Title: Universal, deterministic, and exact protocol to reverse qubit-unitary and qubit-encoding isometry operations

Speakers: Satoshi Yoshida

Series: Quantum Foundations

Date: October 19, 2023 - 11:00 AM

URL: https://pirsa.org/23100102

Abstract: We report a deterministic and exact protocol to reverse any unknown qubit-unitary and qubit-encoding isometry operations. To avoid known no-go results on universal deterministic exact unitary inversion, we consider the most general class of protocols transforming unknown unitary operations within the quantum circuit model, where the input unitary operation is called multiple times in sequence and fixed quantum circuits are inserted between the calls. In the proposed protocol, the input qubit-unitary operation is called 4 times to achieve the inverse operation, and the output state in an auxiliary system can be reused as a catalyst state in another run of the unitary inversion. This protocol only applies only for qubit-unitary operations, but we extend this protocol to any qubit-encoding isometry operations. We also present the simplification of the semidefinite programming for searching the optimal deterministic unitary inversion protocol for an arbitrary dimension presented by M. T. Quintino and D. Ebler [Quantum 6, 679 (2022)]. We show a method to reduce the large search space representing all possible protocols, which provides a useful tool for analyzing higher-order quantum transformations for unitary operations.

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Zoom link https://pitp.zoom.us/j/92900413520?pwd=a1JqU1IzMVdSRGQreWJIbEFCT2hWUT09

# Universal, deterministic, and exact protocol to reverse qubit-unitary and qubit-encoding isometry operations

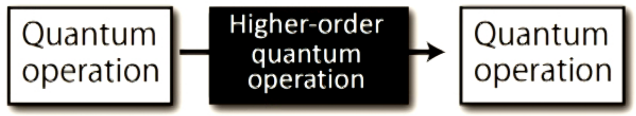
Satoshi Yoshida (UTokyo)
Joint work with Akihito Soeda (NII), Mio Murao (UTokyo)

Phys. Rev. Lett. 131, 120602 (2023)

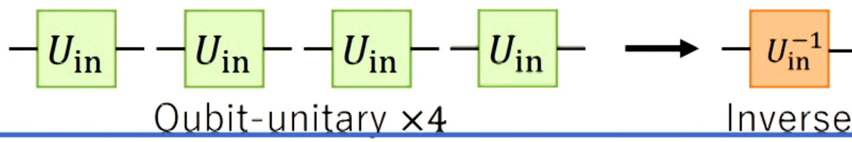
# Outline



- General perspective on higher-order quantum operations

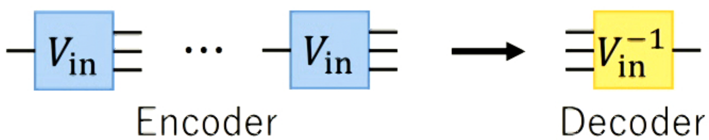



- Result 1: Deterministic exact qubit-unitary inversion




This talk

- Result 2: Isometry inversion



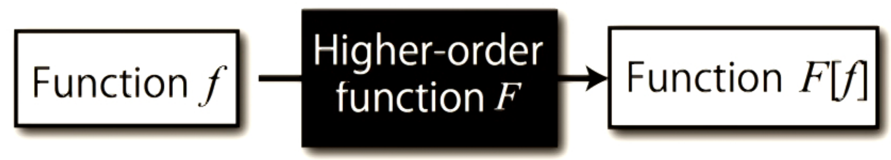

- Future works

# Higher-order quantum operation

- Classical information processing

- Function

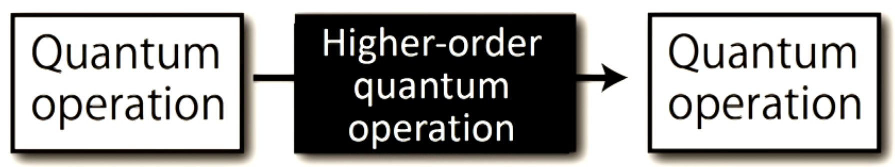

- Higher-order function




- Quantum information processing

- Quantum operation


- Higher-order quantum operation



# Universal transformation of quantum states



- Task

Given ↘ $\rho$


$\sigma = f(\rho)$

Unknown state

This is <u>NOT</u> $|0\rangle \mapsto |\psi\rangle$

- Eg. State cloning

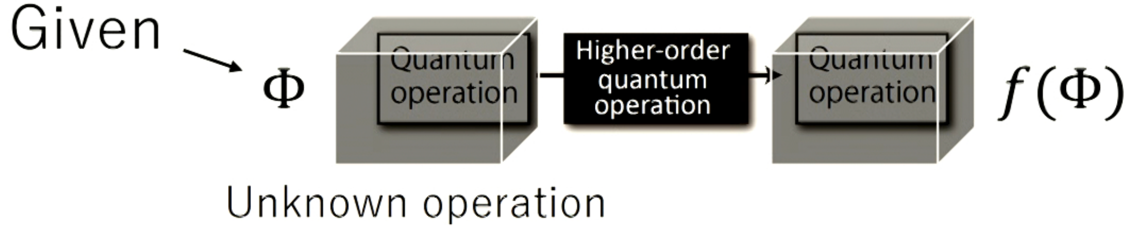
$$\rho \mapsto \rho \otimes \rho$$

W. K. Wootters and W. H. Zurek, Nature 299, 802 (1982).

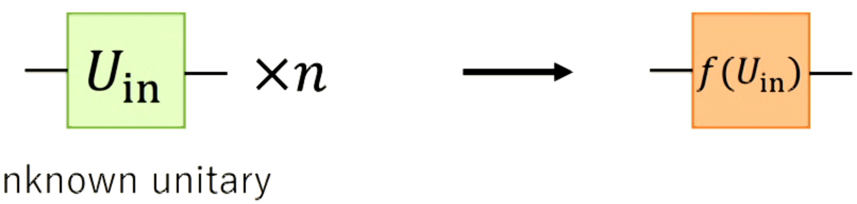
# Universal transformation of quantum operations



- Task




- Eg. Universal transformation of unitary operation



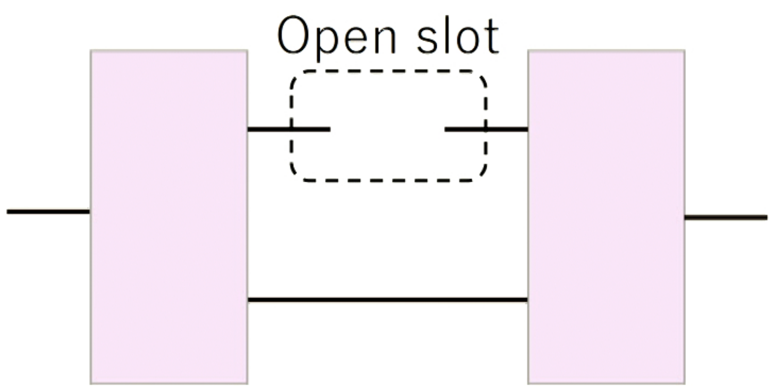

$f(U) = U^{\otimes m}, U^*, U^{-1}, U^T, \text{ctrl} - U, \ldots$

G. Chiribella et al. PRL 101, 180504 (2008). M. Quintino et al. PRL 123, 210502 (2019). D. Trillo et al. PRL 130, 110201 (2023).
J. Miyazaki et al. PRR 1, 013007 (2019). D. Ebler et al. arXiv:2206.00107. Q. Dong at al. arXiv:1911.01645. M. Araujo et al. NJP 16 093026 (2014).

# Quantum combs

- How to implement transformation of quantum operations?

