Title: Carrollian c-functions and flat space holographic RG flows

Speakers: Daniel Grumiller

Series: Quantum Fields and Strings

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Abstract: We discuss c-functions and their holographic counterpart for 2d field theories with Carrollian conformal fixed points in the UV and the IR, and construct asymptotically flat domain wall solutions of 3d Einstein-dilaton gravity that model holographic RG flows. arXiv: 2309.11539

Zoom link https://pitp.zoom.us/j/94223624920?pwd=T3pxOGxnRVFVbGVnZnRuZ2NSTm8wZz09

Carrollian c-functions
and flat space holographic RG flows

Daniel Grumiller

Institute for Theoretical Physics
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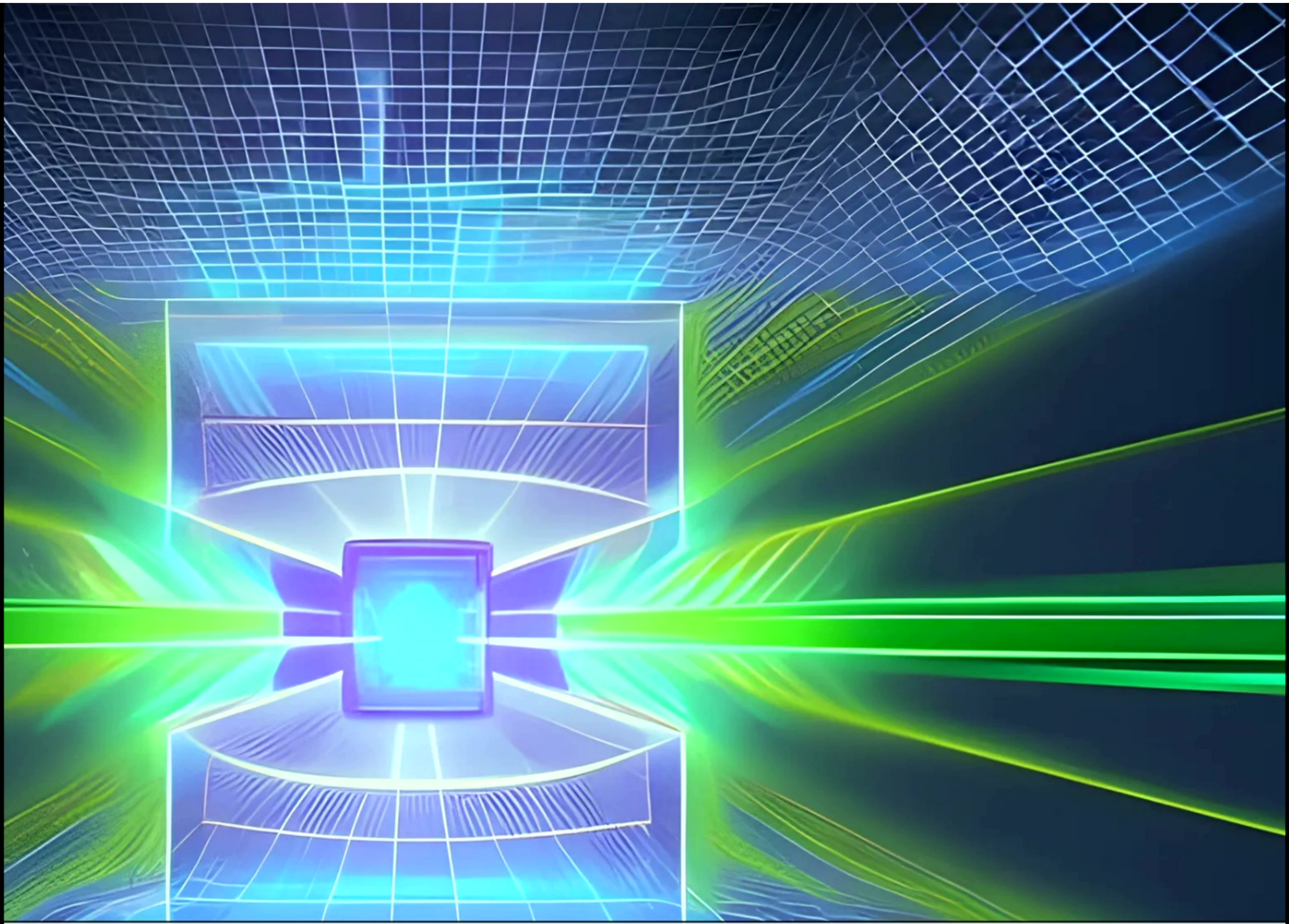
PI, October 2023

with Max Riegler, 2309.11539



Outline

Flat space holography á la Carroll

Holographic c-functions in AdS_3/CFT_2

Flat space domain walls and BMS_3 c-functions

Outlook to Casini–Huerta-like c-function

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Motivation

- How general is holography?
- Does it work beyond AdS/CFT?
- (How) does it work in asymptotically flat spacetimes?

