Title: Relativity Lecture - 110123

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Collection: Relativity 2023/24

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Schwarzschild Solution

- Exact Solution to Vacuum Einstein Eq'

$$R_{\mu\nu} = 0 \qquad (\text{Vacuum} \Rightarrow R = 0)$$

$$ds^2 = -f(r)\, dt^2 + \frac{1}{f(r)}\, dr^2 + r^2\, d\Omega^2 \qquad (d\Omega^2 = d$$

$$f(r) = 1 - \frac{2M}{r}$$

$$\xi_\varphi = \partial_\varphi \qquad \xi_t = \partial_t$$

0)

Birkhoff's theorem:

Any $\boxed{\text{Spherically Symmetric}}$ Solution of $\boxed{\text{Vacuum}}$ Einst. Eqs

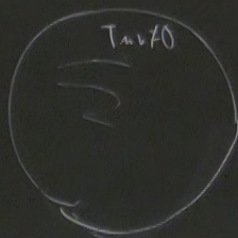
must be $\boxed{\text{Static}}$ and asymptoticall flat

uniquely described by $\boxed{\text{Schwarzchild}}$ metric.

S(O3)

$T_{\mu\nu}=0$

$g_{\mu\nu}(x)$



Proof:

$$ds^2 = -e^{2\chi} f \, dt^2 + \frac{1}{f} dr^2 + r^2 d^2\Omega$$

$f(r,t)$, $\chi(r,t)$, $m(r,t) \equiv \frac{1}{2} r(1-f) \Rightarrow$

$g_{00}, g_{rr}, g_{\Omega} \ldots \longrightarrow R_{\mu\nu} = 0 \rightarrow \ldots$ $\qquad \frac{\partial m}{\partial r} = 4$

$M = m = \text{const}$; $\chi(t)$

$\hookrightarrow f(r)$

$r^2 d^2\Omega$

$) \equiv \frac{1}{2} r(1-f) \Rightarrow f(r,t) = 1 - \frac{2m(r,t)}{r}$

$\frac{\partial m}{\partial r} = 4\pi r^2 (\cancel{-T^t_t})$, $\frac{\partial m}{\partial t} = -4\pi r^2 (\cancel{-T^r_t})$

$\gamma(t)$ $\quad g_{00} dt^2 = -e^{2\gamma(t)} f(r) dt^2$ $\quad dt' = e^{\gamma(t)} dt \Rightarrow ds^2$

$g'_{00} dt'^2 = -f(r) dt'^2$

$$\frac{\partial m}{\partial t} = -4\pi r^2 \left(-\cancel{T^r{}_t}\right)$$

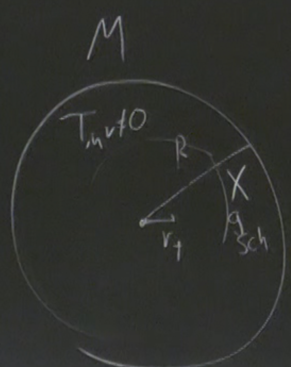
$$\frac{\partial \psi}{\partial r} = \cancel{4\pi f^{-1}\left(-T^t{}_t + T^r{}_r\right)}$$

$$dt' = e^{\psi(t)}\, dt \quad \Rightarrow \quad ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{1}{1 - 2M/r}\, dr^2 + r^2 d\Omega^2$$

- asymptotically flat $\quad \lim_{r\to\infty} f(r) = \lim_{r\to\infty} 1 - \frac{2M}{r} = 1 \Rightarrow$ min

