Title: The quasinormal-mode content of black hole ringdowns

Speakers: Mark Cheung

Series: Strong Gravity

Date: October 19, 2023 - 1:00 PM

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Abstract: In general relativity, the remnant black hole of a binary black hole merger emits "ringdown" radiation: a superposition of quasinormal modes with characteristic complex frequencies. The detection of more than one of these frequencies would serve as smoking-gun evidence of a black hole and would enable a test of the no-hair theorem. In this talk, I will discuss strategies for identifying quasinormal modes within simulated ringdown waveforms both in linear perturbation theory and full general relativity. I will show that a rich spectrum of modes could exist in the ringdown, including overtones, retrograde modes and nonlinear modes, and that interesting quasi-universal relationships exist between some of their amplitudes. I will also discuss the spectral instability of quasinormal modes and its potential imprints on the ringdown waveform. These results could guide the analysis of real ringdown signals detected by gravitational-wave detectors.

Zoom link https://pitp.zoom.us/j/92002625163?pwd=bHNDVVZKYnVNTENLNXJBLzVNKzVSQT09

THE QUASINORMAL MODE CONTENT OF BLACK HOLE RINGDOWNS

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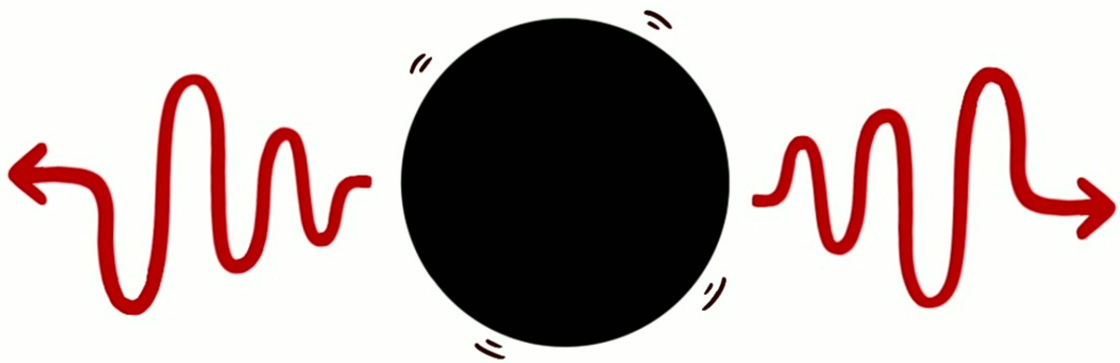
19 Oct 2023

JOHNS HOPKINS UNIVERSITY

Gravitational waves



Gravitational waves

Black Hole Perturbation Theory

Solve the Einstein Field Equations order by order

$$g_{\mu\nu} = g_{\mathrm{Kerr},\mu\nu} + \epsilon h^{(1)}_{\mu\nu} + \epsilon^2 h^{(2)}_{\mu\nu} + O(\epsilon^3)$$

$$G_{\mu\nu}[g] = \epsilon G^{(1)}_{\mu\nu}[h^{(1)}] + \epsilon^2 G^{(1)}_{\mu\nu}[h^{(2)}] + \epsilon^2 G^{(2)}_{\mu\nu}[h^{(1)}, h^{(1)}] + O(\epsilon^3) = 0$$

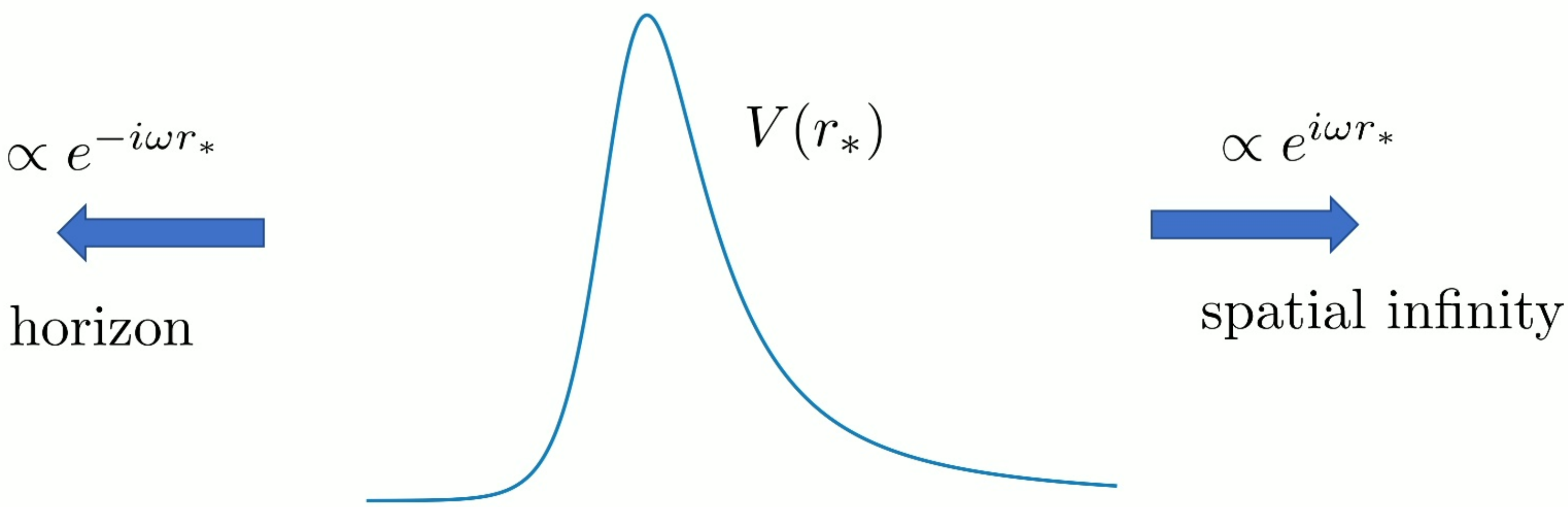
$$\boxed{\begin{aligned} G^{(1)}_{\mu\nu}[h^{(1)}] &= 0 \\ G^{(1)}_{\mu\nu}[h^{(2)}] &= -G^{(2)}_{\mu\nu}[h^{(1)}, h^{(1)}] \end{aligned}} \implies \boxed{\begin{aligned} \mathcal{T}[\Psi_4^{(1)}] &= 0 \\ \mathcal{T}[\Psi_4^{(2)}] &= \mathcal{S}_4^{(2)} \end{aligned}}$$

Teukolsky Equation
(Teukolsky 1973)

Black Hole Perturbation Theory

$$\frac{\partial^2 \psi}{\partial r_*^2} + \left[\omega^2 - V(r_*)\right] \psi = 0$$

Non-rotating case (Schwarzschild)

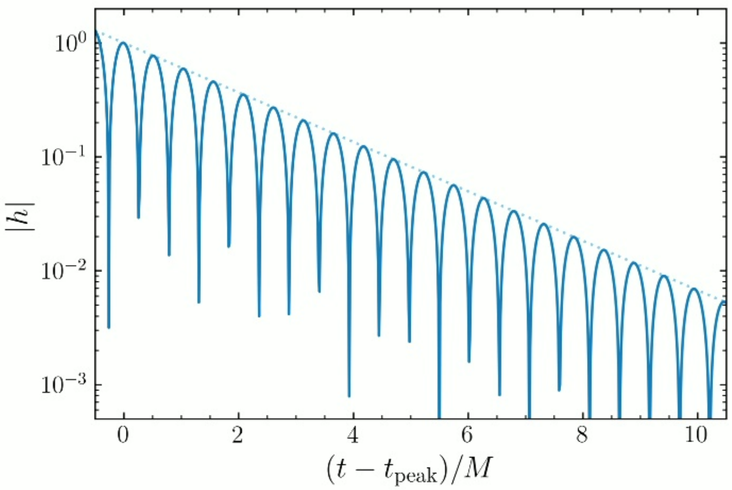


5

Quasinormal modes

$$\mathcal{T}[\Psi_4^{(1)}] = 0$$

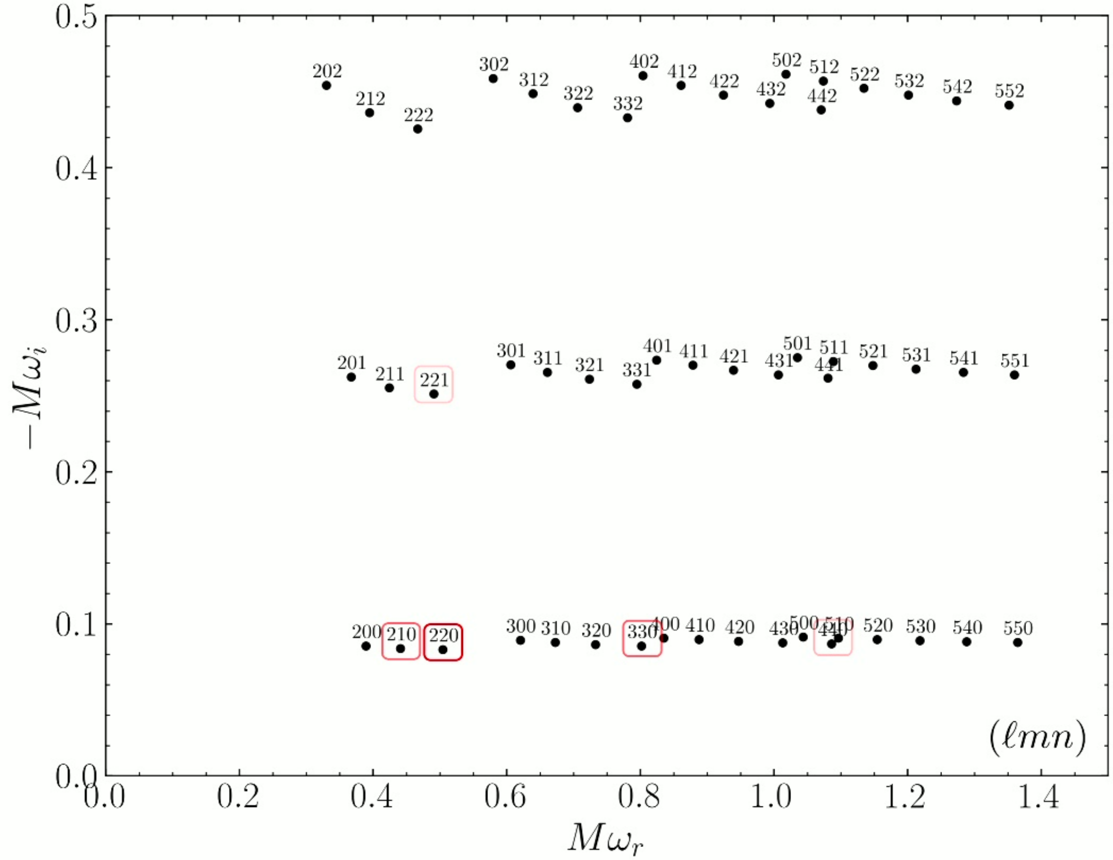
Boundary conditions ⇒ Quasinormal modes

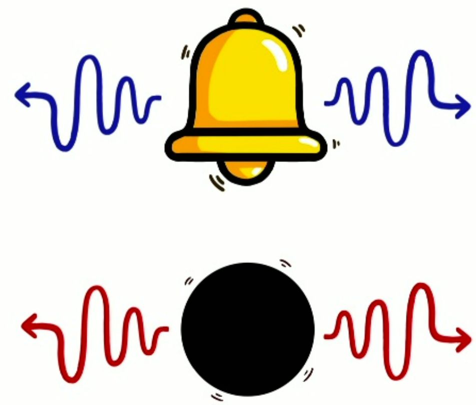


$$\sum_{\ell mn} A_{\ell mn} e^{\omega_{\ell mn,i} t} e^{-i(\omega_{\ell mn,r} t + \phi_{\ell mn})}$$

- ℓmn : mode number
- $\widetilde{\omega}_{\ell mn}$: (complex) quasinormal-mode frequencies

Quasinormal-mode frequencies





7

BH spectroscopy is relatively simple

- QNMs have a simple form $Ae^{-i(\omega t+\phi)}$
- QNM frequencies ω are
  - Only a function of remnant mass and spin $\omega \equiv \omega(M,\chi)$ (no hair theorem)
  - Independent of initial conditions
  - Relatively easy to calculate

BH spectroscopy is powerful

- Smoking gun evidence of black holes

> *"The observation of [the black hole's] resonant frequencies might finally provide direct evidence of black holes with the same certainty as, say, the 21 cm line identifies interstellar hydrogen."*
>
> \- Detweiler 1980

BH spectroscopy is powerful

- Smoking gun evidence of black holes
- Test General Relativity
- Test the no hair theorem $\omega \equiv \omega(M, \chi)$
- Probe the environment surrounding a black hole

$$\omega_{theory} \overset{?}{\leftrightarrow} \omega_{obs}$$

Requires detection of at least two modes

Status of detection

- A single mode detected confidently in most binary BH merger events

  Carullo+ (1902.07527), LVK (1903.04467, 2010.14529, 2112.06861)

- Multi-mode detection studied actively

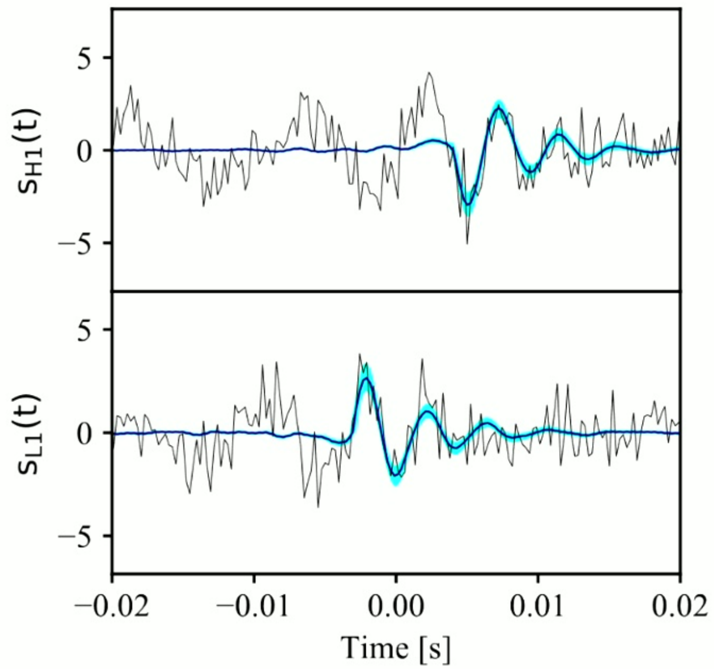
  - GW150914: 220 + 221

    Isi+ (1905.00869), Finch+ (2205.07809), Ma+ (2301.06639),

    Crisostomi+ (2305.18528), Cotesta+ (2201.00822), Isi+ (2202.02941)

  - GW190521: 220 + 330, or 220 + 210 + 320

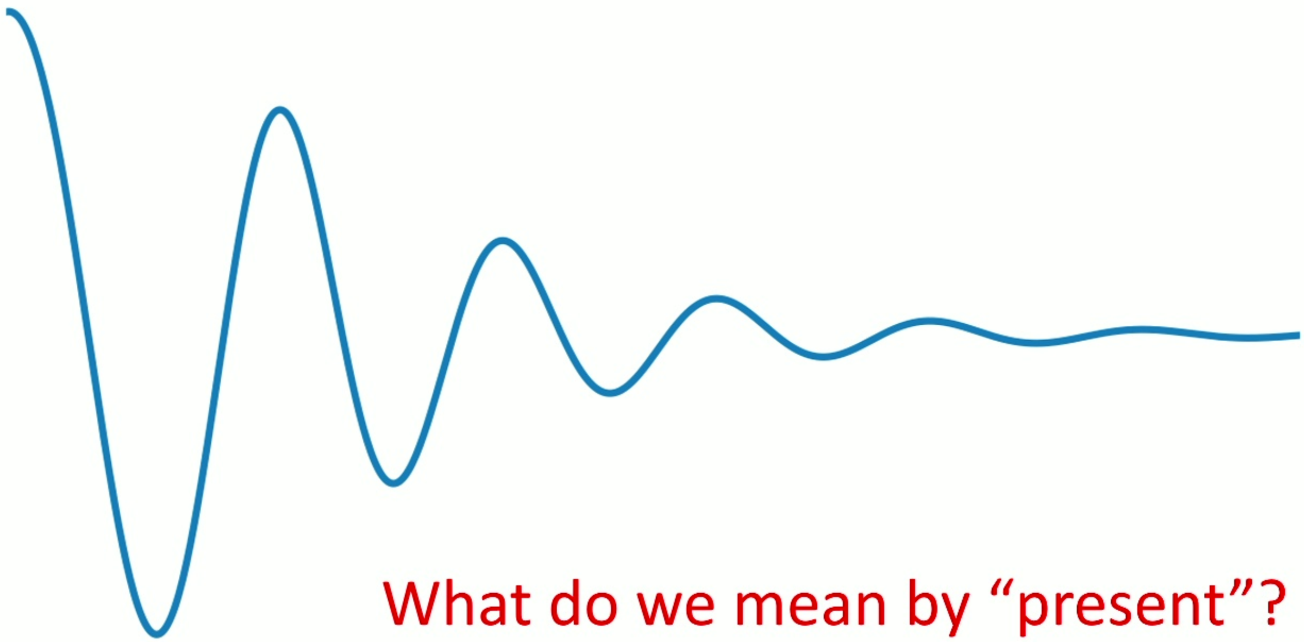
    Capano+ (2105.05238), Siegel+ (2307.11975)



Carullo+ (1902.07527)

Identification of Ringdown Modes

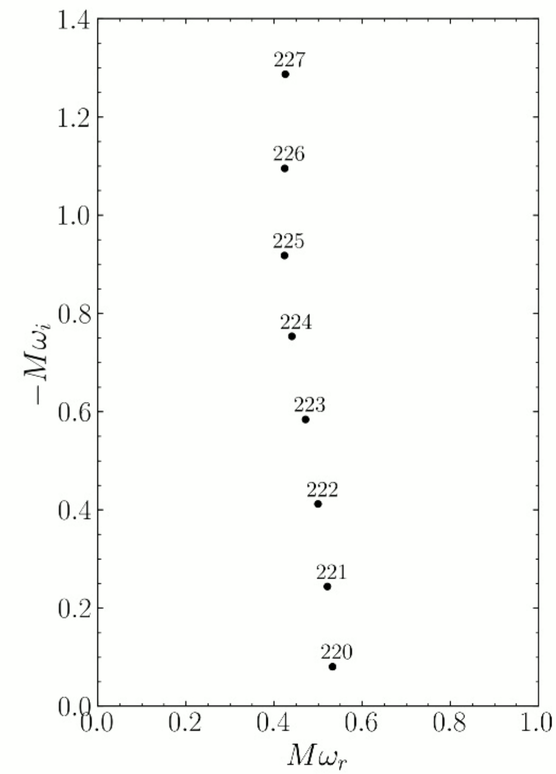
Which modes are "present" in the ringdown?