Two main options:

- take $\Lambda \to 0$ limit of AdS/CFT
- start with $\Lambda = 0$ without taking limits

Two main approaches:

- Carrollian (codimension-1 holography)
- Celestial (codimension-2 holography)

This talk:

Focus on $\text{BMS}_3/\text{CCFT}_2$ correspondence

Holographic wish-list

We want holography to make useful statements about

1. symmetries
2. spectra
3. microstates
4. entanglement
5. correlation functions

Review five examples

Selected checks of $BMS_3/CCFT_2$ correspondence

1. analog of Brown–Henneaux bc's leading to BMS_3 Barnich, Compère '06

$$\mathrm{d}s^2 = \big(\mathcal{O}(1)\ \mathrm{d}u^2 - 2\,\mathrm{d}u\,\mathrm{d}r + \mathcal{O}(1)\ \mathrm{d}u\,\mathrm{d}\varphi + r^2\ \mathrm{d}\varphi^2\big)\,\big(1 + \mathcal{O}(1/r)\big)$$

preserved by asymptotic Killing vectors

$$M_n = ie^{in\varphi}\partial_u + \dots \qquad L_n = ie^{in\varphi}\big(inu\partial_u - inr\partial_r + (1 + \frac{u}{r}\,n^2)\partial_\varphi\big) + \dots$$

whose Lie bracket algebra generates superrotations & -translations

$$[L_n,\, L_m] = (n-m)\,L_{n+m} + \frac{c_L}{12}\,\big(n^3 - n\big)\,\delta_{n+m,\,0}$$

$$[L_n,\, M_m] = (n-m)\,M_{n+m} + \frac{c_M}{12}\,\big(n^3 - n\big)\,\delta_{n+m,\,0}$$

$$[M_n,\, M_m] = 0$$

canonical realization of asymptotic symmetries can lead to central extensions, depending on the bulk theory

algebra above is Carrollian conformal algebra Duval, Gibbons, Horvathy '14 and isomorphic to Galilean conformal algebra in 2d Bagchi '10

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1. analog of Brown–Henneaux bc's leading to BMS_3 Barnich, Compère '06
2. match of linearized spectra & gap in spectrum Bagchi, Detournay, DG '12

e.g. in flat space chiral gravity

$$\mathrm{CCFT}_2 : \psi^{(n)} = L_{-n}|0\rangle \qquad\qquad \mathrm{BMS}_3 : \psi^{(n)}_{uu} = -2ne^{-in\varphi}\dots$$

Virsoro descendants of vacuum mapped to "boundary gravitons"

gap: ground state (global Minkowski vacuum) has Virasoro charge $Q_{L_0} = -k$ while non-perturbative states (flat space cosmologies) have Virasoro charge $Q_{L_0} = k\,\alpha^2$

global Minkowski:

$$-\,\mathrm{d}u^2 - 2\,\mathrm{d}u\,\mathrm{d}r + r^2\;\mathrm{d}\varphi^2$$

flat space cosmologies (with horizon radius r_0):

$$\alpha^2\left(1 - \frac{r_0^2}{r^2}\right)\,\mathrm{d}u^2 - 2\,\mathrm{d}u\,\mathrm{d}r + r^2\left(\mathrm{d}\varphi - \frac{\alpha r_0}{r^2}\;\mathrm{d}u\right)^2$$

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3. microstates á la Cardy Barnich; Bagchi, Detournay, Fareghbal, Simón '12

thermal entropy of flat space cosmologies:

$$S_{\text{macro}} = S_{\text{FSC}} = S_{\text{BH}} = \frac{2\pi\, r_0}{4G}$$

thermal entropy in Carrollian CFT_2 using Cardyology:

$$S_{\text{micro}} = S_{\text{CCFT}_2} = 2\pi\, h_L \sqrt{\frac{c_M}{24 h_M}}$$

fineprint: assumed here vanishing Virasoro central charge and non-vanishing BMS central charge, like in Einstein gravity

explicit computation shows $h_L = \frac{\alpha r_0}{4G}$, $h_M = \frac{\alpha^2}{8G}$, $c_M = \frac{3}{G}$ and thus

$$S_{\text{macro}} = S_{\text{micro}}$$

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4. (holographic) entanglement entropy Bagchi, Basu, DG, Riegler '14

Carrollian CFT_2 calculation yields EE for vacuum on the plane

$$S_{\text{EE}} = \frac{c_L}{6}\,\ln\frac{\Delta x}{\varepsilon_x} + \frac{c_M}{6}\Big(\frac{\Delta u}{\Delta x} - \frac{\varepsilon_u}{\varepsilon_x}\Big)$$

get also EE for other states related to vacuum by uniformization DG, Parekh, Riegler '19

another example: EE for vacuum on the cylinder

$$S_{\text{EE}} = \frac{c_L}{6}\,\ln\frac{2\sin\frac{\Delta\varphi}{2}}{\varepsilon_\varphi} + \frac{c_M}{6}\Big(\frac{\Delta u}{2}\,\cot\frac{\Delta\varphi}{2} - \frac{\varepsilon_u}{\varepsilon_\varphi}\Big)$$

results reproduced on gravity side using Wilson lines Basu, Riegler '15 or swing construction with geodesics Jiang, Song, Wen '17; Apolo, Jiang, Song, Zhong '20

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5. (holographic) stress tensor correlation functions Bagchi, DG, Merbis '15

