Title: Lieb-Schultz-Mattis anomalies as obstructions to gauging - VIRTUAL

Speakers: Sahand Seifnashri

Series: Quantum Matter

Date: October 31, 2023 - 11:00 AM

URL: https://pirsa.org/23100119

Abstract: In this talk, we identify anomalies of 1+1d lattice Hamiltonian systems as 't Hooft anomalies. We consider anomalies in internal symmetries as well as Lieb-Schultz-Mattis (LSM) type anomalies involving lattice translations. Using topological defects, we derive a simple formula for the 'anomaly cocycle' and show it is the obstruction to gauging even on the lattice. We reach this by introducing a systematic procedure to gauge arbitrary internal symmetries on the lattice that may not act on-site. As a by-product of our gauging procedure, we construct non-invertible lattice translation symmetries from LSM anomalies.

Zoom link https://pitp.zoom.us/j/98084408560?pwd=cllSVnpWcEhPK21aVDZubU4yYWNyQT09

Adobe Acrobat for Chrome
View PDFs and use Adobe Acrobat tools in Chrome by enabling the Acrobat extension for Google Chrome.
Enable Extension

Lieb-Schultz-Mattis anomalies as obstructions to gauging

Sahand Seifnashri
Institute for Advanced Study

Quantum Matter Seminar @ Perimeter Institute - Oct 31, 2023

Based on *arXiv:2308.05151*

Introduction: Dynamical constraints from symmetries

Global symmetries can impose powerful constraints on the dynamics of strongly coupled systems both in the continuum and on the lattice

Examples:

1. On the lattice: LSM theorem [Lieb, Schultz, Mattis 1961]
2. In the continuum: 't Hooft anomaly matching ['t Hooft 1980]

- There is a direct connection between the two examples: both are associated to *anomalies* of global symmetries

Continuum: 't Hooft anomaly matching

't Hooft anomaly = obstruction to gauging a global symmetry

1. Anomaly is invariant under RG (anomaly matching)
2. Anomaly is computable[1]

$\Rightarrow$ Anomalous theories cannot be in the trivial phase

For example:

- Anomalies in *generalized* symmetries constrain phases of gauge theories in 3+1d [Gaiotto-Kapustin-Komargodski-Seiberg 2017] and in 1+1d [Komargodski-Ohmori-Roumpedakis-SS 2021]

[1] No systematic method to compute non-perturbative anomalies

Anomalies on the *Lattice*:

1. **LSM theorem:** Spin-$\frac{1}{2}$ chains with SO(3) and lattice translation symmetry

$$H = \sum_j \vec{\sigma}_j \cdot \vec{\sigma}_{j+1} + \Delta H$$

   cannot have a unique gapped ground state

   * (LSM theorem is a consequence of a mixed anomaly between lattice translation and SO(3) symmetries)

2. **Boundary of SPTs**: SPT phases have non-trivial symmetry-protected edge modes, *e.g., topological insulators*

   * (Often taken as the definition of anomaly on the lattice)

- Anomalies on the lattice are *'t Hooft* anomalies

Goal

1. A *microscopic* formula for the anomaly (in internal and translation symmetry) of 1+1d lattice Hamiltonian systems

2. Show that the anomaly is the obstruction to gauging even on the lattice!

Key references

Group cohomology classification of anomalies:

- Chen, Gu, Liu, Wen 2011
- Kapustin, Thorngren 2014
- ...

LSM anomalies:

- Cheng, Zaletel, Barkeshli, Vishwanath, Bonderson 2015
- Cho, Hsieh, Ryu 2017
- Huang, Song, Huang, Hermele 2017
- Ye, Guo, He, Wang, Zou 2021
- ...
- **Cheng, Seiberg 2022**

Computing the anomaly:

- Else, Nayak 2014
- Kawagoe, Levin 2021

Outline

1. Introduction
2. Topological defects
3. Formula for the anomaly and gauging
4. Examples
   - I. Heisenberg chain
   - II. Non-invertible lattice translation symmetry

Symmetries as topological defects

Symmetry is a unitary operator U that:

1. $[U, H] = 0$
2. U maps local operators into local operators

- To manifest the locality property we represent symmetries by *topological defects*
- 'Symmetry operators $\leftrightarrow$ Symmetry defects' on the lattice!

