Title: Probe Fundamental Physics via BH Perturbation Theory

Speakers: Dongjun Li

Series: Strong Gravity

Date: October 12, 2023 - 1:00 PM

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Abstract: Anticipating the launch of several next-generation gravitational wave (GW) detectors in the 2030s, we will be able to more precisely measure spacetime ripples from binary black hole (BH) mergers in a larger parameter space. The forthcoming data will require us to develop more accurate predictions of GWs not only in General Relativity (GR) but also in theories beyond GR and diverse astrophysical environments. Black hole perturbation theory is a cornerstone for making these predictions. In recent years, there have been extensive studies of perturbations of BHs in theories beyond GR, but only for non-rotating or slowly rotating BHs. In this talk, I will present a new formalism, based on Teukolsky's seminal work in the 1970s, to study perturbations of BHs with arbitrary spin in beyond-GR theories and in more complicated astrophysical environments. I will first discuss how to derive a modified Teukolsky equation for BHs deforming perturbatively from their counterparts in GR due to beyond-GR or environmental effects and the necessary techniques to evaluate this equation. Subsequently, I will discuss some applications of this formalism. Specifically, I will prescribe utilizing this formalism to investigate the isospectrality breaking of quasinormal modes (QNMs) in beyond-GR theories, compute the QNM frequency shifts in some specific theories, and efficiently extract these shifts from observation data. Furthermore, I will also show how to apply this formalism to study extreme mass-ratio inspirals beyond GR.

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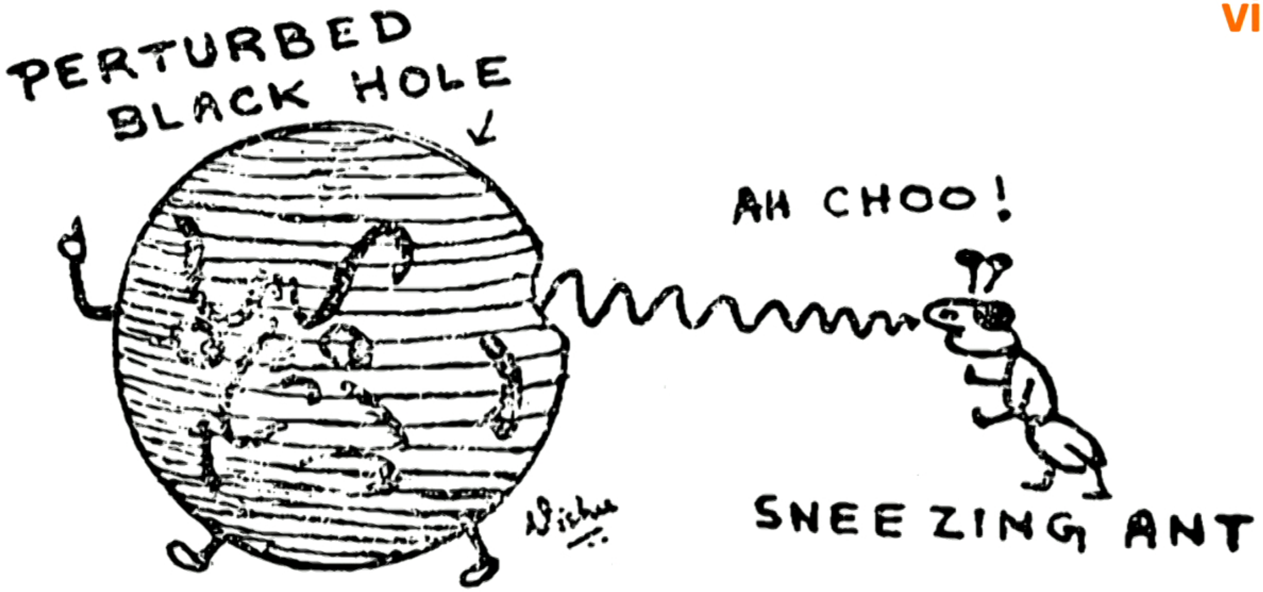
Zoom link: https://pitp.zoom.us/j/99282316326?pwd=REtBSFUxdlgxUGVwZFFvVEVBVnFTUT09

