

Title: Higher-order transformations and the causal structure of quantum processes

Speakers: Hilary Kristjánsson

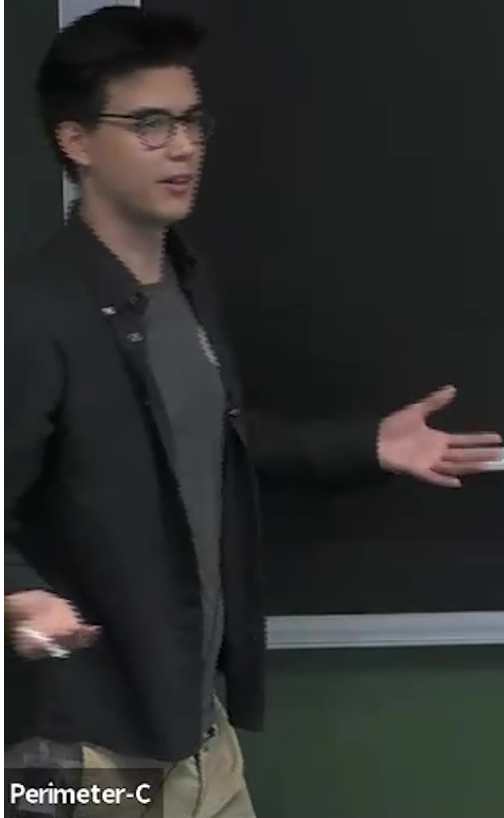
Series: Quantum Foundations

Date: October 06, 2023 - 2:00 PM

URL: <https://pirsa.org/23100106>

Abstract: In this informal talk, I shall give a short introduction to the field of higher-order quantum transformations, including its subfield of indefinite causal order. I shall discuss some of the motivations and important results in the field, current research directions and open problems, as well potential applications in quantum information processing.

Higher-order transformations and the causal
structure of quantum processes



Perimeter-C

Higher-order transformations and the causal structure of quantum processes

quantum states
 $\rho \in \mathcal{L}(\mathcal{H})$



$\rho \geq 0$
 $\text{Tr} \rho = 1$

$$\mathcal{U}(\rho) = X \rho X$$

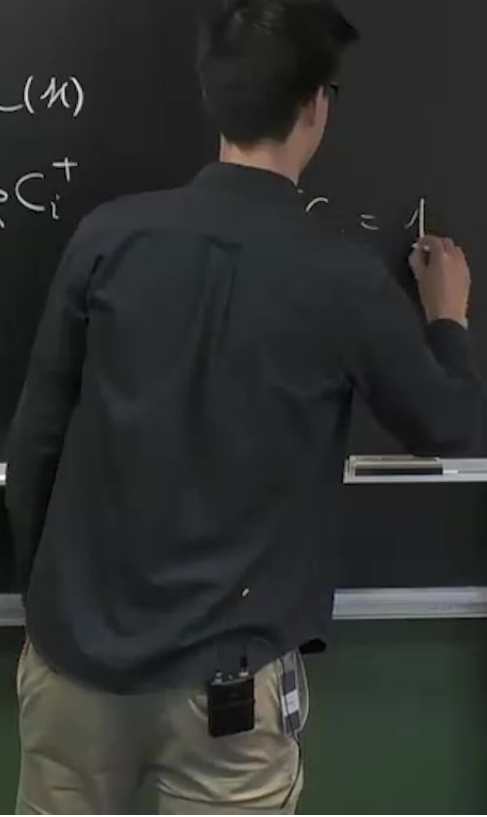
$$x, y \in \mathcal{B}^n$$

$$f(x) = x^2$$

Quantum channels

$$\mathcal{C} : \mathcal{L}(\mathcal{H}) \rightarrow \mathcal{L}(\mathcal{H})$$

$$\mathcal{C}(\rho) = \sum_i C_i \rho C_i^\dagger$$



Higher-order transformations and the causal structure of quantum processes

quantum states
 $\rho \in \mathcal{L}(\mathcal{H})$
 $\rho \geq 0$
 $\text{Tr} \rho = 1$
 $u(\rho) = \langle X, \rho \rangle$



