Title: Relativity Lecture - 103123

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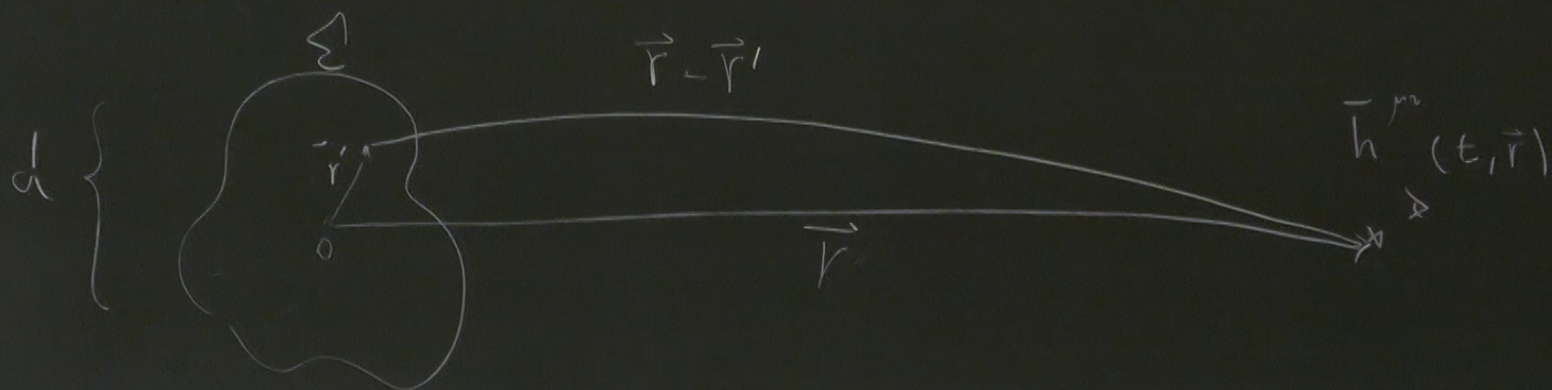
fixing Gauges $\longrightarrow$ allow Calculations $\rightarrow$ extract

$$\Box \bar{h}^{\mu\nu} = -16\pi T^{\mu\nu} \quad , \quad \partial_\mu \bar{h}^{\mu\nu} = 0$$



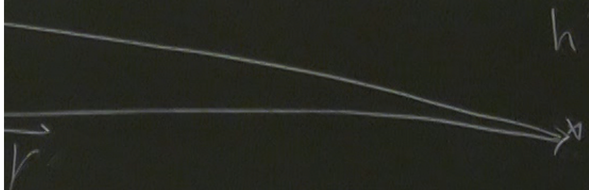
$\rightarrow$ allow calculations $\rightarrow$ extract physics

$\Box\, T^{\mu}$,

$\partial_\mu \bar{h}^{\mu\nu} = 0$

$d \ll r$

$\Rightarrow |\vec{r}'| \ll |\vec{r}|$



$\bar{h}^{\mu\nu}(t,\vec{r})$

$$\bar{h}^{\mu\nu}_{(t,\vec{r})} = 4G \int_{\Sigma} dr'^{3} \, \frac{T_{(t_R,\vec{r}')}}{|r-r'|}$$

$t_R = t - |\vec{r} - \vec{r}'|$

$$\frac{\quad(t_R, \vec{r}\,')}{|r-r'|}$$

$$|\vec{r}-\vec{r}\,'|^2 = r^2 - 2\vec{r}\cdot\vec{r}\,' + \vec{r}\,'^2 = r^2\left(1 - 2\frac{\vec{r}\cdot\vec{r}\,'}{r^2} + \cancel{\frac{r'^2}{r^2}}\right)$$

$$|r-r'| = r\left(1 - \frac{\vec{r}\cdot\vec{r}\,'}{r^2}\right)$$

$$\frac{1}{|r-r'|} = \frac{1}{r}\left(1 + \frac{\vec{r}\cdot\vec{r}\,'}{r^2}\right)$$