→ Quantum circuit with open slot(s): Quantum comb




G. Chiribella et al. PRL 101, 060401 (2008).

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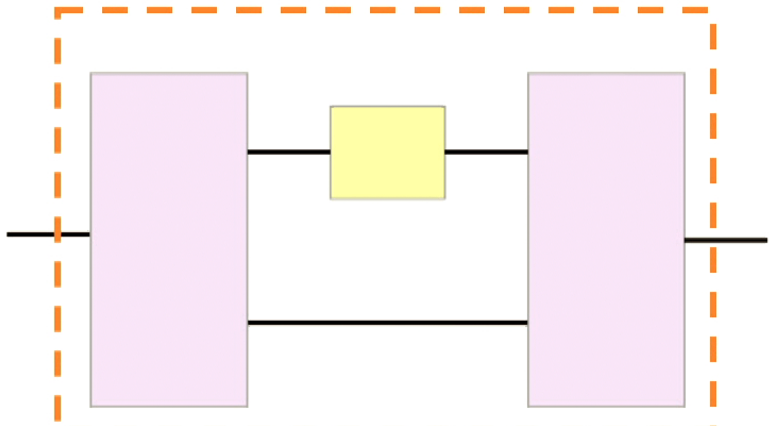


G. Chiribella et al. PRL 101, 060401 (2008).

# Quantum combs

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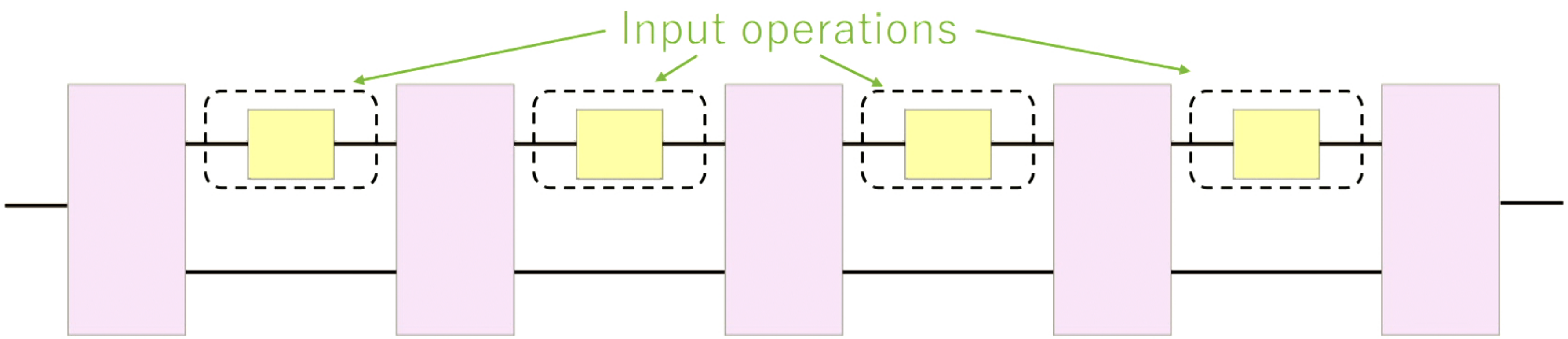
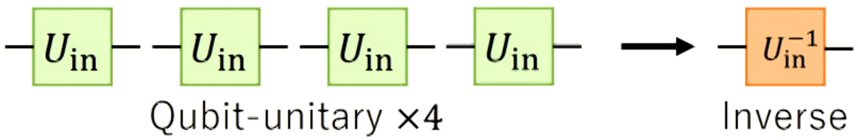



G. Chiribella et al. PRL 101, 060401 (2008).

# Outline

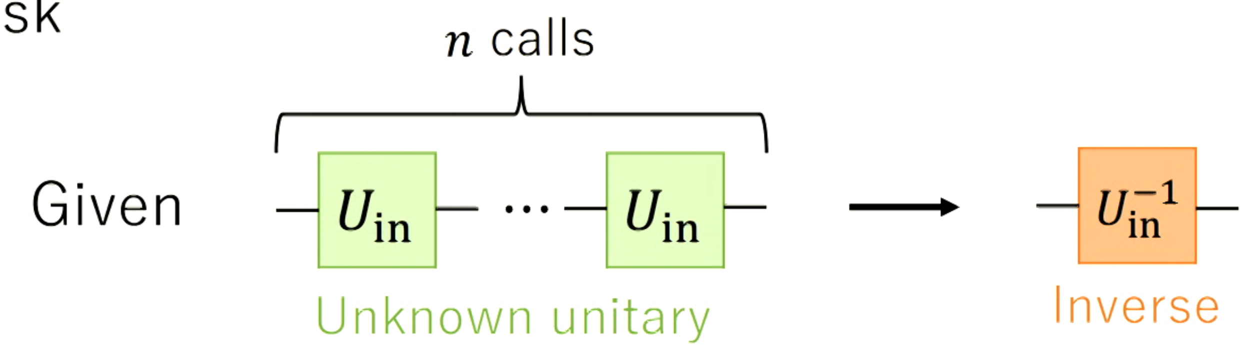



- Result 1: Deterministic exact qubit-unitary inversion



# Unitary inversion

- Task



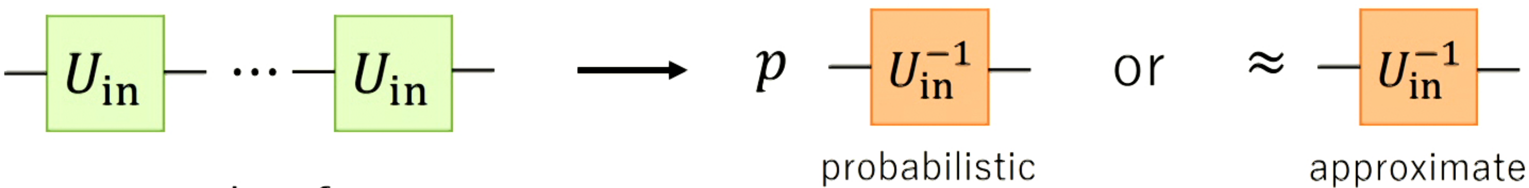

$$U_{\text{in}} = e^{-iHt} \mapsto U_{\text{in}}^{-1} = e^{iHt}$$

→ Simulation of "time inversion"

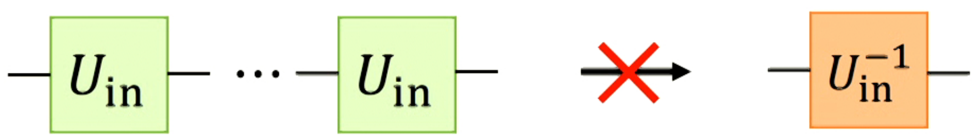
# Unitary inversion

- The fundamental limitation of unitary inversion?
- Previous work:

\- Go results




\- No-go results for some cases

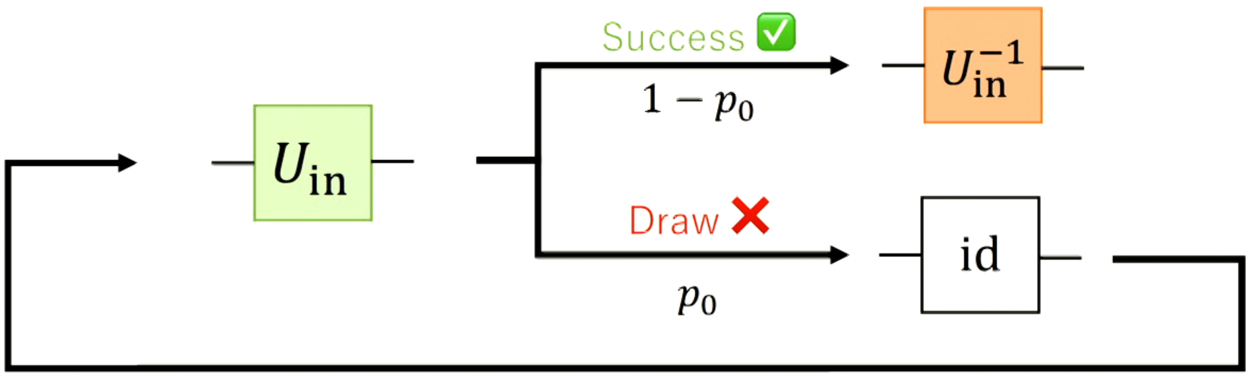



M. Sedlak et al. PRL 122, 170502 (2019), M. Navascues, PRX 8, 031008 (2018), M. Quintino et al. PRL 123, 210502 (2019), M. Quintino et al. PRA 100, 062339 (2019). M. Quintino and D. Ebler Quantum 6, 679 (2022), I. S. Sardharwalla et al. arXiv: 1602.07963, D. Ebler et al. IEEE Trans. Inf. Theo. 69, 8 (2023), D. Trillo et al. Quantum 4, 374 (2020), D. Trillo et al. PRL 130 (11), 110201, P. Schiansky et al. Optica 10, 200 (2023).

