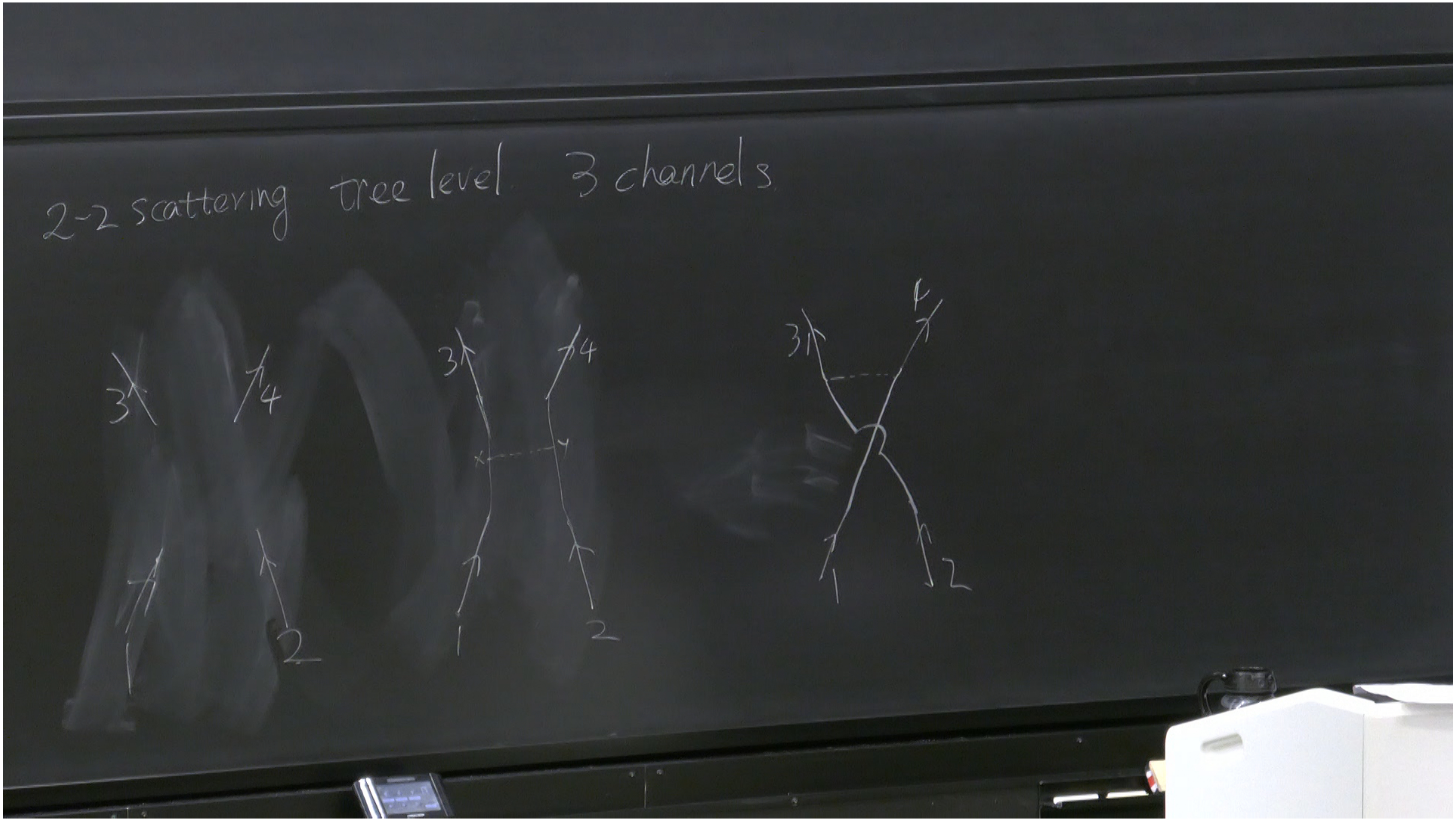
$$\langle 0|T\psi_4\psi_3\bar{\psi}_1\bar{\psi}_2\int d^4x\,\lambda\phi_x\bar{\psi}_x\psi_x|0\rangle$$