\mathcal{B}^A

Quantum channels

$$\mathcal{C} : \mathcal{L}(\mathcal{H}) \rightarrow \mathcal{L}(\mathcal{H})$$

$$\mathcal{C}(\rho) = \sum_i C_i \rho C_i^\dagger, \quad \sum_i C_i^\dagger C_i = \mathbb{1}$$

$$u(p) = XpX$$

$$C(p) = \sum_i C_i p C_i^T, \quad \sum_i C_i^T C_i = \mathbf{1}$$

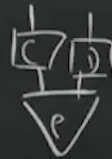
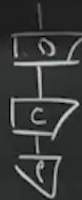
def f(x):
return x+2

def f(g)(x):
return g(x)+3

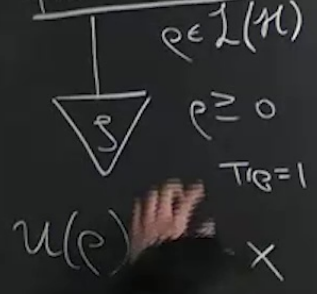
Composition of channels

$$D \circ C(p) = \sum_{i,j} D_i C_j p C_j^T D_i^T$$

$$D_1 \otimes C_2(p_{12})$$



quantum states



structure of quantum processes

$x, y \in \mathcal{B}^n$
 $f(x) = x^2$

Quantum channels

$\mathcal{E} : \mathcal{L}(\mathcal{H}) \rightarrow \mathcal{L}(\mathcal{H})$

$\mathcal{E}(\rho) = \sum_i C_i \rho C_i^\dagger, \quad \sum_i C_i^\dagger C_i = \mathbb{1}$

Completely positive trace-preserving

$$u(\rho) = X\rho X$$

$$C(\rho) = \sum_i C_i \rho C_i^\dagger, \quad \sum_i C_i^\dagger C_i = \mathbb{1}$$

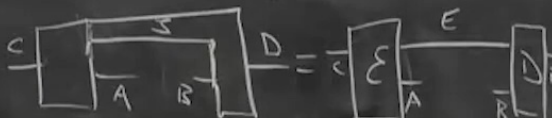
Completely positive trace-preserving

Quantum Superchannels

$$\mathcal{S}: [\mathcal{L}(H) \rightarrow \mathcal{L}(H)] \longrightarrow [\mathcal{L}(H) \rightarrow \mathcal{L}(H)]$$

- completely -
- Completely positive - preserving
- trace - preserving - preserving

Pauli Theorem



$$u(\rho) = X\rho X$$

$$C(\rho) = \sum_i C_i \rho C_i^\dagger, \quad \sum_i C_i^\dagger C_i = \mathbb{1}$$

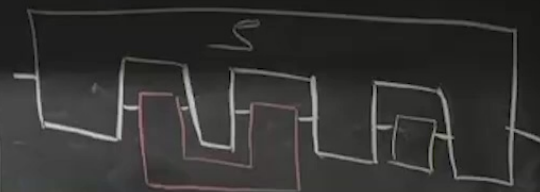
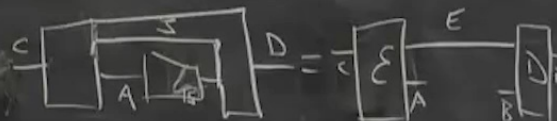
Completely positive trace-preserving

Quantum Superchannels

$$\mathcal{S}: [\mathcal{L}(H) \rightarrow \mathcal{L}(H)] \longrightarrow [\mathcal{L}(H) \rightarrow \mathcal{L}(H)]$$

- completely -
- Completely positive - preserving
- trace - preserving - preserving

Pauli Theorem



Q: what about



$$U(\rho) = X\rho X$$

$$C(\rho) = \sum_i C_i \rho C_i^\dagger, \quad \sum_i C_i^\dagger C_i = \mathbb{1}$$

