Title: Loop-corrected soft photon theorems and large gauge transformations

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Abstract: In the last few years, a remarkable link has been established between the soft theorems and asymptotic symmetries of quantum field theories: soft theorems are Ward identities of the asymptotic symmetry generators. In quantum electrodynamics, Weinberg's soft photon theorem is nothing but the Ward identity of a gauge transformation whose parameter is non-trivial at infinity. Likewise, Low's tree-level subleading soft photon theorem is the Ward identity of a gauge transformation whose parameter diverges linearly at infinity. More recently, it has been shown that Low's theorem receives loop corrections that are logarithmic in soft photon energy. Then, it is natural to ask whether such corrections are associated with some asymptotic symmetry of the S-matrix. There have been proposals for conserved charges whose Ward identities yield the loop-corrected soft theorems, but a clear symmetry interpretation remains elusive. We explore this question in the context of scalar QED, in hopes of shedding light on the connection between asymptotic symmetries and loop-corrected soft theorems.

Zoom link https://pitp.zoom.us/j/94420835190?pwd=dEpOSHluRzFpVTg3Qm10OS9PTTU3dz09

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Loop-Corrected Soft Photon Theorems and Large Gauge Transformations

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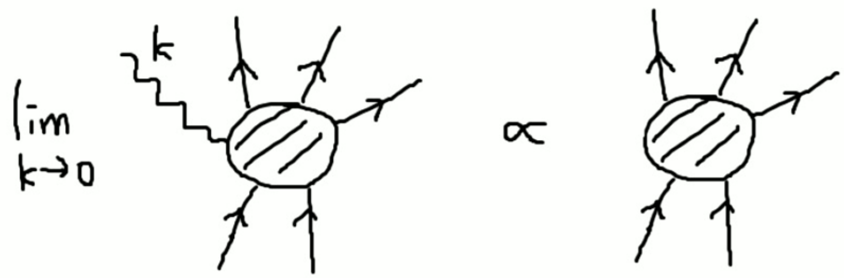
Soft theorem

- Soft theorem is a theorem in QFT that derives from studying Feynman diagrams.



$$\lim_{\omega\to 0} A_{n+1}(k,\pm) \;=\; \lim_{\omega\to 0}\left(\frac{1}{\omega}S_0^{\pm} + S_1^{\pm} + O(\omega)\right) A_n$$

- $\frac{1}{\omega}S_0^{\pm}$ is of order $1/\omega$ and diverges as $\omega \to 0$.
  $S_1^{\pm}$ is independent of ω and is convergent.

Soft theorem

$$\lim_{\omega\to 0} A_{n+1}(k,\pm) = \lim_{\omega\to 0}\left(\frac{1}{\omega}S_0^{\pm} + S_1^{\pm} + O(\omega)\right)A_n$$

- Leading soft photon theorem

$$\frac{1}{\omega}S_0^{\pm} = \sum_i Q_i \frac{p_i\cdot\epsilon^{\pm}(k)}{p_i\cdot k}$$

- Subleading soft photon theorem (tree-level)

$$S_1^{\pm} = -i\sum_i Q_i \frac{k\cdot J_i\cdot\epsilon^{\pm}(k)}{p_i\cdot k} \qquad J_i^{\mu\nu} = L_i^{\mu\nu} + S_i^{\mu\nu}$$

- Also leading/subleading soft graviton theorems, soft gluon theorem, soft photino theorem...

Asymptotic symmetry

- What is an asymptotic symmetry?

$$\text{Asymptotic Symmetry} = \frac{\text{Allowed Gauge Symmetry}}{\text{Trivial Gauge Symmetry}}$$

  There are exceptions, but it's a useful way to think about this.