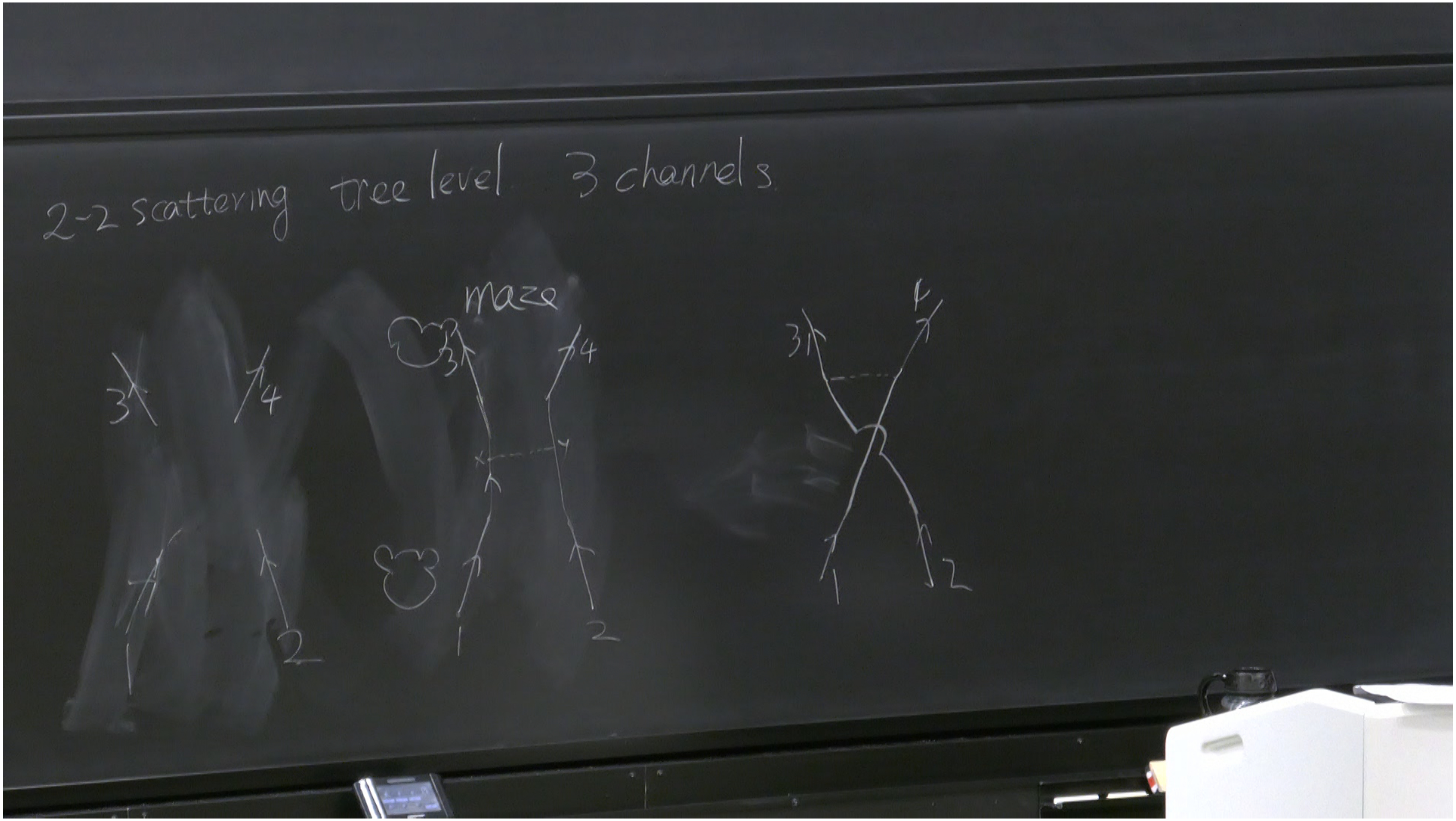
$$\langle 0|T\psi_4\psi_3\bar{\psi}_1\bar{\psi}_2\int d^4x\,\lambda\phi_x\bar{\psi}_x\psi_x\int d^4y\,\lambda\phi_y\bar{\psi}_y\psi_y|0\rangle$$