Caltech The Division of Physics, Mathematics and Astronomy

Probe Fundamental Physics via BH Perturbation Theory

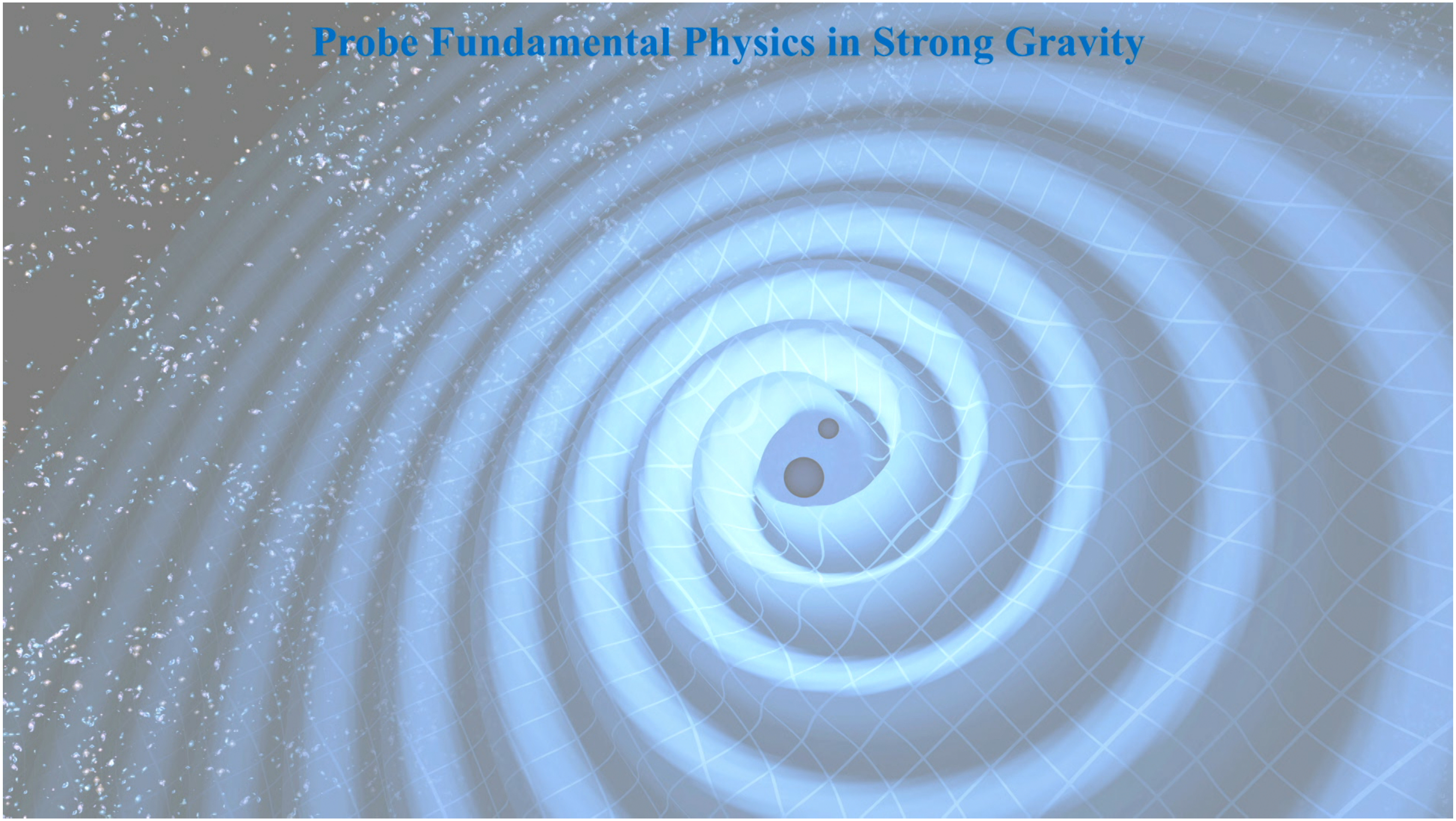


(Image credit: C.V. Vishveshwara)

Dongjun Li (Caltech)
Strong Gravity Seminar
Perimeter Institute,
October 12th, 2023