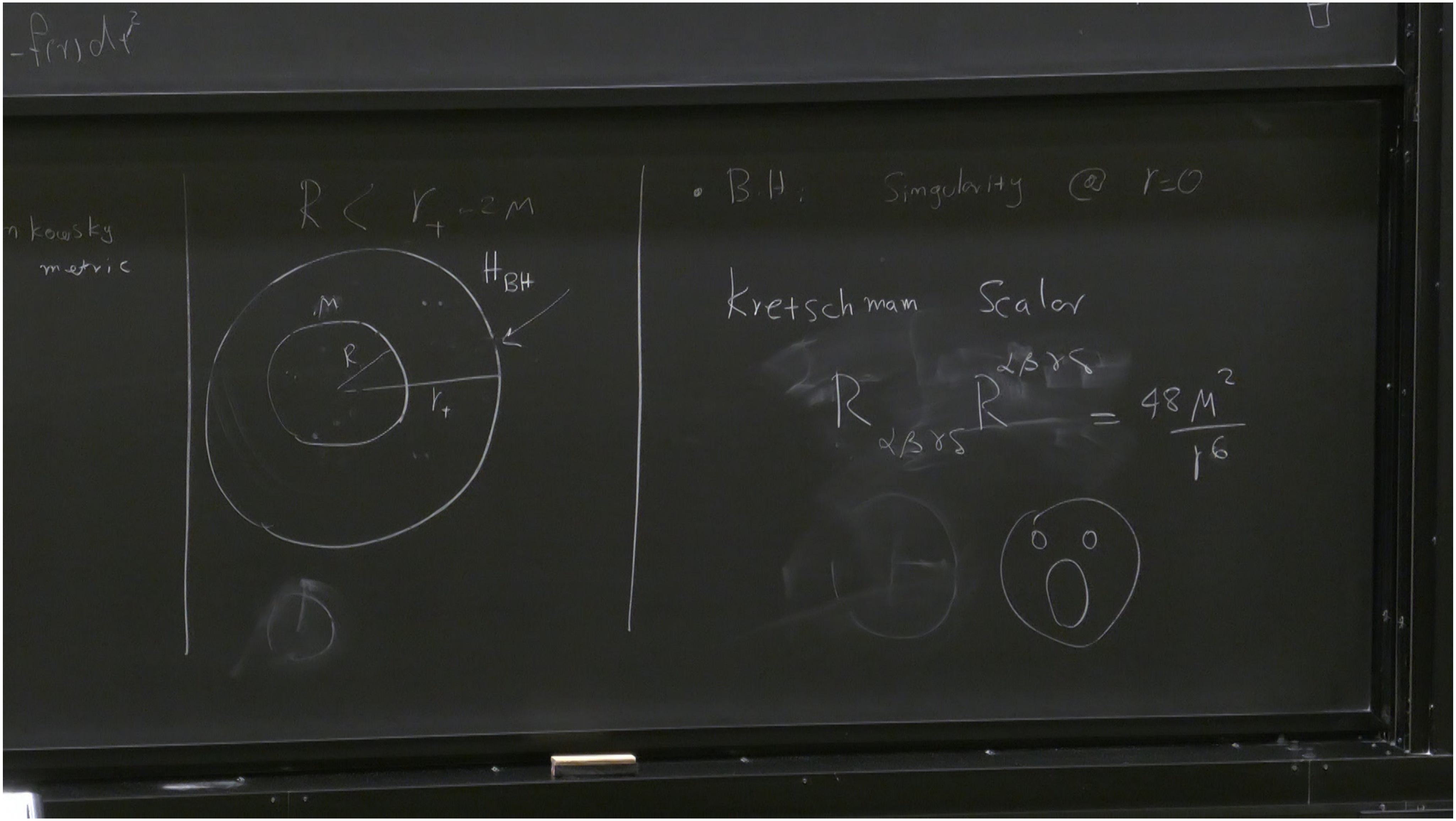
- Coordinate Singularity

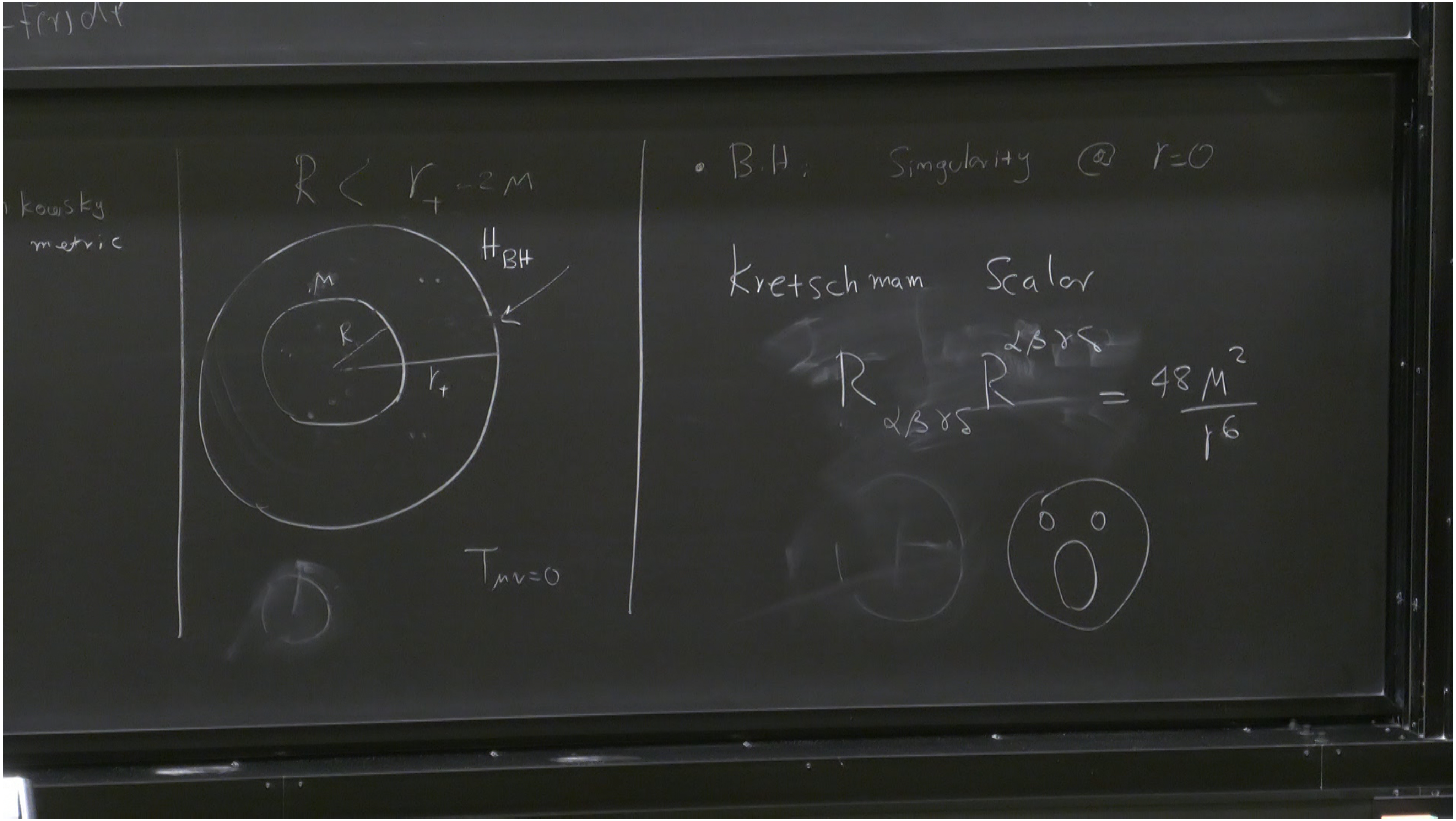
$$f(r) = 0 \quad @ \quad r = r_+ = 2M \quad \Rightarrow g_{00} = 0 \ \& \ g_{rr} \to \infty$$



