Title: Relativity Lecture - 110723

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Collection: Relativity 2023/24

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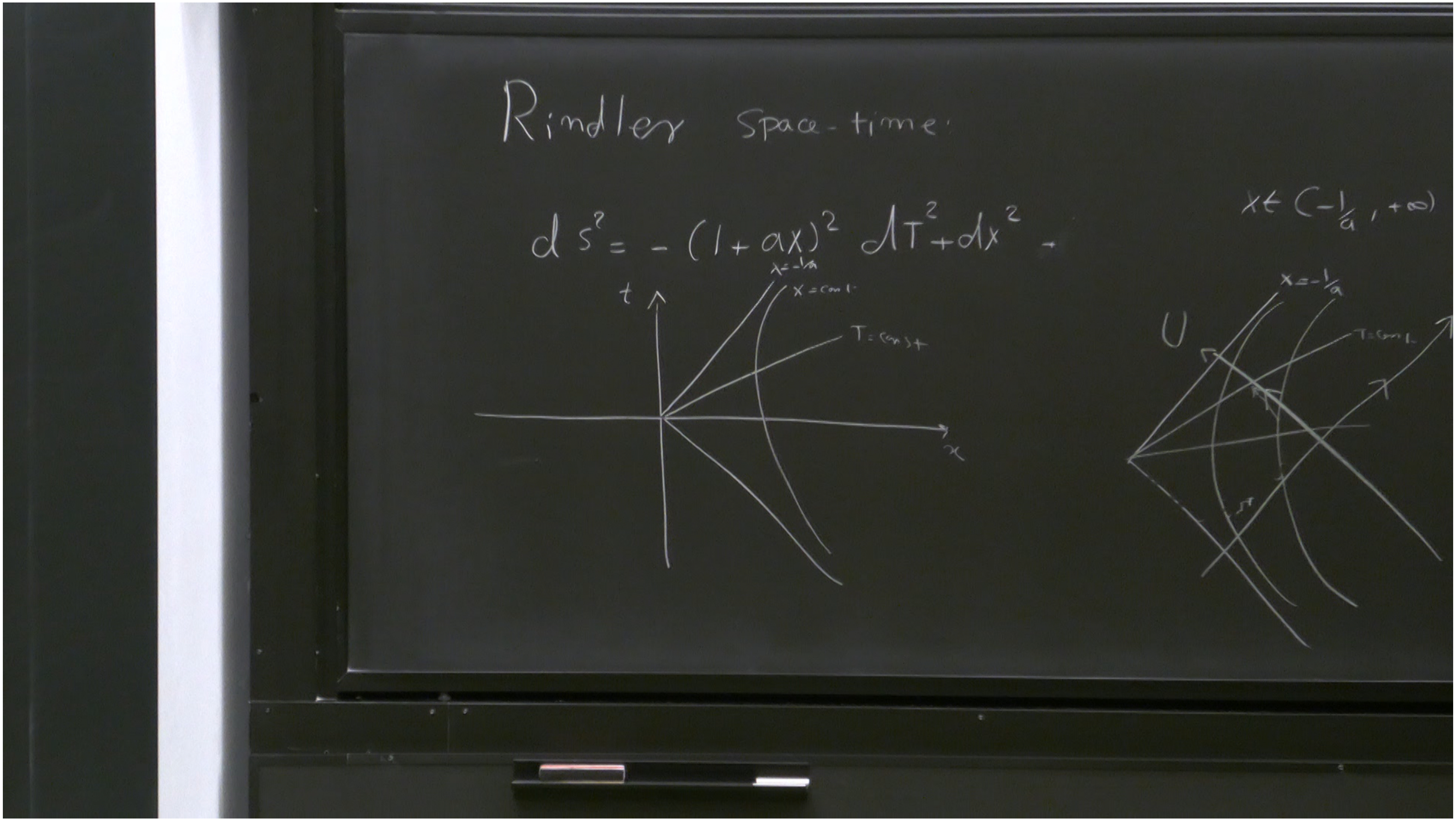
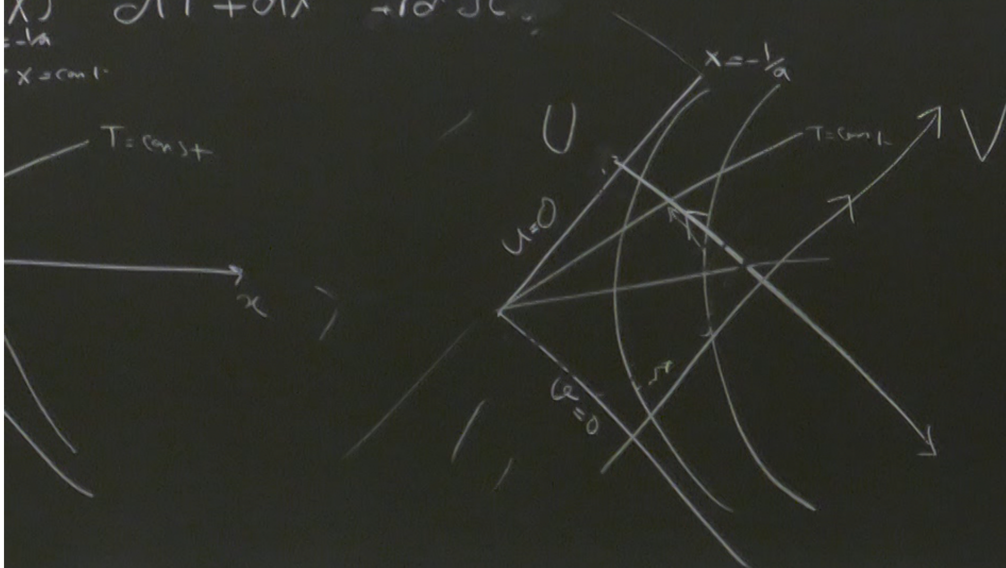