# Unitary inversion

- Previous work: Go results

Best known：Success-or-draw

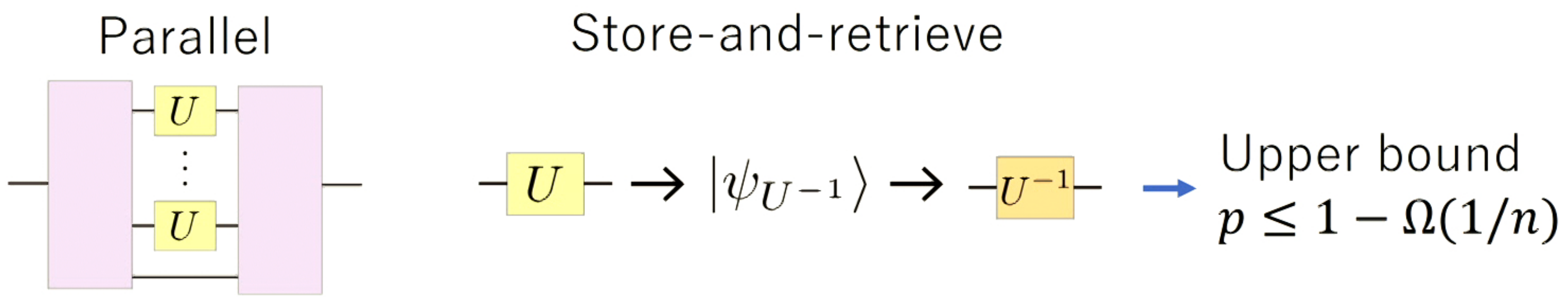



→ Success probability $p = 1 - p_0^{-O(n)} < 1$

M. Quintino et al. PRL 123, 210502 (2019), M. Quintino et al. PRA 100, 062339 (2019). M. Quintino and D. Ebler Quantum 6, 679 (2022). D. Trillo et al. PRL 130 (11), 110201.

# Unitary inversion

- Previous work: No-go results

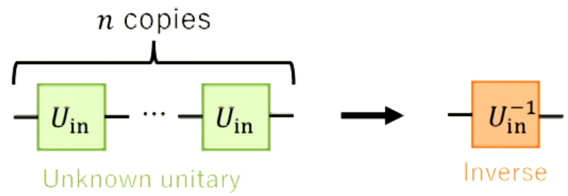



Numerics: $p_{\mathrm{opt}}(d,n), F_{\mathrm{opt}}(d,n)$ for small $d,n$

→ Still less than 1

M. Sedlak et al. PRL 122, 170502 (2019), M. Quintino et al. PRL 123, 210502 (2019), M. Quintino et al. PRA 100, 062339 (2019). M. Quintino and D. Ebler Quantum 6, 679 (2022).

# Unitary inversion






- Previous work

| | Probabilistic | Deterministic |
|---|---|---|
| Approximate | ✓ | ✓ |
| Exact | ✓ | ??? |

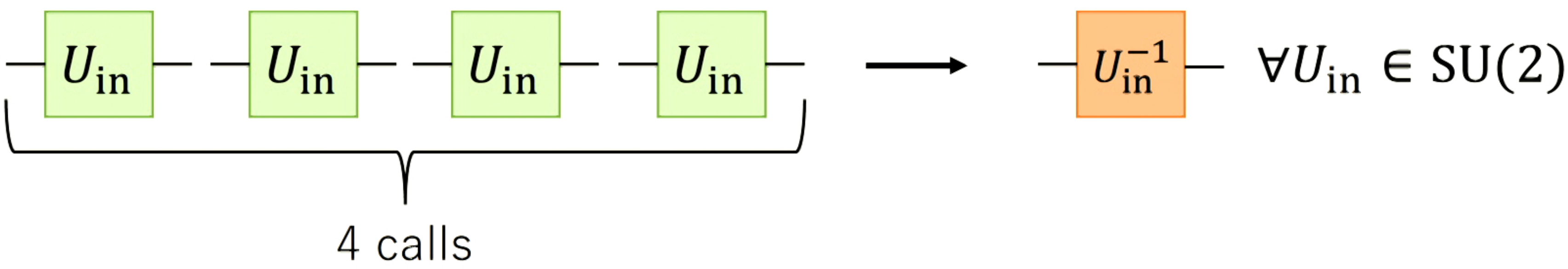
Open problem

We answer the open problem positively for $d = 2$!

# Unitary inversion

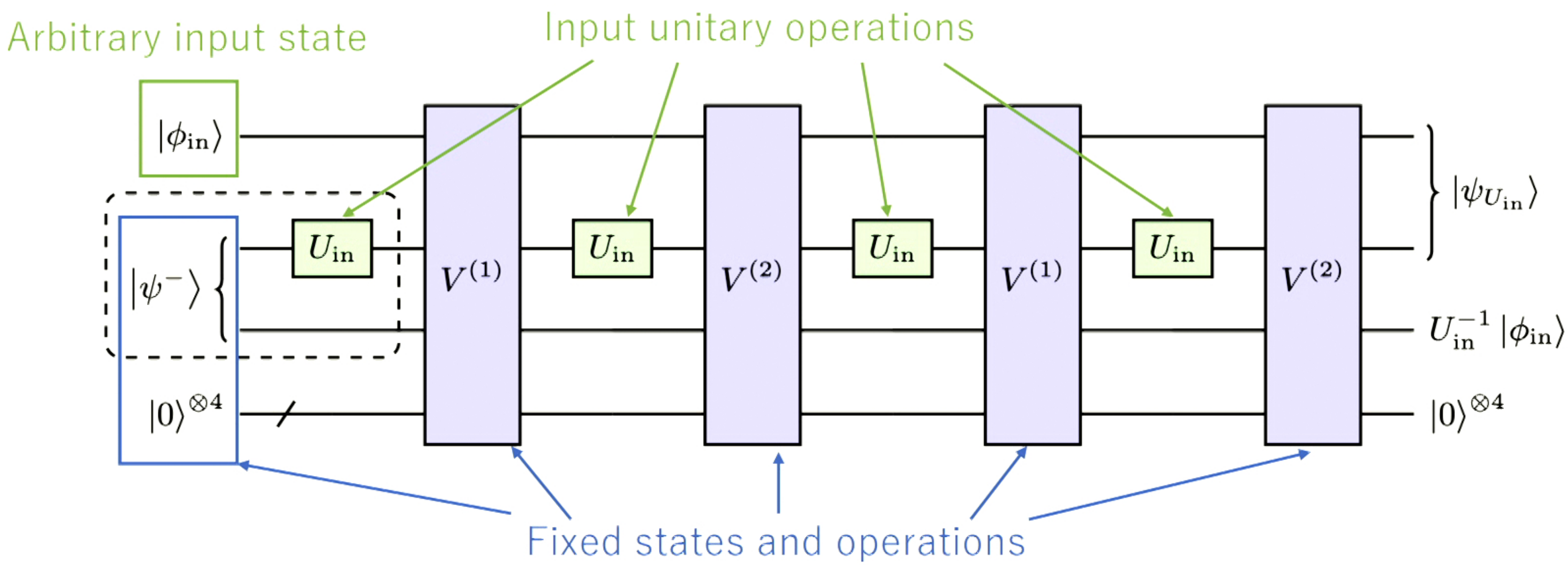
- Main result:

There exists a deterministic and exact qubit-unitary inversion protocol.