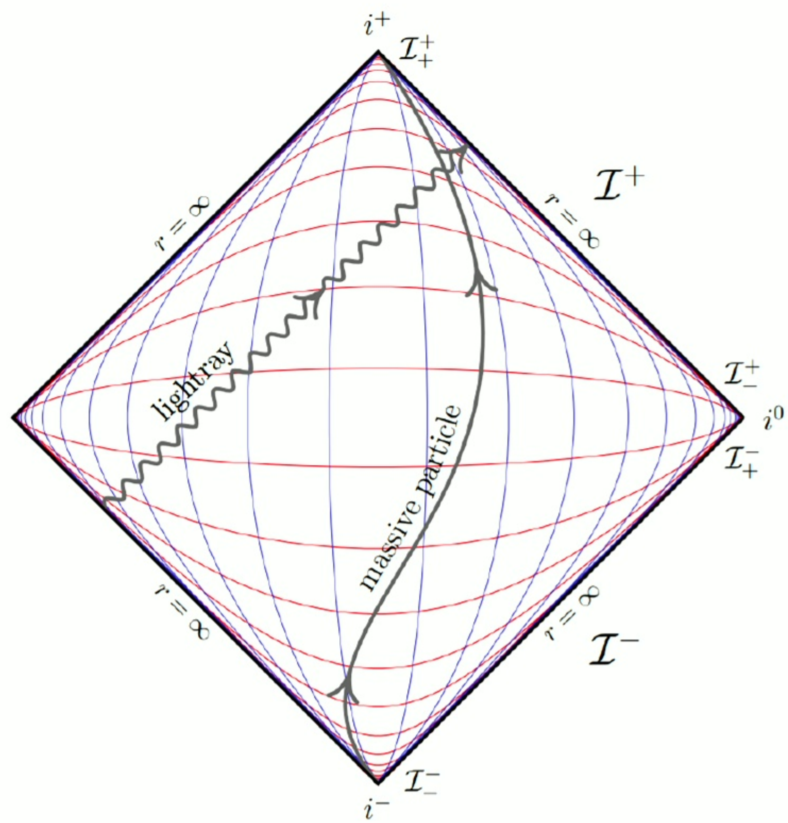
- Aren't all gauge transformations trivial?
  No. If you follow the Noether procedure, you notice that there is a falloff condition on the parameter for the symmetry to be trivial.

- This asymptotic symmetry group is often referred to as large gauge transformations (LGT). This definition allows for topologically trivial LGTs.

  Sometimes this is referred to as improper gauge transformation.

Asymptotic symmetry

Penrose diagram of Minkowski spacetime [Strominger 1703.05448]



Asymptotic symmetry

- The asymptotic symmetry of QED is the set of $U(1)$ gauge transformations whose gauge parameter does not vanish at infinity.

$$A_\mu \to A_\mu + \partial_\mu \alpha, \qquad \phi \to e^{iQ\alpha}\phi, \qquad \lim_{r\to\infty} \alpha \neq 0$$

- The LGT generator does not vanish on shell; these are physical transformations.

$$Q^+[\alpha] = \int_{\mathcal{I}^+_-} \alpha * F \qquad Q^-[\alpha] = \int_{\mathcal{I}^-_+} \alpha * F$$

- Asymptotic symmetries are symmetries of the S-matrix

$$\langle \text{out}|[Q,S]|\text{in}\rangle = 0 \ , \qquad [Q,S] \equiv Q^+ S - S Q^-$$

This is the Ward identity of the asymptotic symmetry generators $Q^\pm$.

Asymptotic symmetries and soft theorems

- There is a remarkable link that have been established between many asymptotic symmetries and soft theorems.

$$\langle \text{out}|[Q,S]|\text{in}\rangle = 0 \qquad \Longleftrightarrow \qquad \text{soft theorem}$$

- LGT in QED $\Longleftrightarrow$ Leading soft photon theorem
  [He, Mitra, Porfyriadis, Strominger]
- Divergent LGT in QED $\Longleftrightarrow$ Tree-level subleading soft photon theorem
  [Campiglia, Laddha]
- BMS supertranslations $\Longleftrightarrow$ Leading soft graviton theorem
  [He, Mitra, Lysov, Strominger]
- BMS superrotations $\Longleftrightarrow$ Tree-level subleading soft graviton theorem
  [Kapec, Lysov, Pasterski, Strominger]
- This even extends to higher dimensions [He, Mitra]
- $\cdots$

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Divergent LGT $\iff$ Tree-level subleading soft theorem

- Consider a gauge transformation $\delta A_\mu = \partial_\mu \alpha$ where $\alpha = O(r)$ for large r. The Lorenz gauge condition $\nabla^2 \alpha = 0$ dictates its form,

$$\alpha(u, r, \hat{x}) = r\lambda(\hat{x}) + u(1 + \tfrac{D^2}{2})\lambda(\hat{x}) + \cdots$$