Rindler space-time:
$ds^2 = -(1+aX)^2 dT^2 + dX^2$.
$X \in (-\frac{1}{a}, +\infty)$
t
x
X = -1/a
X = const
T = const
U
X = -1/a
T = const

...ce-time:

$$\ldots x)^2\, dT^2 + dx^2 + r^2 d\Omega^2$$

$x \in (-\tfrac{1}{a}, +\infty)$



$$ds^2 = -e^{a(V-U)}\, dU\, dV + r^2 d\Omega^2$$

$$\downarrow \quad u = -\frac{1}{a} e^{-aU} \qquad v = \frac{1}{a} e^{aV}$$

$$ds^2 = -du\, dv + r^2 d\Omega^2$$

$u \in (0, +\infty) \rightarrow u \in (-\infty, +\infty)$

$v \in (-\infty, +\infty)$

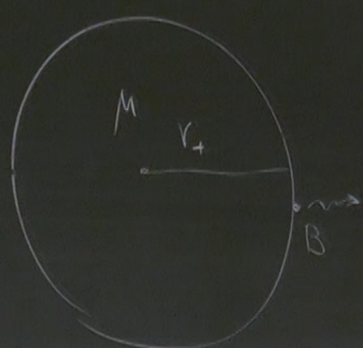
Schwarzchild Black hole:

$$ds^2 = -f_{(r)}\,dt^2 + f^{-1}_{(r)}\,dr^2 + r^2 d\Omega^2$$

$$f_{(r)} = 1 - \frac{2M}{r}$$

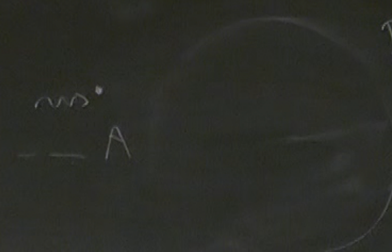
$R_\oplus \sim 6400\,km$

$r_+ \sim 9\,mm$



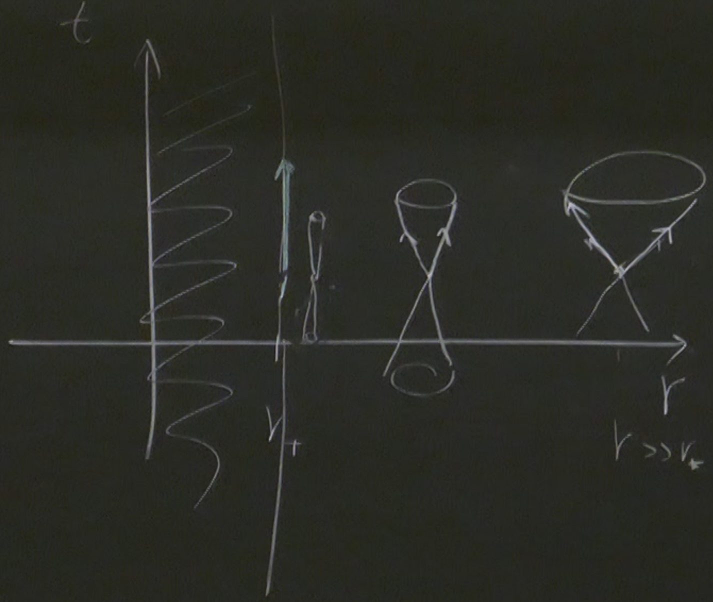
$r_+ = 2M$

H_{BH}



$R_\odot \sim 8 \times 10^5\,km$

$r_+ \sim 3\,km$



$$ds^2=0 \quad (d\Omega^2=0)$$

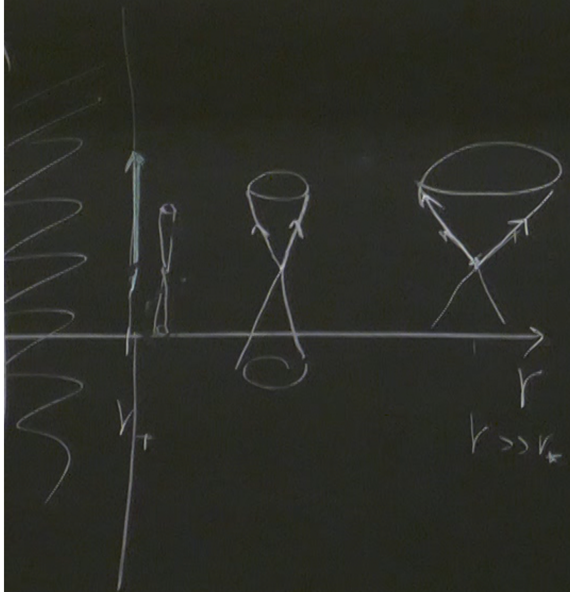
$$dt^2 = \frac{1}{f^2}$$

$$\frac{dt}{dr} = \pm\frac{1}{f(r)} = \pm\frac{1}{1-\frac{2M}{r}}$$

$$R = g^{\mu\nu}R_{\mu\nu} = 0$$

$$R_{\mu\nu}R^{\mu\nu} = 0$$

$$ds^2=0 \quad (d\Omega^2=0) \qquad dt^2 = \frac{1}{f^2}\,dr^2 \Rightarrow dt =$$



$$\frac{dt}{dr} = \pm\frac{1}{f(r)} = \pm\frac{1}{1-\frac{2M}{r}}$$

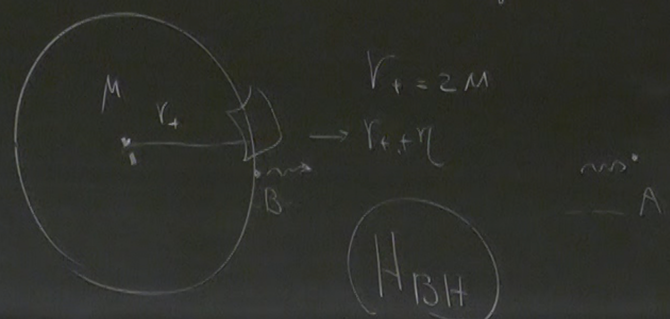
$$R = g^{\mu\nu}R_{\mu\nu} = 0 \qquad R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} \longrightarrow +\infty$$

$$R_{\mu\nu}R^{\mu\nu} = 0 \qquad r \longrightarrow 0$$

Schwarzchidd Black hole:

$$ds^2 = -f_{(r)}\,dt^2 + f_{(r)}^{-1}\,dr^2 + r^2 d\Omega^2$$

$$f(r) = 1 - \frac{2M}{r}$$

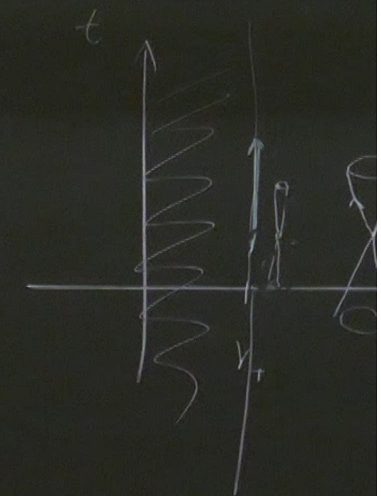


$R_\oplus \sim 6400\ km$

$r_+ \sim 9\ mm$

$R_\odot \sim 8\times10^5\ km$

$r_+ \sim 3\ km$



$R = g^{\mu\nu} R$

$R_{\mu\nu}R^{\mu\nu} = 0$

$r \gg r_*$

$$ds^2 = 0 \quad (d\Omega^2 = 0) \qquad dt^2 = \frac{1}{f^2}\, dr^2 \Rightarrow dt = \pm \frac{1}{f(r)}\, dr$$