What do we mean by "present"?

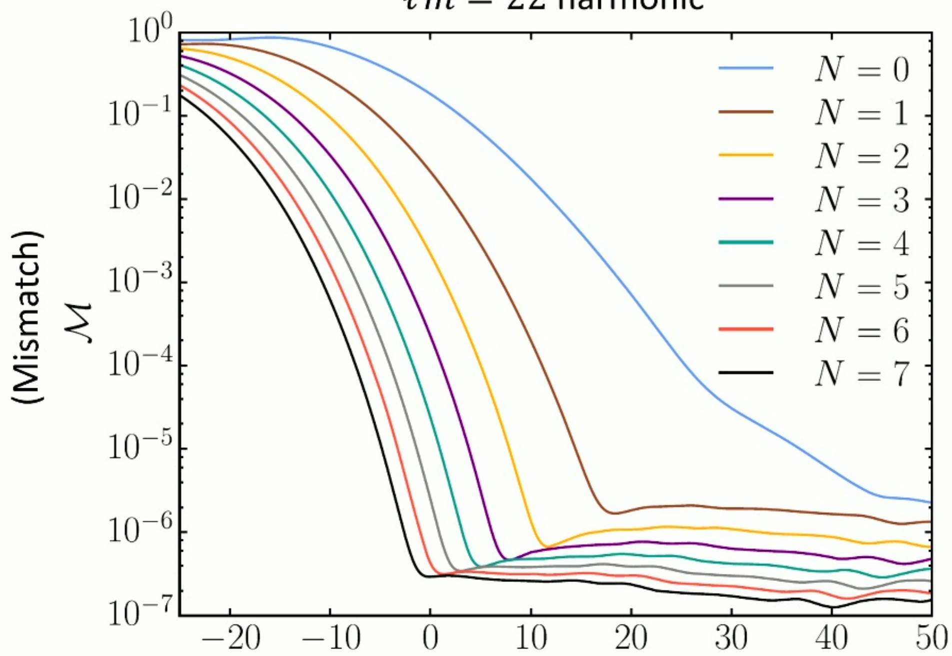
"Present" = "the modes fit the ringdown well"?

- Some modes are "expected to be there"
  - E.g. the overtones
- See if these modes fit the ringdown waveform well



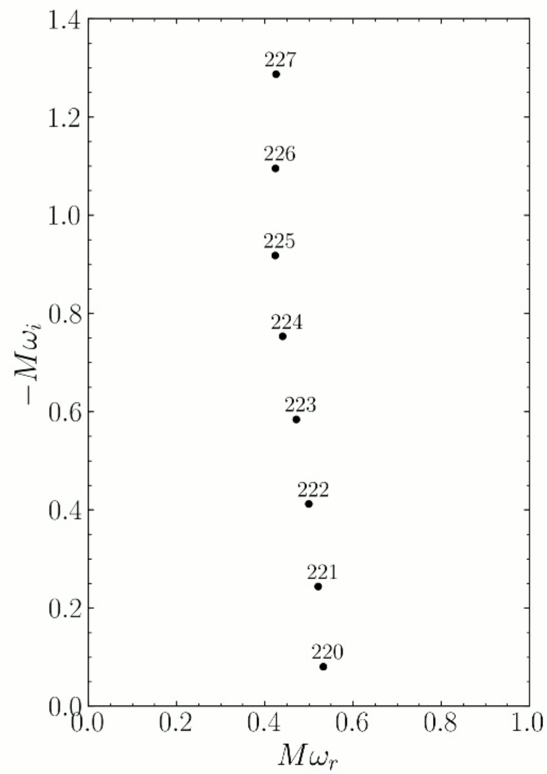
“Present” = “the modes fit the ringdown well”?

SXS:BBH:0305, a GW150914-like NR waveform
$\ell m = 22$ harmonic



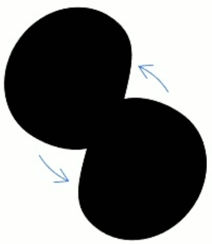
Giesler+ (1903.08284)

(Starting time of fit)



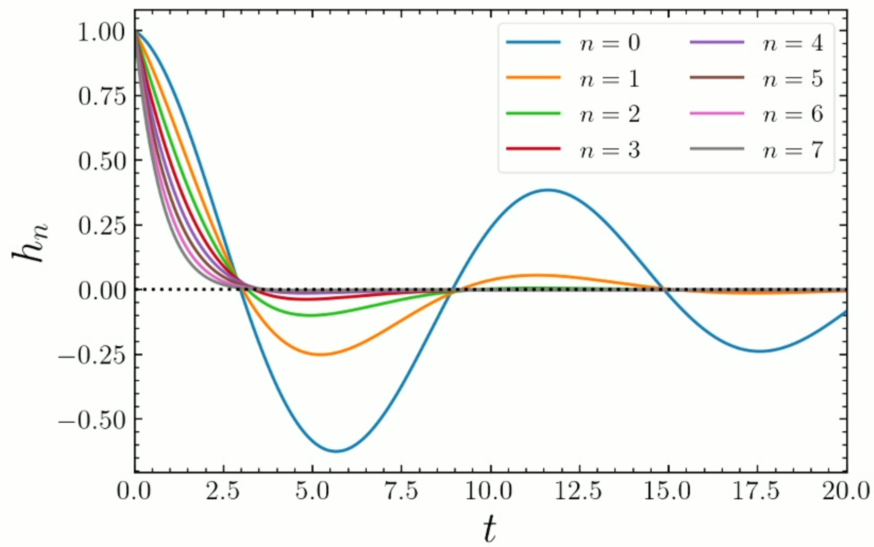
Potential issues

- Physics:
  - Nonlinearities are expected
  - The background spacetime is evolving
- Fitting:
  - More overtones, higher risk of overfitting



Potential issues

- Physics:
  - Nonlinearities are expected
  - The background spacetime is evolving
- Fitting:
  - More overtones, higher risk of overfitting
  - Overtones decay rapidly, only the early part matters in a fit



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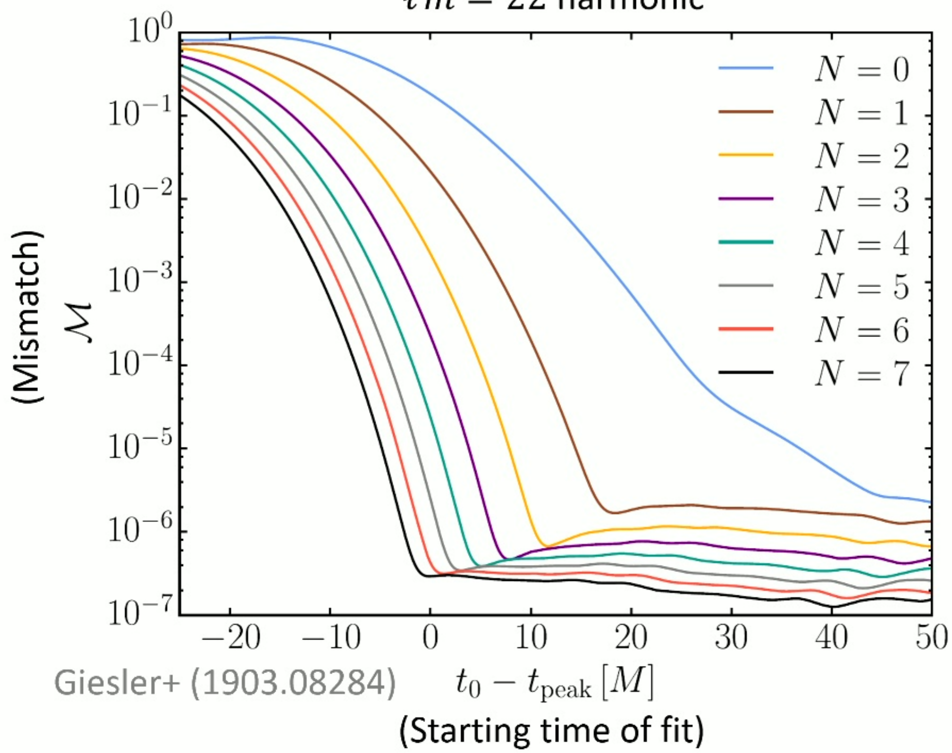

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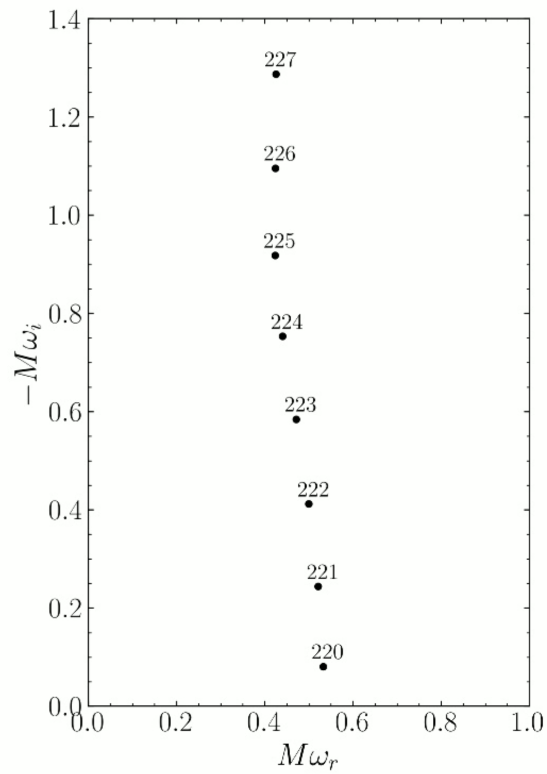
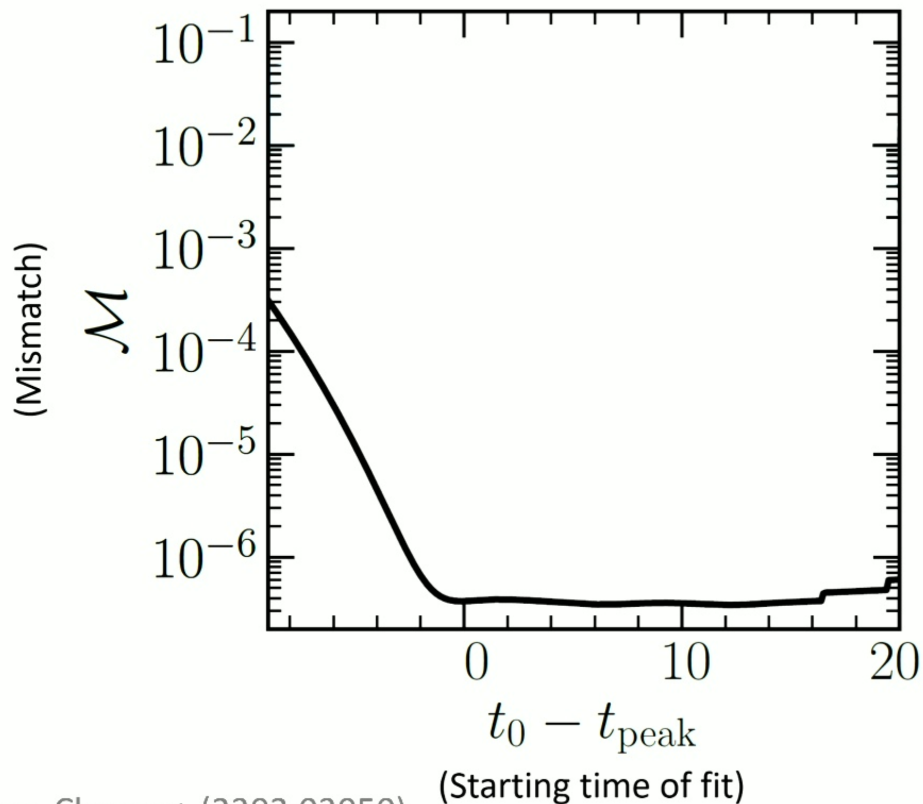
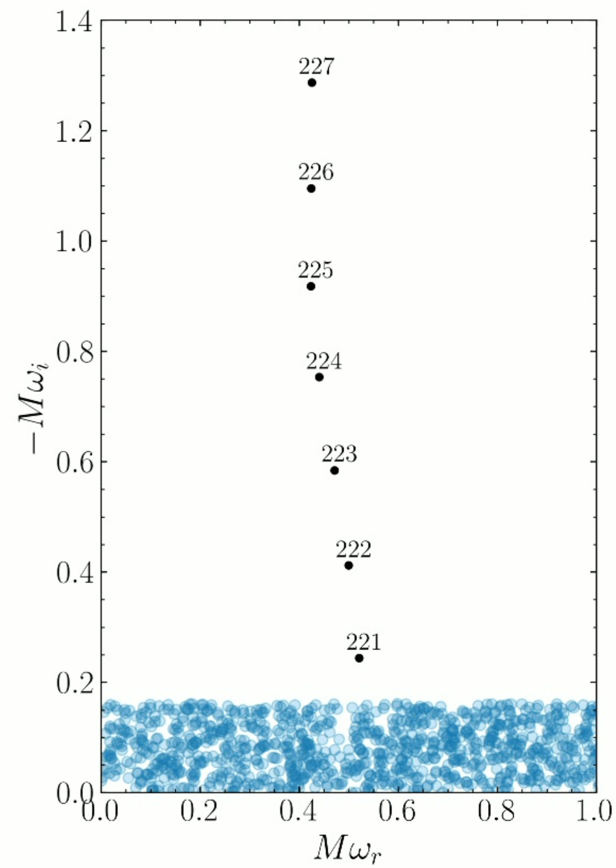


Can any random QNM fit it well?





Baibhav, Cheung+ (2302.03050)

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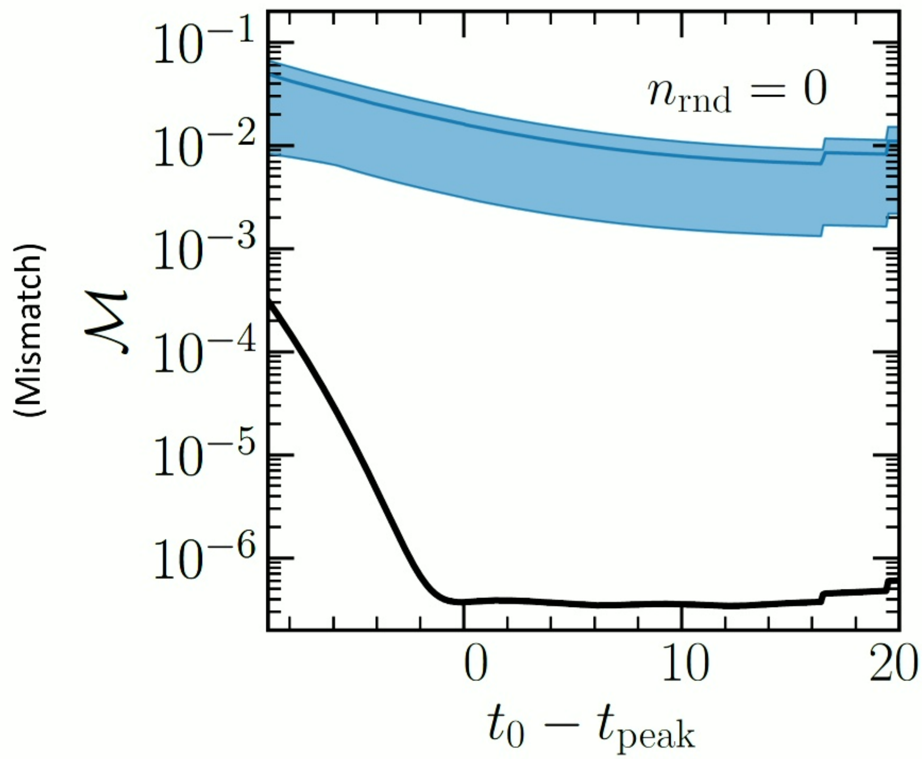
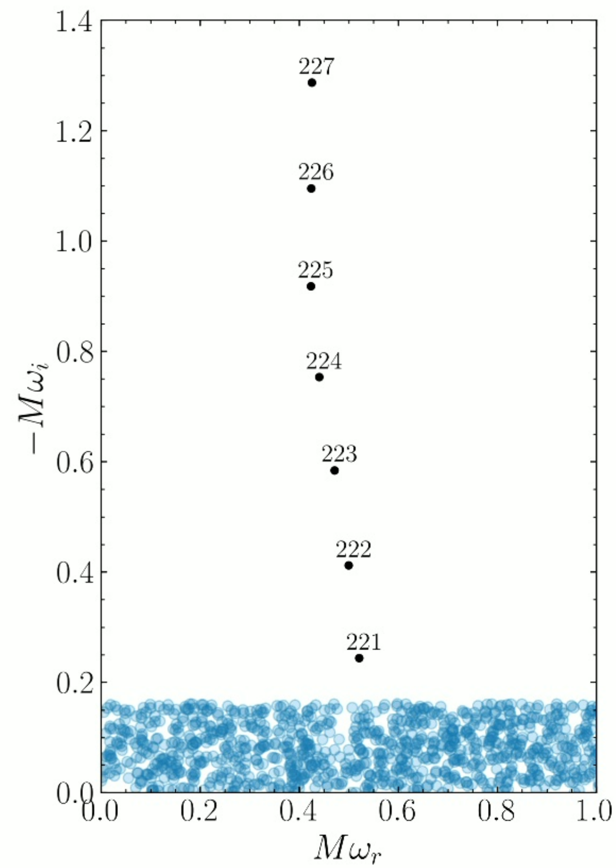