- If we compute the charge on a constant time slice Σ_t, the divergent term organizes into $t = u + r$ times the convergent LGT charge,

$$Q_t[\alpha] = tQ_t[\lambda] + \int_{\Sigma_t} dV(\cdots) \cong \int_{\Sigma_t} dV(\cdots)$$

- Sending $t \to \infty$ while keeping $u = t - r$ fixed, we get

$$Q[\alpha] \cong \frac{1}{2}\int_{\mathcal{I}^+} du\, d^2\hat{x}\, \lambda(\hat{x}) \left[D^A \overset{2}{j}_A - uD^2 \overset{2}{j}_u + u\partial_u D^2 D^A \overset{0}{A}_A \right]$$

The Ward identity of this charge $\langle [Q, S] \rangle = 0$ is equivalent to the tree-level subleading soft theorem. [Lysov, Pasterski, Strominger] [Campiglia, Laddha]

Loop corrections to subleading soft theorem

- It turns out that the subleading soft theorem receives one-loop corrections that are infrared-divergent that have to be regularized [Bern, Davies, Nohle]

$$\lim_{\omega\to 0} A_{n+1}(k,\pm) \overset{\text{1-loop}}{=} \lim_{\omega\to 0}\left(\frac{1}{\omega}S_0^{\pm} + S_1^{\pm} + \frac{1}{\epsilon}S_{1,\text{div}}^{\pm} + O(\omega)\right)A_n$$

- This seems to be a backlash of restricting to power-law expansions. Allowing non-analytic terms of order $\ln\omega$ in the soft photon energy yields logarithmic soft factors [Sahoo, Sen]

$$\lim_{\omega\to 0} A_{n+1}(k,\pm) \overset{\text{1-loop}}{=} \lim_{\omega\to 0}\left(\frac{1}{\omega}S_0^{\pm} + \ln\omega\, S_{\ln}^{\pm} + \cdots\right)A_n$$

There is a logarithmic soft theorem that is more leading compared to the subleading soft theorem.

Loop corrections to subleading soft theorem

- Here is what the logarithmic soft photon factor looks like:

$$S_{\ln}^{\pm} = \frac{i}{4\pi} \sum_{\substack{i\neq j \\ \eta_i\eta_j=1}} \frac{Q_i^2 Q_j p_i^2 p_j^2 \left[(p_i\cdot\epsilon^{\pm})(p_j\cdot k) - (i\leftrightarrow j)\right]}{(p_i\cdot k)((p_i\cdot p_j)^2 - p_i^2 p_j^2)^{\frac{3}{2}}}$$

$$- \frac{i}{8\pi^2} \sum_{i\neq j} \frac{Q_i^2 Q_j}{p_i\cdot k} (\epsilon^{\pm}\cdot J_i\cdot k) \frac{(p_i\cdot p_j)}{\sqrt{(p_i\cdot p_j)^2 - p_i^2 p_j^2}} \ln\left(\frac{p_i\cdot p_j + \sqrt{(p_i\cdot p_j)^2 - p_i^2 p_j^2}}{p_i\cdot p_j - \sqrt{(p_i\cdot p_j)^2 - p_i^2 p_j^2}}\right)$$

- What is the asymptotic symmetry whose Ward identity is the logarithmic soft theorem?

- We aim to shed light on this question in massive scalar QED.

Massive scalar QED with long-range interactions

- We work with massive scalar QED.
- We employ two coordinate systems, a hyperbolic coordinate $(\tau, \rho, \hat{x})$ for timelike infinity i^+ and retarded time coordinate $(u, r, \hat{x})$ for null infinity.

$$\tau = \sqrt{t^2 - r^2}, \qquad \rho = \frac{r}{\sqrt{t^2 - r^2}}, \qquad u = t - r$$

- $\tau \to \infty$ with ρ fixed follows the future trajectory of massive particles. $r \to \infty$ with u fixed follows the future trajectory of massless particles.