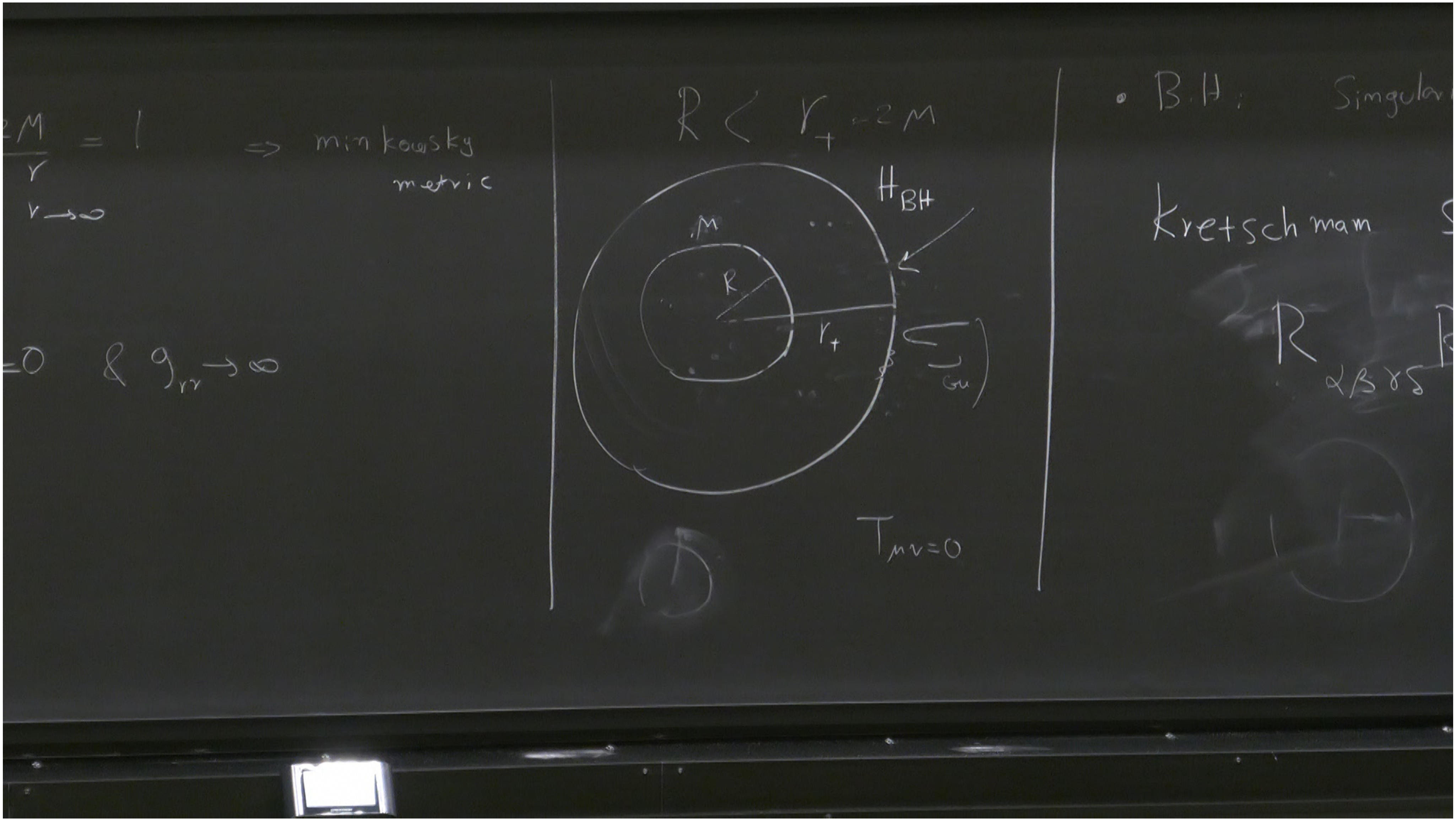
g_{sch} ✓

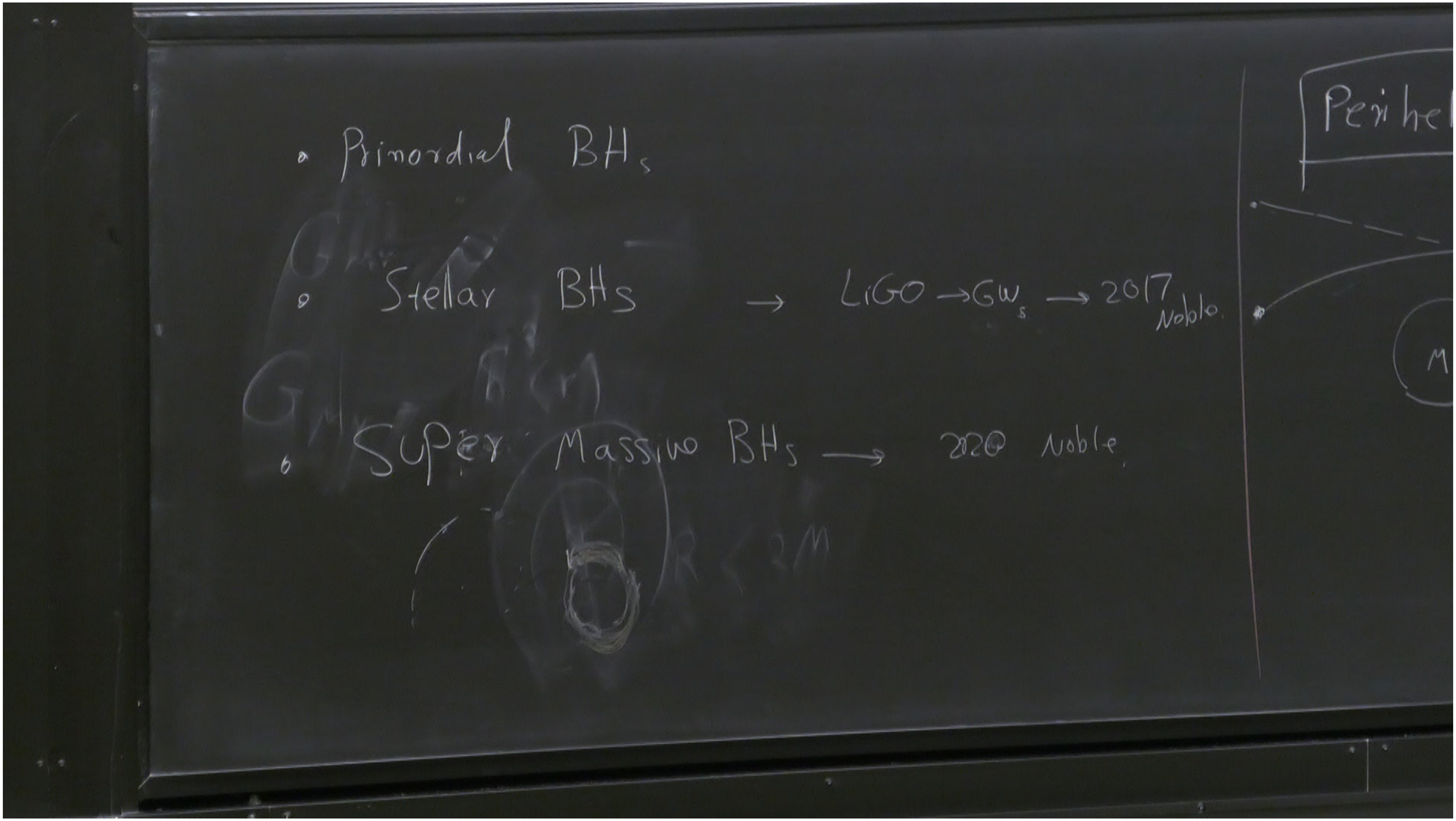