$CCFT_2$ conservation equations (M: supertranslations, L: superrotations)

$$\partial_u \langle M\, \mathcal{O}\rangle = 0 \qquad\qquad \partial_u \langle L\, \mathcal{O}\rangle = \partial_\varphi \langle M\, \mathcal{O}\rangle$$

yield BPZ-like recursion relations ($s_{ij} = 2\sin[(\varphi_1 - \varphi_2)/2]$, $c_{ij} = \cot[(\varphi_1 - \varphi_2)/2]$)

$$\langle M_1 L_2 \dots L_n\rangle = \sum_{i=2}^{n} \left(\frac{2}{s_{1i}^2} + \frac{c_{1i}}{2}\,\partial_{\varphi_i}\right) \langle M_2 L_3 \dots L_n\rangle$$

$$\langle L_1 L_2 \dots L_n\rangle = \frac{c_L}{c_M}\,\langle M_1 L_2 \dots L_n\rangle + \sum_{i=1}^{n} u_i \partial_{\varphi_i}\,\langle M_1 L_2 \dots L_n\rangle$$

reproduced on gravity side (in Chern–Simons formulation)

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FWF
Der Wissenschaftsfonds.

7-point function of the stress-tensor from 3D holography.

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Wiedner Hauptstrasse 8-10/136 A-1040 Wien, AUSTRIA

$$\langle T(z_1)T(z_2)T(z_3)T(z_4)T(z_5)T(z_6)T(z_7)\rangle =$$

Based on work presented in "Stress tensor correlators in three-dimensional gravity" with A. Bagchi and D. Grumiller. Special thanks to Friedrich Schöller for Mathematica wizardry

BMS_3 in Einstein gravity

Asymptotic symmetries in asymptotically flat space for 3d Einstein gravity:
Ashtekar, Bicak, Schmidt '96; Barnich, Compère '06

$$[L_n,\, L_m] = (n-m)\, L_{n+m}$$
$$[L_n,\, M_m] = (n-m)\, M_{n+m} + \frac{c_M}{12}\, n(n^2-1)\, \delta_{n+m,\,0}$$
$$[M_n,\, M_m] = 0$$

numerous holographic checks & results based on BMS_3 symmetries

- flat space chiral gravity Bagchi, Detournay, DG '12
- cardyology Bagchi, Detournay, Fareghbal, Simón; Barnich '12
- phase transitions Bagchi, Detournay, DG, Simón '13
- entanglement entropy Bagchi, Basu, DG, Riegler '14; Jiang, Song, Wen '17
- holographic dictionary & 1-point fct.'s Detournay, DG, Schöller, Simón '14
- 1-loop partition fct. & BMS characters Barnich,González,Maloney,Oblak '15
- all stress tensor correlators Bagchi, DG, Merbis '15
- BMS bootstrap Bagchi, Gary, Zodinmawia '16
- BMS blocks Hijano '18
- ...

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- L_n: superrotations (diff(S^1))
- M_n: supertranslations
- $c_M = 3/G$: BMS_3-central charge

Note: c_M dimensionful $\Rightarrow$ dimensionless ratios still meaningful

$$\frac{c_M}{h_M}\,, \qquad \frac{c_M^{\text{UV}}}{c_M^{\text{IR}}}\,, \qquad c_M \times \text{length}$$

where

$$M_0|\Psi\rangle = h_M|\Psi\rangle$$

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- L_n: superrotations (diff(S^1))
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- $c_M = 3/G$: BMS_3-central charge
- c_L: non-zero for TMG, but zero in Einstein gravity
- BMS_3 emerges as UR limit of CFT_2 symmetries $\Rightarrow$ Carrollian CFT_2

$$L_n := \mathcal{L}_n^+ - \mathcal{L}_{-n}^- \qquad\qquad M_n := \frac{1}{\ell}\,(\mathcal{L}_n^+ + \mathcal{L}_{-n}^-)$$

with Virasoros $[\mathcal{L}_n^\pm,\, \mathcal{L}_m^\pm] = (n-m)\,\mathcal{L}_{n+m}^\pm + \frac{c^\pm}{12}\,(n^3-n)\,\delta_{n+m,\,0}$
yields $\mathrm{CCFT}_2 \simeq \mathrm{BMS}_3$ with $c_L = c^+ - c^-$ and $c_M = \frac{1}{\ell}\,(c^+ + c^-)$

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Zamolodchikov c-function

Consider RG-flow from UV to IR in QFT_2

- coupling constants g^i have β-functions

$$\dot{g}^i = \beta^i(g)$$

- there exists a function $c(g)$ with the properties
  1. Monotonicity

$$\dot{c}(g) := \beta^i(g)\,\frac{\partial c(g)}{\partial g^i} \leq 0$$

  2. Saturation at fixed points

$$\dot{c}(g_*) = 0 \qquad \leftrightarrow \qquad \beta^i(g_*) = 0$$

  Note: at fixed points enhancement to CFT_2 Virasoro symmetries

$$[L_n,\, L_m] = (n-m)\, L_{n+m} + \frac{c_{\text{Vir}}(g_*)}{12}\, n(n^2-1)\,\delta_{n+m,\,0}$$

Zamolodchikov c-function

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  3. Equality to Virasoro central charge

$$c(g_*) = c_{\text{Vir}}(g_*)$$

- Consequence:

$$c_{\text{Vir}}^{\text{UV}} > c_{\text{Vir}}^{\text{IR}} \qquad \Rightarrow \qquad \text{more dof in UV}$$

Zamolodchikov c-function

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- Explicit construction of a c-function using stress tensor and its 2-point correlators Zamolodchikov '86