Massive scalar QED with long-range interactions

- At large times $\tau \to \infty$, one usually approximates matter fields to be free, ie as a solution of the Klein-Gordon equation

$$(\nabla^2 - m^2)\phi = 0$$

But actually, interactions with the gauge field lead to the equation as $\tau \to \infty$

$$(\mathcal{D}^2 - m^2)\phi = (\nabla^2 - m^2 - 2ieA_\tau \partial_\tau + \cdots)\phi = 0$$

- This means the phase space does not consist of free massive fields at i^+; it consists of the dressed fields [Campiglia, Laddha]

$$\phi(\tau, \rho, \hat{x}) \overset{\tau\to\infty}{=} \tau^{ie\overset{1}{A}_\tau} \phi_{\text{free}}(\tau, \rho, \hat{x})$$

where the gauge field has falloff $A_\tau = \frac{1}{\tau}\overset{1}{A}_\tau + \cdots$.

Massive scalar QED with long-range interactions

- This introduces logarithmic terms in the large-τ expansion of the matter current. The current, via Maxwell's equations, introduces logarithmic terms to the large-r expansions of the gauge field and field strength,

$$F_{ur} = \frac{1}{r^2}\overset{2}{F}_{ur} + \frac{\ln r}{r^3}\overset{\ln}{F}_{ur} + \frac{1}{r^3}\overset{3}{F}_{ur} + \cdots$$
$$F_{rA} = \frac{\ln r}{r^2}\overset{\ln}{F}_{rA} + \frac{1}{r^2}\overset{2}{F}_{rA} + \cdots$$
$$A_A = \overset{0}{A}_A + \frac{\ln r}{r}\overset{\ln}{A}_A + \frac{1}{r}\overset{1}{A}_A + \cdots$$

and to the large-u expansions as well

$$\overset{2}{F}_{rA} \overset{u\to\pm\infty}{=} u\overset{2,-1}{F}{}^{\pm}_{rA} + \ln u\overset{2,\ln}{F}{}^{\pm}_{rA} + \cdots$$
$$\overset{0}{A}_A \overset{u\to\pm\infty}{=} \overset{0,0}{A}{}^{\pm}_{A} + \frac{1}{u}\overset{0,1}{A}{}^{\pm}_{A} + \cdots$$

Divergent LGT with long-range interactions

- Using the new falloffs, we reconsider the divergent LGT in Lorenz gauge

$$\delta A = \partial_\mu \alpha, \qquad \alpha(u, r, \hat{x}) = r\lambda(\hat{x}) + u(1 + \tfrac{D^2}{2})\lambda(\hat{x}) + \cdots$$

- The charge has two contributions

$$Q[\alpha] = \underbrace{\int_{\mathcal{I}^+_+} d^2\hat{x}\, \lambda(\hat{x}) \left[-\overset{3}{F}_{ur} - \frac{u}{2} D^2 \overset{2}{F}_{ur} \right]}_{\text{hard charge } Q_H} + \underbrace{\frac{1}{2} \int_{\mathcal{I}+} du\, d^2\hat{x}\, \lambda(\hat{x})\, u\partial_u D^2 D^A \overset{0}{A}_A}_{\text{soft charge } Q_S}$$

Contributions from the current j_μ on $\mathcal{I}^+$ have shifted to F_{rA} on $\mathcal{I}^+_+$ because the charges are massive.

- One finds that this expression is the same as what you would get from free massive fields, so has anything changed?

Divergent LGT with long-range interactions

- Maxwell's equations imply $\overset{3}{F}_{ur} = D^A \overset{2}{F}_{rA}$ and $D^2 \overset{2}{F}_{ur} = -2\partial_u D^A \overset{2}{F}_{rA}$, so the hard charge written in terms of $\overset{2}{F}_{rA}$ is

$$Q_H = \int_{\mathcal{I}^+_+} d^2\hat{x}\, \lambda(\hat{x})\, (u\partial_u - 1) D^A \overset{2}{F}_{rA}$$

- Recall that long-range interaction gives rise to the new terms

$$\overset{2}{F}_{rA} \overset{u\to\pm\infty}{=} u \overset{2,-1}{F}{}^{\pm}_{rA} + \ln u \overset{2,\ln}{F}{}^{\pm}_{rA} + \cdots$$

- Notice that the projector $(u\partial_u - 1)$ in the hard charge cancels the leading term linear in u and picks up the $\ln u$ term,

$$Q_H = -\lim_{u\to+\infty} \int_{\mathcal{I}^+_+} d^2\hat{x}\, \lambda(\hat{x}) \ln u\, D^A \overset{2,\ln}{F}{}^{+}_{rA} + (\text{finite})$$

so the new term makes the hard charge diverge logarithmically.