Example: Transverse-field Ising model

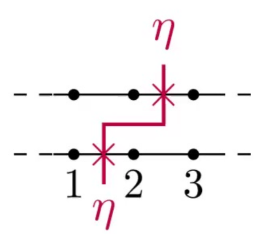
$$H = -\sum_j \sigma_j^z \sigma_{j+1}^z - h \sum_j \sigma_j^x \quad \rightarrow \quad \text{has a } \mathbb{Z}_2 \text{ symmetry } \eta$$

Symm operator : $U_\eta = \prod_j \sigma_j^x : \quad \sigma^z \mapsto -\sigma^z$

Symm defect : $H_\eta^{(1,2)} = -\sum_{j \neq 1} \sigma_j^z \sigma_{j+1}^z + \sigma_1^z \sigma_2^z - h \sum_j \sigma_j^x$

- The defect is topological since

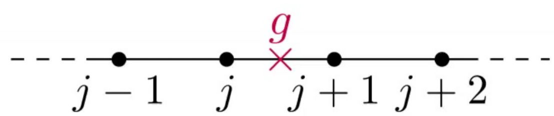
$$H_\eta^{(2,3)} = \sigma_2^x \, H_\eta^{(1,2)} \, \sigma_2^x \quad \rightarrow \quad \sigma_2^x \equiv$$



Topological defects for symmetry group G

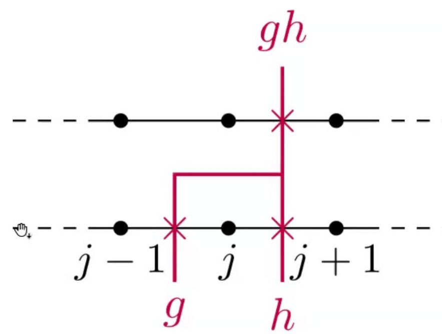
Defect Hamiltonian for a $g \in G$ twist on link $(j, j+1)$

$$H_g^{(j,j+1)} \equiv$$



Group multiplication → fusion rule for the defects:

$$\lambda^j(g,h) \equiv$$



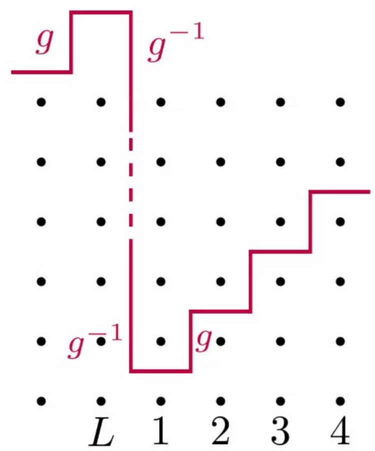
$$: H_{g;h}^{(j-1,j);(j,j+1)} \mapsto H_{gh}^{(j,j+1)}$$

$$H_{gh}{}^{(j,j+1)} = \lambda^j(g,h)\, H_{g;h}^{(j-1,j);(j,j+1)} \left(\lambda^j(g,h)\right)^{-1}$$

Symmetry operators $\leftrightarrow$ Symmetry defects

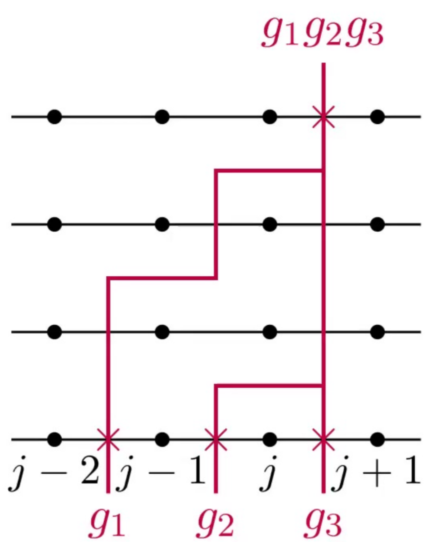
Operators $\to$ Defects : $H_g^{(j,j+1)} = U_g^{\leq j}\, H\, \left(U_g^{\leq j}\right)^{-1}$

Defects $\to$ Operators:

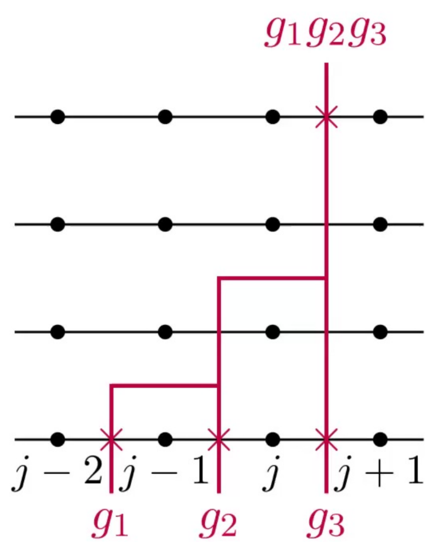
$U_g =$  $: \; H \mapsto H_{g^{-1};g}^{(L,1);(1,2)} \mapsto \cdots \mapsto H$

$$U_g = \lambda^L(g, g^{-1})\, \lambda^{L-1}(g,1) \cdots \lambda^3(g,1)\, \lambda^2(g,1)\, \left(\lambda^1(g^{-1}, g)\right)^{-1}$$

Microscopic formula for the anomaly (F-moves)



$$= F(g_1, g_2, g_3)$$



In equation:[2]

$$\boxed{\lambda^j(g_1, g_2g_3)\lambda^{j-1}(g_1, 1)\lambda^j(g_2, g_3) = F(g_1, g_2, g_3)\,\lambda^j(g_1g_2, g_3)\lambda^{j-1}(g_1, g_2)}$$

[2] F is just a phase given that there is no exact 1-form symmetry

Conditions on $F : G \times G \times G \to U(1)$

1. F satisfies a *modified* pentagon/cocycle equation[3]:

$$\frac{F(g_2, g_3, g_4)F(g_1, g_2g_3, g_4)F(g_1, g_2, g_3)}{F(g_1, g_2, g_3g_4)F(g_1g_2, g_3, g_4)} = F(g_1, g_2, 1)$$

2. $\lambda^j(g, h)$ has phase ambiguity[4] that propagates into F:

$$F(g_1, g_2, g_3) \sim F(g_1, g_2, g_3)\frac{\gamma^j(g_2, g_3)\gamma^{j-1}(g_1, 1)\gamma^j(g_1, g_2g_3)}{\gamma^{j-1}(g_1, g_2)\gamma^j(g_1g_2, g_3)}$$

---

[3] Analog of the Wess-Zumino consistency conditions in the continuum

[4] Corresponds to local counterterms in the continuum

Anomaly cocycles and group cohomology

1. F defines anomaly *cocycles*:

$$\omega(g_1, g_2, g_3) = \frac{F(g_1, g_2, g_3)}{F(g_1, g_2, 1)}\,, \qquad \alpha(g_1, g_2) = F(g_1, g_2, 1)$$

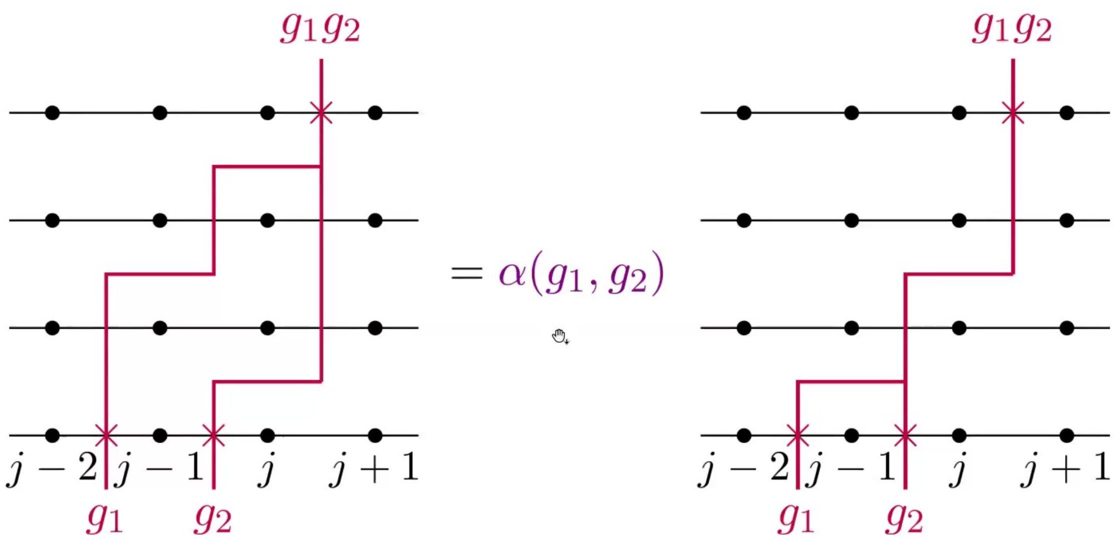
2. Modding out by j-independent phase ambiguities we get *cohomology classes*

$$[\omega] \in H^3(G, U(1))\,, \qquad [\alpha] \in H^2(G, U(1))$$

(Modding out by j-dependent phases trivializes α)