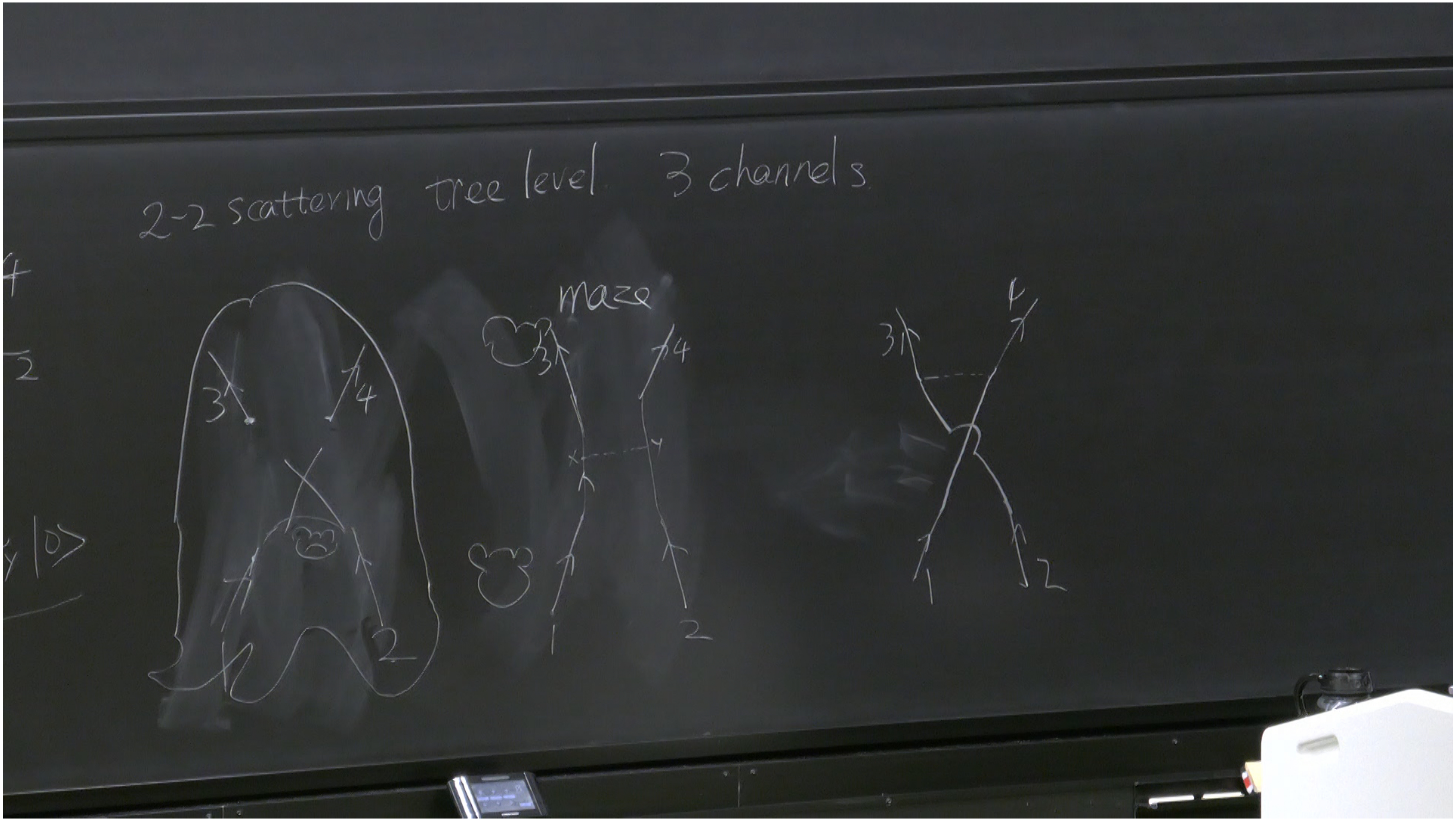

2-2 scattering tree level. 3 channels
3
4
1
2
3
4
1
2

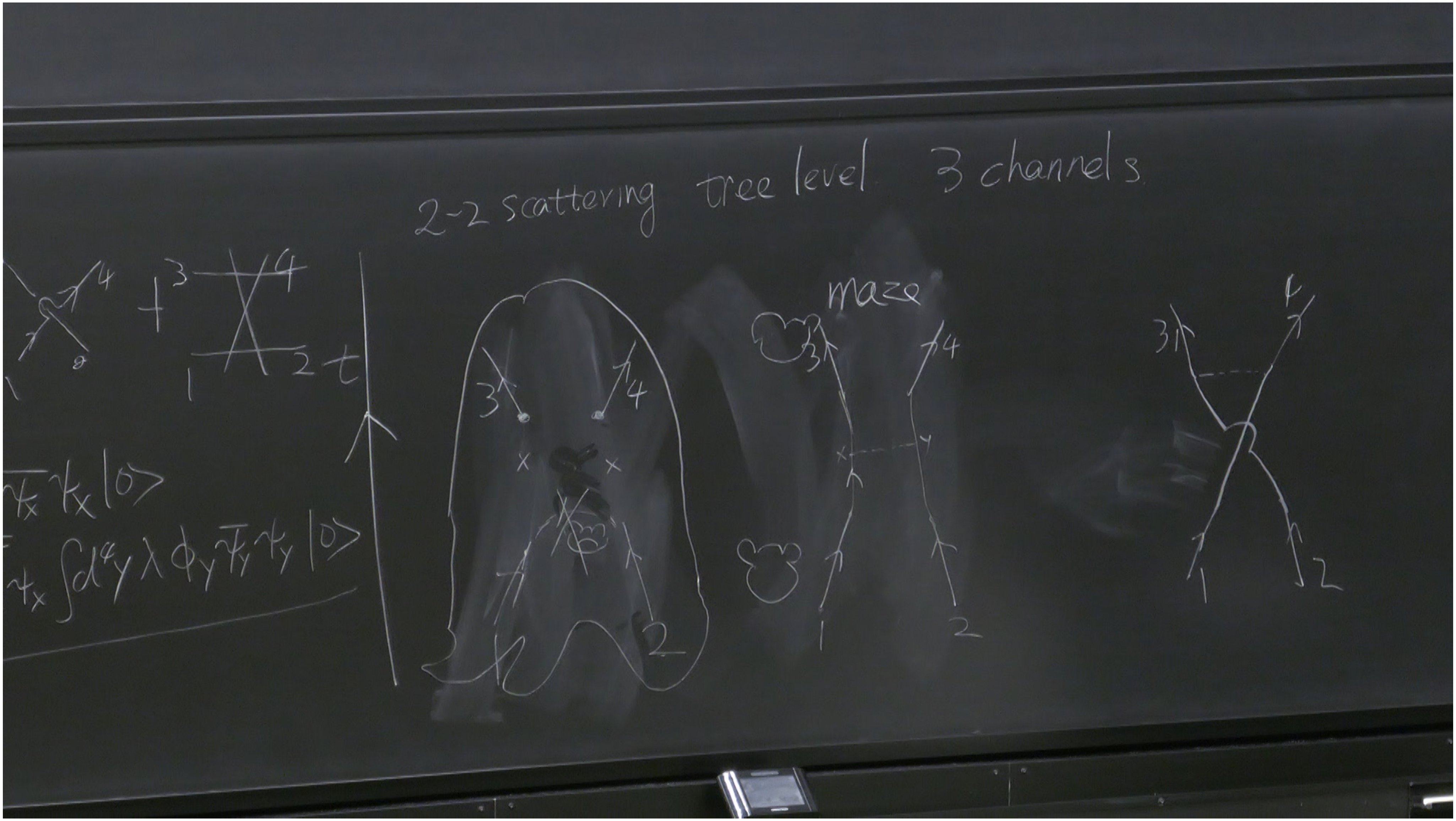

2-2 scattering tree level 3 channels
3
4
1
2
3
4
1
2
3
4
1
2


2-2 scattering tree level 3 channels


2-2 scattering tree level 3 channels


2-2 scattering tree level 3 channels
maze
3
4
1
2

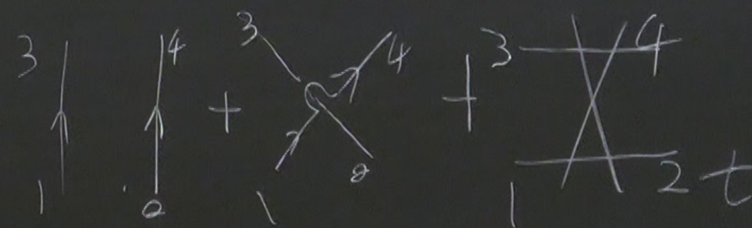

2-2 scattering tree level 3 channels
maze
3
4
1
2

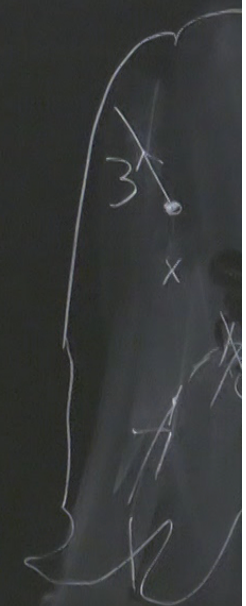

2-2 scattering tree level 3 channels
maze

$\langle 0 | T\, \psi_4 \psi_3 \bar{\psi}_1 \bar{\psi}_2 | 0 \rangle$ 0th order

2-2 scat

$bb \to bb$





$$\langle 0 | T\, \psi_4 \psi_3 \bar{\psi}_1 \bar{\psi}_2 \int d^4x \, \lambda \phi_x \bar{\psi}_x \psi_x | 0 \rangle$$