$R > r_+ = 2M$

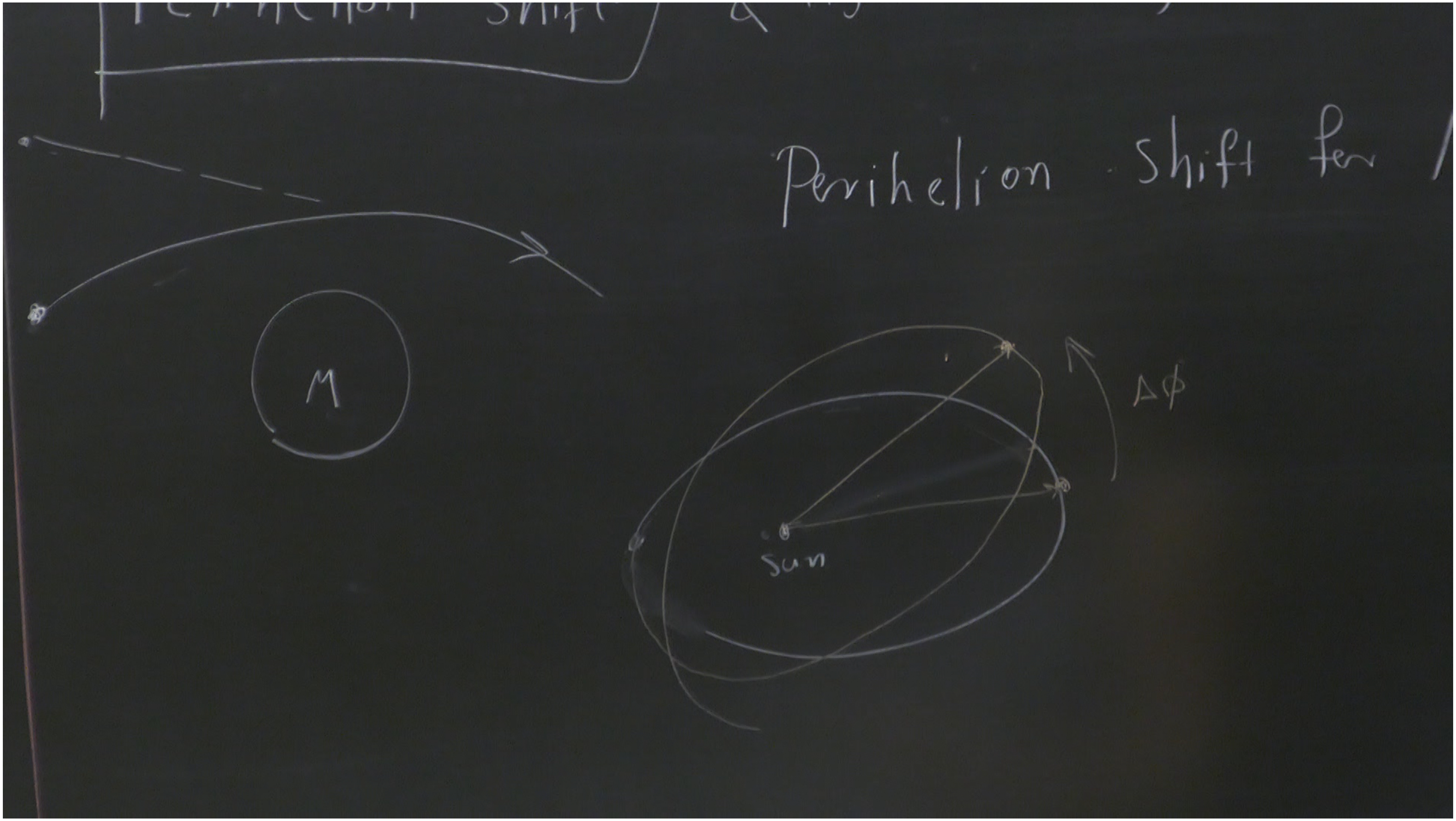

-f(r) dt²
nkowsky
metric
R < r+ ~ 2M
M
R
r+
H_BH
• B.H: Singularity @ r=0
Kretschman Scalar
R_αβγδ R^αβγδ = 48 M² / r⁶