(Starting time of fit)



Baibhav, Cheung+ (2302.03050)

15

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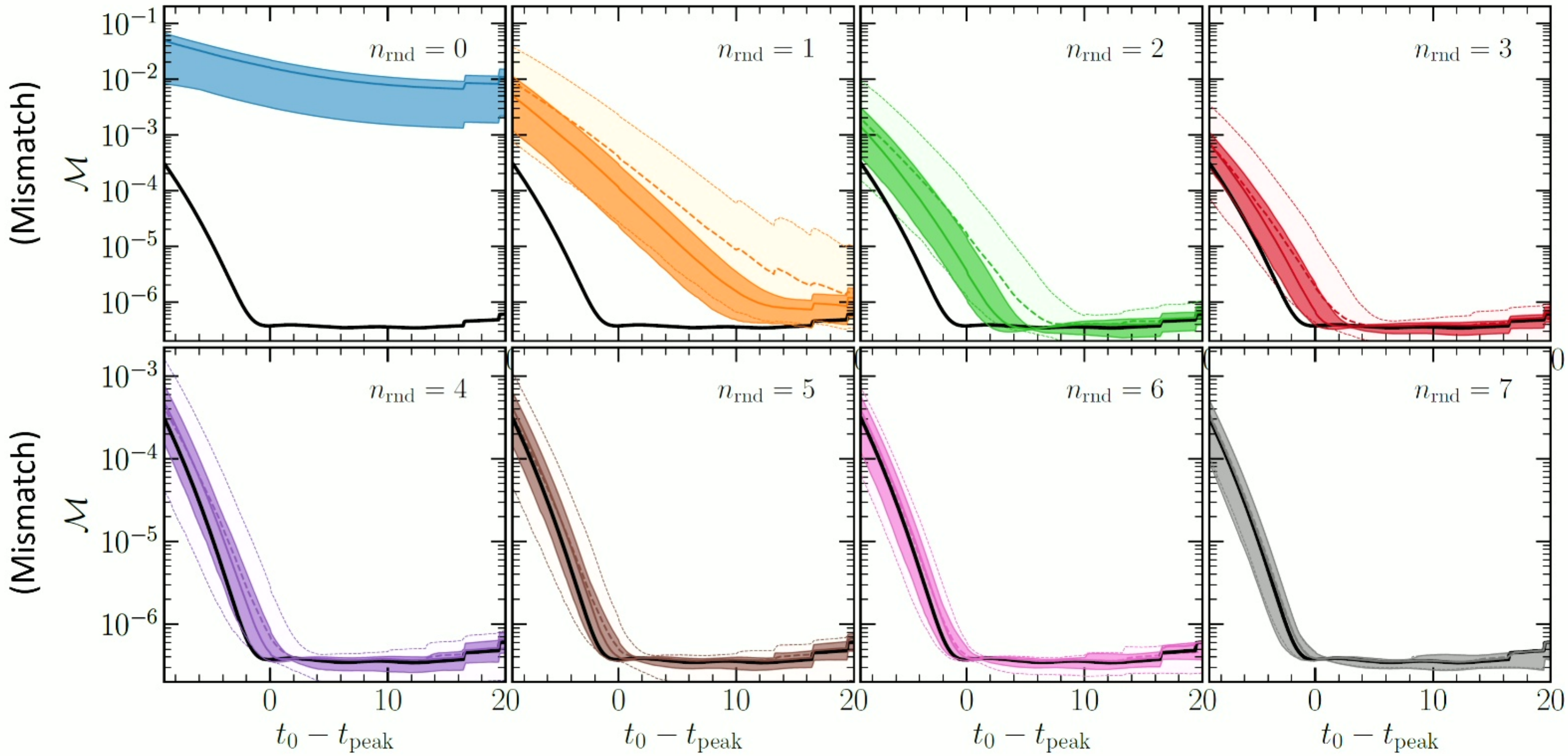


Baibhav, Cheung+ (2302.03050)

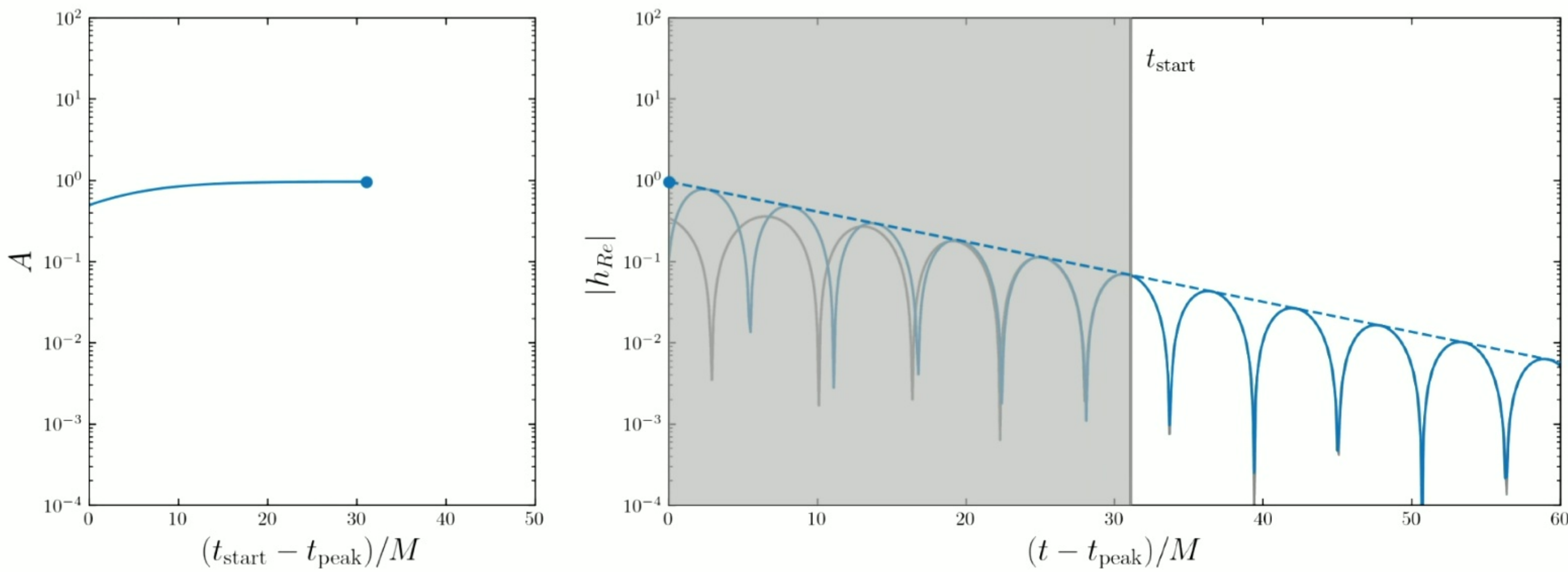
Is the fitted mode amplitude constant?

$$\boxed{A}e^{-i(\omega t+\phi)}$$

constant in time

Baibhav, Cheung+ (2302.03050)

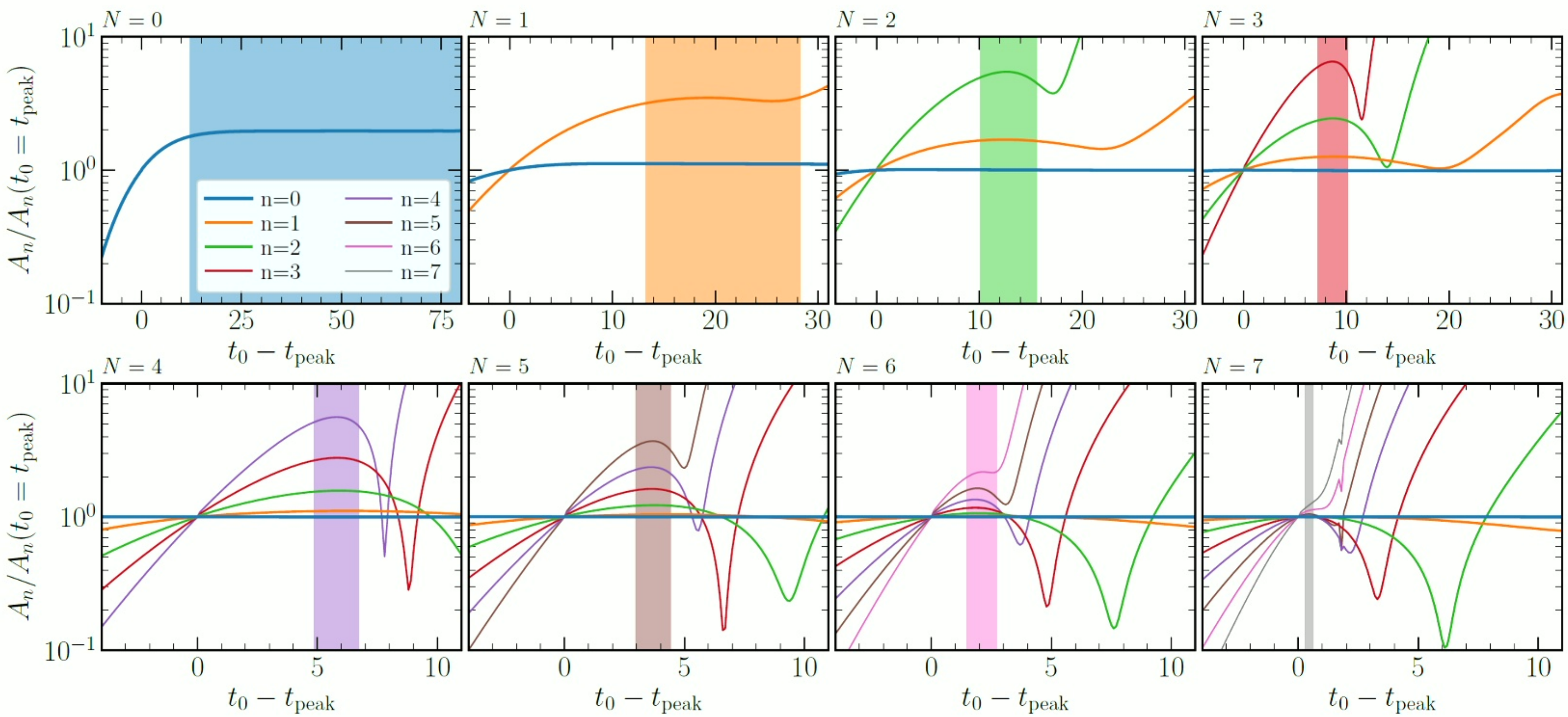
Is the fitted mode amplitude constant?



Baibhav, Cheung+ (2302.03050)

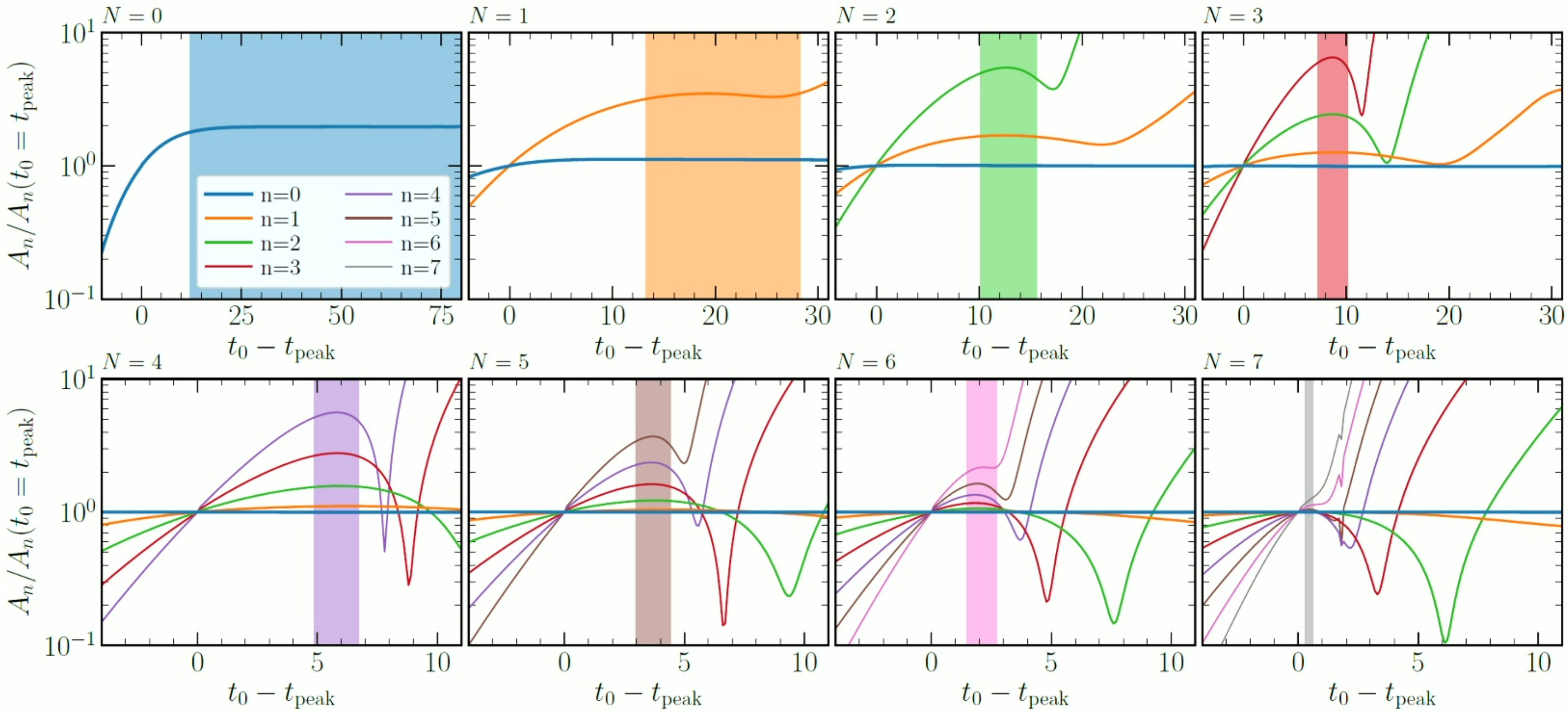
18

Is the fitted mode amplitude constant?



Baibhav, Cheung+ (2302.03050)

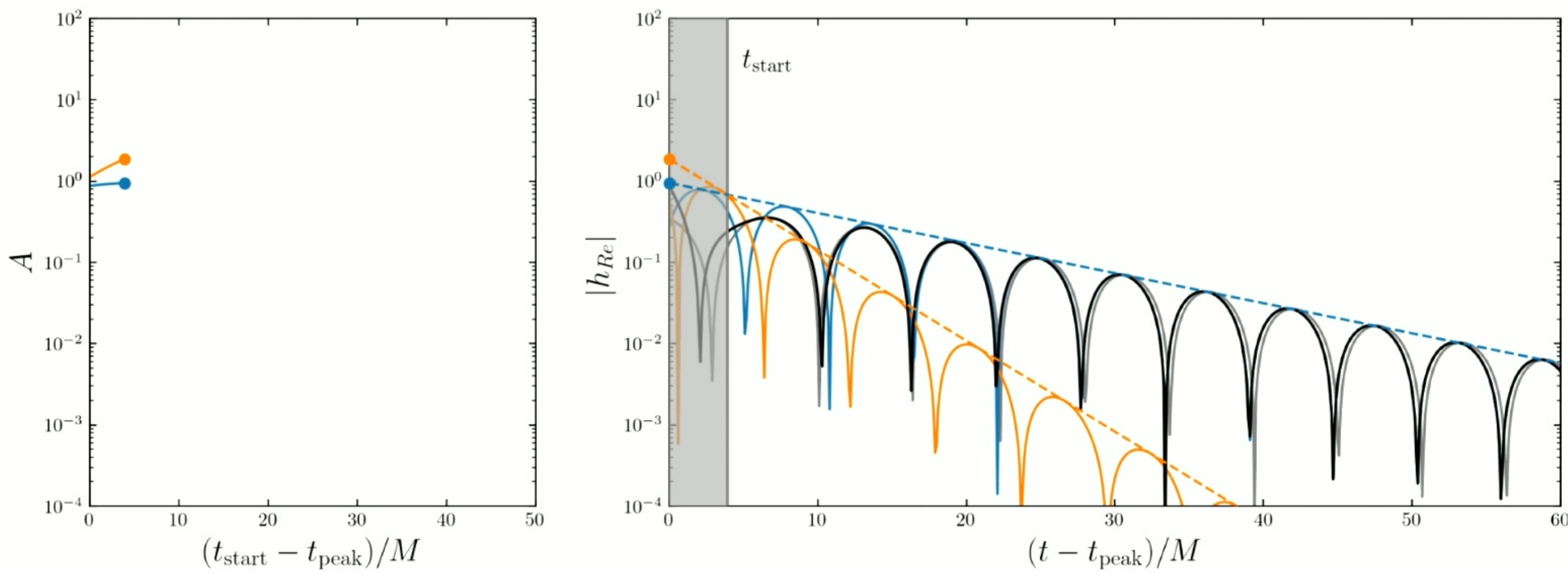
Is the fitted mode amplitude constant?



Baibhav, Cheung+ (2302.03050)

$n \geq 2$: Not really

Is the fitted mode amplitude constant?