Domain wall c-function in $\text{AdS}_3/\text{CFT}_2$ (Freedman, Gubser, Pilch, Warner '99)

Two interesting alternative c-functions in holography context

domain walls in AdS_3 (set AdS radius to unity)

$$\mathrm{d}s^2 = \mathrm{d}\rho^2 + e^{2A(\rho)}\big(-\mathrm{d}t^2 + \mathrm{d}x^2\big) \qquad \lim_{\rho\to\infty} A(\rho) = \rho + \dots$$

holographic model for RG flow (UV: $\rho \to \infty$)

- $\rho = \text{const.}$ slices: Poincaré invariant

$$\text{KVs}: \qquad \partial_t \qquad \partial_x \qquad x\partial_t + t\partial_x$$

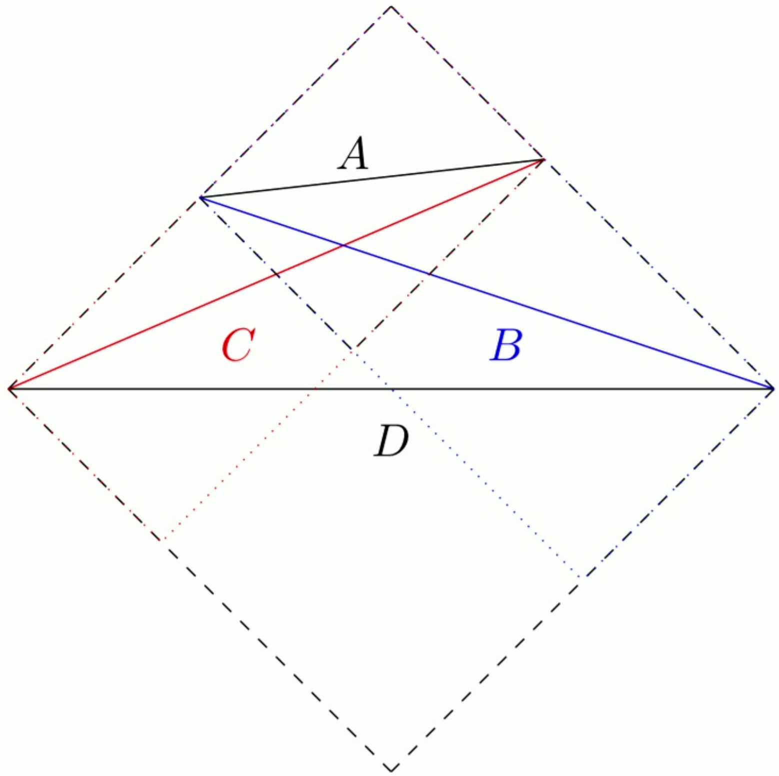
conformal KVs: infinitely many (CFT_2 symmetries)

- holographic domain wall c-function

$$c_{\text{dw}}(\rho) = \frac{c^{\text{UV}}}{A'(\rho)}$$

monotonicity implied by reality of bulk scalar field [see later!]

Casini–Huerta (CH) c-function (CH '06)



$$AD = BC \Rightarrow C = \lambda A,\ D = \lambda B,\ \lambda > 1$$

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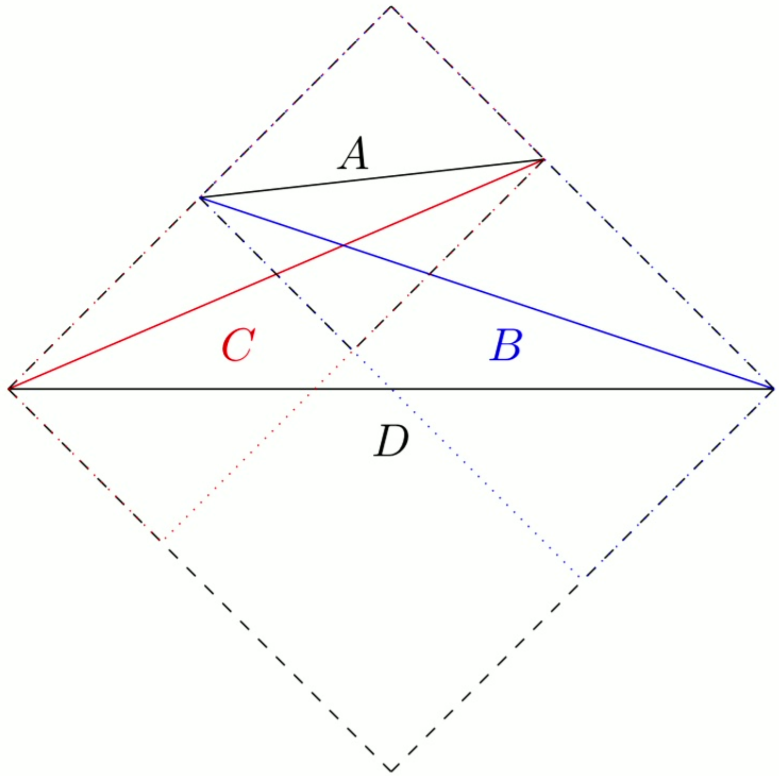