In collaboration with
Caltech: Colin Weller, Yanbei Chen
Perimeter: Michael LaHaye, Huan Yang
UIUC: Pratik Wagle, Nicolas Yunes
UT Austin: Asad Hussain, Aaron Zimmerman

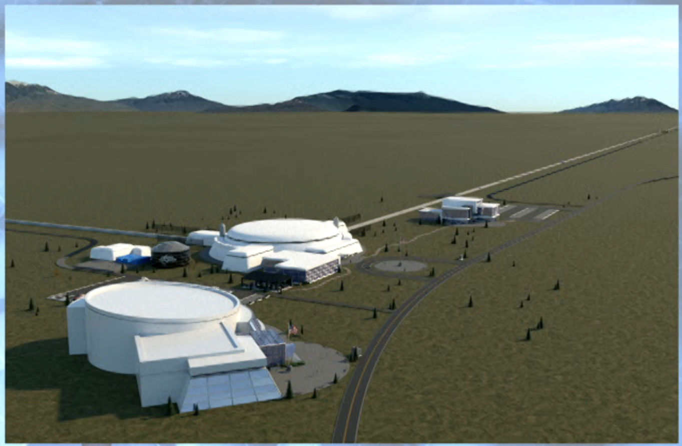
Probe Fundamental Physics in Strong Gravity



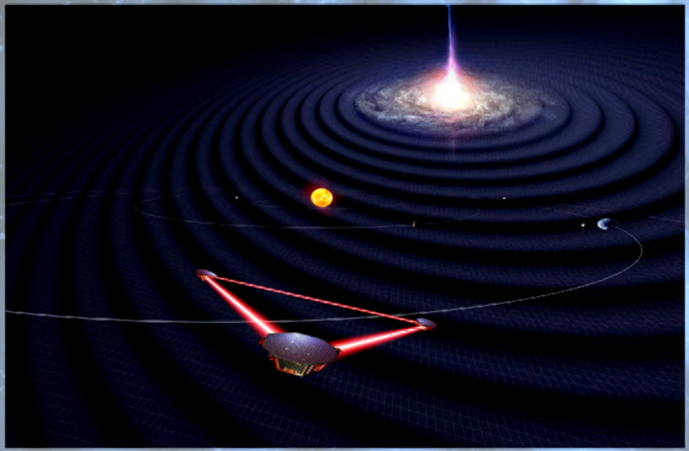
Probe Fundamental Physics in Strong Gravity

LVK Collaboration

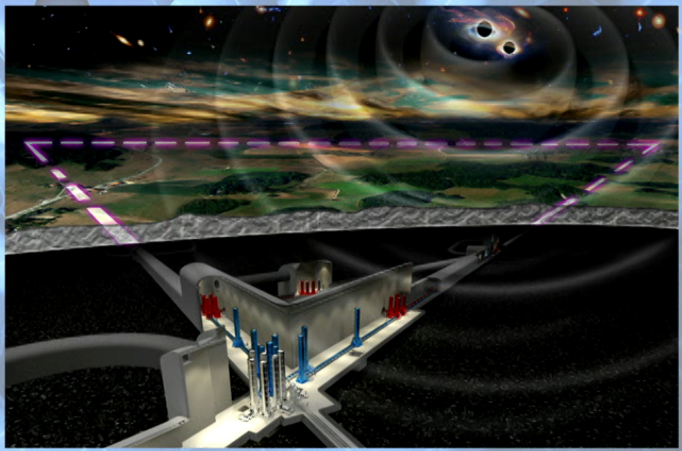
Cosmic Explorer (Image credit: Eddie Anaya)



LISA (Image credit: Simon Barke)



Einstein Telescope

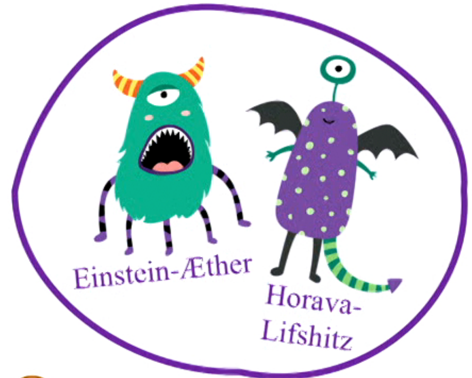


Beyond GR effects

- ❑ Incompatibility between GR and QM → Unified theories, e.g., loop gravity, string theory
- ❑ Observational anomalies, e.g., matter-antimatter asymmetry → Modify GR