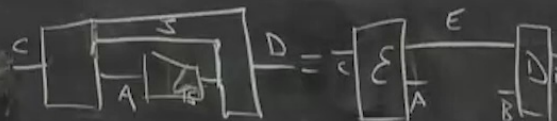
Completely positive trace-preserving

Quantum Superchannels

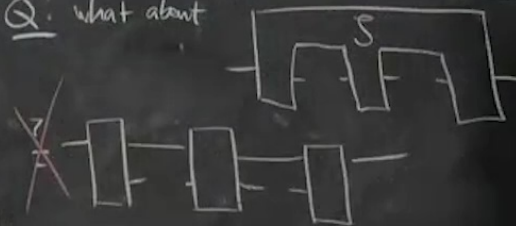
$$\mathcal{S}: [\mathcal{L}(H) \rightarrow \mathcal{L}(H)] \longrightarrow [\mathcal{L}(H) \rightarrow \mathcal{L}(H)]$$

- completely -
- Completely positive - preserving
- trace - preserving - preserving

Pauli theorem



Q: what about

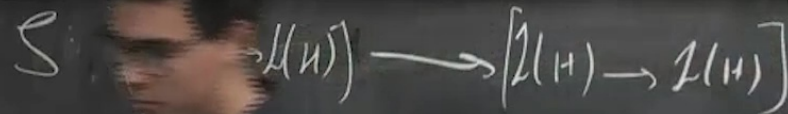


$$U(\rho) = X\rho X$$

$$C(\rho) = \sum_i C_i \rho C_i^\dagger, \quad \sum_i C_i^\dagger C_i = \mathbb{1}$$

Completely positive trace-preserving

Quantum channels

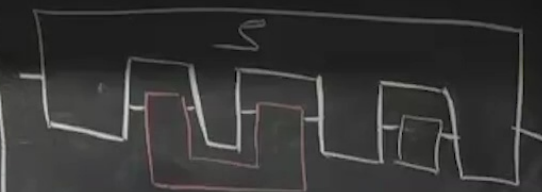
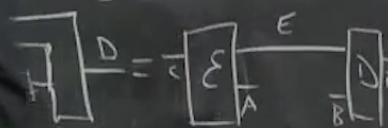


C_i

trace-preserving

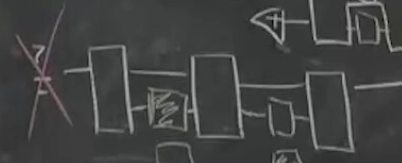
completely positive

Partial trace



Q. what about

quantum switch



$$u(\rho) = X\rho X$$

$$C(\rho) = \sum_i C_i \rho C_i^\dagger, \quad \sum_i C_i^\dagger C_i = \mathbb{1}$$

Completely positive trace-preserving

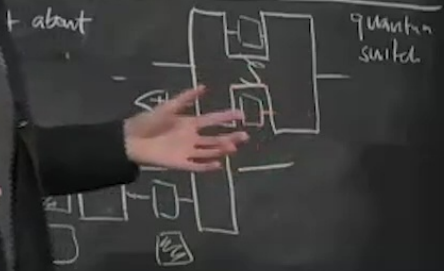
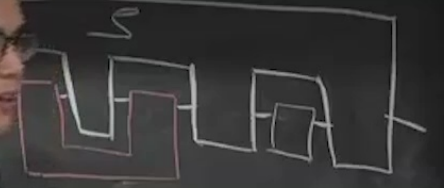


Quantum Superchannels

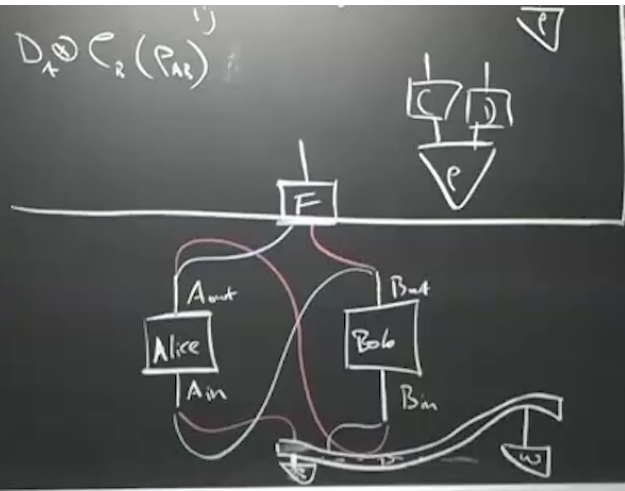
$$\mathcal{S}: [\mathcal{L}(H) \rightarrow \mathcal{L}(H)] \rightarrow [\mathcal{L}(H) \rightarrow \mathcal{L}(H)]$$