$$T_{\mu\nu}(t_R, \vec{r}\,') = T_{\mu\nu}\Big(t - r + \underbrace{\frac{\vec{r}\cdot\vec{r}\,'}{r}}_{O(d)}, \vec{r}\,'\Big) = T_{\mu\nu}(t-r, \vec{r}\,') + \dot{T}_{\mu\nu}(t-r, r')\left(\frac{\vec{r}\cdot\vec{r}\,'}{r}\right)$$

$$\dot{T} d \sim \omega \cancel{T} d \ll \cancel{T}$$

$$\omega d \ll 1 \qquad d \ll T$$

$$\bar{h}_{\mu\nu}(t,r) \simeq \frac{4G}{r}\int_{\Sigma} d\vec{r}'\; T_{\mu\nu}(t-r, r')$$

$$\bar{h}_{00}(t,r) = \frac{4G}{r}\int_{\Sigma} \underbrace{d\vec{r}'\; T_{00}(t-r, r')}_{E \sim M} = \frac{4GM}{r}$$

$$\uparrow$$

$$-4\phi$$

$$h_{ij}(t,r) = \frac{4G}{r}\int_{\Sigma'} d^3r' \; T_{ij}(t-r, r') =$$

$$T^{ij} = T^{(ij)}$$

$$T^{ij} = \partial_k\left(T^{ik} r'^j\right) - \left(\partial_k T^{ik}\right) r'^j = \partial_0(\qquad) + \left(\partial_0 T^{0i}\right) r'^j$$

$$T^{(0i} r'^{j)} = \frac{1}{2}\,\partial_k\left(T^{0k} r'^i r'^j\right) - \frac{1}{2}\left(\partial_k T^{0k}\right) r'^i r'^j$$

$$T^{0k}\delta^i_k r'^j + T^{0k} r'^i \delta^j_k$$

$$= T^{0i} r'^j + T^{0j} r'^i$$

$$T^{(ij)} = \partial_0 \left(T^{0(i} r'^{j)} \right) + \partial(\quad) = \partial(\quad) - \tfrac{1}{2}\, \partial_0 \left(\partial_k T^{0k} \right) r'^i r'^j$$

$$\partial_k T^{0k} + \partial_0 T^{00} = 0$$

$$\boxed{I_{ij}(\underset{\substack{\downarrow \\ t-r}}{t_r}) \equiv \int_{\Sigma} dr'^3\, T^{00}(t_r, \vec{r}')\, r'^i r'^j}$$

$$= T^{0i} r'^j + T^{0j} r'^i$$

$$\tfrac{1}{2}\, \partial_0 \left(\partial_k T^{0k} \right) r'^i r'^j = \partial(\quad) + \frac{1}{2} \left(\partial_0^2 T^{00} \right) r'^i r'^j$$

$$\boxed{\int dr'^3\, T^{00}(t_r, \vec{r}') \, r'^i r'^j}$$

Quadrupole moment

$$\bar{h}_{ij}(t,\vec{r}) = \frac{4G}{r} \int_{\Sigma'} d r'^3 \, \frac{1}{2} \partial_0^2 T^{00}(t_r, r') \, r'^i r'^j$$

$$\boxed{\bar{h}_{ij}(t,\vec{r}) = \frac{2G}{r} \frac{d}{dt^2} I_{ij}(t_r)}$$

$$\int d r'^3 \, \frac{1}{2} \partial_0^2 T^{00}(t_r, r') \, r'^i r'^j$$

$$\Sigma'$$

$$\boxed{\frac{d}{dt^2} I_{ij}(t_r)}$$

$$\xrightarrow{F.T} \tilde{h}_{ij}(\omega, \vec{r}) = -\frac{2G\omega^2}{r} e^{i\omega r} \tilde{I}_{ij}(\omega)$$

$$\bar{h}_{ij}(t,\vec{r}) = \frac{4G}{r}\int_{\Sigma'} dr'^3\, \frac{1}{2}\partial_0^2 T^{00}(t_r, r')\, r'^i r'^j$$

$$\boxed{\bar{h}_{ij}(t,\vec{r}) = \frac{2G}{r}\frac{d}{dt^2} I_{ij}(t_r)} \xrightarrow{f.T} \tilde{h}_{ij}(\omega,\vec{r}) = -\frac{2G\omega}{r}$$