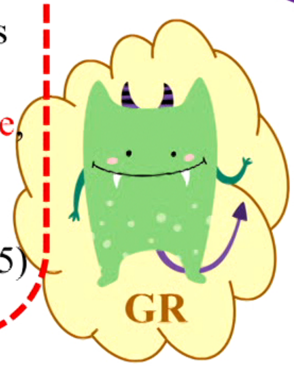
Higher Derivative

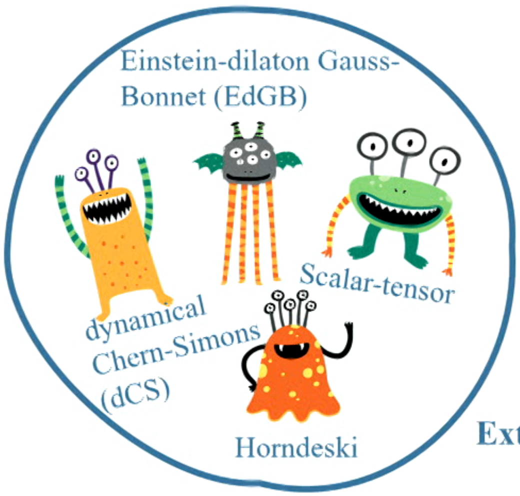
Lorentz-violating



Lovelock's theorem:

In four spacetime dimensions,
the only divergence-free symmetric
rank-2 tensor constructed
solely from the metric and its derivatives
up to second differential order,
and preserving diffeomorphism invariance,
is the Einstein tensor
plus a cosmological term.
(Lovelock, 1971 & 1972; Berti et al., 2015)



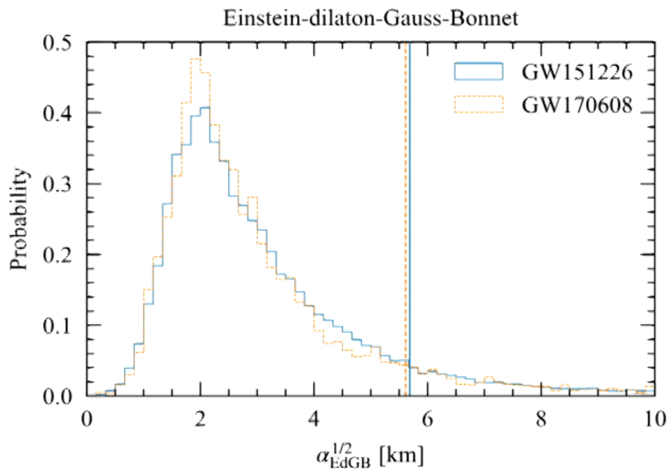


Extra Scalar Field

Massive Gravity

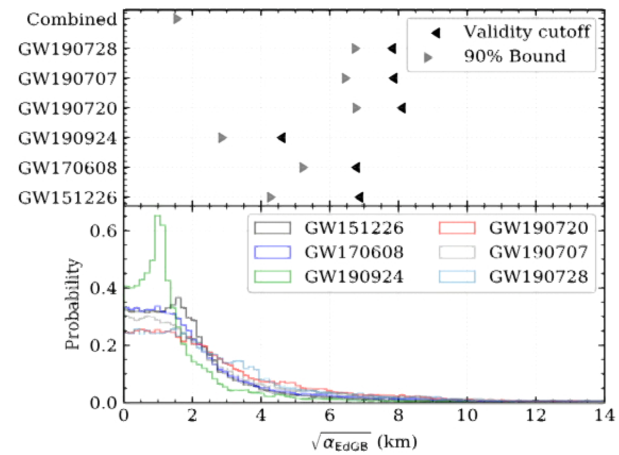
Theory	Parameter	Current bound	Most (Least) Stringent Forecasted Bound
Generic Dipole	$\delta\dot{E}$	1.1×10^{-3} [44, 45]*	10^{-11} (10^{-6})
Einstein-dilaton-Gauss-Bonnet	$\sqrt{\alpha_{\rm EdGB}}$	1 km [46] 3.4 km [47]*	10^{-3} (1) km
Black Hole Evaporation	$\dot{M}$	–	10^{-8} (10^{2}) $M_\odot$/yr
Time Varying G	$\dot{G}$	$10^{-13}-10^{-12}$ yr^{-1} [48–52]	10^{-9} (10) yr^{-1}
Massive Graviton	m_g	10^{-29}eV [53–56] 10^{-23}eV [3, 57]*	10^{-26} (10^{-24}) eV
dynamic Chern Simons	$\sqrt{\alpha_{\rm dCS}}$	5.2 km [58]	10^{-2} (10) km
Non-commutative Gravity	$\sqrt{\Lambda}$	2.1 l_p [59]*	10^{-3} (10^{-1}) l_p