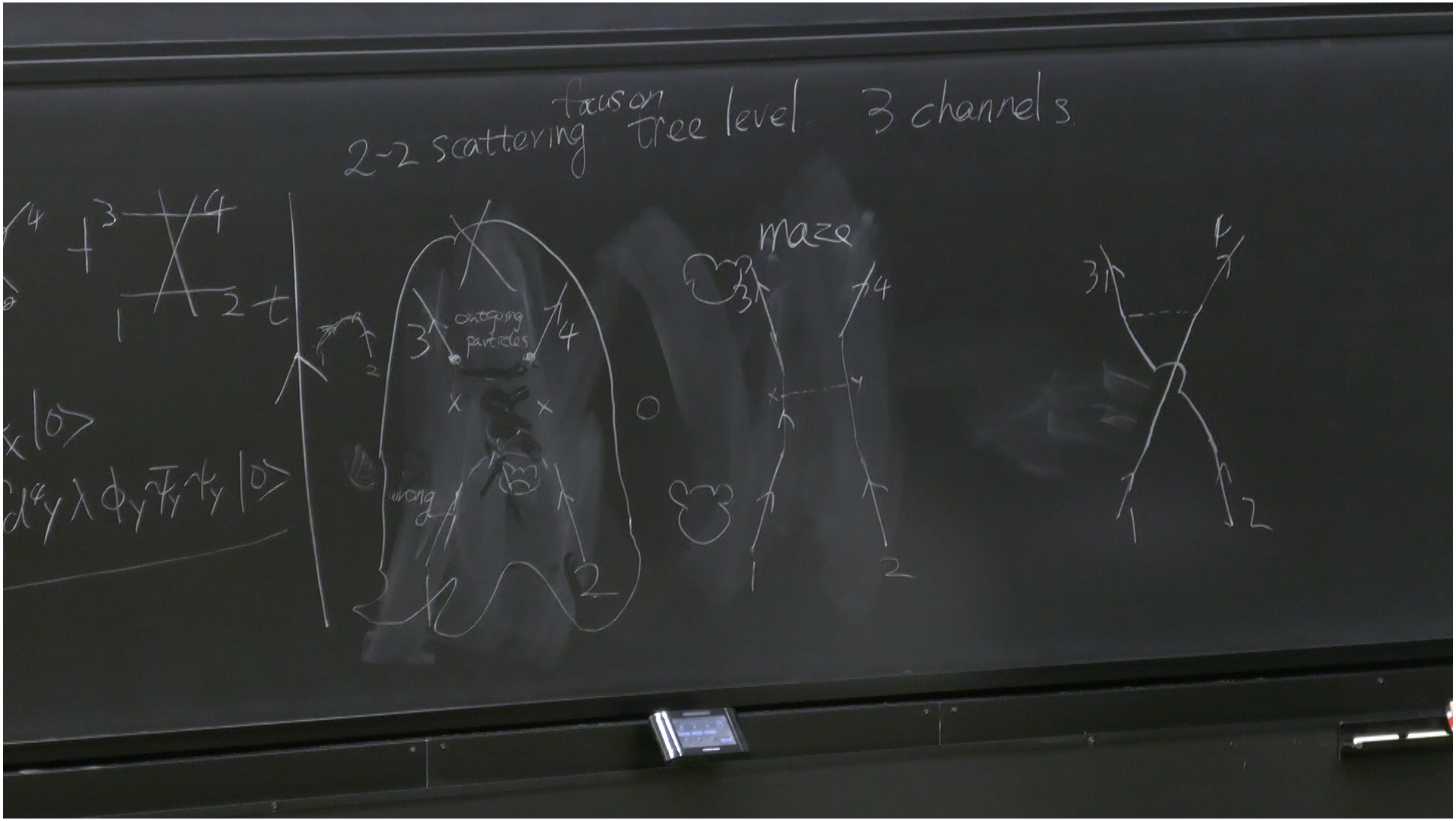
$$\langle 0 | T\, \psi_4 \psi_3 \bar{\psi}_1 \bar{\psi}_2 \int d^4x \, \lambda \phi_x \bar{\psi}_x \psi_x \int d^4y \, \lambda \phi_y \bar{\psi}_y \psi_y | 0 \rangle$$