- completely -
- Completely positive - preserving
- trace - preserving - preserving

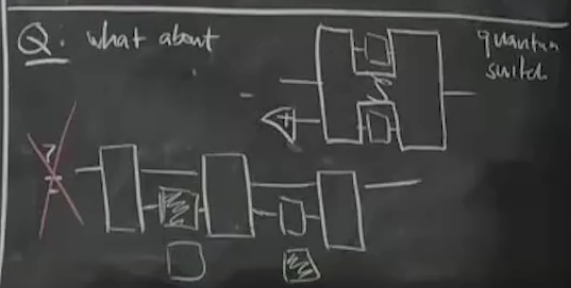
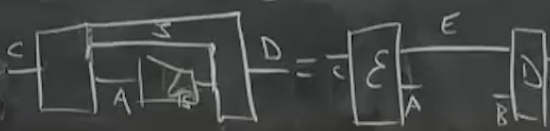
Pauli theorem



def $f(g|x)$.
 return $g'(x+z)+3$



completely -
 completely positive - preserving
 trace - preserving - preserving



der transformations and the causal
 structure of quantum processes

$\mathcal{X}, \mathcal{Y} \in \mathcal{B}^n$ Quantum channels \mathcal{B}

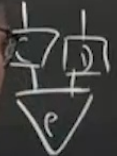
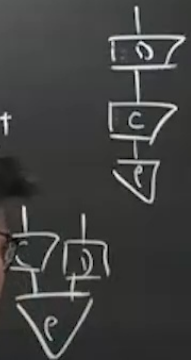
def f(x):
return x+2

def f(g)(x):
return g(x+2)+3

Composition of channels

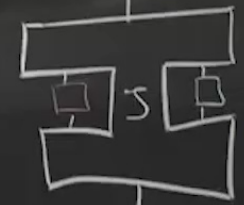
$$D \circ C(p) = \sum_{ij} D_i C_j p + D_j^t$$

$$D_1 \otimes C_2(p_{AB})$$



Current research

Yokojima: unitary S



Higher-order transformation

the causal

quantum states
 $\rho \in \mathcal{L}(\mathcal{H})$

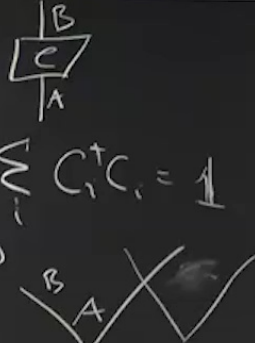
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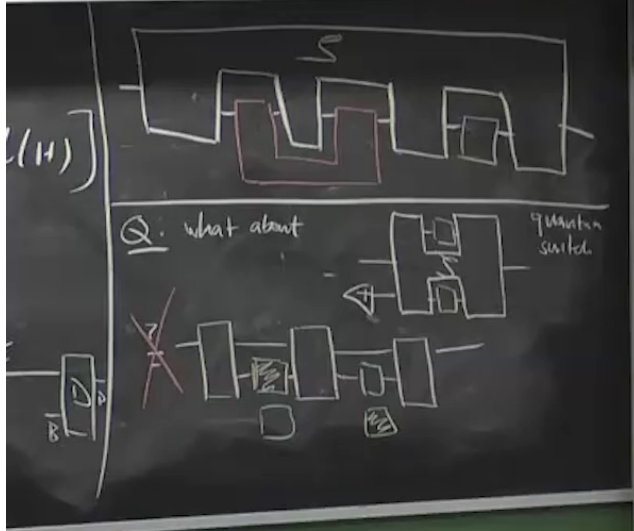
quantum processes