- $[\omega]$ is the 't Hooft anomaly in G
- $[\alpha]$ is the mixed anomaly between G and translation (LSM)

The LSM anomaly



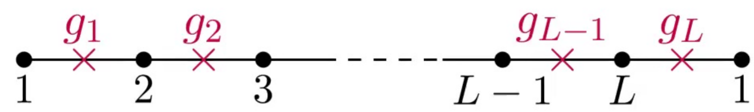
In equation:

$$\lambda^j(g_1, g_2)\lambda^{j-1}(g_1, 1)\lambda^j(g_2, 1) = \alpha(g_1, g_2)\, \lambda^j(g_1 g_2, 1)\lambda^{j-1}(g_1, g_2)$$

Gauging: (1) Add dynamical gauge "fields"

Extend $\mathcal{H}$ by adding $|G|$-dim Hilbert spaces on links:

$$\mathcal{H} \;\to\; \boxed{\widetilde{\mathcal{H}} = \bigoplus_{g_1,\ldots,g_L \in G}}$$



and

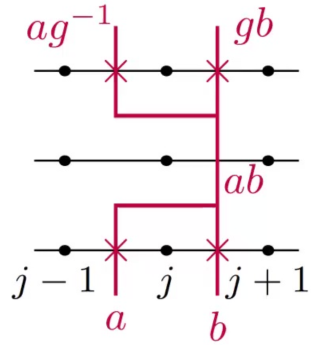
$$\widetilde{H} = \sum_{g_1,\ldots,g_L \in G} H_{g_1;\ldots;g_L} \otimes |g_1,\ldots,g_L\rangle \langle g_1,\ldots,g_L|_{\text{links}}$$

$$\widetilde{\lambda}^j(g,h) = \lambda^j(g,h) \otimes |1\rangle \langle g|_{(j-1,j)} \otimes |gh\rangle \langle h|_{(j,j+1)}$$

$$\Rightarrow \left[\widetilde{\lambda}^j(g,h), \widetilde{H}\right] = 0$$

Gauging: (2) Imposing Gauss's law

$$\mathcal{G}^j(g) = \sum_{a,b} \left(\widetilde{\lambda}^j(ag^{-1}, gb)\right)^\dagger \widetilde{\lambda}^j(a,b) \sim \sum_{a,b}$$



$\widetilde{H}$ has a local G symmetry: $\mathcal{G}^j(g)\mathcal{G}^j(h) = \mathcal{G}^j(gh)$

The Gauss's law at site j is : $\quad P_j \equiv \frac{1}{|G|}\sum_{g\in G} \mathcal{G}^j(g) = 1$

- P_j commutes with P_{j+1} iff $F(g_1, g_2, g_3) = 1$:

$\Rightarrow$ $[F]$ is the 't Hooft anomaly

LSM anomaly for “on-site symmetries”

Consider a symmetry acting projectively on each site:

$$U(g) = \prod_j U^j(g)$$

where U^j is a projective representation of G acting on site j.
The fusion operators are:

$$\lambda^j(g,h) = U^j(g)$$

Using the formula for the anomaly we get

$$U^j(g_1)U^j(g_2) = \alpha(g_1,g_2)\, U^j(g_1g_2)$$

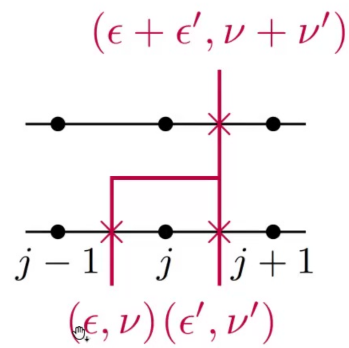
- $[\alpha] \in H^2(G,U(1))$ determines the projective representation of G per site