$$\frac{dt}{dr} = \pm \frac{1}{f(r)} = \pm \frac{1}{1 - \frac{2M}{r}}$$

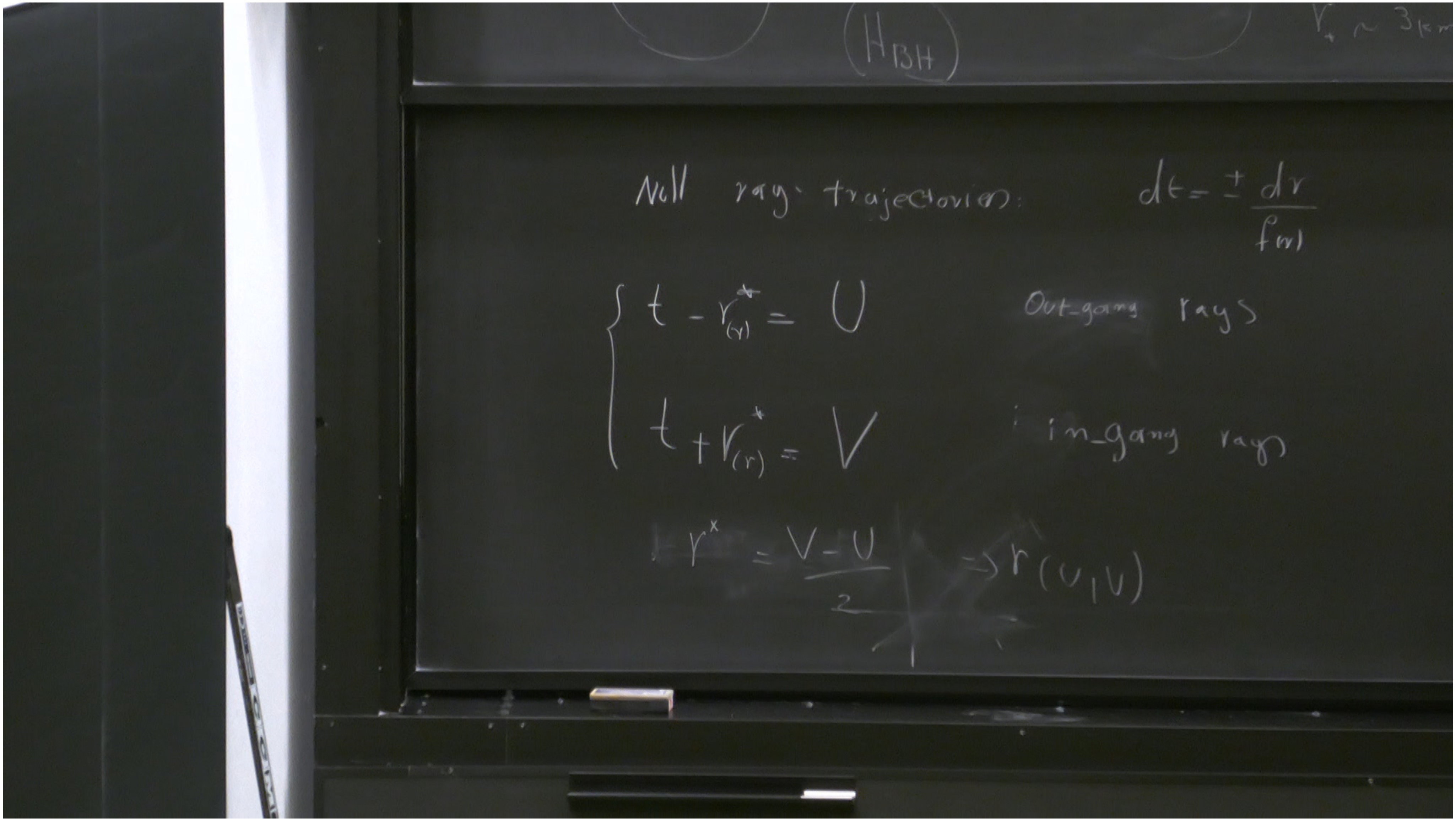
Kruskal Extension, 1960

$$R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} \longrightarrow +\infty$$

$$r \longrightarrow 0$$

External $M_{Sch} \longrightarrow$

- Following Null geodesics
- Switching coordinates to affine parametrization


H_BH
Null ray trajectories: dt = ± dr / f(r)
t - r*(r) = U   Out-going rays
t + r*(r) = V   in-going rays
r* = (V - U)/2 → r(U, V)

$$t = \pm \int_a^r \frac{dr}{f(r)} + \text{const} \Rightarrow t = \pm \underbrace{\left(r + 2M \ln \left| \frac{r}{2M} - 1 \right| \right)}_{r^*(r)} + \text{Const}$$

$$\dashrightarrow ds^2 = -\underset{\substack{\downarrow \\ 1 - \frac{2M}{r}}}{f(r)}\, dU dV + r^2 d\Omega^2$$

(H_{BH}) $\quad r_+ \sim 3\,km$

trajectories: $dt = \pm \frac{dr}{f(r)}$ $\qquad t = \pm \int_a^r \frac{dr}{f(r)} + \text{const} \Rightarrow t = \pm$

U : out-going rays

$$\dashrightarrow ds^2 = -\underbrace{f(r)}_{1-\frac{2M}{r}}\, dU\, dV + r^2 d\Omega^2$$

V : in-going rays

$\frac{V-U}{2} \Rightarrow r(U,V)$ $\qquad r_* \to -\infty$, $r \to r_+$ $\Rightarrow \begin{cases} U \to +\infty \\ V \to -\infty \end{cases}$

$$\int_a^r \frac{dr}{f(r)} + \text{const} \Rightarrow t = \pm \underbrace{\left(r + 2M \ln\left|\frac{r}{2M} - 1\right|\right)}_{r^*(r)} + \text{Const}$$

$$> \quad ds^2 = -f(r)\, dU\, dV + r^2 d\Omega^2$$

$$f(r) = 1 - \frac{2M}{r}$$

$$(U, 0, 0, 0)$$

$$\ln|x| \qquad x \to 0$$

$+\infty$

$-\infty$

Next : affine parametrization.

$$u = -e^{-U/4M} \qquad v = e^{V/4m}$$

$$ds^2 = -\frac{32M^3}{r} e^{-r/2M} \, du \, dv + r^2 d\Omega^2$$

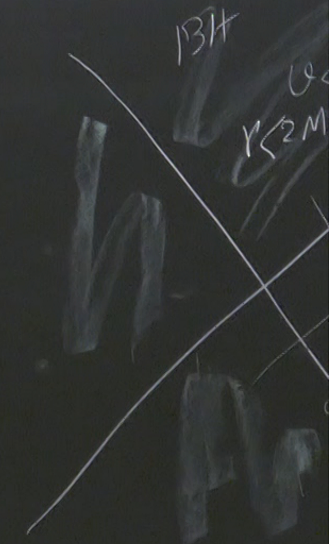
$u \longrightarrow 0$, $v \longrightarrow 0$