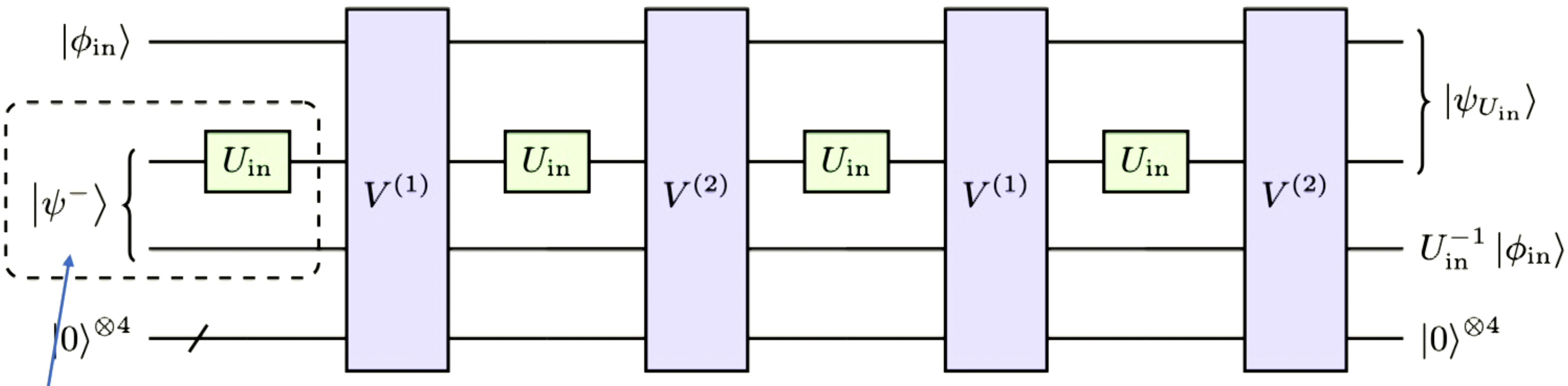
# Qubit-unitary inversion protocol





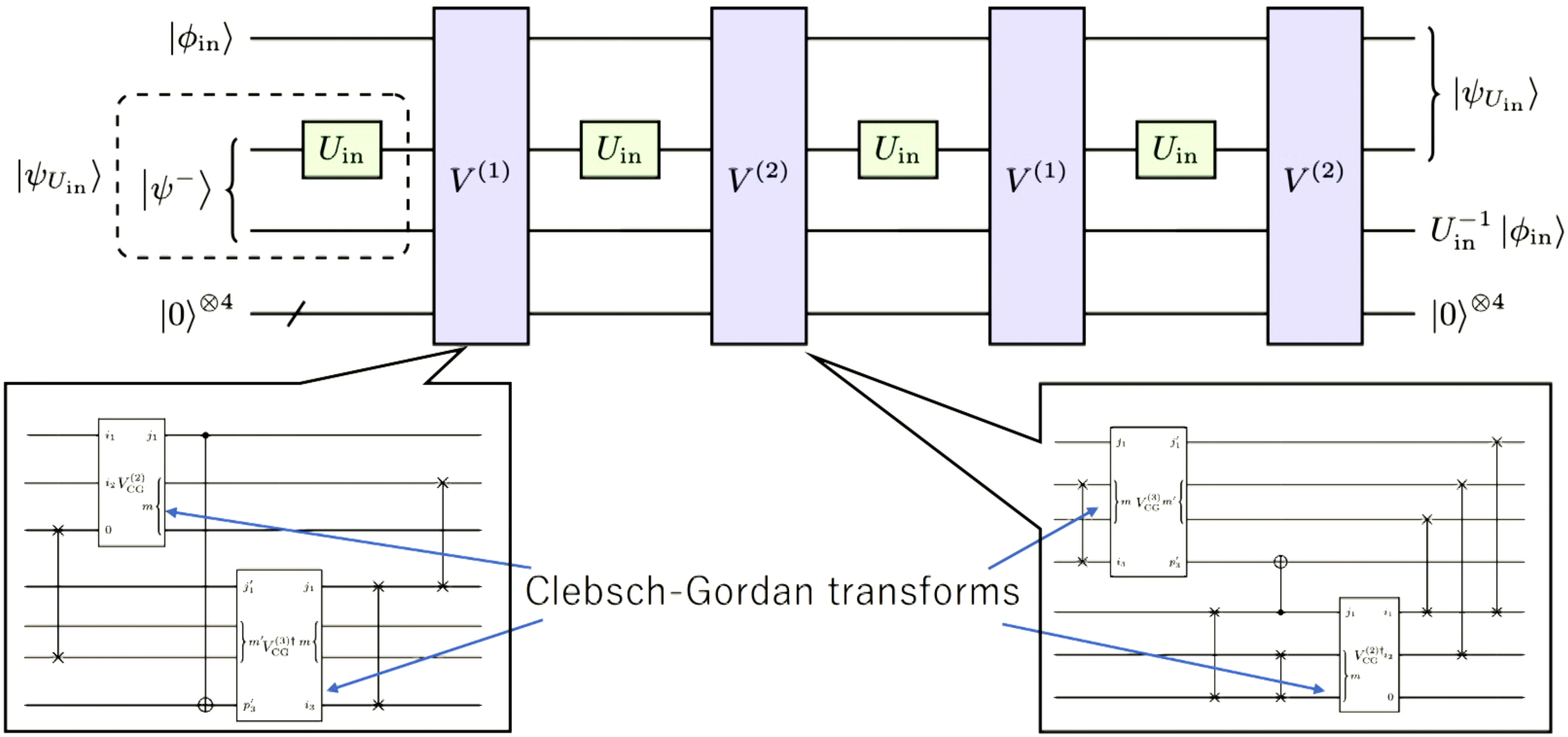
# Qubit-unitary inversion protocol



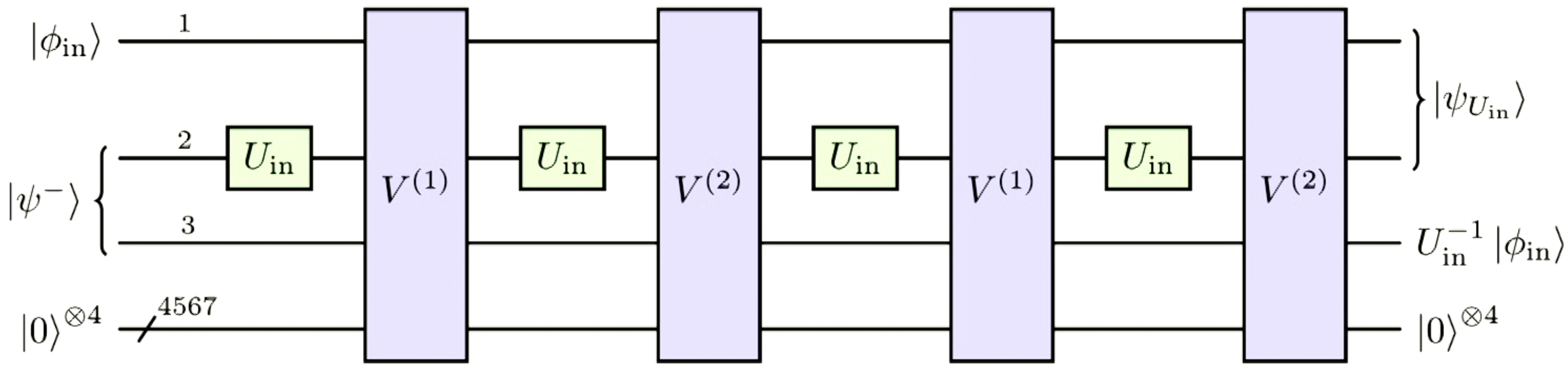



$$|\psi^-\rangle \coloneqq \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