$$\langle f | S | i \rangle =$$

$$\int d^4x\, d^4y \int \prod_{j=3}^{4} dx_j \int \prod_{j=1}^{2} d^4x_i \, e^{ik_j x_j} \, \bar{u}_j (i \not{\partial}_j - m)$$

$$\langle 0 | T \psi_4 \psi_3 \bar{\psi}_1 \bar{\psi}_2 \phi_x \bar{\psi}_x \psi_x \phi_y \bar{\psi}_y \psi_y | 0 \rangle$$

$$(i \overleftarrow{\not{\partial}}_i + m) u_i e^{-ik_i x_i}$$


focus on
2-2 scattering tree level 3 channels
maze
outgoing particles
wrong

$$\langle f|S|i\rangle =$$

$$\int d^4x\, d^4y \int \prod_{j=3}^{4} dx_j \int \prod_{j=1}^{2} d^4x_i\, e^{ik_jx_j}\, \bar{u}_j (i\overleftarrow{\not\partial}_j - m)$$

$$\langle 0|T\, \psi_4 \psi_3 \bar\psi_1 \bar\psi_2\, \phi_x \bar\psi_x \psi_x\, \phi_y \bar\psi_y \psi_y|0\rangle$$

$$(i\overrightarrow{\not\partial}_i + m) u_i e^{-ik_i\cdot x_i}$$

$$\psi_3 \bar\psi_1 \bar\psi_2 \bar\psi_x \psi_x$$

$$\psi_3 \bar\psi_x \psi_x \bar\psi_1$$

$$\langle f|S|i\rangle = \int d^4x\, d^4y \int \prod_{j=3}^{4} dx_j \int \prod_{j=1}^{2} d^4x_i \; \overbrace{e^{ik_j x_j}\, \bar{u}_j (i\not{\partial}_j - m)}^{I_{3,f}}$$

$$\langle 0| T\, \psi_4 \psi_3 \bar{\psi}_1 \psi_2\, \phi_x \bar{\psi}_x \psi_x\, \phi_y \bar{\psi}_y \psi_y |0\rangle$$

$$(i\overleftarrow{\not{\partial}}_i + m) u_i e^{-ik_1 \cdot x_i}$$

$\psi_3 \bar{\psi}_1 \psi_2$

$\psi_3 \bar{\psi}_x$

$$\psi_3 \bar{\psi}_1 \bar{\psi}_2 \bar{\psi}_x \psi_x \to \psi_3 \bar{\psi}_x \bar{\psi}_1 \bar{\psi}_2 \psi_x \quad \text{2 times}$$

$$\psi_3 \bar{\psi}_x \, \psi_x \bar{\psi}_1 \qquad \bar{\psi}_1 \bar{\psi}_2 \psi_x \quad \text{2 times}$$

$$\int d^4x_3\, d^4x_1\, e^{ik_3\cdot x_3}\, \bar{u}_{3a} (i\partial\!\!\!/_3 - m)_{ab}\, \psi_{3b}\, \bar{\psi}(x)_e\, \psi(x)_e\, \bar{\psi}_{1d}\, (i\overleftarrow{\partial\!\!\!/}_1 + m)_{dc}\, u_{1c}\, e^{-ik_1\cdot x_1}$$

$$S_F(x_3 - x)_{be} \qquad S_F(x - x_1)_{ed}$$

$$I_{1,f} = b_1(+\infty) - b_1(-\infty)$$

$$i\delta(x_3 - x)\,\delta_{ae} \qquad (-i)\,\delta(x - x_1)\,\delta_{ec}$$

$$= \bar{u}_3 \cdot u_1\, e^{i(k_3 - k_1)x}$$

$$\int d^4x_3\, d^4x_1\, e^{ik_3\cdot x_3}\, \bar{u}_{3a} (i\not{\partial}_3 - m)_{ab}\, \psi_{3b}\, \bar{\psi}(x)_e\, \psi(x)_e\, \bar{\psi}_{1d}\, (i\overleftarrow{\not{\partial}}_1 + m)_{dc}\, u_{1c}\, e^{-ik_1\cdot x_1}$$