$x, y \in \mathcal{B}^n$

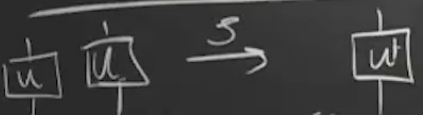
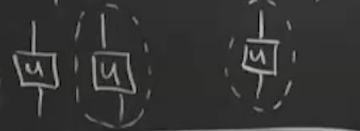
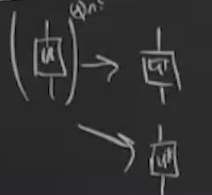
\mathcal{B}

$\mathcal{L}(\mathcal{K}) \rightarrow \mathcal{L}(\mathcal{K})$
 $= \sum_i C_i \rho C_i^\dagger$
 completely positive trace-preserving
 $\sum_i C_i^\dagger C_i = \mathbb{1}$



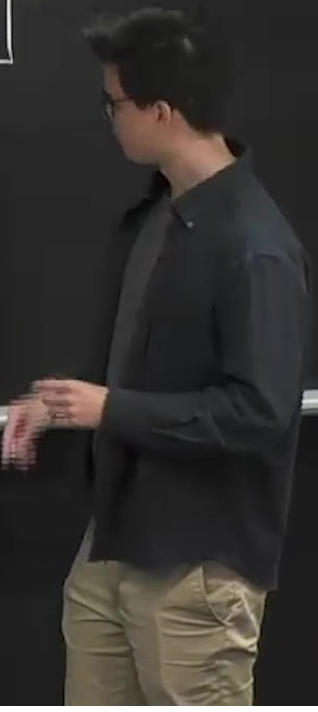


Tasks using superchannels

prob harmful exact	det exact
prob harmful approx	det approx

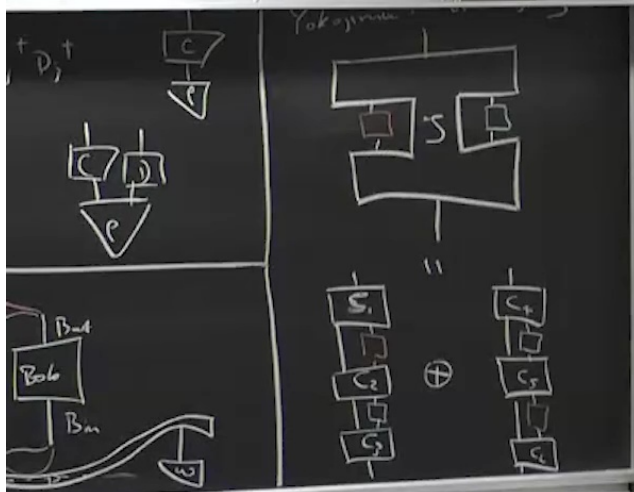
← qubit u



10)

$$= \sum_i C_i \rho C_i^\dagger, \quad \sum_i C_i^\dagger C_i = \mathbb{1}$$

ely positive trace-preserving



Tasks using superchannels

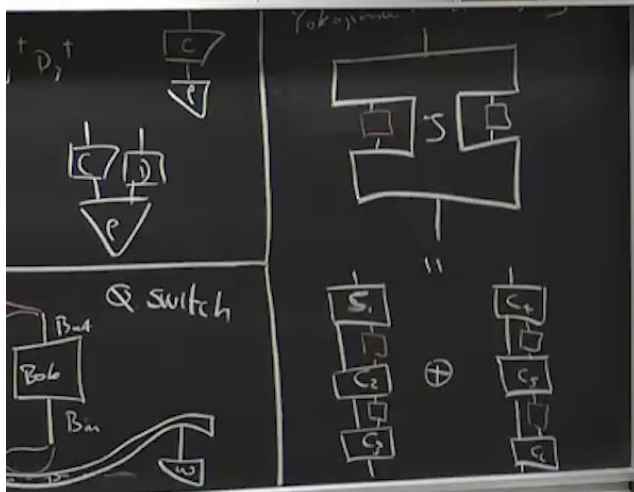
prob harmful exact	det exact	← qubit U
prob harmful approx	det approx	

Controlisation

10)

$$= \sum_i C_i \rho C_i^\dagger, \quad \sum_i C_i^\dagger C_i = 1$$

ely positive trace-preserving



Tasks using superchannels

prob harmal exact	det exact	← quit U
prob harmal approx	det approx	

Controlisation