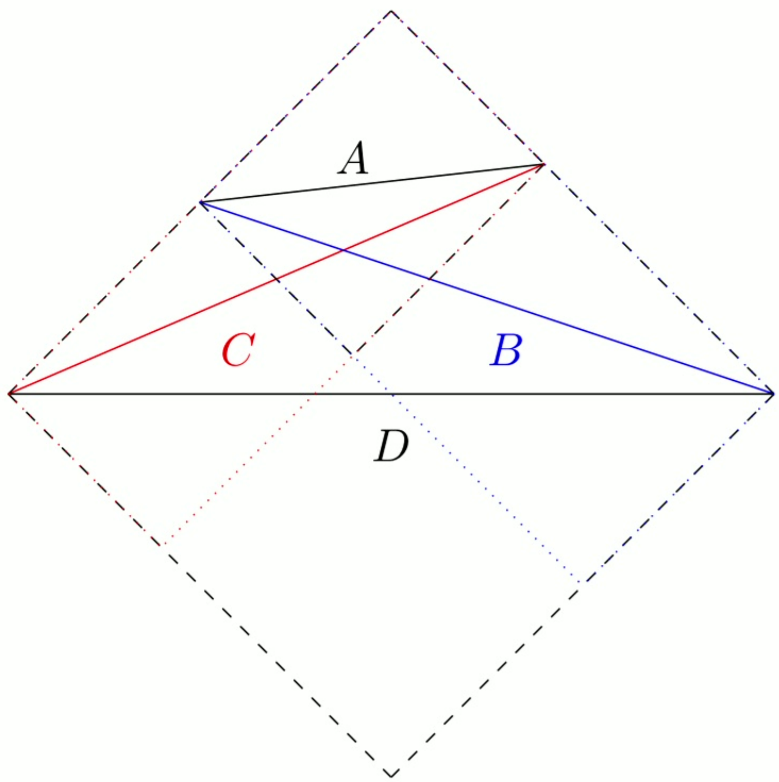
- SSA of EE

$$S(C) + S(B) \geq S(A) + S(D)$$

Lieb, Ruskai '73; Kiefer '59

$$AD = BC \;\Rightarrow\; C = \lambda A,\; D = \lambda B,\; \lambda > 1$$

Casini–Huerta (CH) c-function (CH '06)



- SSA of EE

$$S(C) + S(B) \geq S(A) + S(D)$$

- Use Minkowski diagram

$$S(B) - S(A) \geq S(\lambda B) - S(\lambda A)$$

- Differential instead difference

$$c_{\text{CH}}(L) = \# \, L \, S'(L)$$

non-increasing under dilatations!

$$AD = BC \;\; \Rightarrow \;\; C = \lambda A, \; D = \lambda B, \; \lambda > 1$$

Properties of CH c-function

- Monotonicity

$$c'_{\text{CH}}(L) = 3L\, S''(L) + 3S'(L) \leq 0$$

- Fixed point values

$$\lim_{L\to 0} c_{\text{CH}}(L) = c^{\text{UV}} \qquad \lim_{L\to\infty} c_{\text{CH}}(L) = c^{\text{IR}}$$

$\Rightarrow$ is indeed a c-function different from previous ones!

- Monotonicity of CH c-function

$$\frac{c'_{\text{CH}}(L)}{3L} = S''(L) - \frac{S'(L)}{L} + \frac{6}{c_{\text{CH}}(L)}\,(S'(L))^2 \leq 0$$

implies ground state QNEC [see next slide]

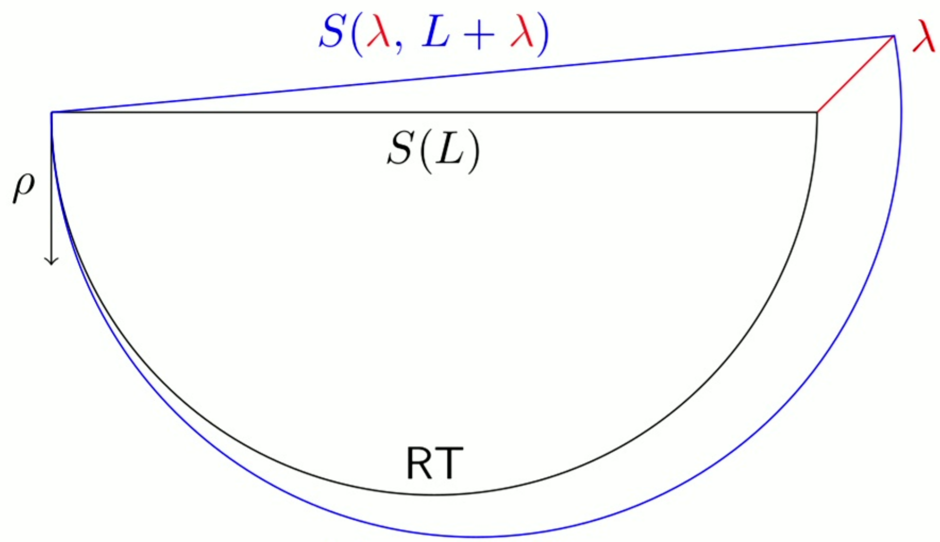
$$S''(L) - \frac{S'(L)}{L} + \frac{6}{c_{\text{UV}}}\,(S'(L))^2 \leq 0$$