Baibhav, Cheung+ (2302.03050)

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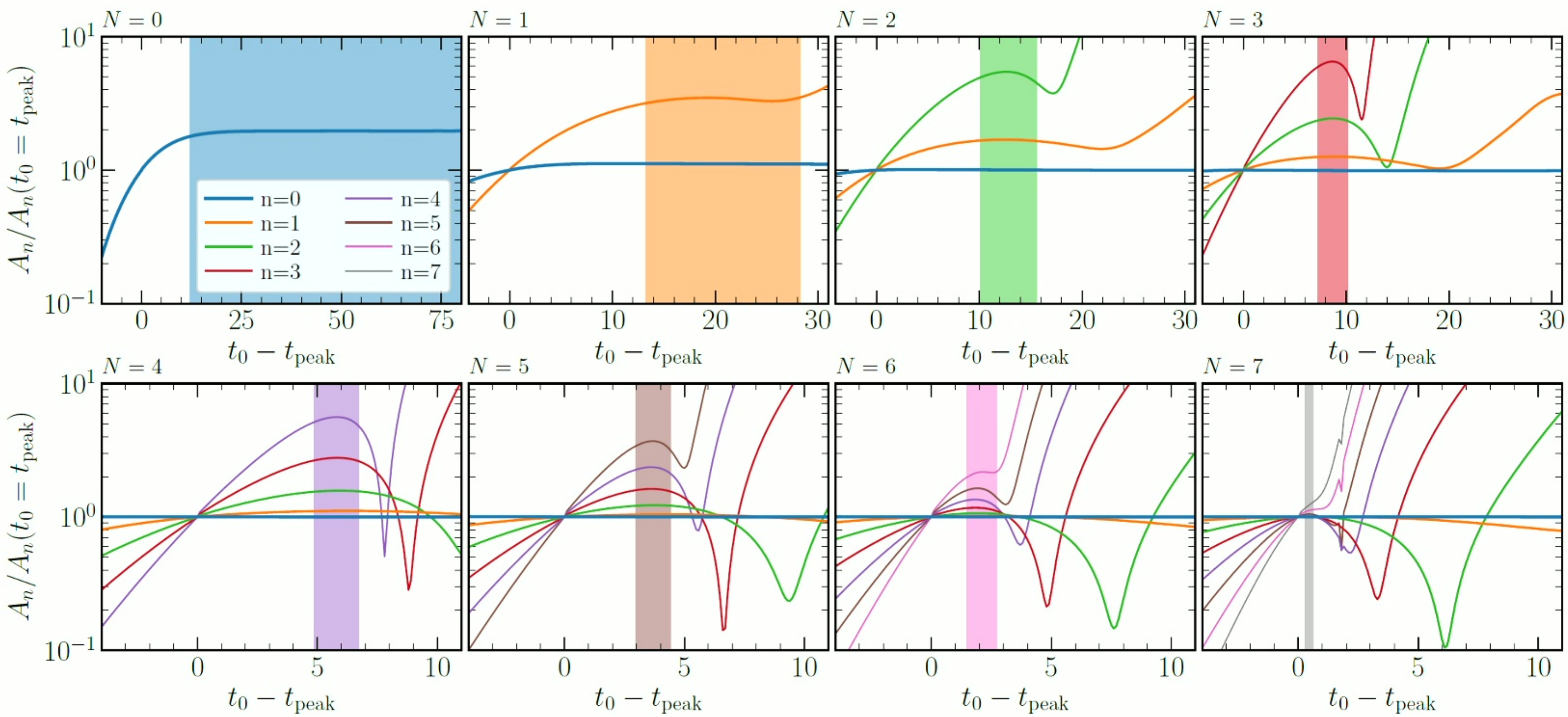
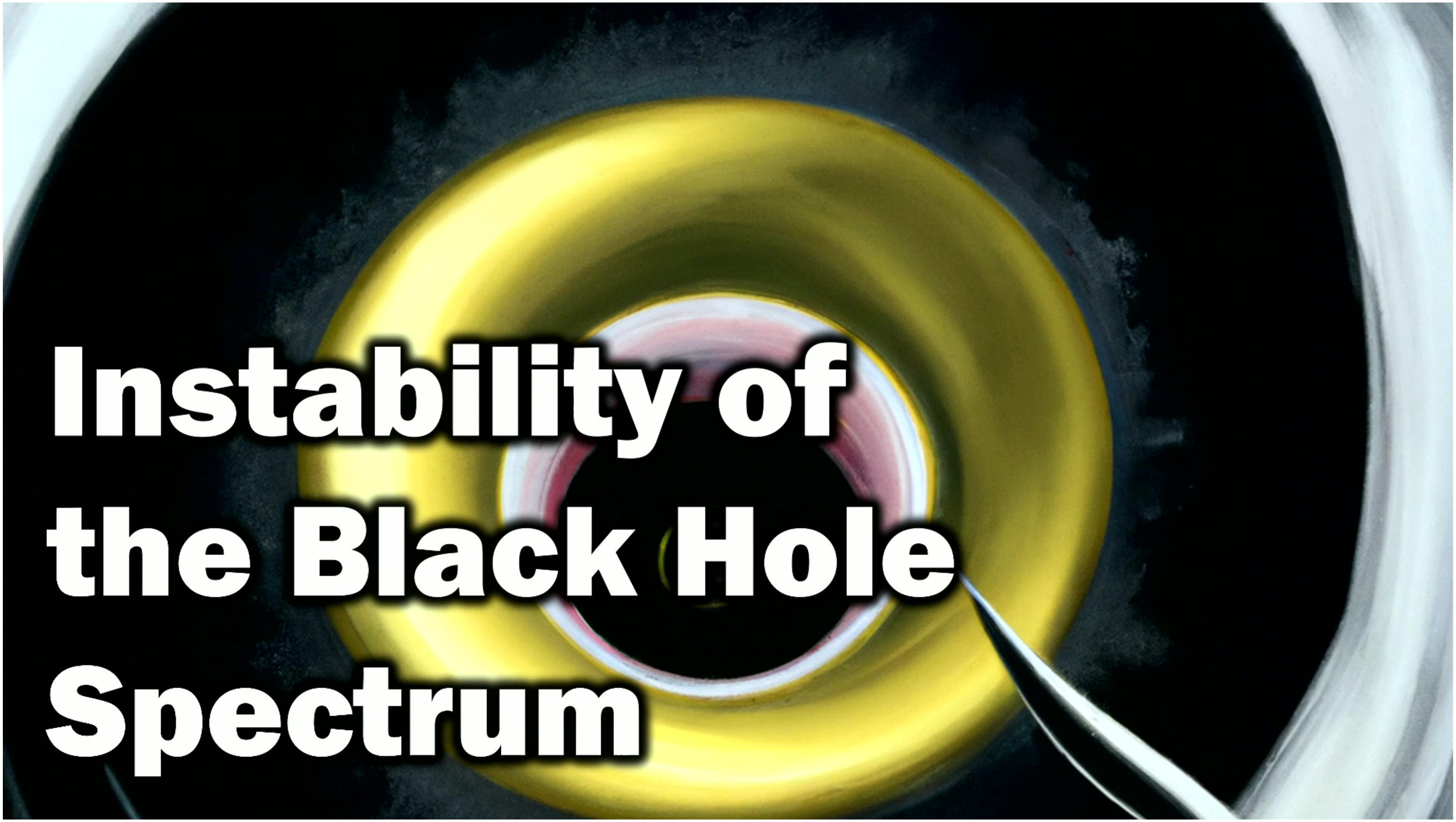


$n \geq 2$: Not really

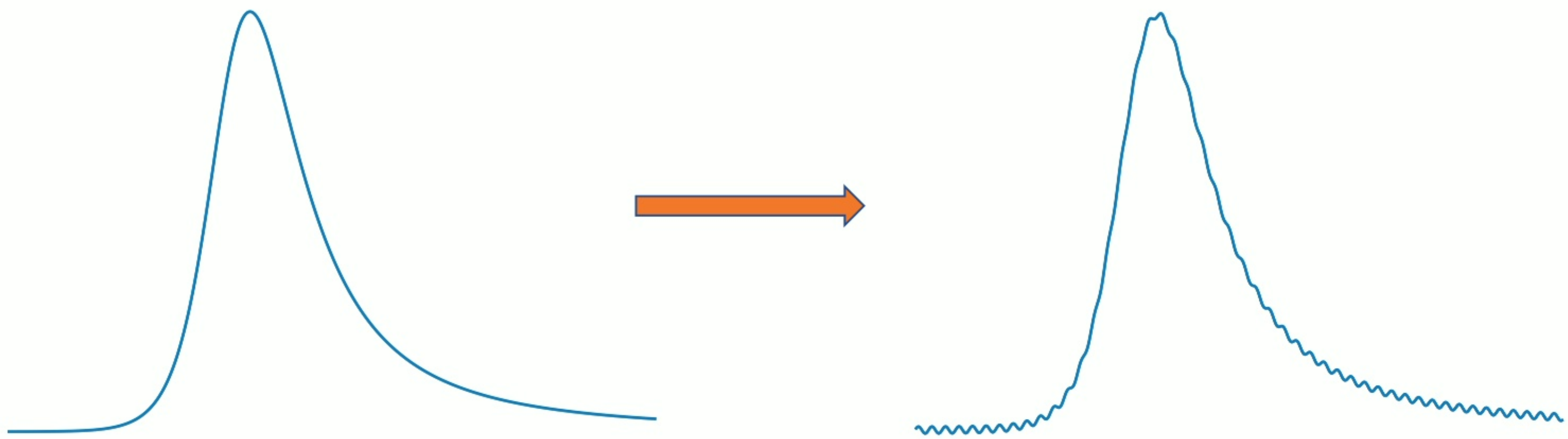
Baibhav, Cheung+ (2302.03050)

Some lessons

- The "many overtones" model fits well, but might not be physical
  - The overtones could be overfitting the merger
- More stringent criteria might be required to test whether a mode is "physically present"
  - Check whether the frequency of a mode is the best-fit frequency
  - Check whether the amplitudes (and phases) are constant
- Questions
  - "We cannot see the overtones" or "the overtones are not there"?
  - Is the ringdown fully linear?


Instability of
the Black Hole
Spectrum

What if we perturb the potential?

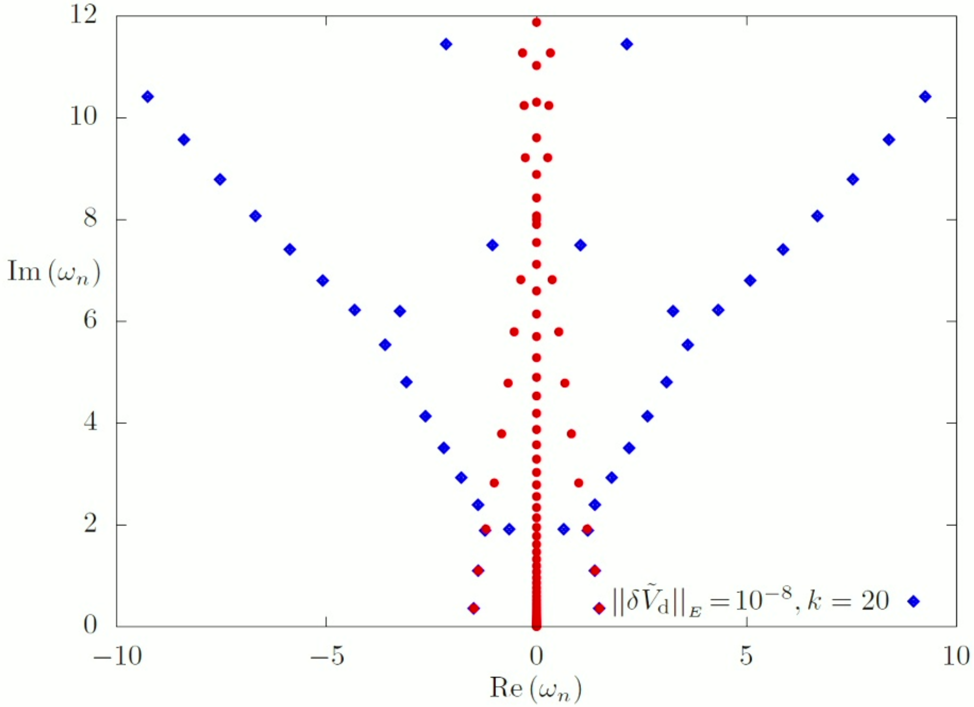


$V(r_*)$ $\qquad\qquad$ $V(r_*) + \epsilon \sin(k r_*)$

Spectral instability

With respect to an "energy norm"

See also: Yang+ (2210.01724)

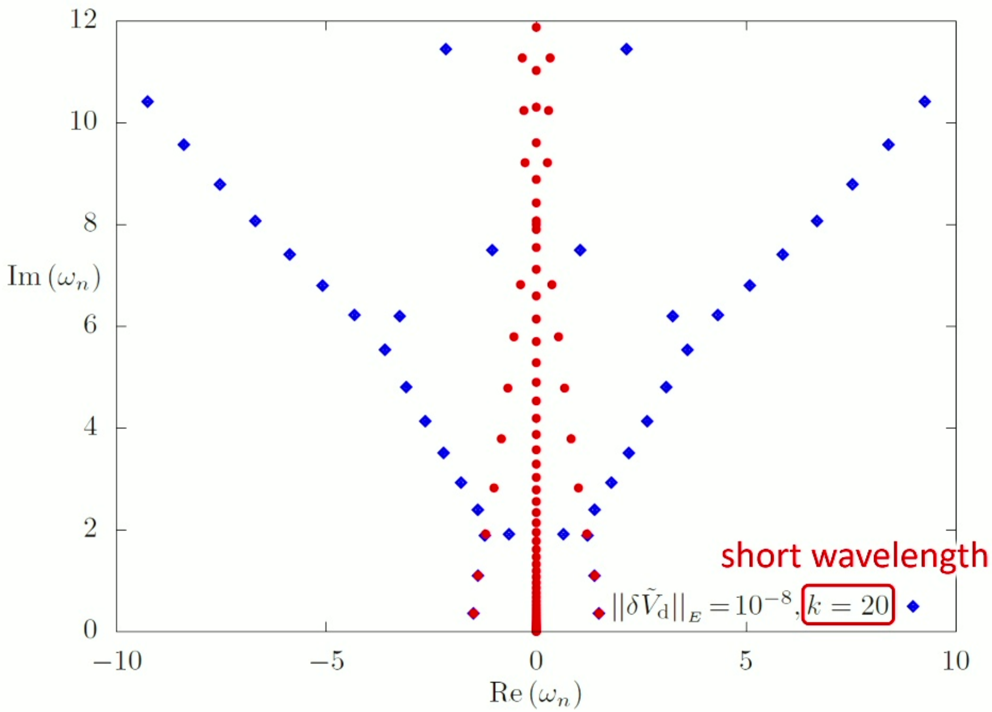


Jaramillo+ (2004.06434)

Spectral instability

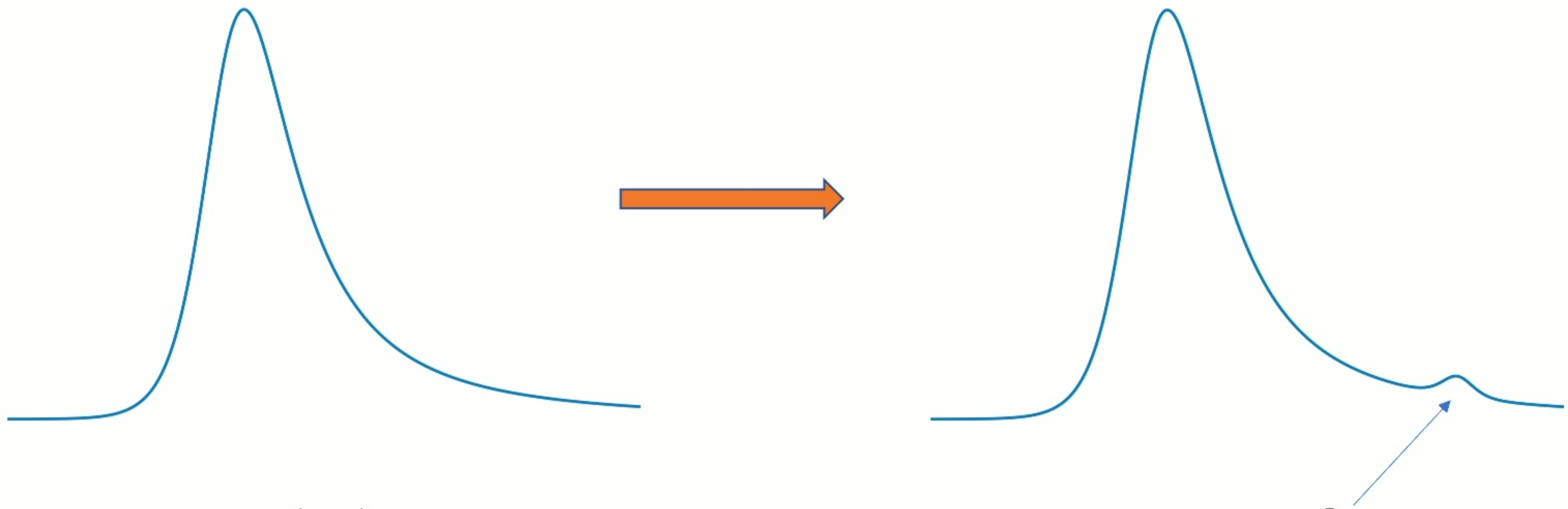
With respect to an "energy norm"

See also: Yang+ (2210.01724)



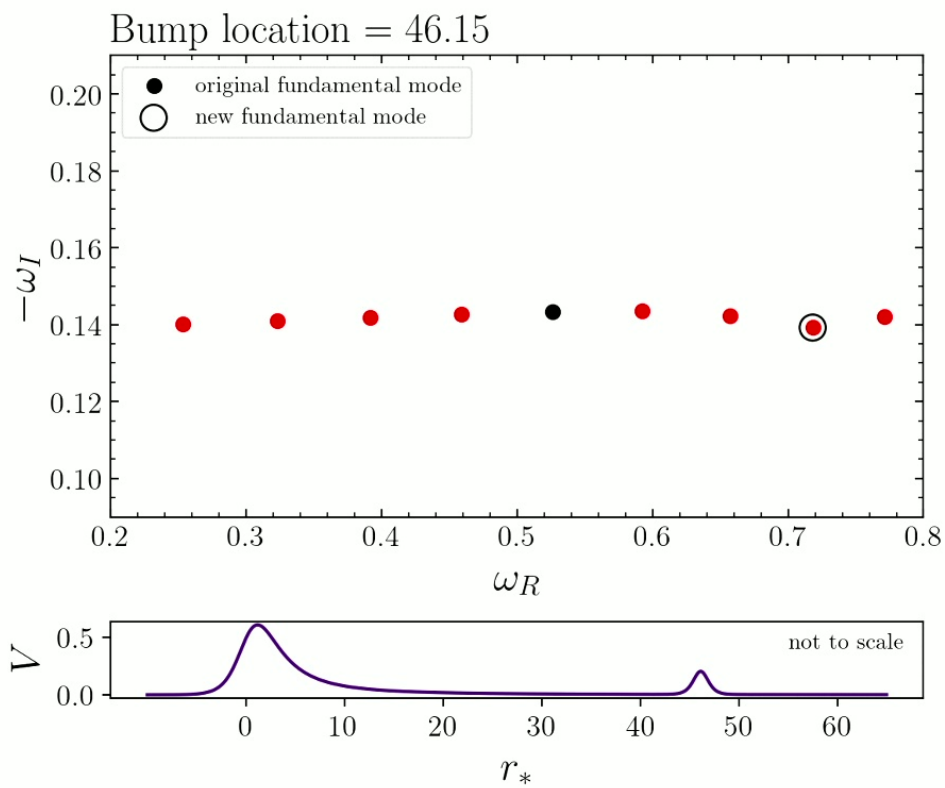
Jaramillo+ (2004.06434)

What if we add a small bump?