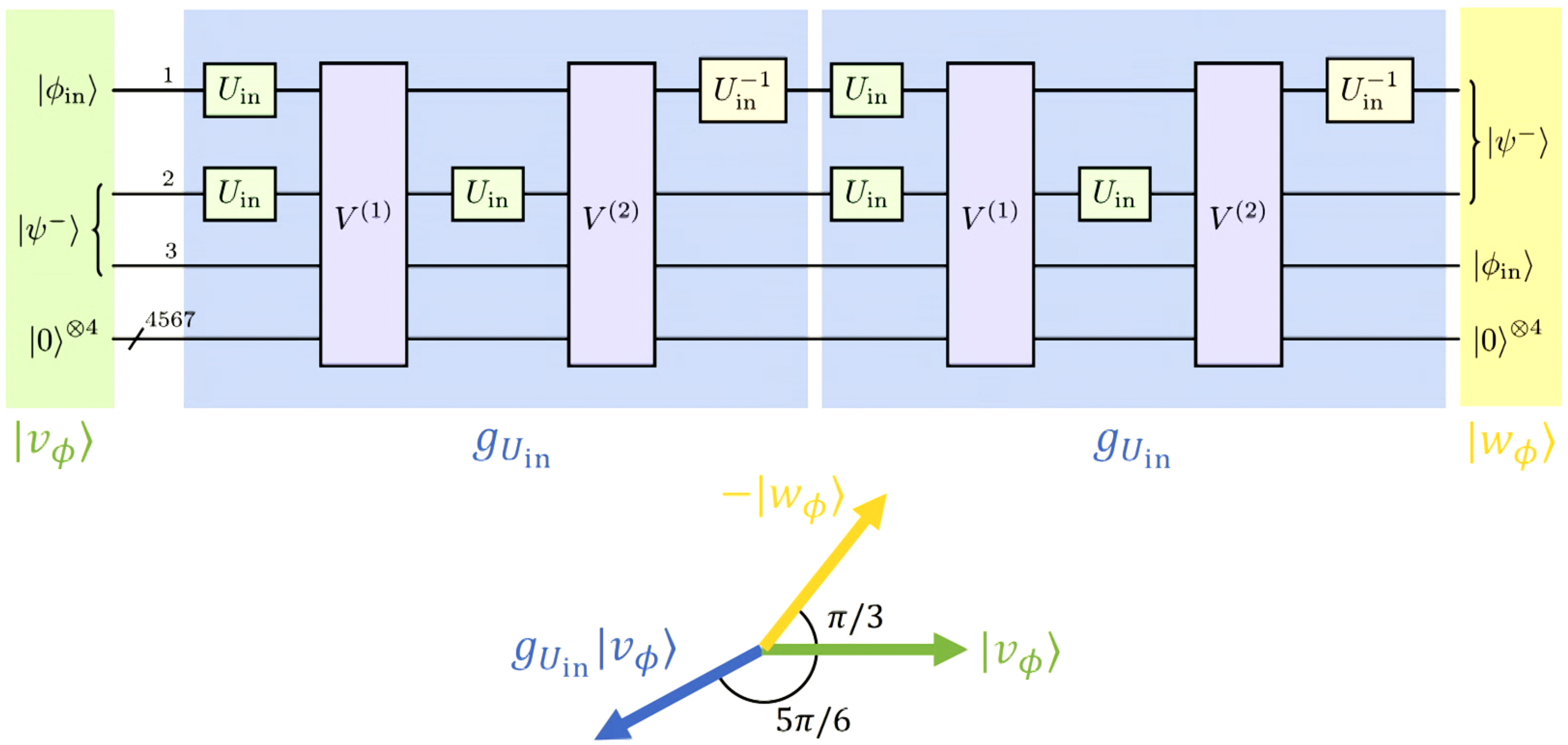
# Qubit-unitary inversion protocol



# Proof sketch



# Proof sketch

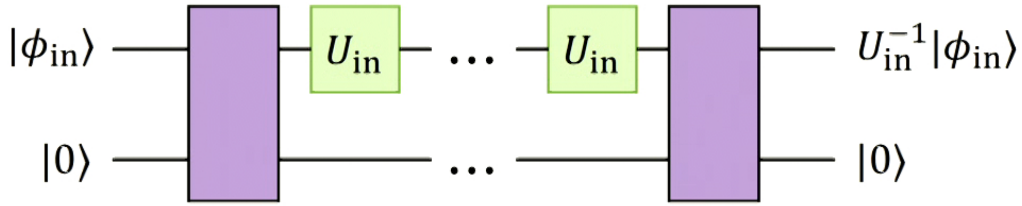


# Characteristics of this protocol



- Catalytic use of $|\psi_{U_{\mathrm{in}}}\rangle$


- Cleanness of the protocol



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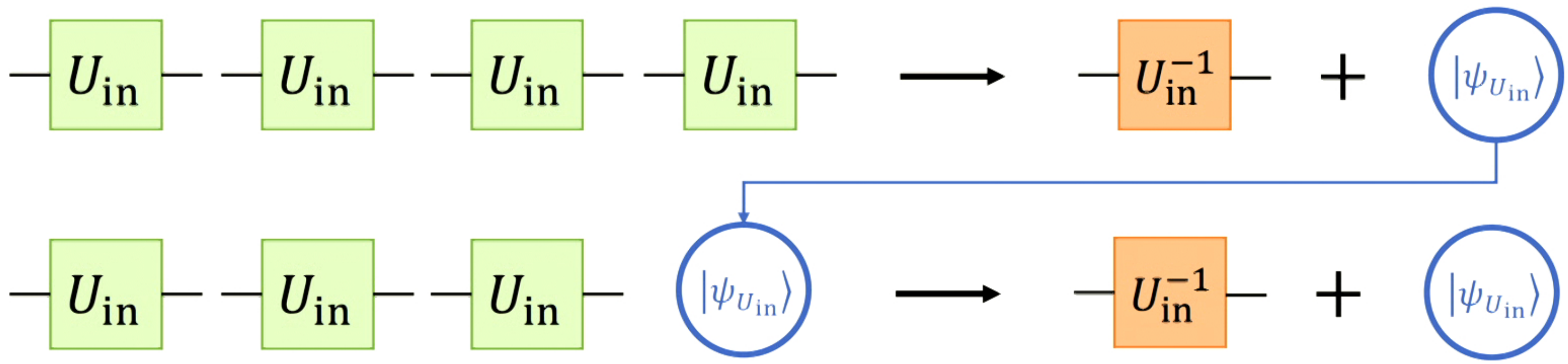

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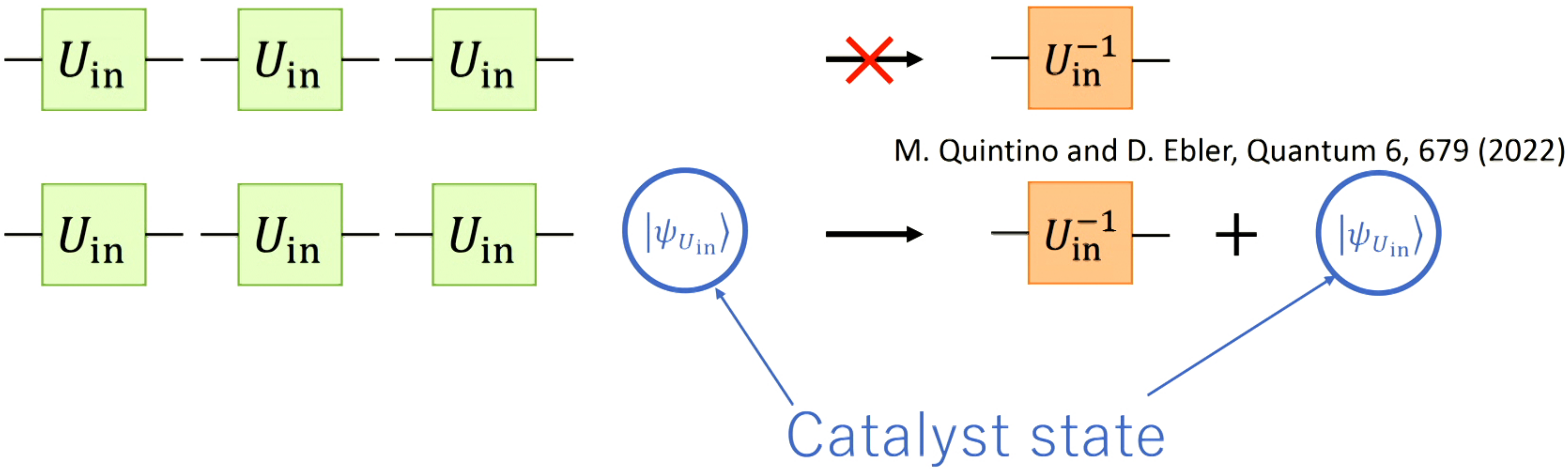
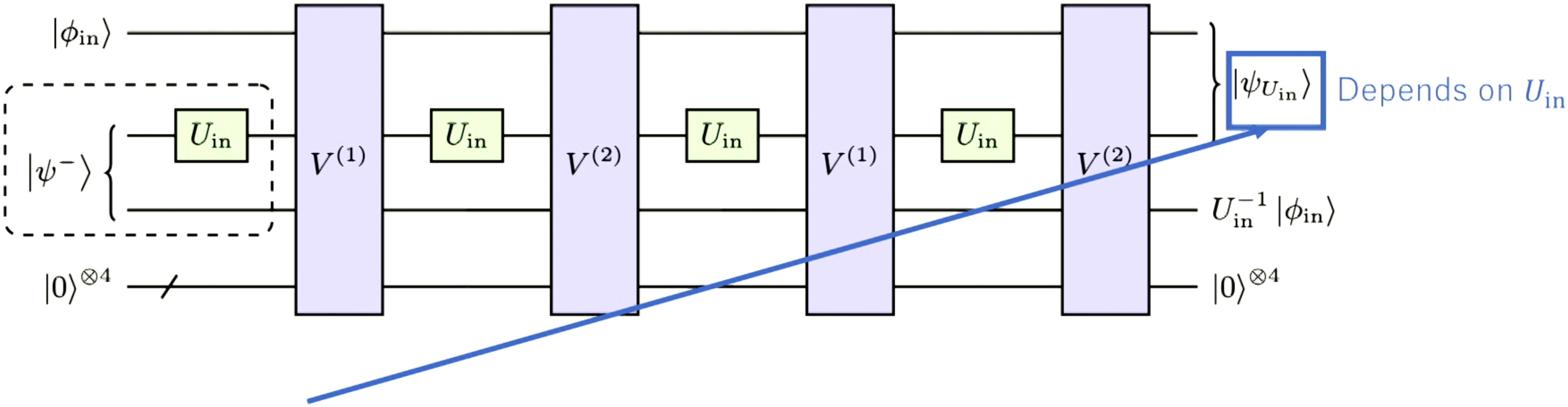