$r \to r_+$ $\qquad$ $r \to r_+$

$u \in (0, +\infty)$

$v \in (0, +\infty$ $\rightarrow$ $u \in (-\infty, +\infty)$

$v \in (-\infty, +\infty$



$$dt = \pm \frac{dr}{f(r)} \qquad t = \pm \int_a^r \frac{dr}{f(r)} + \text{const} \quad \Rightarrow \quad t = \pm \Big(r + \underbrace{2M \ln \Big| \frac{r}{2M}}_{r^*_{(r)}}$$

rays

$$\dashrightarrow \; ds^2 = -\underset{\substack{\downarrow \\ 1 - \frac{2M}{r}}}{f(r)}\, dU\, dV + r^2 d\Omega^2$$

$(U, 0, 0, 0)$

ln

oing rays

$r_* \to -\infty$

) $r \to r_+$ $\Rightarrow \begin{cases} U \to +\infty \\ V \to -\infty \end{cases}$

$r^* \to +\infty \begin{cases} U \underset{r \to \infty}{\longrightarrow} -\infty \\ V \to \end{cases}$

x

$$dt = \pm \frac{dr}{f(r)} \qquad t = \pm \int_a \frac{dr}{f(r)} + \text{const} \implies t = \pm ($$

Out-going rays

in-going rays

$$\dashrightarrow ds^2 = -f(r)\, dU dV + r^2 d\Omega^2$$

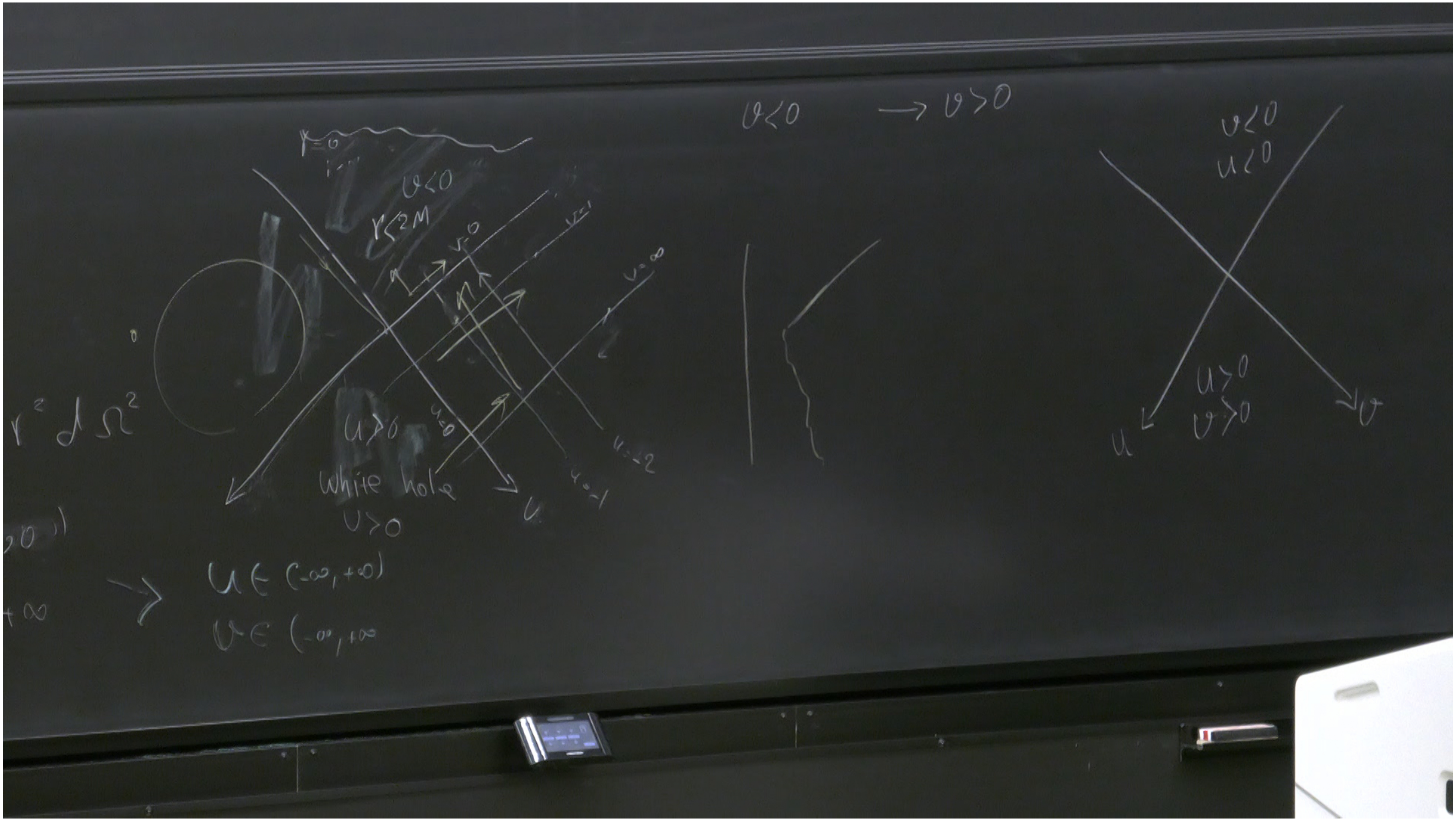
$$\downarrow$$

$$1 - \frac{2M}{r}$$

$(U, 0$

(U, V) $\quad r_* \to -\infty$, $r \to r_+$ $\quad \Rightarrow \begin{cases} U \to +\infty \\ V \to -\infty \end{cases}$

$r_* \to +\infty$ $\begin{cases} U \xrightarrow{\;} -\infty \\ \quad r \to \infty \\ V \longrightarrow +\infty \end{cases}$



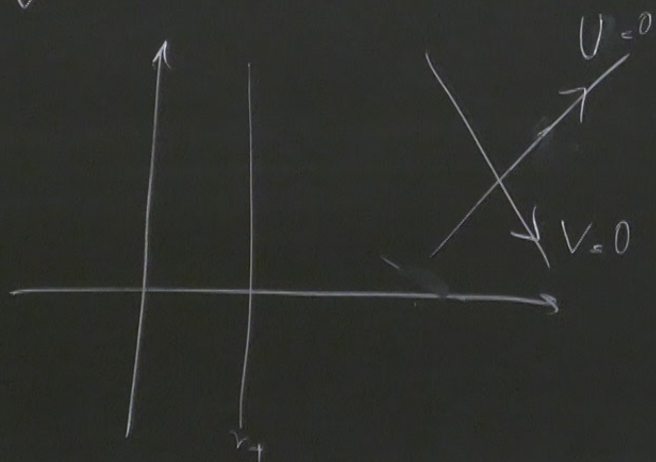
v<0
r≤2M
white hole
v>0
u ∈ (-∞, +∞)
v ∈ (-∞, +∞
v<0 → v>0
v<0
u<0
u>0
v>0

$$t - r^* = U \qquad \Rightarrow t = r^* + U = r + 2M \ln\left|\frac{r}{2M} - 1\right| + U$$