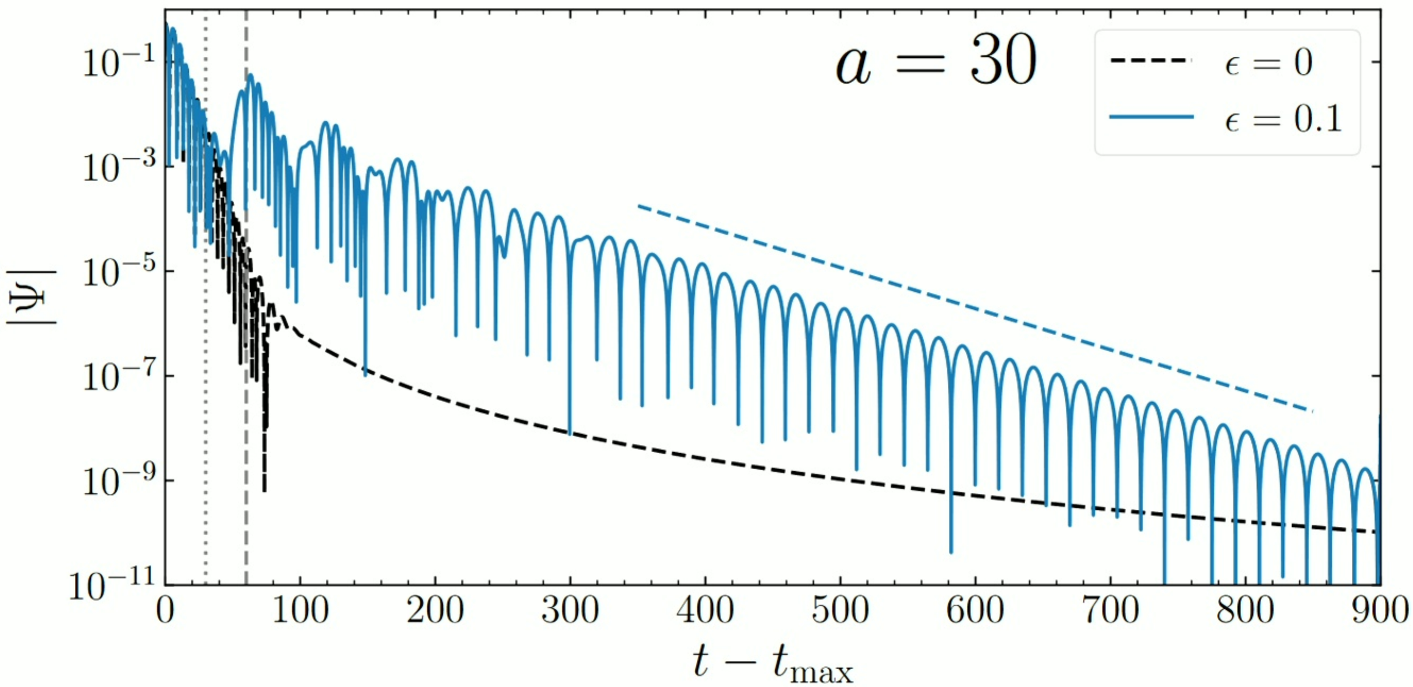
$$V(r_*) \qquad \rightarrow \qquad V(r_*) + \epsilon\,\mathrm{sech}^2(r_* - a)$$

The fundamental mode is also unstable



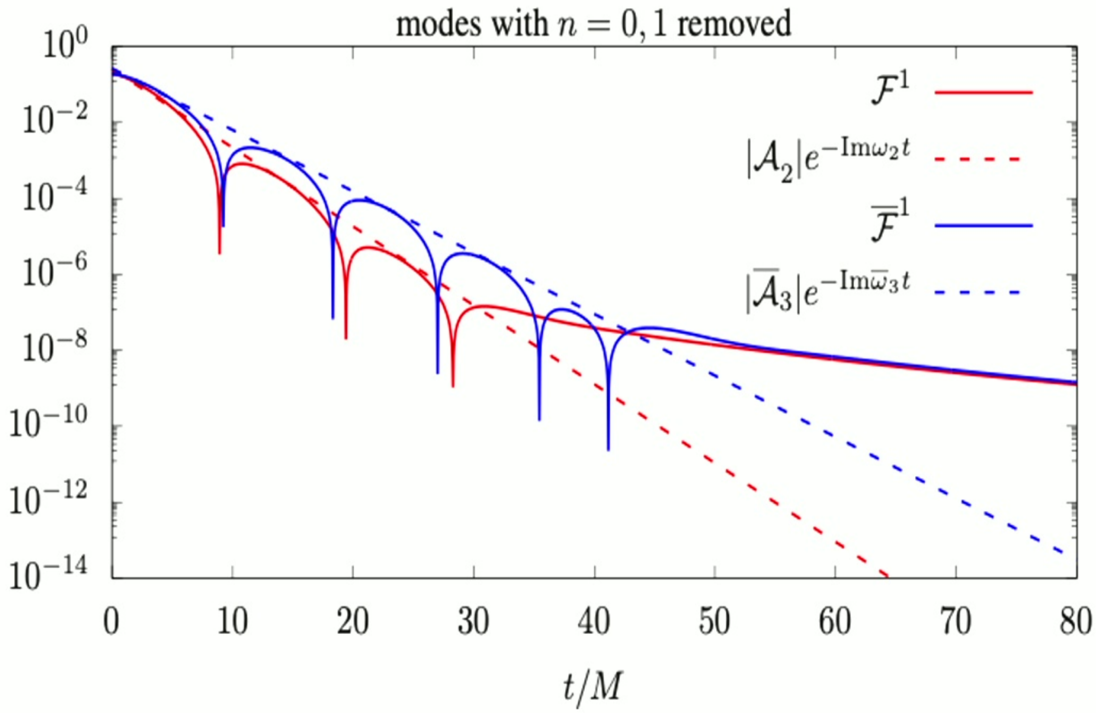
Cheung+ (2111.05415)

Instability in the time domain



Berti, Cardoso, Cheung+ (2205.08547)

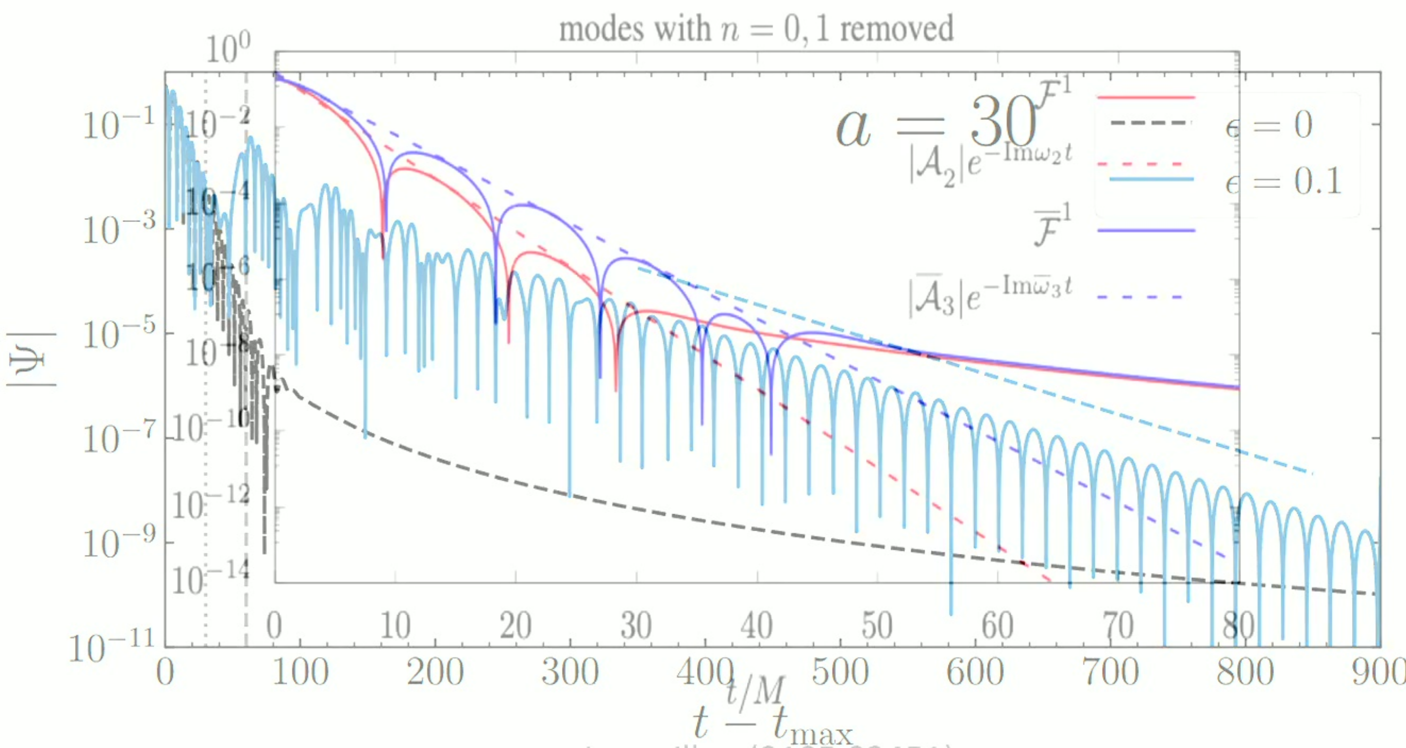
Overtone instability



Jaramillo+ (2105.03451)

Overtone instability

Instability in the time domain



Jaramillo+ (2105.03451)
Berti, Cardoso, Cheung+ (2205.08547)

Why do we care?

- QNM frequencies might be drastically different from the “clean” ones
- Importance of time domain verification
- Motivating agnostic mode search procedures

Related work

- Nonlinear QNMs found in simulations
  - Tentative evidence: London+ (1404.3197)
  - Explicitly: Ma+ (2207.10870), Mitman+ (2208.07380)
- Other nonlinearities
  - Turbulent black holes: Yang+ (1402.4859)
  - Absorption-induced excitation: Sberna+ (2112.11168)
  - Gravitational-wave memory: Zertuche+ (2110.15922), Mitman+ (2007.11562)

Black Hole Perturbation theory (2nd order)

Same Teukolsky operator

$$\mathcal{T}[\Psi_4^{(2)}] = \mathcal{S}_4^{(2)} \propto \ (A^{(1)})^2 e^{-i(2\omega^{(1)}t + 2\phi^{(1)})}$$

$$\Psi_4^{(2)} = \underbrace{\Psi_{4,\mathrm{homogeneous}}^{(2)}}_{\sim \Psi_4^{(1)}} + \underbrace{\Psi_{4,\mathrm{particular}}^{(2)}}_{A^{(2)} e^{-i(\omega^{(2)}t + \phi^{(2)})}}$$

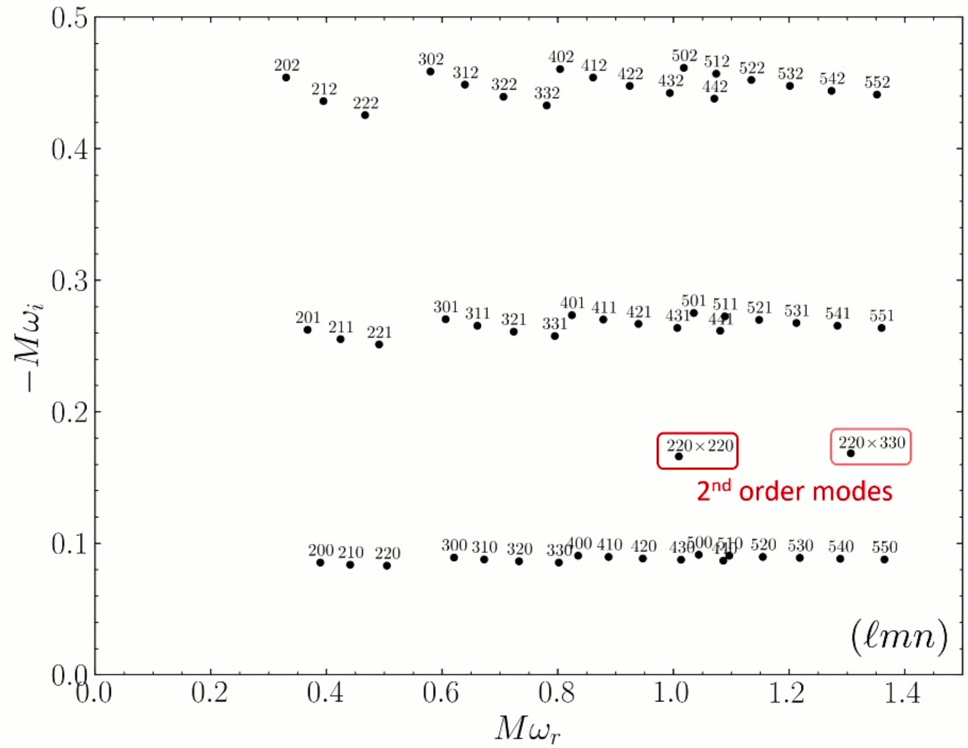
general

2nd order quasinormal mode

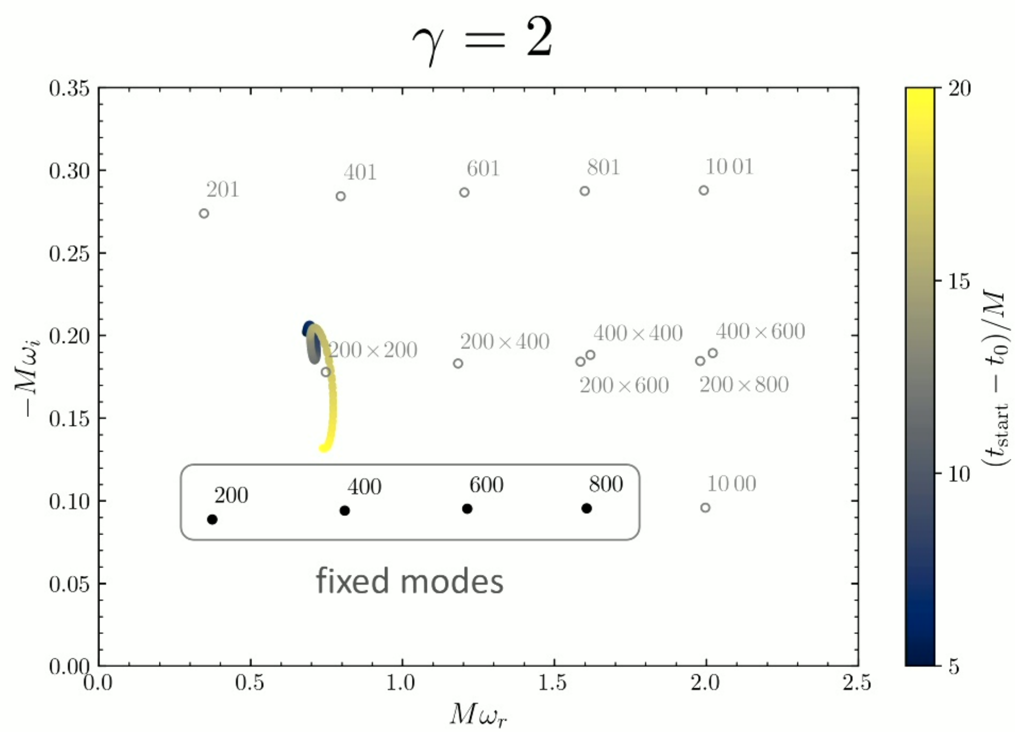
$$\omega_{i\times j}^{(2)} = \omega_i^{(1)} + \omega_j^{(1)}$$
$$A_{i\times j}^{(2)} \propto A_i^{(1)} A_j^{(1)}$$
$$\phi_{i\times j}^{(2)} = \phi_i^{(1)} + \phi_j^{(1)} + \mathrm{constant}$$

Ioka+ (0704.3467), Nakano+ (0708.0450), Pazos+ (1009.4665), London+ (1404.3197)

Quasinormal-mode frequencies



Mode search (head-on mergers)