D. Jonathan, D and M. Plenio, M. B, PRL, 83, 3566 (1999).

# Characteristics of this protocol

- Non-clean protocol



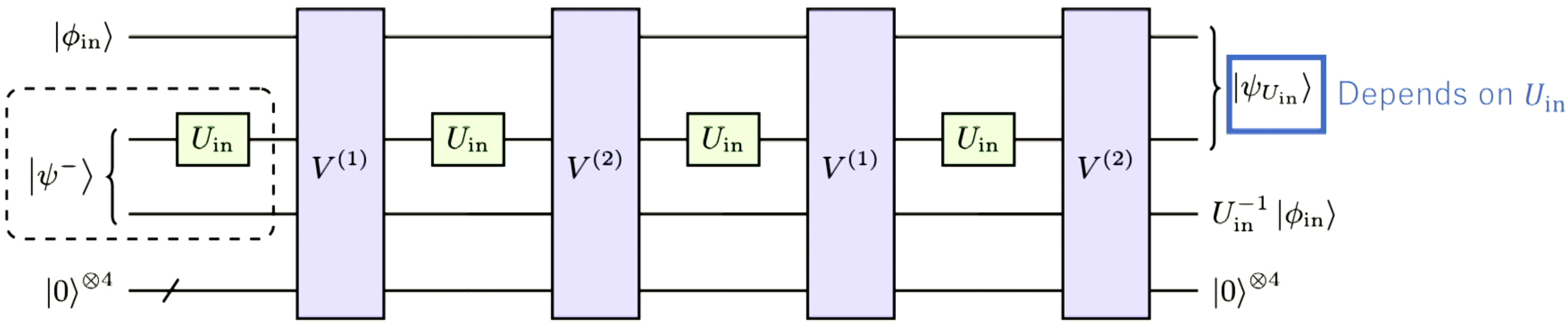

$\rho = \int_{SU(2)} dU_{\rm in} |\psi_{U_{\rm in}}\rangle\langle\psi_{U_{\rm in}}| = \frac{I \otimes I}{4}$ → Initialization cost $W = k_B T H(\rho) = 2 k_B T \ln 2$

R. Landauer, IBM journal of research and development 5, 183 (1961).
F. Meier and H. Yamasaki, arXiv:2305.11212 (2023).

# Characteristics of this protocol



- Non-clean protocol

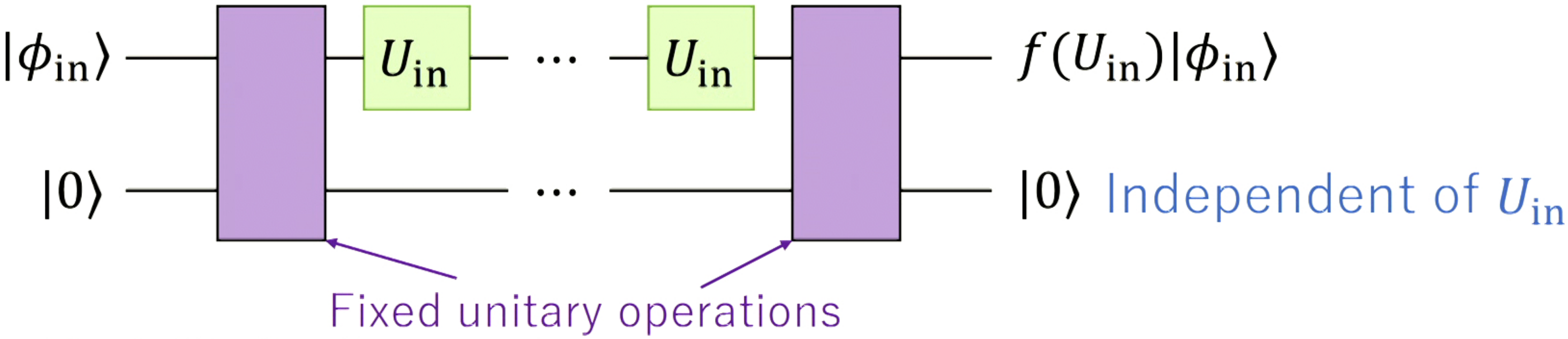



$$(I \otimes U_{\text{in}})|\psi_{U_{\text{in}}}\rangle = U_{\text{in}}^{\otimes 2}|\psi^-\rangle = |\psi^-\rangle$$

# Characteristics of this protocol



- Clean protocol for $f: \mathrm{SU}(d) \to \mathrm{SU}(d)$



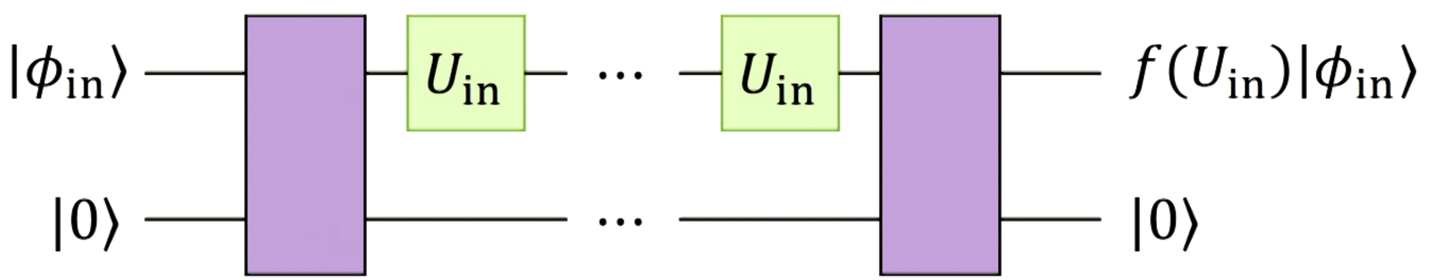

- No initialization cost

- $\mathrm{ctrl} - U_{\mathrm{in}} \to \mathrm{ctrl} - f(U_{\mathrm{in}})$

Z. Gavorová et al. arXiv:2011.10031 (2020).

# Characteristics of this protocol

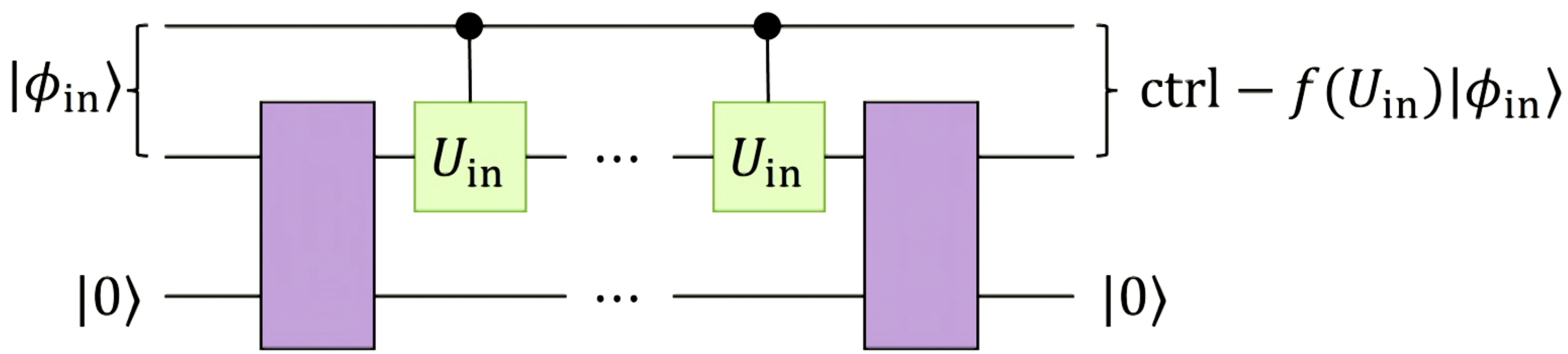
- $\mathrm{ctrl} - U_{\mathrm{in}} \rightarrow \mathrm{ctrl} - f(U_{\mathrm{in}})$



# Characteristics of this protocol



- $\text{ctrl} - U_{\text{in}} \to \text{ctrl} - f(U_{\text{in}})$



# How to find this protocol?
# : Numerical search + symmetry



- SDP to optimize approximation error

$$\max F_{\text{ave}} := \int_{\text{SU}(d)} dU_{\text{in}}\, F\left[U_{\text{in}}^{-1}, \mathcal{C}\left(U_{\text{in}}^{\otimes n}\right)\right]$$

$$\text{s.t.}\ \mathcal{C}\ \text{is a quantum comb}$$

M. Quintino and D. Ebler, Quantum 6, 679 (2022)

Choi representation:

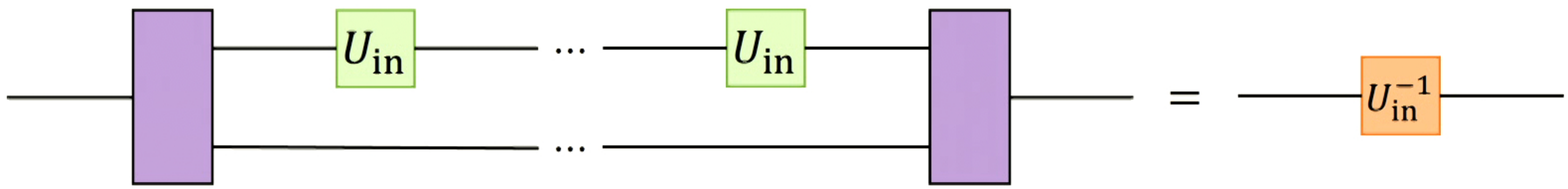
| **CPTP map** | | **Choi matrix** |
|---|---|---|
| $\mathcal{C}(\Lambda_{\text{in}}) = \Lambda_{\text{out}}$ | $\Leftrightarrow$ | $\text{Tr}_{\text{in}}(C J_{\Lambda_{\text{in}}}^{T}) = J_{\Lambda_{\text{out}}}$ |

# Reduction of SDP using $\mathbf{SU}(d)\times\mathbf{SU}(d)$ symmetry



M. Quintino et al. PRA 100, 062339 (2019)

- Symmetry in unitary inversion protocol



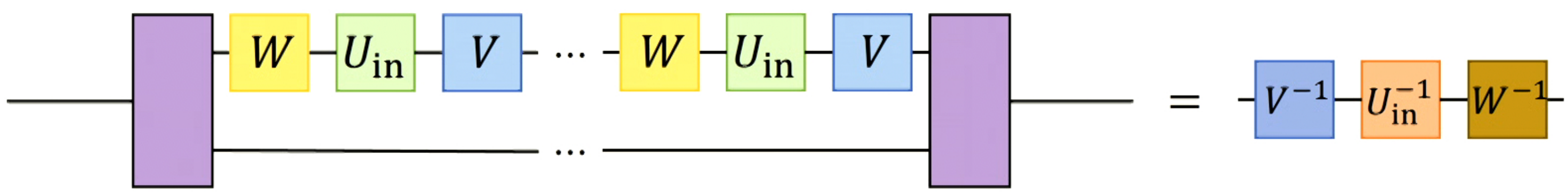
# Reduction of SDP using $\mathbf{SU}(d)\times\mathbf{SU}(d)$ symmetry



M. Quintino et al. PRA 100, 062339 (2019)

- Symmetry in unitary inversion protocol

① $U_{\mathrm{in}} \mapsto V U_{\mathrm{in}} W$ for $V, W \in \mathrm{SU}(d)$



# Reduction of SDP using $\mathbf{SU}(d)\times\mathbf{SU}(d)$ symmetry

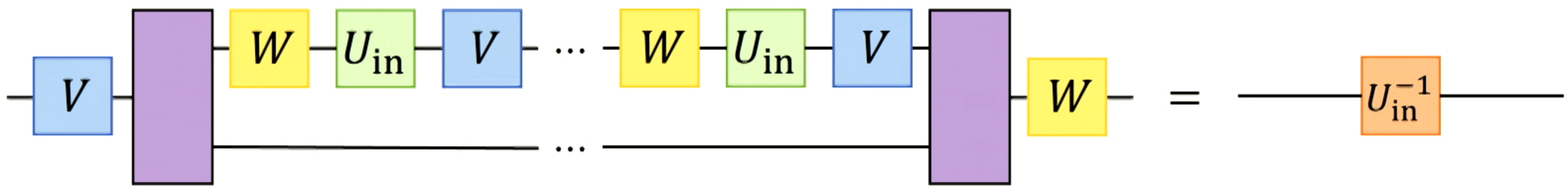


M. Quintino et al. PRA 100, 062339 (2019)

- Symmetry in unitary inversion protocol

① $U_{\text{in}} \mapsto V U_{\text{in}} W$ for $V, W \in \text{SU}(d)$

② Insert $V$ and $W$ to the whole circuit




→ $[C, V^{\otimes n+1} \otimes W^{\otimes n+1}] = 0 \quad \forall V, W \in \text{SU}(d)$

# Reduction of SDP using $\mathbf{SU}(d)\times\mathbf{SU}(d)$ symmetry



- Schur-Weyl duality：

An operator $X$ commutes with $V^{\otimes n+1}$ for all $V \in \mathrm{SU}(d)$

$\Leftrightarrow X$ is a linear combination of permutation operators $P_\sigma$, where

$$P_\sigma |i_1, \cdots, i_{n+1}\rangle := \left|i_{\sigma^{-1}(1)}, \cdots, i_{\sigma^{-1}(n+1)}\right\rangle$$

- $\left[C, V^{\otimes n+1} \otimes W^{\otimes n+1}\right] = 0 \quad \forall V, W \in \mathrm{SU}(d)$

$\Longrightarrow C = \sum_{\sigma,\tau\in\mathfrak{S}_{n+1}} c_{\sigma\tau} P_\sigma \otimes P_\tau,$ where $c_{\sigma\tau} \in \mathbb{C}$

<u>Remark</u>

In the paper, we utilize the Young-Yamanouchi basis which forms the orthogonal basis of $\mathrm{span}\{P_\sigma\}$.

# Numerical calculation of the SDP



| | $d=2$ | $d=3$ | $d=4$ | $\cdots$ |
|---|---|---|---|---|
| $n=2$ | ✓ | ✓ | | |
| $n=3$ | ✓ | | | |
| $n=4$ | | ??? | | |
| $\vdots$ | | | | |

→

| | $d=2$ | $d=3$ | $d=4$ | $\cdots$ |
|---|---|---|---|---|
| $n=2$ | ✓ | ✓ | ✓ | ✓ |
| $n=3$ | ✓ | ✓ | ✓ | ✓ |
| $n=4$ | ✓ | ✓ | ✓ | ✓ |
| $n=5$ | ✓ | ✓ | ✓ | ✓ |

Deterministic exact unitary inversion

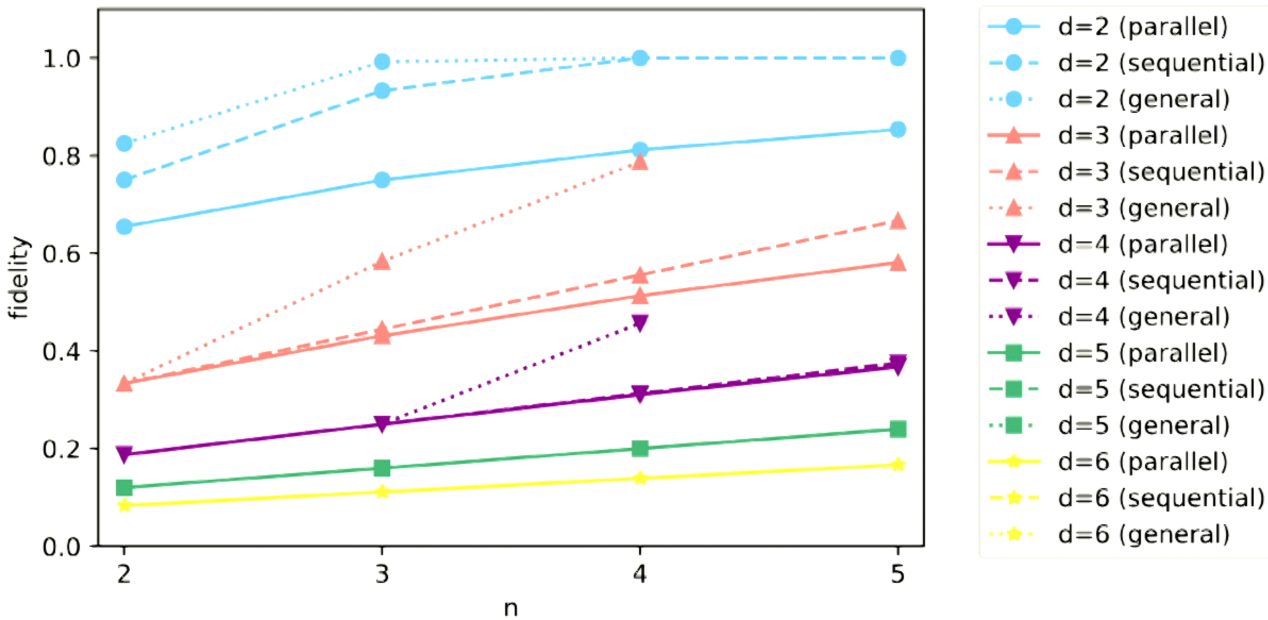
- Matrix representation of quantum comb → Quantum circuit