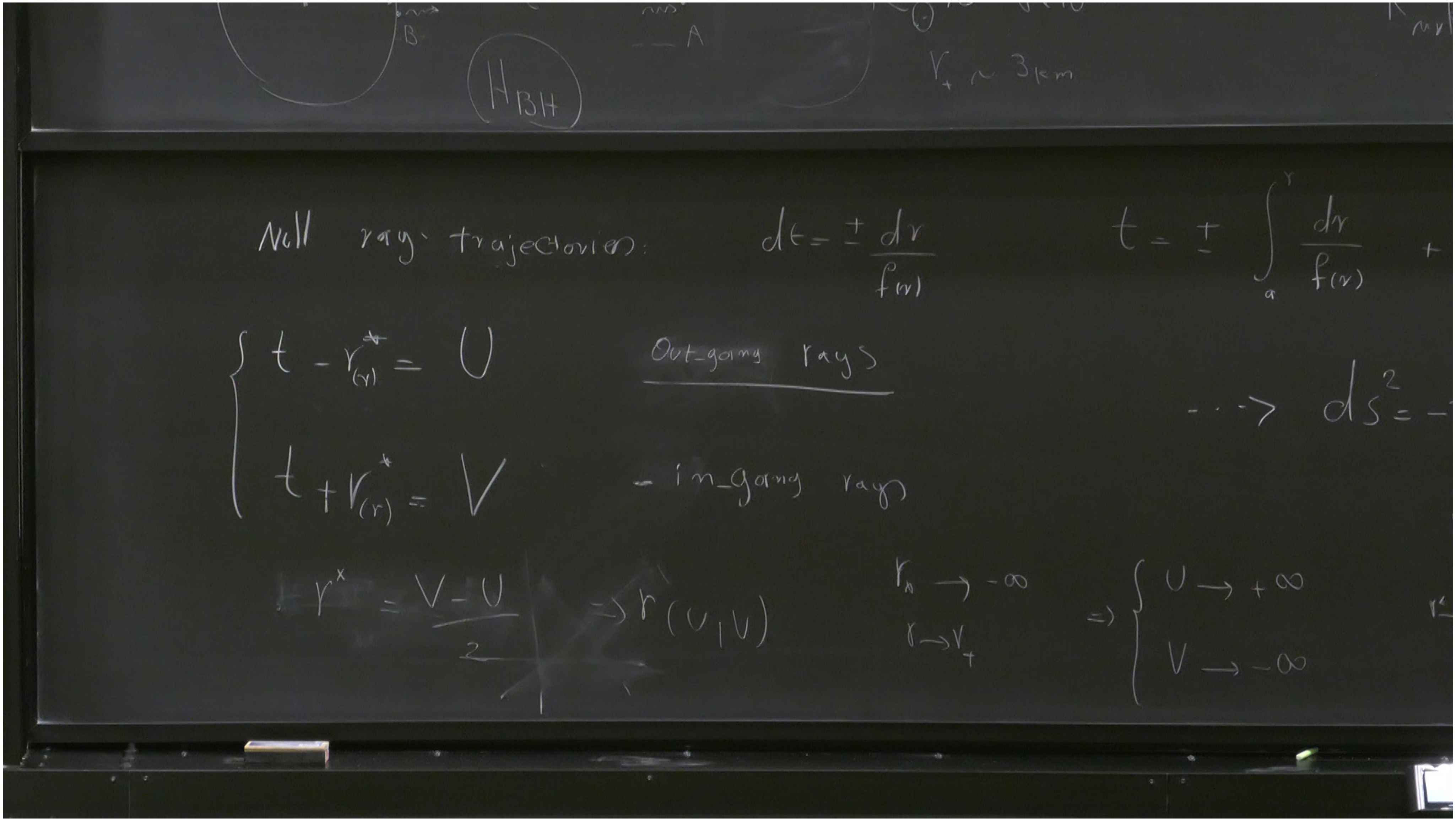
$$t + r^* = V$$

$r > 2M$

$\ln \nearrow$



$$V = 0 \Rightarrow t = -r - 2M \ln\left(\frac{r}{2M} - 1\right|$$


B
A
H_{BH}
$r_+ \sim 3\,km$
Null ray trajectories: $dt = \pm \frac{dr}{f(r)}$
$t = \pm \int_a^r \frac{dr}{f(r)} +$
$t - r^*_{(r)} = U$
Out-going rays
$t + r^*_{(r)} = V$
- in-going rays
$\dashrightarrow ds^2 = -$
$r^* = \frac{V-U}{2}$
$\rightarrow r(U,V)$
$r_* \rightarrow -\infty$
$r \rightarrow r_+$
$\Rightarrow U \rightarrow +\infty$
$V \rightarrow -\infty$