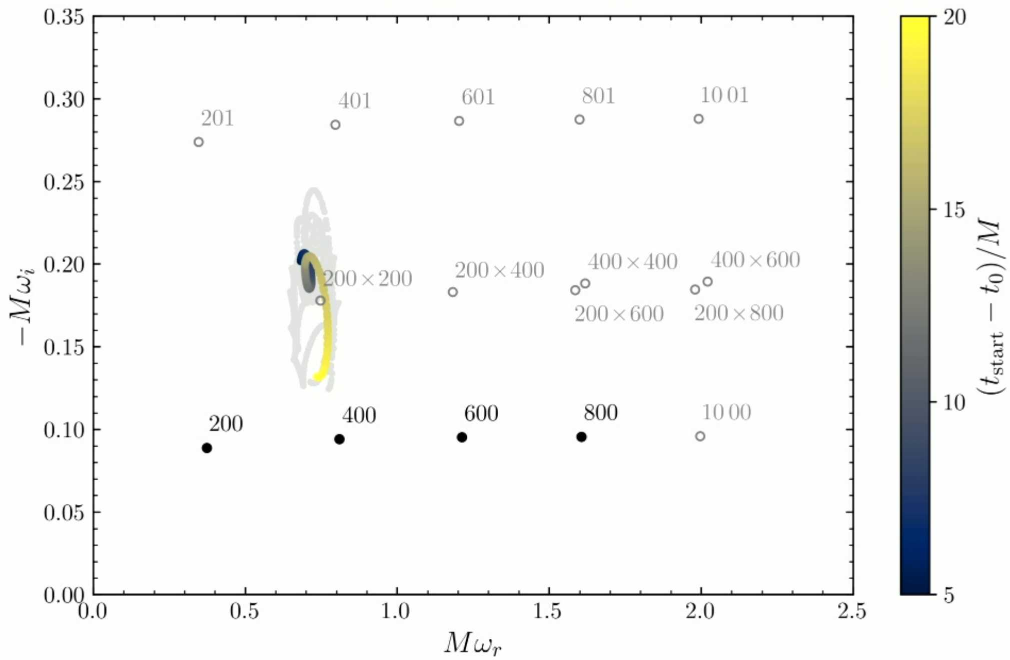


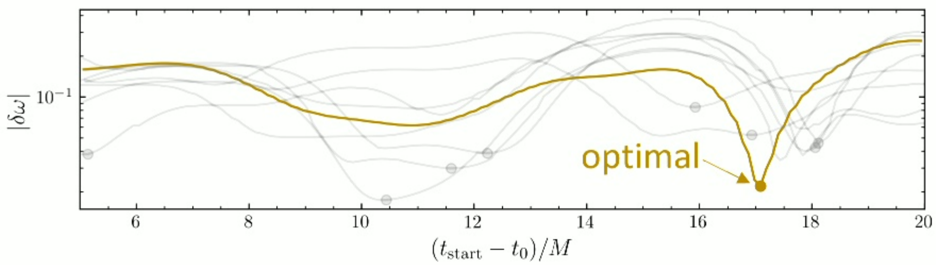
Fit model: $\sum_{\ell mn} \boxed{A_{\ell mn}} e^{\omega_{\ell mn,i} t} e^{-i(\omega_{\ell mn,r} t + \boxed{\phi_{\ell mn}})}$

Cheung+ (2208.07374) 36

Mode search (head-on mergers)

Highlighted: $\gamma = 2$





Fit model: $\sum_{\ell mn} A_{\ell mn} e^{\omega_{\ell mn,i} t} e^{-i(\omega_{\ell mn,r} t + \phi_{\ell mn})}$

Cheung+ (2208.07374)

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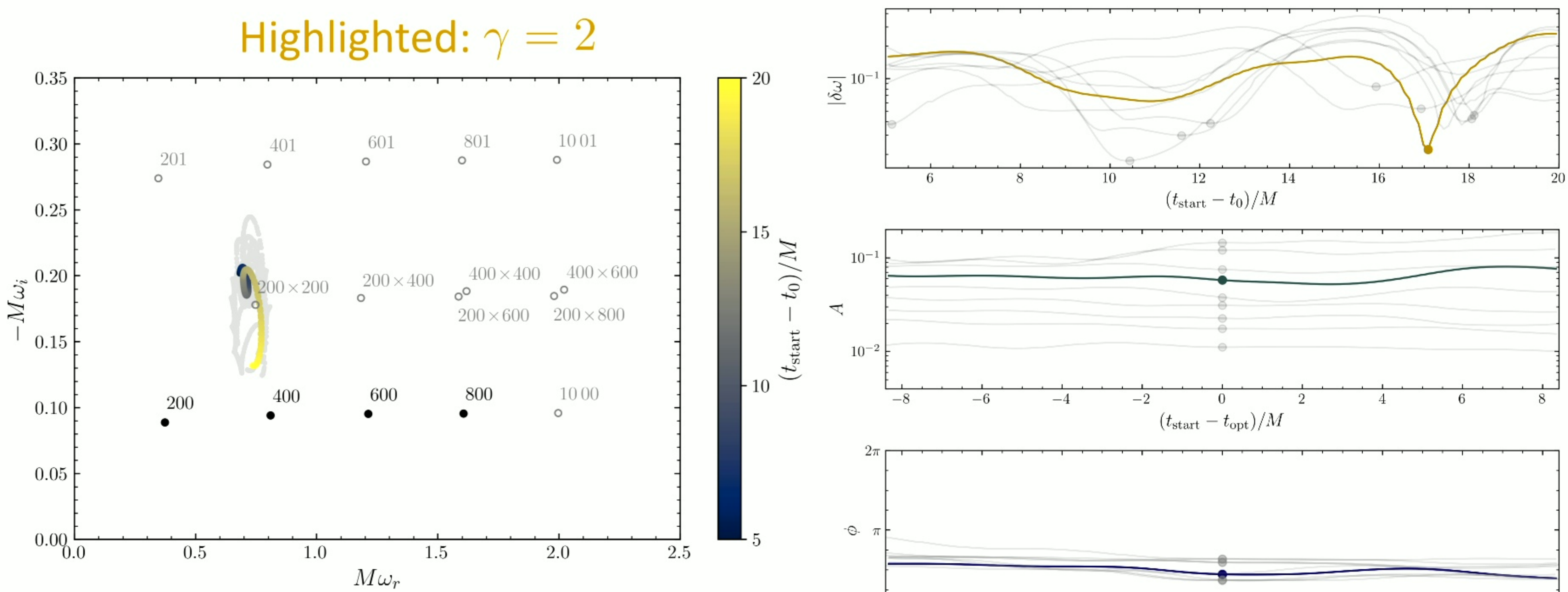

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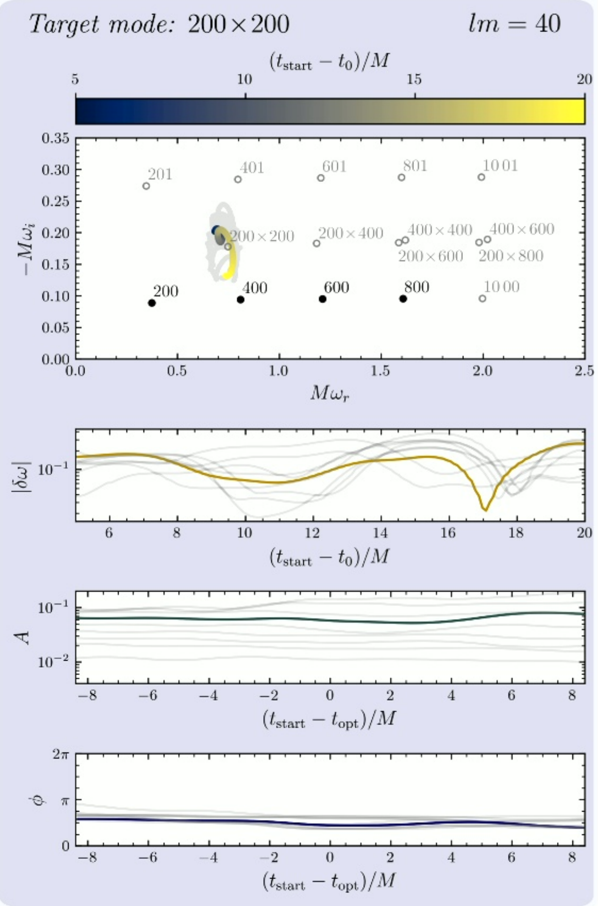
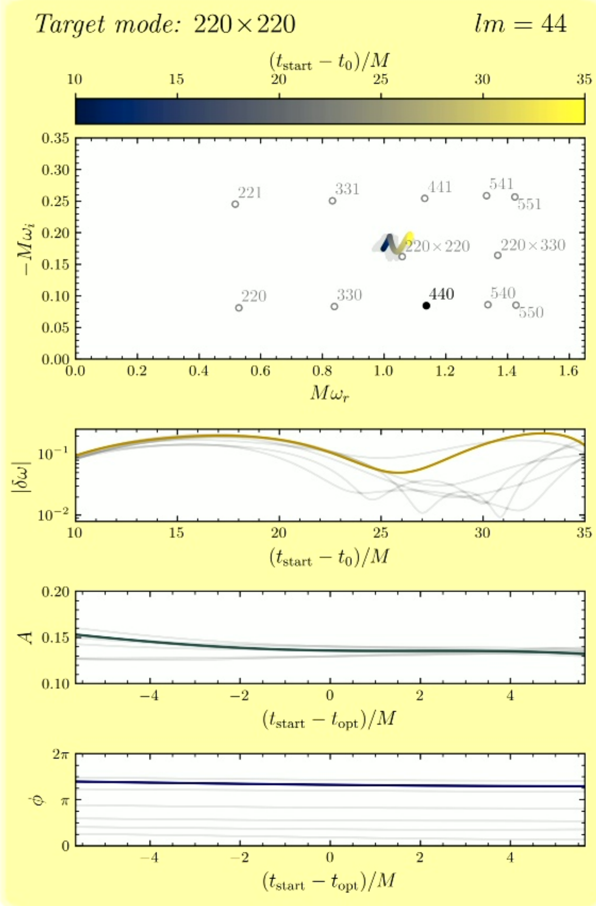
Cheung+ (2208.07374) 36

Head-on mergers

Quasicircular mergers

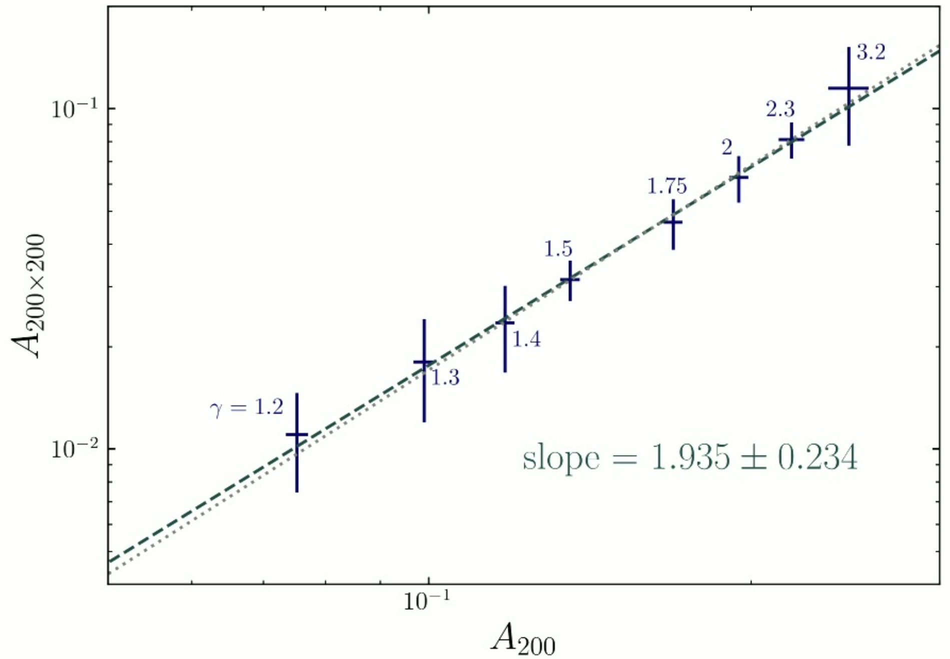




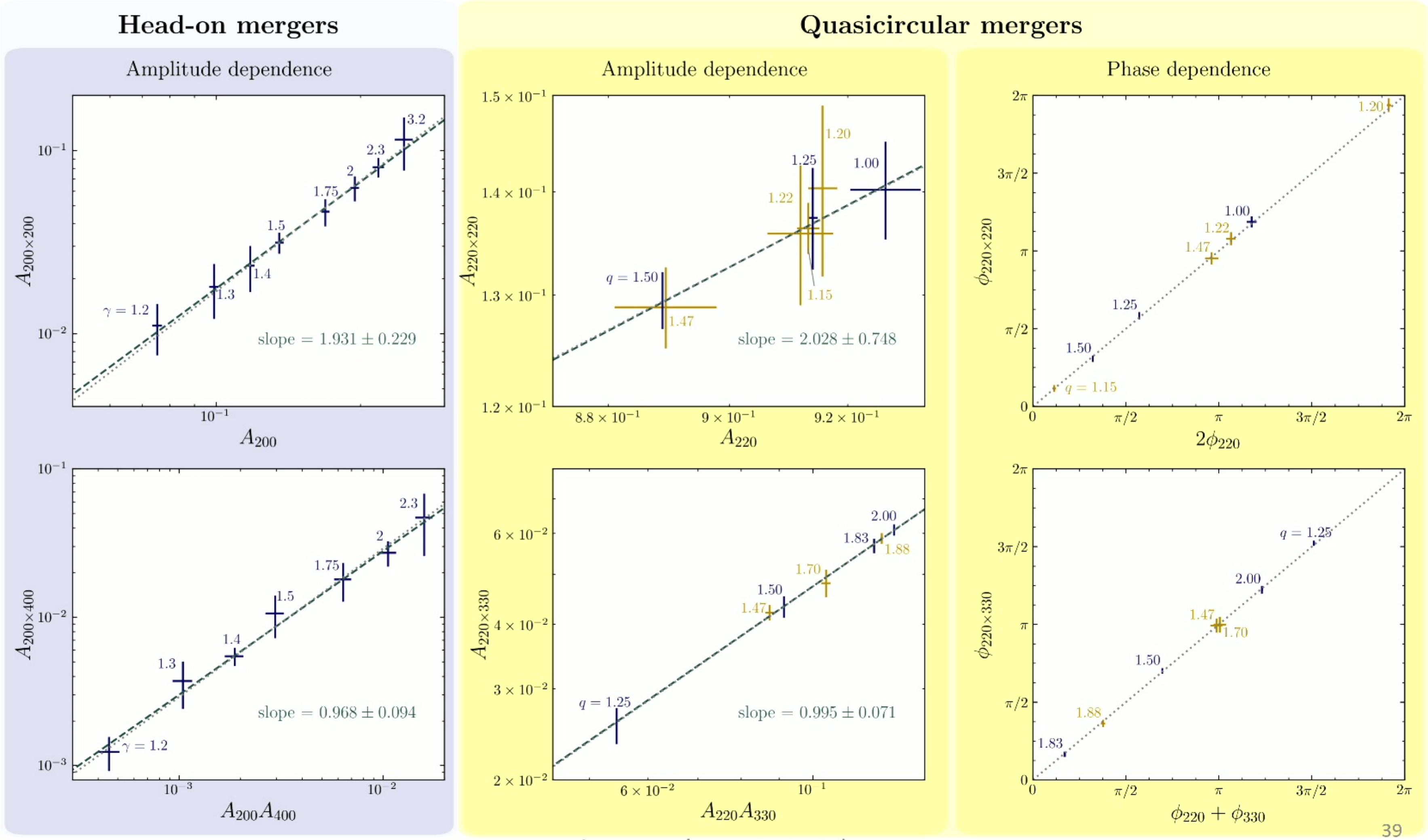
Cheung+ (2208.07374)

Amplitude dependence (head-on mergers)

$$A_{i\times j} \propto A_i A_j$$



Cheung+ (2208.07374)

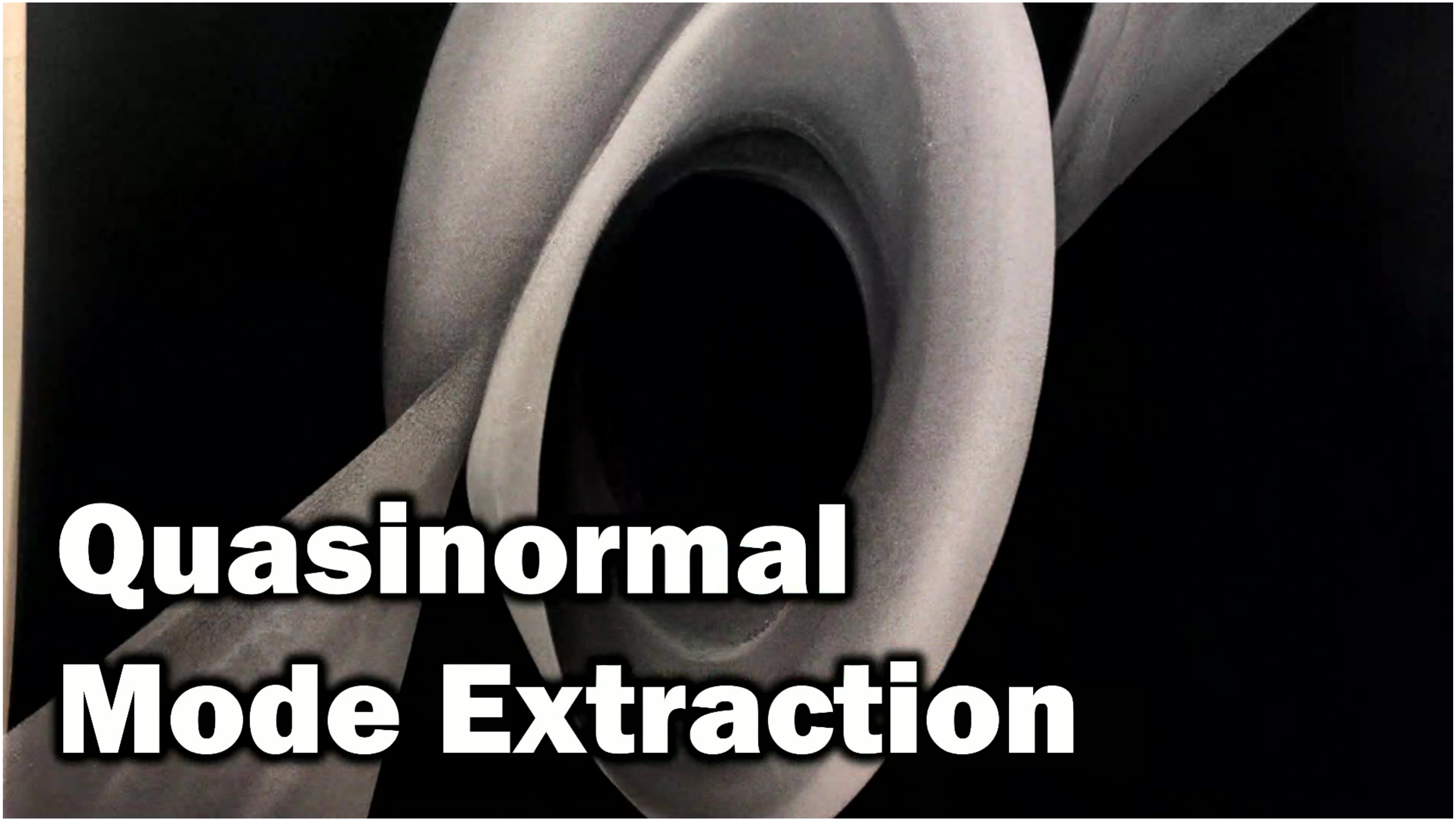


Cheung+ (2208.07374)

Why do we care?