A. Bisio et al. PRA 83, 022325 (2011)

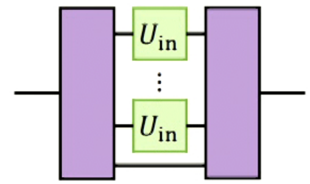
- Note: Reduction of SDP using unitary group symmetry

D. Grinko and M. Ozols, arXiv:2207.05713

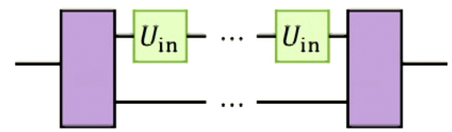
# Numerical calculation of the SDP




parallel




sequential



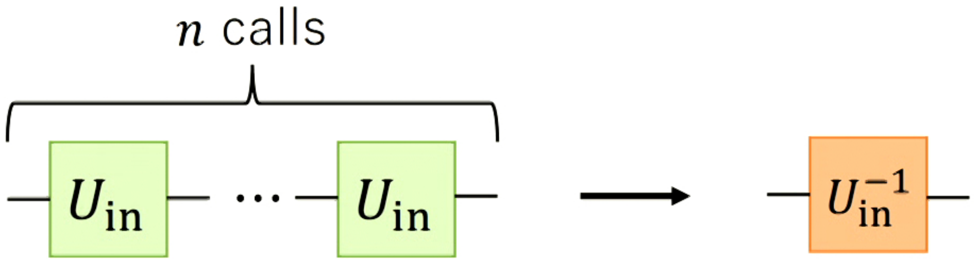

general ∋ indefinite causal order


beyond circuit model
but preserves CPTP maps

Sci. Adv., 3: e1602589 (2017)

# Future works

- Deterministic exact unitary inversion for $d > 2$



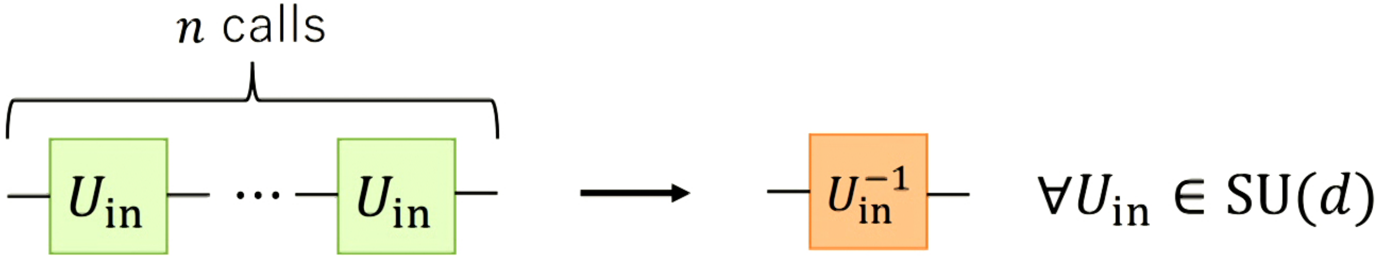

$\forall U_{\mathrm{in}} \in \mathrm{SU}(d)$

- Is it possible for arbitrary $d$?
- If so, minimum number of $n$?

# Future works



- Deterministic exact unitary inversion for $d > 2$

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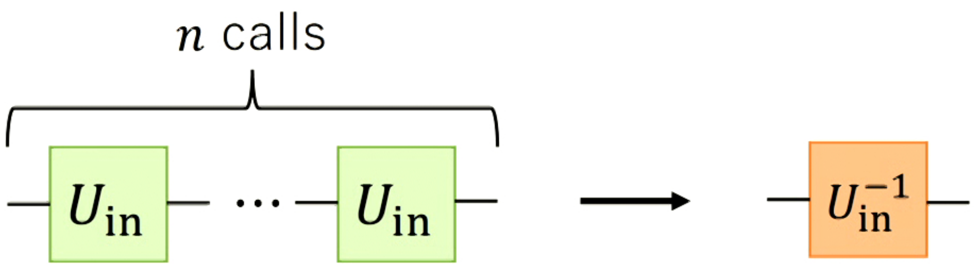
# Future works

- Deterministic exact unitary inversion for $d > 2$



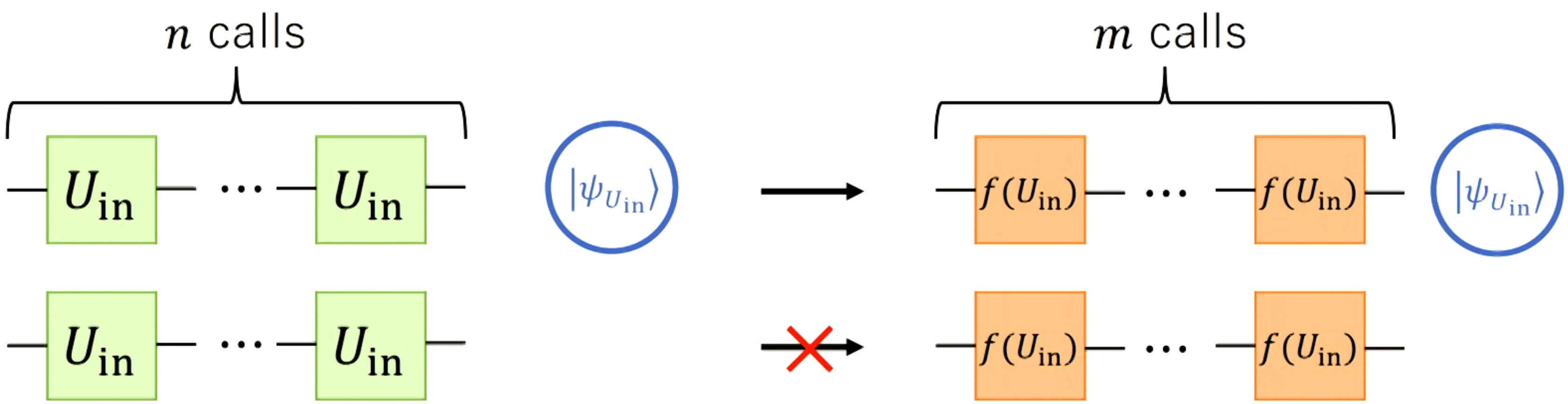

$\forall U_{\text{in}} \in \text{SU}(d)$

- Is it possible for arbitrary $d$?
- If so, minimum number of $n$?

Conjecture $n = d^2$? → 
- Further simplification of SDP
- Systematic understanding

# Future works

- Catalytic higher-order quantum operations

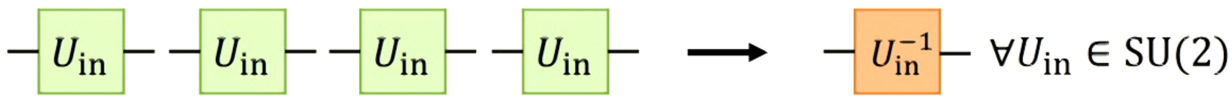



- How catalyst helps in other tasks?
- Relationship to asymptotic setting?


T. Kondra et al. PRL 127, 150503 (2021).
N. Shiraishi and T. Sagawa, PRL 126, 150502 (2021).
H. Wilming, PRL 127, 260402 (2021).

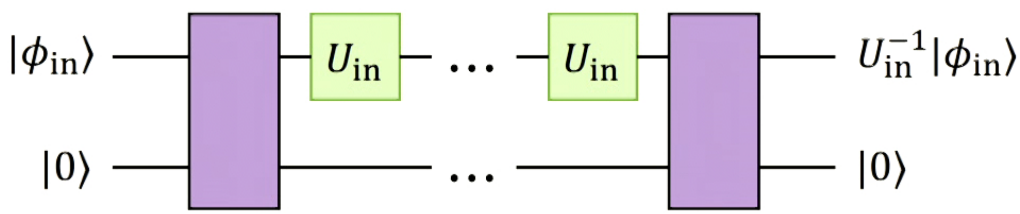
# Summary

- Deterministic exact qubit-unitary inversion




- Catalyst


- Clean-version protocol




- Extension to isometry inversion (in preparation)

Phys. Rev. Lett. 131, 120602 (2023)