Ecker, DG, Soltanpanahi, Stanzer '20

QNEC in 2d

Quantum Null Energy Condition (QNEC) in 2d:

$$2\pi \langle T_{\pm\pm} \rangle \geq \frac{\mathrm{d}^2 S}{\mathrm{d}\lambda^2}\bigg|_{\lambda=0} + \frac{6}{c^{\mathrm{UV}}} \left(\frac{\mathrm{d}S}{\mathrm{d}\lambda}\right)^2\bigg|_{\lambda=0}$$



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- both sides of QNEC transform with Schwarzian derivative Wall '11

  under bulk diffeos (or boundary conformal trafos):

$$\delta_\xi S = \underbrace{\xi S' - \frac{c}{12}\,\xi'}_{\text{anomalous scalar}}$$

implies

$$\delta_\xi Q = \underbrace{\xi Q' + 2\xi' Q - \frac{c}{12}\,\xi'''}_{\text{infinitesimal Schwarzian}}$$

with $Q := S'' + \frac{6}{c}\,(S')^2$

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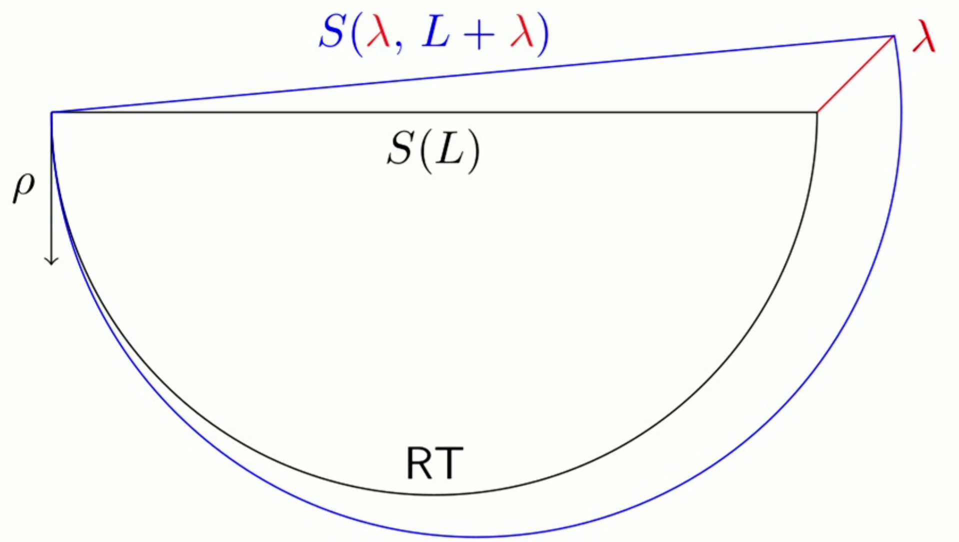

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Bousso, Fisher, Leichenauer, Wall '15

- both sides of QNEC transform with Schwarzian derivative Wall '11
- QNEC saturates for states dual to vacuum Einstein solutions
  Khandker, Kundu, Li '18; Ecker, DG, Sheikh-Jabbari, Stanzer, van der Schee '20
- for boost invariant ground states:

$$0 \geq \frac{\mathrm{d}^2 S}{\mathrm{d}\lambda^2}\bigg|_{\lambda=0} + \frac{6}{c^{\mathrm{UV}}} \left(\frac{\mathrm{d}S}{\mathrm{d}\lambda}\right)^2\bigg|_{\lambda=0} = S''(L) - \frac{S'(L)}{L} + \frac{6}{c^{\mathrm{UV}}}\,(S')^2$$

since $S(\lambda,\, L+\lambda) = S(0,\, \sqrt{(L+\lambda)^2 - \lambda^2})$
Ecker, DG, Sheikh-Jabbari, Stanzer '20

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  since $S(\lambda,\, L+\lambda) = S(0,\, \sqrt{(L+\lambda)^2 - \lambda^2})$
  Ecker, DG, Sheikh-Jabbari, Stanzer '20
- ground state QNEC necessary for monotonicity of CH c-function

Flat space domain walls

$$\mathrm{d}s^2 = -e^{A(r)}\, 2\,\mathrm{d}u\,\mathrm{d}r + e^{2A(r)}\;\mathrm{d}x^2$$

as solutions of Einstein–Klein–Gordon bulk theory

$$I_{\text{bulk}}[g_{\mu\nu},\,\phi] = \frac{1}{16\pi G}\int \mathrm{d}^3x\sqrt{-g}\,\Big(R - \frac{1}{2}\,(\partial\phi)^2 - V(\phi)\Big)$$

- $r = r_0 = const.$ slices have degenerate metric $0\;\mathrm{d}u^2 + 0\;\mathrm{d}u\,\mathrm{d}x + e^{2A(r_0)}\,\mathrm{d}x^2$ whose conformal KVs form BMS$_3$

$$\text{conformal KVs:} \qquad \big(\xi_M(x) + u\,\xi_L'(x)\big)\,\partial_u + \xi_L(x)\,\partial_x$$

  $\xi_L(x)$: generates diff(S^1)
  $\xi_M(x)$: generates supertranslations

Flat space domain walls

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- $r = r_0 = const.$ slices have degenerate metric $0\,\mathrm{d}u^2 + 0\,\mathrm{d}u\,\mathrm{d}x + e^{2A(r_0)}\,\mathrm{d}x^2$ whose conformal KVs form BMS_3
- Ricci tensor $R_{\mu\nu} = 0$ except $R_{rr} = -A''$ (VSI spacetime)
- Einstein–Klein–Gordon: EOM hold for any function $A(r)$ provided
  1. $V(\phi) = 0$
  2. $\frac{1}{2}\,(\phi')^2 = -A''$