$\rightarrow 3M_0$

Energy loss in GWs: (Back-Reaction)

Vacuum $\quad G^{\mu\nu}=0$

$$g_{\mu\nu} = \eta_{\mu\nu} + \epsilon h_{\mu\nu} + \epsilon^2 \underbrace{h^{(2)}_{\mu\nu}}_{O(h^2)} + O(h^3)$$

- linear order

$$G^{\mu\nu}_{Lin}(h) = G^{\mu\nu}(\eta+\epsilon h)^{(1)} = 0$$

$$\partial_\mu G^{\mu\nu}_{Lin}(h) = 0 \quad \Rightarrow h^{GW}_{+},\ h^{GW}_{\times}$$

Quadratic Order:

$$G^{\mu\nu}(\eta + \epsilon h^{GW} + \epsilon^2 h^{(2)}) = \cancel{\epsilon G^{\mu\nu}_{lin}(h^{GW})} + \epsilon^2 \Big(\underbrace{G^{\mu\nu}_{quad}(h^{GW}) + G^{\mu\nu}(h^{(2)})}_{0} \Big) = 0$$

$$G^{\mu\nu}(h^{(2)}) = - \underbrace{G^{\mu\nu}_{quadratic}(h^{GW})}_{8\pi G\, \tau^{\mu\nu}_{GW}}$$

$$\partial_\mu T^{\mu\nu} = 0$$

$$\tau^{\mu\nu}_{GW} = -\frac{1}{8\pi G} G^{\mu\nu}_{quad}(h^{GW})$$

$= \{ h_+ , h_\times$

$$\begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 2h_+ & 2h_\times \\ 0 & 2h_\times & -2h_+ \end{matrix}\Bigg)$$

$$T^{00} = T^{11} = -T^{01} = \frac{1}{4\pi}\left(2 h_+ \ddot{h}_+ + \dot{h}_+^2 + 2 h_\times \ddot{h}_\times + \dot{h}_\times^2\right)$$

$$O(\omega^2 h_{+/\times}^2)$$

$$\langle T^{00} \rangle = \frac{1}{32} \langle \dot{h}_{ij}^{TT} \dot{h}_{ij}^{TT} \rangle = \frac{1}{4\pi} \langle \dot{h}_+^2 + \dot{h}_\times^2 \rangle$$