$I.$ spin-$\frac{1}{2}$ antiferromagnetic Heisenberg chain

$$H = \frac{1}{2}\sum_j (\sigma_j^x \sigma_{j+1}^x + \sigma_j^y \sigma_{j+1}^y + \sigma_j^z \sigma_{j+1}^z)$$

Consider the $\mathbb{Z}_2^X \times \mathbb{Z}_2^Y \subset SO(3)$ symmetry generated by $X = \prod_j \sigma_j^x$ and $Y = \prod_j \sigma_j^y$:

$$\lambda^j(\epsilon, \nu; \epsilon', \nu') = (\sigma_j^x)^\epsilon (\sigma_j^y)^\nu =$$



we get the anomaly cocycles:

$$\boxed{\alpha(\epsilon, \nu; \epsilon', \nu') = (-1)^{\epsilon' \nu}, \qquad \omega = 1}$$

II. Non-invertible lattice translation symmetry

What happens to the lattice translation symmetry after gauging an internal symmetry with LSM anomaly?

II. Gauging $\mathbb{Z}_2 \times \mathbb{Z}_2$ of the Heisenberg (XYZ) chain

$$\widetilde{H} = \frac{1}{2}\sum_{j=1}^{2L}\left(J_x \sigma_j^x \tau_{j,j+1}^x \sigma_{j+1}^x + J_y \sigma_j^y \mu_{j,j+1}^x \sigma_{j+1}^y + J_z \sigma_j^z \tau_{j,j+1}^x \mu_{j,j+1}^x \sigma_{j+1}^z\right)$$

Consider the $\mathbb{Z}_2^X \times \mathbb{Z}_2^Y \subset SO(3)$ symmetry with:

$$\lambda^j(\epsilon,\nu;\epsilon',\nu') = (-1)^{j\nu\epsilon'}(\sigma_j^x)^\epsilon(\sigma_j^y)^\nu$$

This leads to *modified* Gauss's laws:

$$\mathcal{G}_j(X) = (\tau_{j-1,j}^x)^{j+1}\mu_{j-1,j}^z \sigma_j^x \mu_{j,j+1}^z$$
$$\mathcal{G}_j(Y) = \tau_{j-1,j}^z \sigma_j^y \tau_{j,j+1}^z (\mu_{j,j+1}^x)^j$$

- $T\mathcal{G}_j T^{-1} \neq \mathcal{G}_{j+1}$ but $T^2\mathcal{G}_j T^{-2} = \mathcal{G}_{j+2}$

II. Gauging $\mathbb{Z}_2 \times \mathbb{Z}_2$ of the Heisenberg (XYZ) chain

After gauge-fixing we find:

$$\widetilde{H} = \frac{1}{2} \sum_{j=1}^{L} \Big(J_x(\tilde{\tau}_j^x + \tilde{\mu}_{j-1}^z \tilde{\mu}_j^z) + J_y(\tilde{\mu}_j^x + \tilde{\tau}_{j-1}^z \tilde{\tau}_j^z) + J_z(\cdots) \Big)$$

Translation symmetry T becomes a *non-invertible* symmetry $\tilde{T}$ after gauging which acts as

$$\tilde{\mu}_{j-1}^x \mapsto \tilde{\tau}_{j-1}^z \tilde{\tau}_j^z \mapsto \tilde{\mu}_j^x$$

$$\tilde{\tau}_{j-1}^x \mapsto \tilde{\mu}_{j-1}^z \tilde{\mu}_j^z \mapsto \tilde{\tau}_j^x$$

with the fusion rule

$$\tilde{T} \times \tilde{T} = T^2 (1 + \tilde{X})(1 + \tilde{Y})$$

- $\tilde{T}$ can map a local operator into a disorder/twist operator:

$$\tilde{\mu}_j^z \to \tilde{\tau}_j^x \, \tilde{\tau}_{j-1}^x \, \tilde{\tau}_{j-2}^x \, \tilde{\tau}_{j-3}^x \, \tilde{\tau}_{j-4}^x \cdots$$

Summary

- We found a *microscopic* formula for 1+1d anomalies
- Generalized LSM theorem beyond "on-site" symmetries
- Introduced a method to gauge non-on-site symmetries
- Non-invertible translation symmetry from LSM anomalies
- The method works for mixed anomalies involving any lattice symmetry: $(-1)^F$, time-reversal, reflection, ...
- Generalization to higher dimensions?