Flat space domain walls

$$\mathrm{d}s^2 = -e^{A(r)}\, 2\, \mathrm{d}u\, \mathrm{d}r + e^{2A(r)}\ \mathrm{d}x^2$$

as solutions of Einstein–Klein–Gordon bulk theory

$$I_{\text{bulk}}[g_{\mu\nu},\, \phi] = \frac{1}{16\pi G} \int \mathrm{d}^3x\sqrt{-g}\, \Big(R - \frac{1}{2}\, (\partial\phi)^2 - V(\phi)\Big)$$

- $r = r_0 = const.$ slices have degenerate metric $0\ \mathrm{d}u^2 + 0\ \mathrm{d}u\, \mathrm{d}x + e^{2A(r_0)}\, \mathrm{d}x^2$ whose conformal KVs form BMS_3
- Ricci tensor $R_{\mu\nu} = 0$ except $R_{rr} = -A''$ (VSI spacetime)
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  1. $V(\phi) = 0$
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Domain wall c-function Fareghbal, Naseh, Rouhani PLB (2017)

Is holographic domain wall function

$$c_{\text{dw}}(r) = \frac{c_M^{\text{UV}}}{A'(r)}$$

c-function for Carrollian QFTs with BMS_3 invariant UV & IR fixed points?

- reality of scalar field $\frac{1}{2}(\phi')^2 = -A''$ implies monotonicity

$$A'' \leq 0$$

- asymptotic flatness implies correct UV value

$$\lim_{r\to\infty} c_{\text{dw}}(r) = \lim_{r\to\infty} \frac{c_M^{\text{UV}}}{A'(r)} = c_M^{\text{UV}}$$

- if the corresponding AdS_3 domain wall flows to an IR fixed point also the flat space domain wall flows to an IR fixed point and we have

$$\frac{c^{\text{UV}}}{c^{\text{IR}}} = \frac{c_M^{\text{UV}}}{c_M^{\text{IR}}} \qquad \text{with } c_M^{\text{UV}} > c_M^{\text{IR}}$$

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Domain wall example

Take flat space domain wall related to AdS domain wall with super-potential $W(\phi) = -2 - \frac{1}{4}\,\phi^2 - \frac{\alpha}{8}\,\phi^4$ (mass $m^2 = -\frac{3}{4}$ above BF)

- yields profile function

$$A(r) = r + A_0 - \frac{j^2}{16}\left(\frac{1}{e^r - \alpha j^2} + \frac{r - \ln\left(e^r - \alpha j^2\right)}{\alpha j^2}\right)$$

- for positive α: hit singularity (no IR fixed point)
- for negative α: flow to IR fixed point

$$A(r \to -\infty) = \left(1 - \frac{1}{16\alpha}\right) r + A_0 + \frac{1 + \ln\left(-\alpha j^2\right)}{16\alpha} + \mathcal{O}(e^{2r})$$

- can read off BMS$_3$ central charges at fixed points

$$c_M^{\mathrm{IR}} = \frac{c_M^{\mathrm{UV}}}{1 - \frac{1}{16\alpha}} < c_M^{\mathrm{UV}}$$