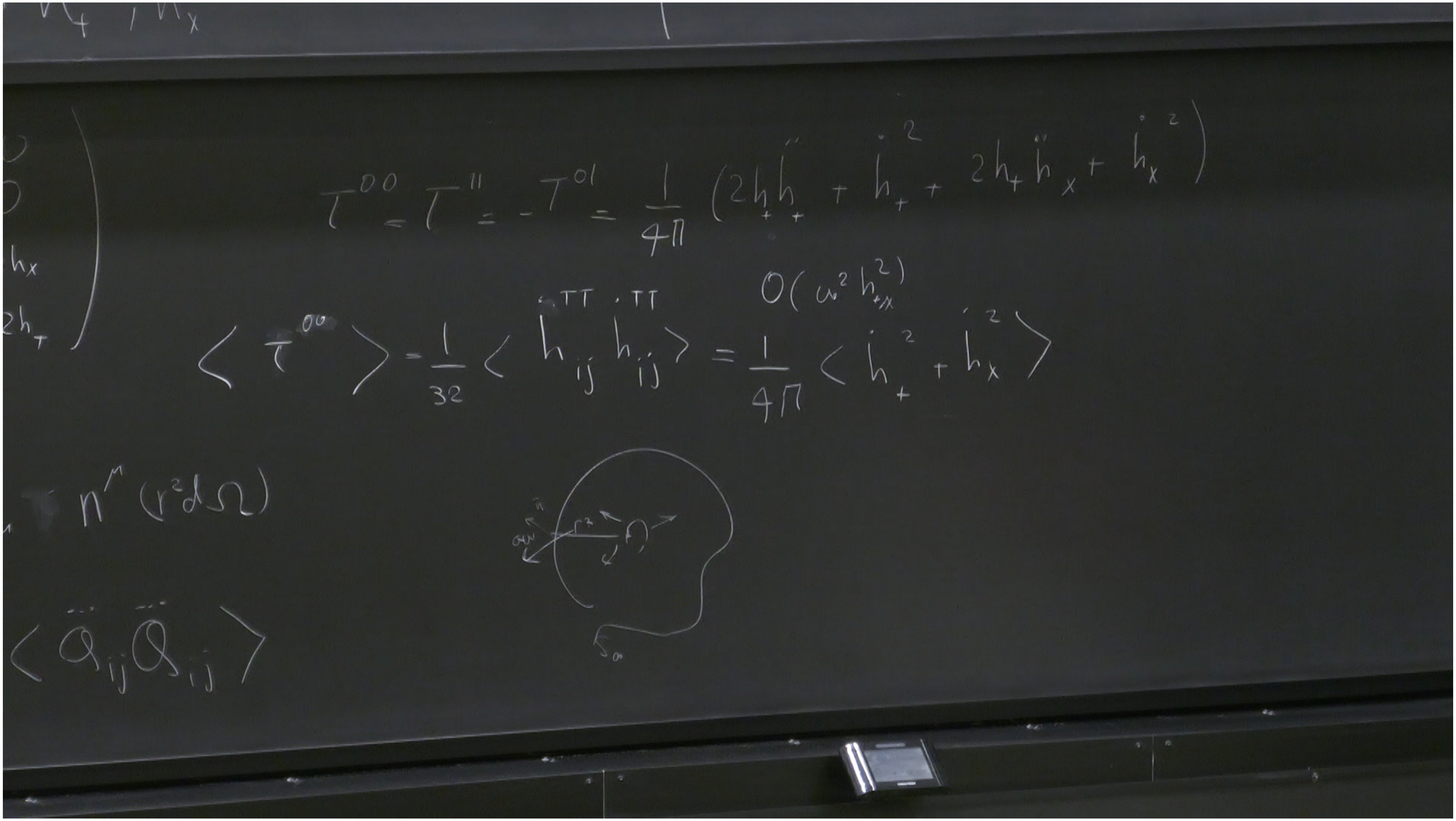
$$= T^{0i} x^{,j} + T^{0j} x^{,i}$$

$$x^{,i} x^{,j}) = \partial(\quad) + \frac{1}{2} (\partial_0^2 T^{00}) x^{,i} x^{,j}$$

$$x^{,i} x^{,j}$$

Quadrupole moment

$$Q_{ij} = I_{ij} - \frac{1}{3} \delta_{ij} \delta^{kl} I_{kl}$$


$T^{00} = T^{11} = -T^{01} = \frac{1}{4\pi}\left(2h_+\ddot{h}_+ + \dot{h}_+^2 + 2h_+\ddot{h}_x + \dot{h}_x^2\right)$
$O(\omega^2 h_{+/x}^2)$
$\langle T^{00} \rangle = \frac{1}{32}\langle \dot{h}_{ij}^{TT}\dot{h}_{ij}^{TT} \rangle = \frac{1}{4\pi}\langle \dot{h}_+^2 + \dot{h}_x^2 \rangle$
$n^\mu (r^2 d\Omega)$
$\langle \dddot{Q}_{ij}\dddot{Q}_{ij} \rangle$