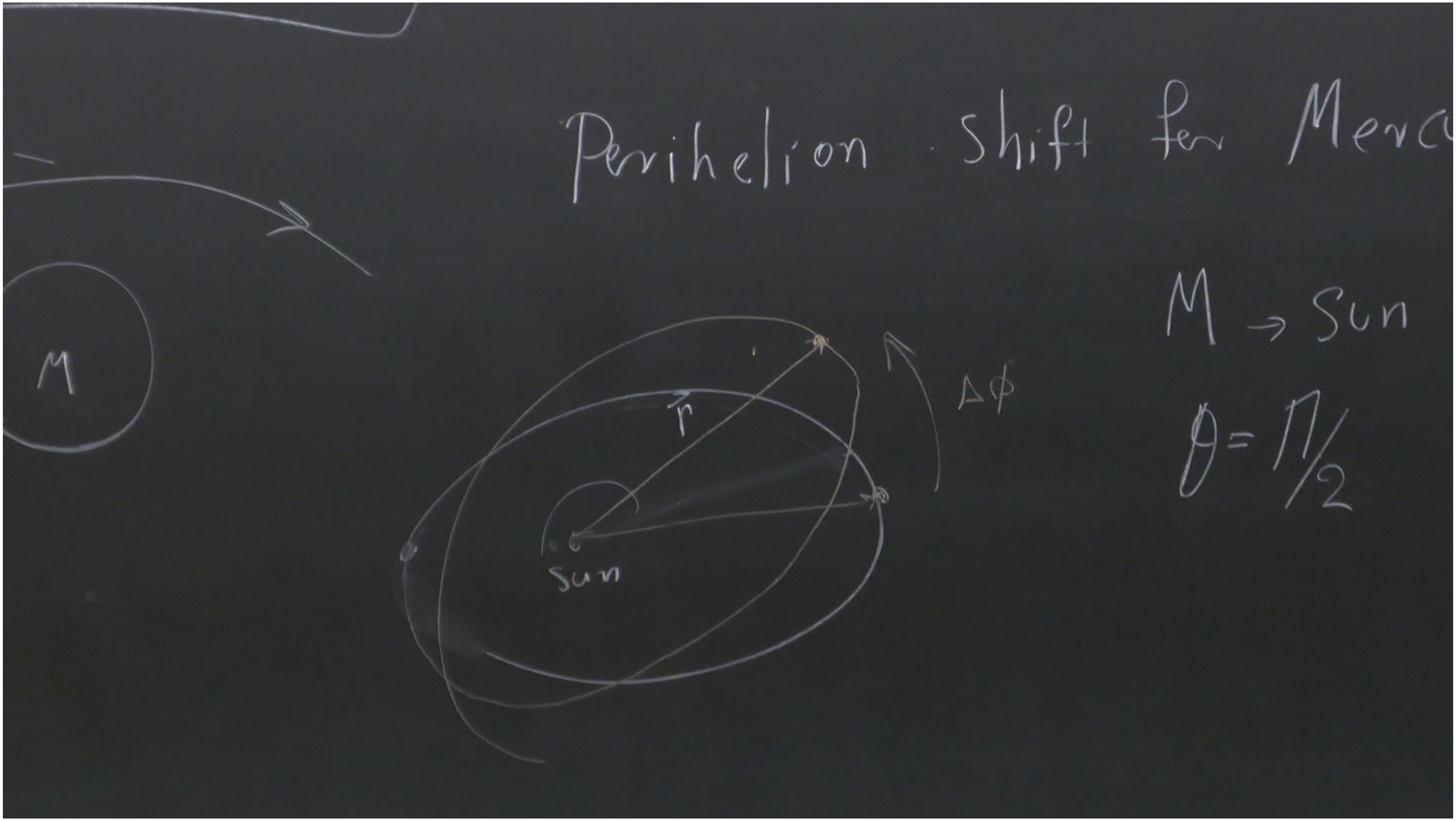


metric
R < r+ = 2M
HBH
M
R
r+
Tμν=0
• B.H: Singularity @ r=0
Kretschmam Scalar
Rαβγδ Rαβγδ = 48 M2 / r6