- Nonlinearities are important for modeling the ringdown
- New way to test general relativity
  - Do nonlinear modes exist in detected gravitational waves?
    - Require next generation detectors (Yi, …, Cheung+ in prep.)
  - If they are identified, do their amplitudes/phases follow the expected relationships?

$$A_{i\times j} = c_{A,i\times j} A_i A_j$$
$$\phi_{i\times j} = \phi_i + \phi_j + c_{\phi,i\times j}$$


Quasinormal
Mode Extraction

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$$\phi_{i\times j} = \phi_i + \phi_j + \boxed{c_{\phi,i\times j}}$$

might depend on the theory of gravity

How could we extract a quasinormal mode?

$$Ae^{-i(\boxed{\omega}t+\phi)}$$

- Is the frequency "present"?
  - Is the frequency of the mode the best fit frequency?

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$$A e^{-i(\omega t + \phi)}$$

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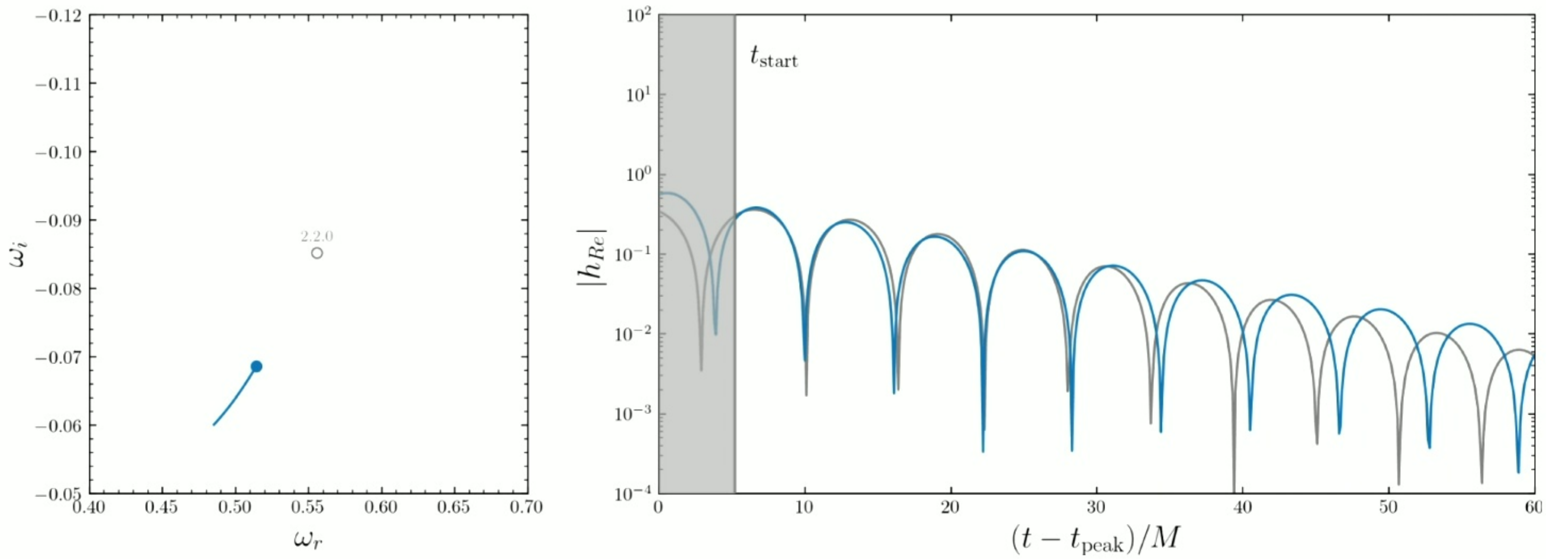
How could we extract a quasinormal mode?

$$Ae^{-i(\omega t+\phi)}$$

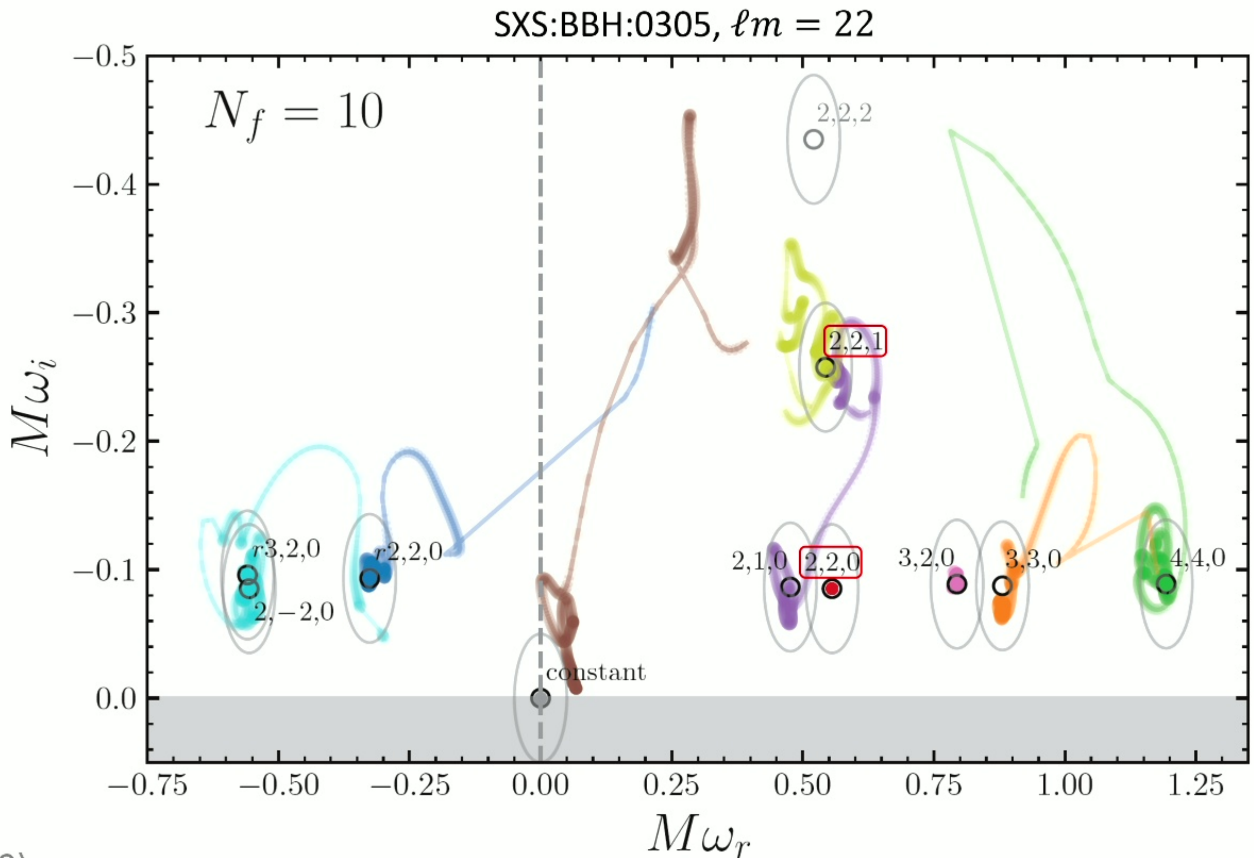
- Is the frequency "present"?
  - Is the frequency of the mode the best fit frequency?
- Are the amplitude and phases constant?
- Do we see the mode in a long enough time window (not just briefly)?

Is the frequency "present"?

$$Ae^{-i(\omega t+\phi)}$$

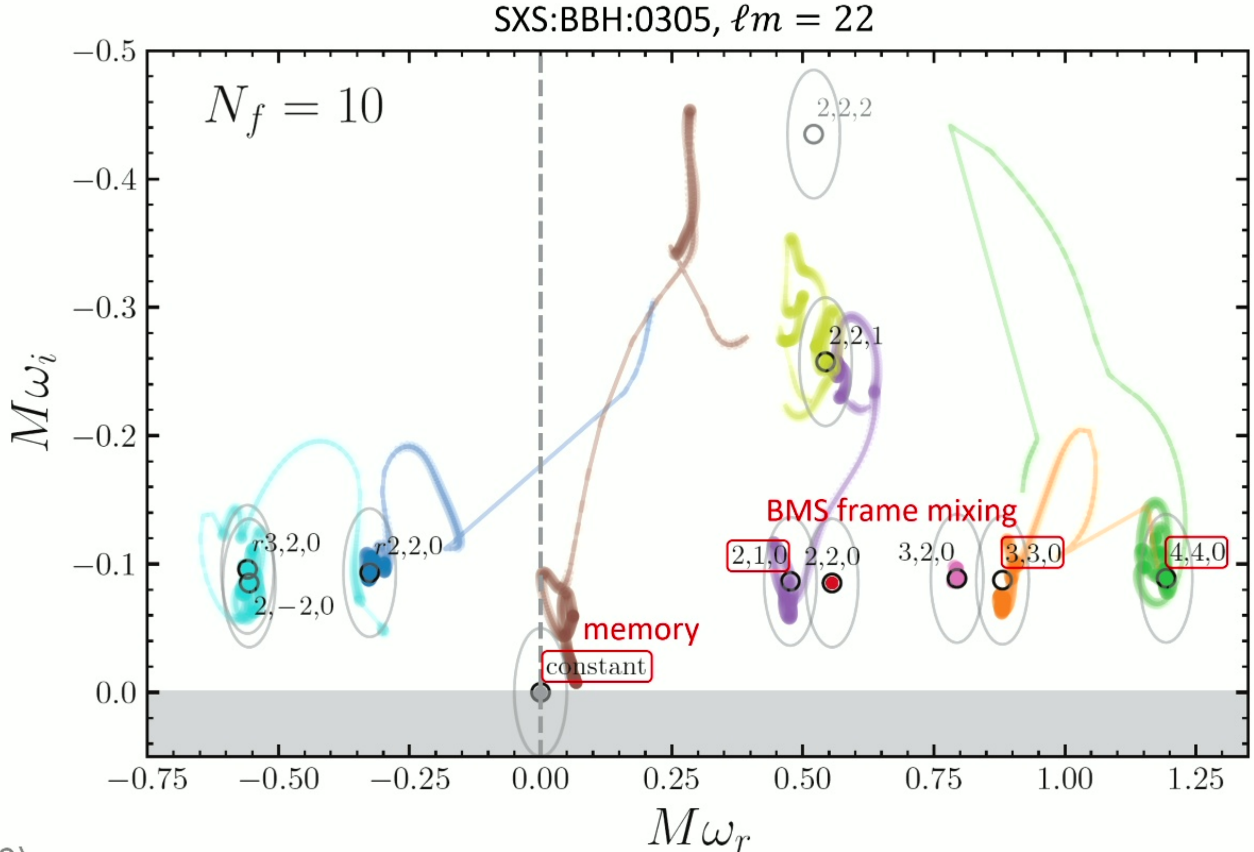


A rich spectrum



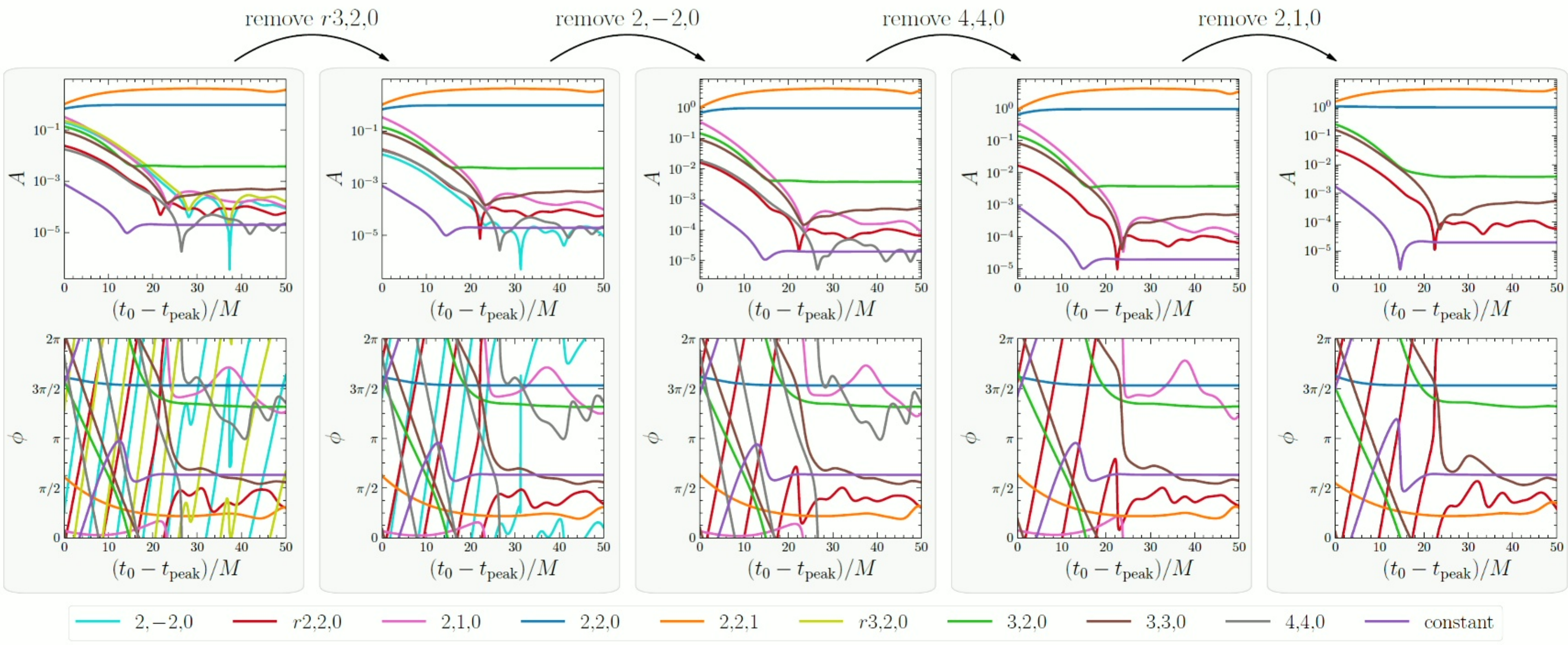
Cheung+ (2310.04489)

A rich spectrum



Cheung+ (2310.04489)

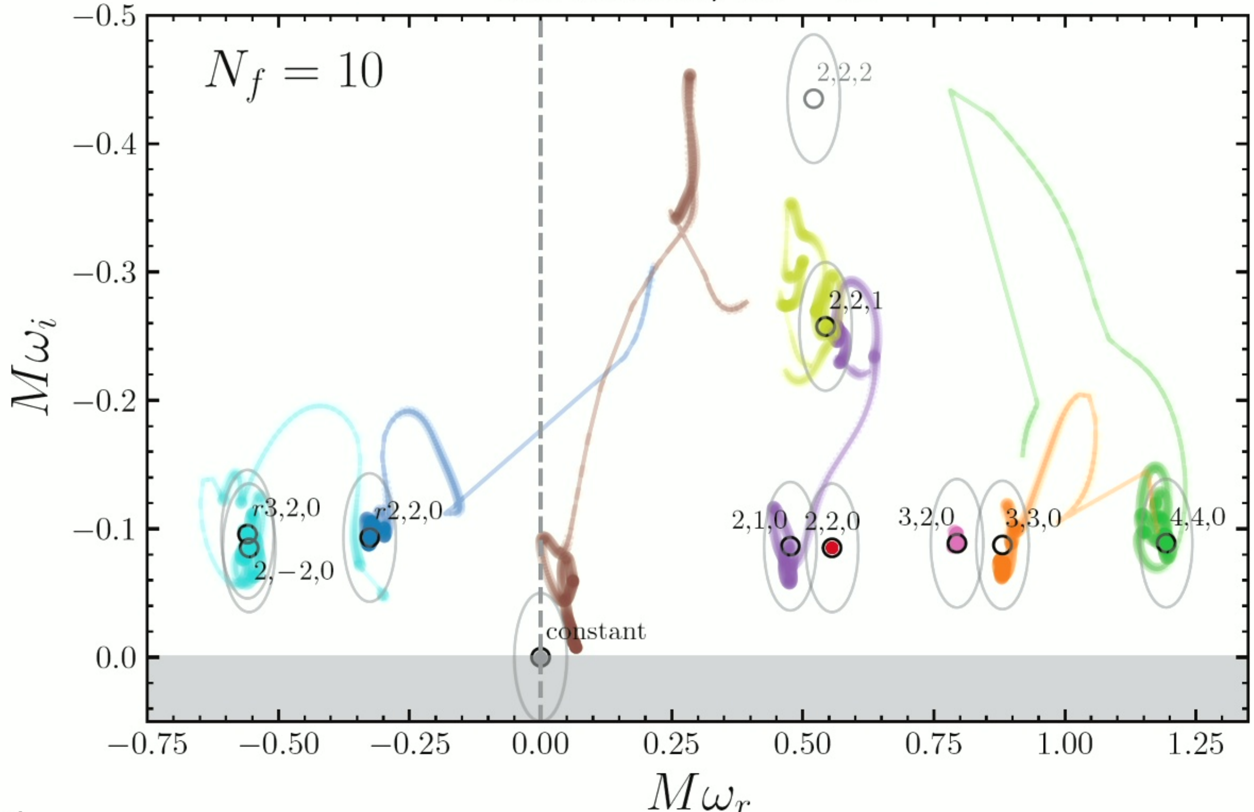
Are the amplitudes and phases constant?