Divergent LGT with long-range interactions

- The soft charge and the relevant new term introduced to $\overset{0}{A}_A$ are

$$Q_S = \frac{1}{2}\int_{\mathcal{I}^+} du\, d^2\hat{x}\, \lambda\, u\partial_u D^2 D^A \overset{0}{A}_A, \qquad \overset{0}{A}_A \overset{u\to\pm\infty}{=} \overset{0,0}{A}{}^{\pm}_{A} + \frac{1}{u}\overset{0,1}{A}{}^{\pm}_{A} + \cdots$$

- We can choose a suitably large but finite u_0 and write

$$\begin{aligned}\int_{-\infty}^{\infty} du\, u\partial_u \overset{0}{A}_A &= \lim_{U\to+\infty}\left(\int_{-U}^{-u_0} + \int_{u_0}^{U}\right) du\, u\partial_u \overset{0}{A}_A + (\text{finite}) \\ &= \lim_{U\to+\infty} \ln(U^{-1})\left(\overset{0,1}{A}{}^{+}_{A} - \overset{0,1}{A}{}^{-}_{A}\right) + (\text{finite}) \\ &= \lim_{U\to+\infty} \ln(U^{-1})\int_{-\infty}^{\infty} du\,(-\partial_u u^2 \partial_u \overset{0}{A}_A) + (\text{finite})\end{aligned}$$

Thus, the new u^{-1} term in $\overset{0}{A}_A$ makes the soft charge diverge.

Divergent LGT with long-range interactions

- The total charge becomes

$$Q[\alpha] = \lim_{U\to\infty} \ln(U^{-1}) \left[\int_{\mathcal{I}^+_+} d^2\hat{x}\, \lambda\, D^A \overset{2,\ln}{F}{}^{+}_{rA} - \frac{1}{2} \int_{\mathcal{I}^+} du\, d^2\hat{x}\, \lambda\, \partial_u u^2 \partial_u D^2 D^A \overset{0}{A}_A \right] + (\text{finite terms})$$

It turns out that the coefficient of $\ln(U^{-1})$ is exactly the "logarithmic charge" proposed by Campiglia and Laddha whose Ward identity yields the logarithmic soft theorem.

- This implies that the Ward identity of the divergent LGT is the log soft theorem.

Summary

- Therefore, we have the correspondence
  Divergent LGT $\Longleftrightarrow$ Logarithmic soft theorem

- More precisely, the charge $Q[\alpha]$ that generates linearly divergent gauge transformations $\alpha(u, r, \hat{x}) = r\lambda(\hat{x}) + \cdots$ has the following Ward identities:
  - Free matter fields: $Q[\alpha]$ is finite, and

$$\langle \text{out}|[Q[\alpha], S]|\text{in}\rangle = 0 \iff \text{tree-level subleading soft photon theorem}$$

  - Dressed matter fields: $Q[\alpha]$ has a logarithmically divergent piece, and

$$\langle \text{out}|[Q[\alpha], S]|\text{in}\rangle = 0 \iff \text{loop-corrected subleading soft photon theorem}$$

- The asymptotic symmetry that corresponds to tree-level soft theorem leads to loop corrections once long-range interactions are taken into account.

Some loose ends

The story is not complete yet!

- The hard charge can be written as an integral over i^+ [Campiglia, Laddha]

$$Q_H = \int_{\mathcal{I}^+_+} d^2\hat{x}\, \lambda(\hat{x})\, D^A \overset{2,\ln}{F}{}^+_{rA}(\hat{x}) = \int_{i^+} d^3y\, V^\alpha(y) |\varphi|^2 \partial_\alpha A_\tau$$

  A_τ is sourced by φ; the charge is quartic in the matter fields.
  $\implies$ Phase space of dressed matter fields? What is its action on phase space?

- On the soft side, the divergent gauge transformation is projected down to a phase space of convergent fields. What is the action on this phase space?
  - ▶ In gravity, this problem will not be present for superrotation.

- What is the celestial CFT generator? The log soft factor should be related to the residue of a double pole at zero conformal weight $\Delta = 0$...

Thank you for your attention!