more info in plot of flat space domain wall c-function

Outline

Flat space holography á la Carroll

Holographic c-functions in $\text{AdS}_3/\text{CFT}_2$

Flat space domain walls and BMS_3 c-functions

Outlook to Casini–Huerta-like c-function

Domain wall example

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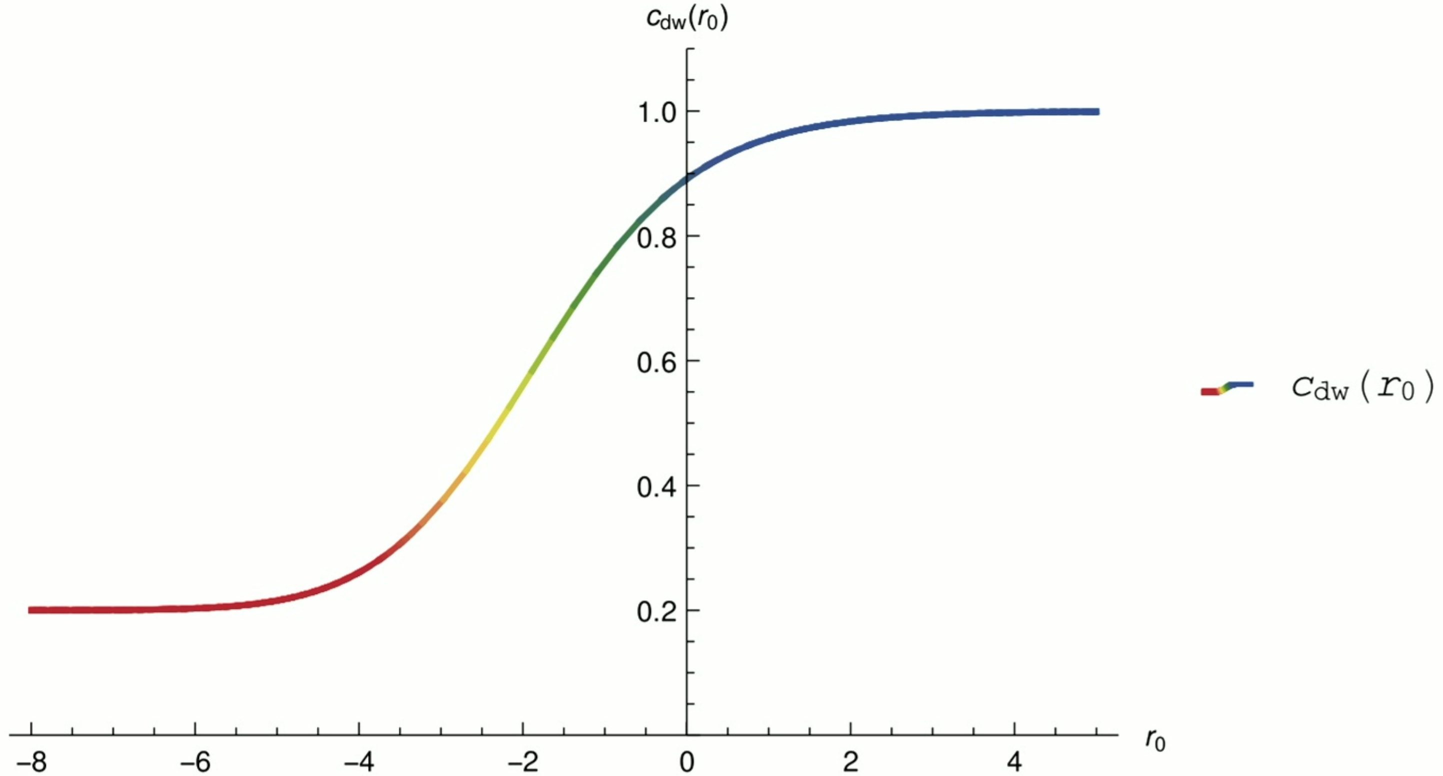

Quantum energy conditions in CCFT$_2$ DG, Parekh, Riegler '19

- split EE into L- and M-parts:

$$S_{\text{EE}} = S_L + S_M \qquad S_L = \frac{c_L}{6} \ln \frac{\Delta x}{\epsilon_x} \qquad S_M = \frac{c_M}{6} \left(\frac{\Delta u}{\Delta x} - \frac{\epsilon_u}{\epsilon_x}\right)$$

- for $c_L \neq 0 = c_M$ QEC like chiral half of QNEC

$$2\pi \langle L \rangle \geq S_L'' + \frac{6}{c_L} S_L'^2$$

prime means spatial variation of the entangling region

- for $c_M \neq 0 = c_L$ QEC is

$$2\pi \langle M \rangle \geq \dot{S}_M' + \frac{6}{c_M} \dot{S}_M^2$$

dot means temporal variation of the entangling region

QECs at our disposal — Casini–Huerta-like c-function conceivable!

Tentative proposal for Casini–Huerta-like c-function

- for $c_L \neq 0 = c_M$ CH-like c-function postulated as

$$c_L(\Delta x) = 6\Delta x \, S'_L$$

- sanity checks: correct normalization; does monotonicity imply QEC?

$$\frac{c'_L}{6\Delta x} = S''_L + \frac{6}{c_L} S'^2 \leq 0$$

yields QNEC combination (like chiral half of CFT_2)

- for $c_M \neq 0 = c_L$ instead postulate c-function

$$c_M(\Delta x) = 6\Delta x \, \dot{S}_M$$

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$$\frac{c_M'}{6\Delta x} = \dot{S}_M' + \frac{6}{c_M} \, \dot{S}^2 \leq 0$$

yields indeed other QEC

> Monotonicity of conjectured c-functions yields ground state QEC

Summary

Take-away lessons from this talk:

- Carrollian approach to flat space holography makes sharp predictions (symmetries, spectrum, microstates, entanglement, correlators, ...)
- flat space domain walls from AdS_3 domain walls (exact solutions of 3d Einstein-dilaton gravity, in flat space without potential, in AdS_3 with potential)
- proved domain wall c-function is a $CCFT_2$ c-function with all the right properties (monotonicity from bulk unitarity, correct fixed-point values of BMS_3 central charges)
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All results in 3 bulk dimensions. Do not know how/if all results generalize to other dimensions.

Conclusions by ChatGPT-3.5 (abridged)

Ladies and gentlemen, esteemed colleagues, and fellow enthusiasts of the Carrollian cosmos,
As we prepare to wrap up our journey through the whimsical wonderland of ''Carrollian c-functions and flat space holographic RG flows,'' I can't help but feel a bit like Alice herself, exploring the curious corners of spacetime where BMS symmetries, Carrollian CFTs, and c-functions come together in a harmonious dance.
Our adventure today has taken us down the rabbit hole of flat space holography. But as we prepare to return from this captivating journey, let us remember that in science, as in Wonderland, curiosity is our guiding star. As Carrollian characters once said, ''Begin at the beginning, and go on till you come to the end: then stop.''

My conclusion

Carrollian holography is a lot of fun — feel free to join!

AI prompt: Carroll and Zamolodchikov meet 't Hooft and Susskind at the rabbit hole to wonderland