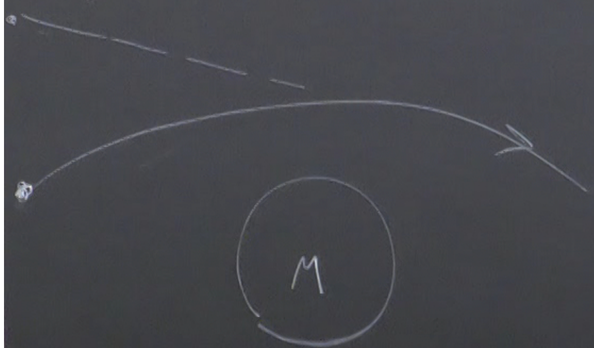

2M / r = 1
r→∞
⇒ minkowsky
metric
=0 & g_rr → ∞
R < r_+ = 2M
H_BH
M
R
r_+
T_μν=0
• B.H:
Singular
Kretschmam
R_αβγδ


Primordial BHs
Stellar BHs → LiGO → GWs → 2017 Noble
Super Massive BHs → 2020 Noble

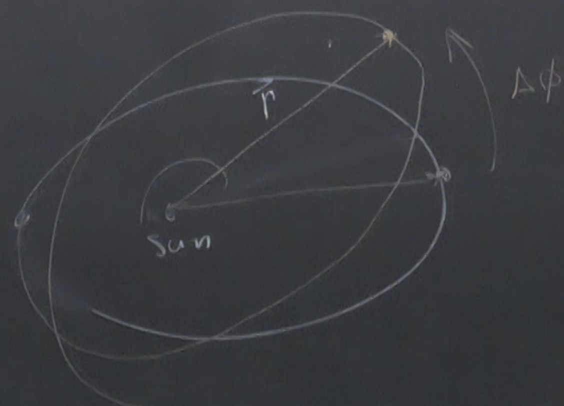

Perihelion Shift for
M
Δφ
sun


Perihelion shift for Merc
M
r
Sun
Δφ
M → Sun
θ = π/2

Perihelion shift & light bending



Perihelion shift for Mercury $\rightarrow \Delta\phi = 43$



$M \rightarrow$ Sun

$\theta = \pi/2$

$$u^\mu = \frac{dx^\mu}{d\lambda} = (\dot{t}, \dot{r}, 0, \dot{\varphi})$$

$K = \partial_t$ $\quad m = \partial_\varphi$

$$E = -k_\mu u^\mu = -g_{\mu\alpha} k^\alpha u^\mu = -g_{\mu 0} u^\mu = -g_{00} u^0 = f\dot{t}$$

$$L = m_\mu u^\mu = g_{\phi\phi} \dot{\phi} = r^2 \dot{\phi}$$

$$r^2 \underbrace{d\Omega^2}_{\sin\theta d\theta^2 + d\varphi^2} = r^2 \sin\theta^2 d\theta^2 + \underbrace{r^2}_{g_{\varphi\varphi}} d\varphi^2$$

$$-k = u^\mu u_\mu = -f\dot{t}^2 + \frac{1}{f}\dot{r}^2 + r^2\dot{\phi}^2$$

$\boxed{k = 1 \quad \text{for timelike}}$

$k = 0$ for null

$$V' = 0 \Rightarrow r = \frac{1 \pm \sqrt{1 - 12\, M^2/L^2}}{2M/L^2}$$

$$\begin{cases} L^2 = 12M^2 \Rightarrow r = \dfrac{L^2}{2M} = 6M \\ L^2 < 12M^2 \rightarrow \text{no roots} \\ L^2 > 12M^2 \Rightarrow \text{Two roots} \end{cases}$$