Cheung+ (2310.04489)

A rich spectrum

SXS:BBH:0305, ℓm = 22



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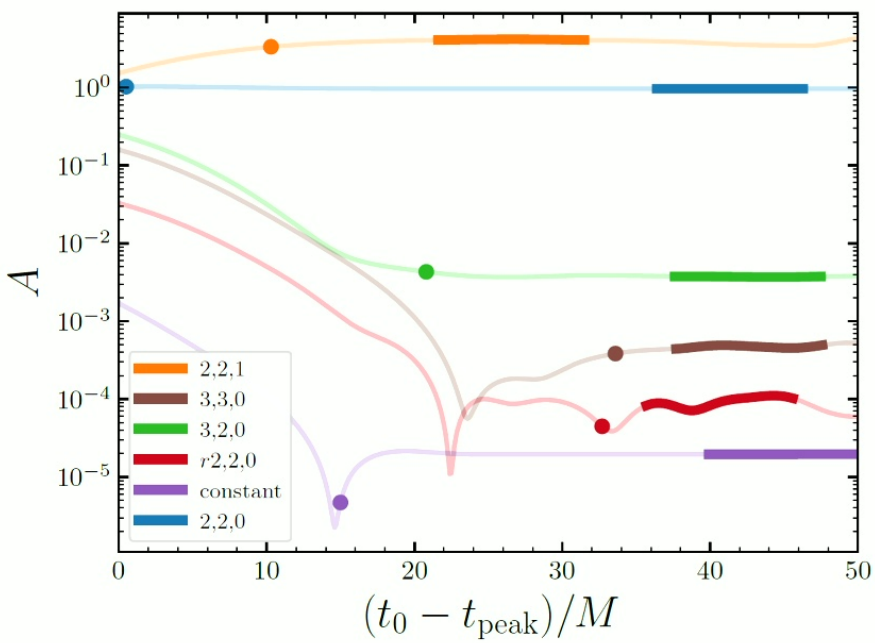
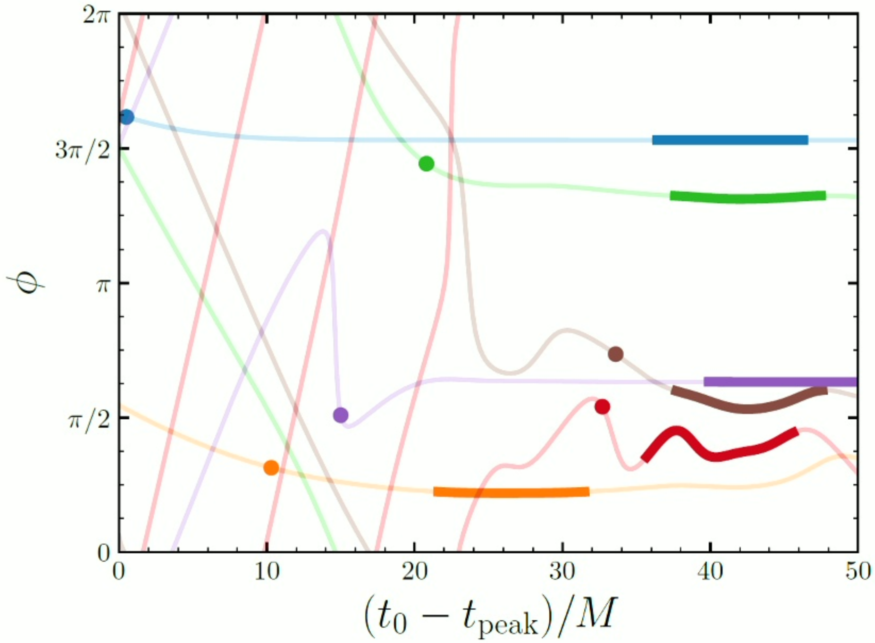
Amplitude extraction





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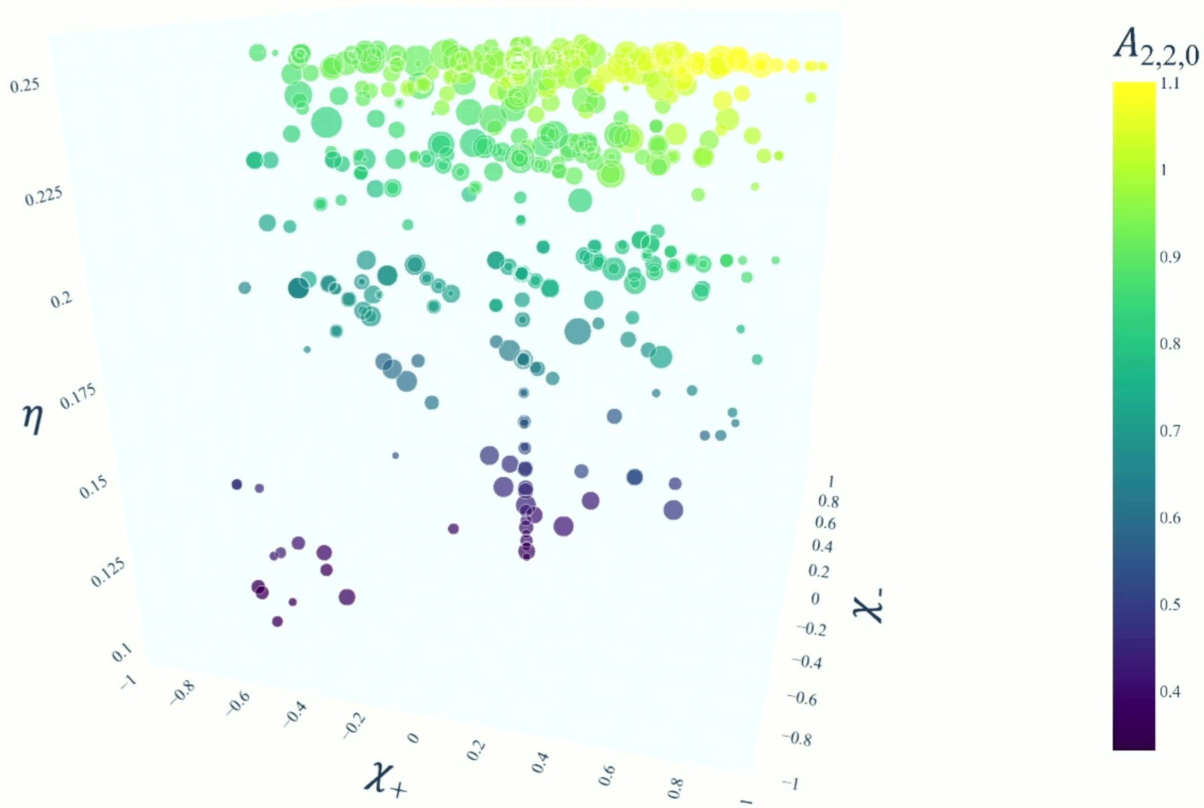
mhycheung.github.io/jaxqualin/

GitHub

Hyper-model of amplitudes



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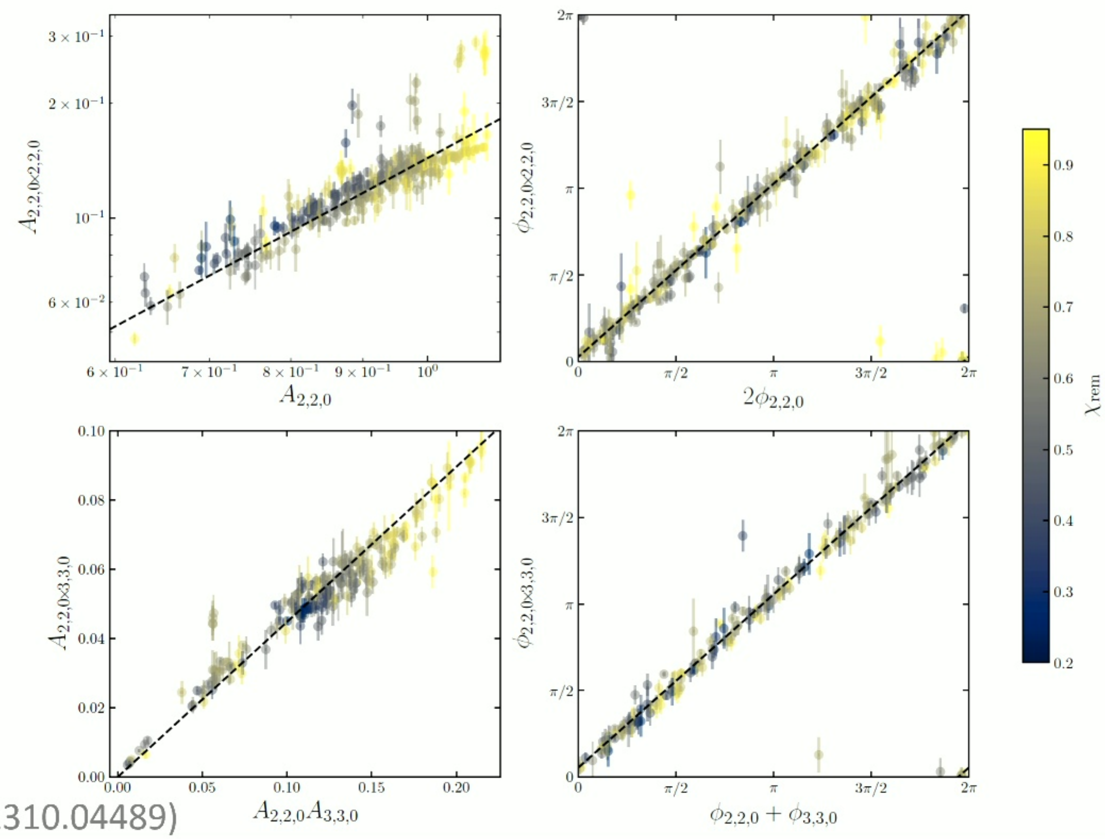
Quadratic modes, revisited

$$A^{(2)}_{i\times j} \propto A^{(1)}_i A^{(1)}_j \qquad \phi^{(2)}_{i\times j} = \phi^{(1)}_i + \phi^{(1)}_j + \text{constant}$$



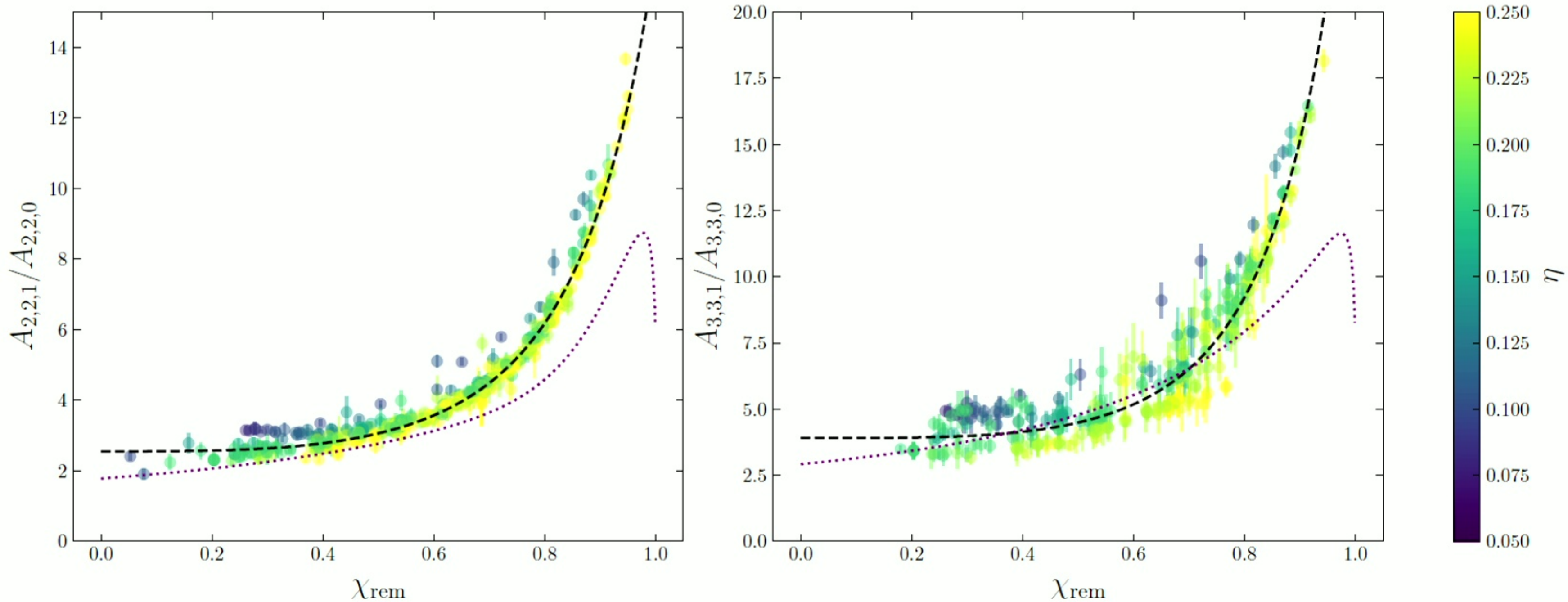
Also found by:
Ma+ (2207.10870), Mitman+ (2208.07380)

Tentative evidence:
London+ (1404.3197)

Gaussian scattering:
Redondo-Yuste+ (2308.14796)

Cheung+ (2310.04489)

Overtones



mode excitation: $C_{\ell mn} = B_{\ell mn} I_{\ell mn}$,

Purple dotted lines: $B_{\ell m1}/B_{\ell m0}$

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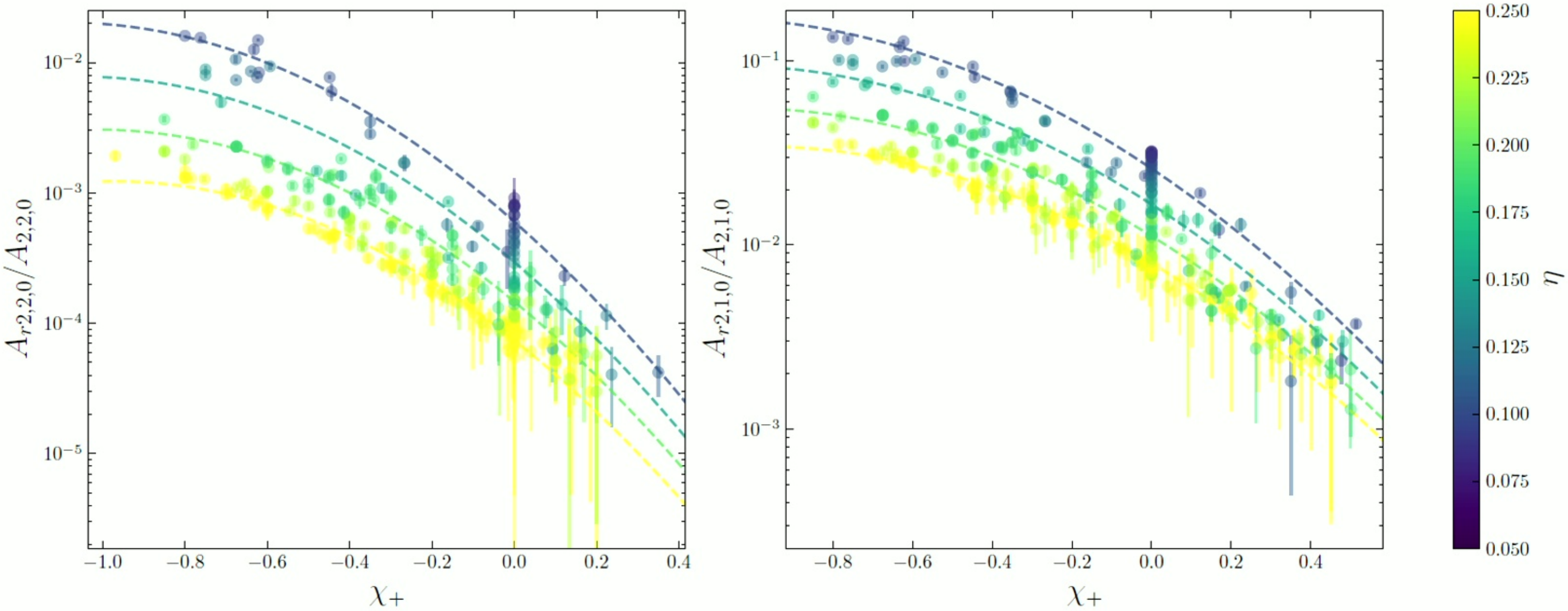
Retrograde modes



Cheung+ (2310.04489)

mhycheung.github.io/jaxqualin/

Conclusions

- A frequency agnostic fit + amplitude stability check could be a good strategy for extracting quasinormal modes
- The black hole spectrum is spectrally unstable
- A rich spectrum of modes exists in the ringdown
  - Fundamental modes, overtones, retrograde modes, quadratic modes
- Quasi-universal relationships between mode amplitudes warrant further investigation
- Our amplitude hyper-models can be used to guide data analysis