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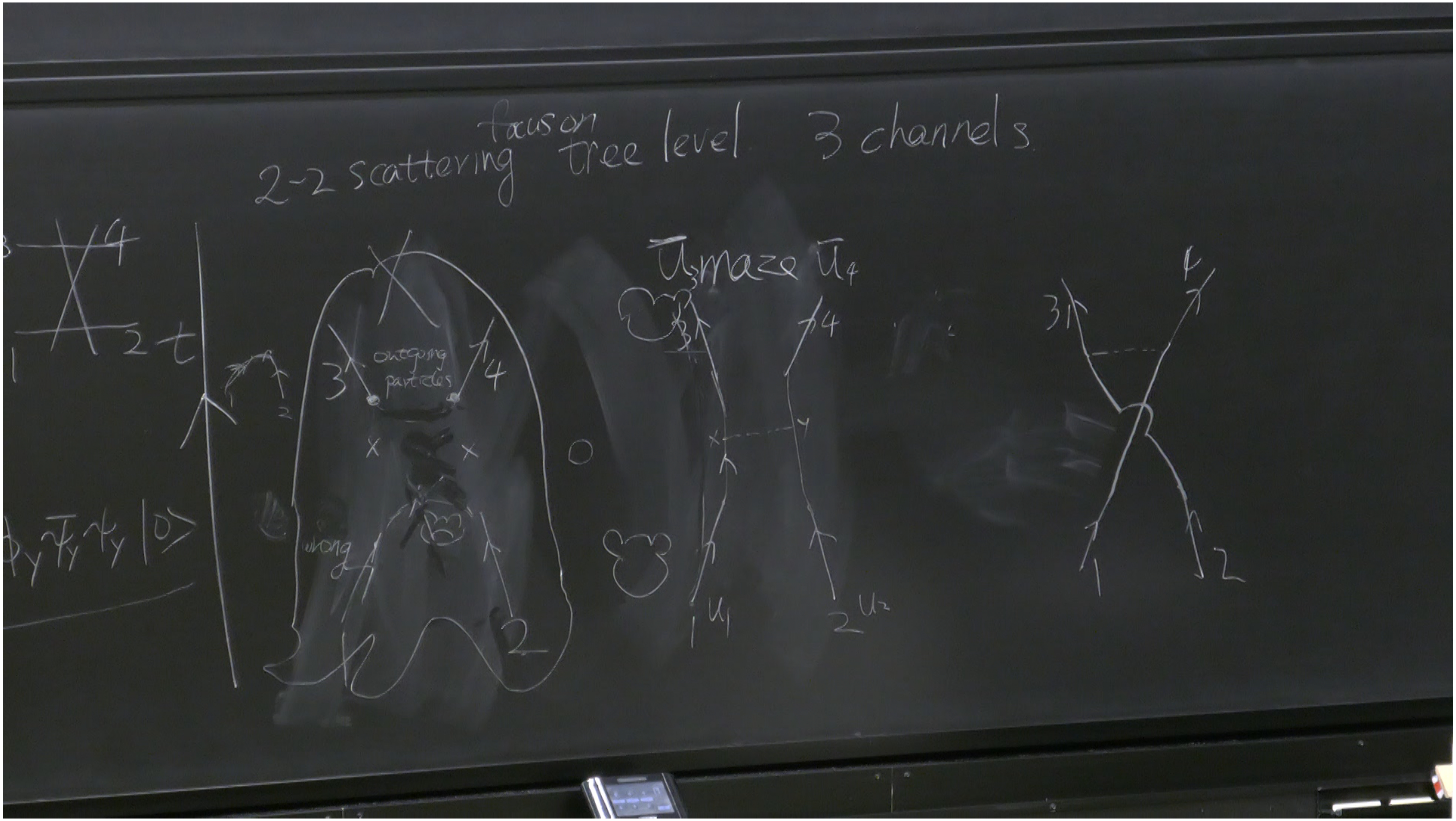
$$I_{1,f} = b_1(+\infty) - b_1(-\infty)$$

$$= \bar{u}_3 \cdot u_1\, e^{i(k_3 - k_1)x}$$

$$\bar{u}_4 \cdot u_2\, e^{i(k_4 - k_2)y}$$


focus on
2-2 scattering tree level 3 channels
outgoing particles
wrong
ū₃ maze ū₄
u₁
u₂

$$\psi_4 \bar\psi_2 \bar\psi_y \psi_y$$

$$\psi_4 \bar\psi_y \; \bar\psi_2 \psi_y$$

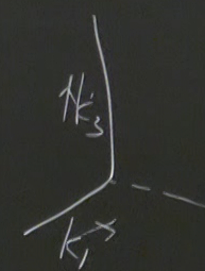
$$\psi_y \bar\psi_2$$

$$\int d^4x\, d^4y\; e^{i(k_3-k_1)x}\, e^{i(k_4-k_2)y}$$

$$(\bar u_3 \cdot u_1)(\bar u_4 \cdot u_2) \int d^4p \frac{i}{p^2-m^2+i\epsilon}\, e^{-ip(x-y)}$$

$$= \delta^4(k_3-k_1-p)\; \delta^4(k_4-k_2+p)$$

$$= \frac{i\, \bar u_3 \cdot u_1\; \bar u_4 \cdot u_2}{(k_3-k_1)^2-m^2+i\epsilon}\, \delta^4(\Sigma k)$$



$I_{3,f}$

$$e^{ik_j x_j}\, \bar{u}_j (i\cancel{\partial}_j - m)$$

$$\psi_4 \psi_3 \bar\psi_1 \bar\psi_2 \phi_x \bar\psi_x \psi_x \phi_y \bar\psi_y \psi_y |0\rangle$$

$$(i\cancel{\partial}_i + m)\, u_i\, e^{-ik_i x_i}$$

$$\psi_4 \psi_3 \bar\psi_1 \bar\psi_2 \bar\psi_x \psi_x$$

$$\psi_3 \bar\psi_1 \bar\psi_2 \bar\psi_x \psi_x \to \underline{\psi_3 \bar\psi_x}\, \bar\psi_1 \bar\psi_2 \psi_x \quad \text{2 times}$$

$$\psi_3 \bar\psi_x \psi_x \bar\psi_1 \qquad \bar\psi_1 \bar\psi_2 \psi_x \quad \text{2 times}$$

$\psi_x \rightarrow \underline{\psi_3 \bar{\psi}_x} \bar{\psi}_1 \bar{\psi}_2 \psi_x$ 2 times

$$= \frac{\bar{u}_4 \cdot u_1 \; \bar{u}_3 \cdot u_2}{(k_4 - k_1)^2 - m^2 + i\epsilon}$$

$\bar{\psi}_1 \bar{\psi}_2 \psi_x$ 2 times incoming fermion

$p \rightarrow$ vertex $= u^r(p)$

outgoing fermion

p vertex $= \bar{u}^r(\vec{p})$

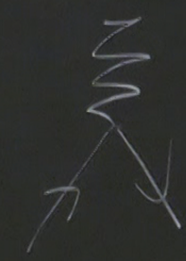
rule: fermion line

go against the arrow.

$-e\bar{\psi}\gamma^\mu\psi A_\mu$

A_μ — vector field

$-ie\gamma^\mu$



$$\langle f|S|i\rangle = \int d^4x\, d^4y \int \prod_{j=3}^{4} dx_j \int \prod_{j=1}^{2} d^4x_i \; \underbrace{e^{ik_j x_j}\, \bar{u}_j (i\not{\partial}_j - m)}_{I_{3,f}}$$

$$\langle 0|T\, \psi_4 \psi_3 \bar{\psi}_1 \bar{\psi}_2\, \phi_x \bar{\psi}_x \psi_x\, \phi_y \bar{\psi}_y \psi_y |0\rangle$$

$$(i\not{\partial}_i + m)\, u_i\, e^{-ik_i x_i}$$

$$\psi_4 \psi_3 \bar{\psi}_1 \bar{\psi}_2 \bar{\psi}_x \psi_x$$

$$\psi_3 \bar{\psi}_1 \bar{\psi}_2 \bar{\psi}_x \psi_x \rightarrow \underline{\psi_3 \bar{\psi}_x} \bar{\psi}_1 \bar{\psi}_2 \psi_x \quad \text{2 times}$$

$$= \frac{\bar{u}_4 \cdot u_1 \; \bar{u}_3 \cdot u_2}{(k_4 - k_1)^2 - M^2 + i\epsilon}$$

$$\psi_3 \bar{\psi}_x \psi_x \bar{\psi}_1 \qquad \bar{\psi}_1 \bar{\psi}_2 \psi_x \quad \text{2 times}$$