(Perkins et al., arXiv 2010.09010)



(Nair et al., arXiv 1906.00870)

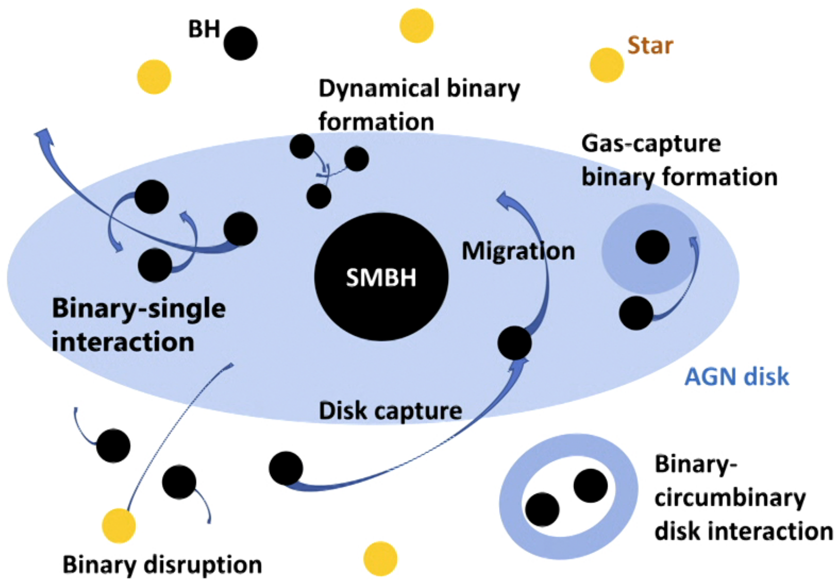
Better Constraints!



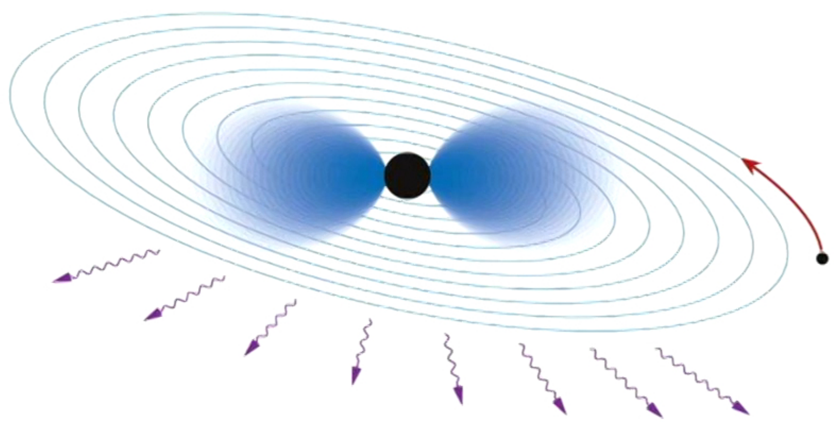
(Perkins et al., arXiv 2104.11189)

Non-vacuum environment

- ❑ Ordinary matter, such as gaseous accretion disk, plasma, other compact objects
- ❑ Dark matter, such as ultralight boson clouds, dark matter spikes, primordial BHs

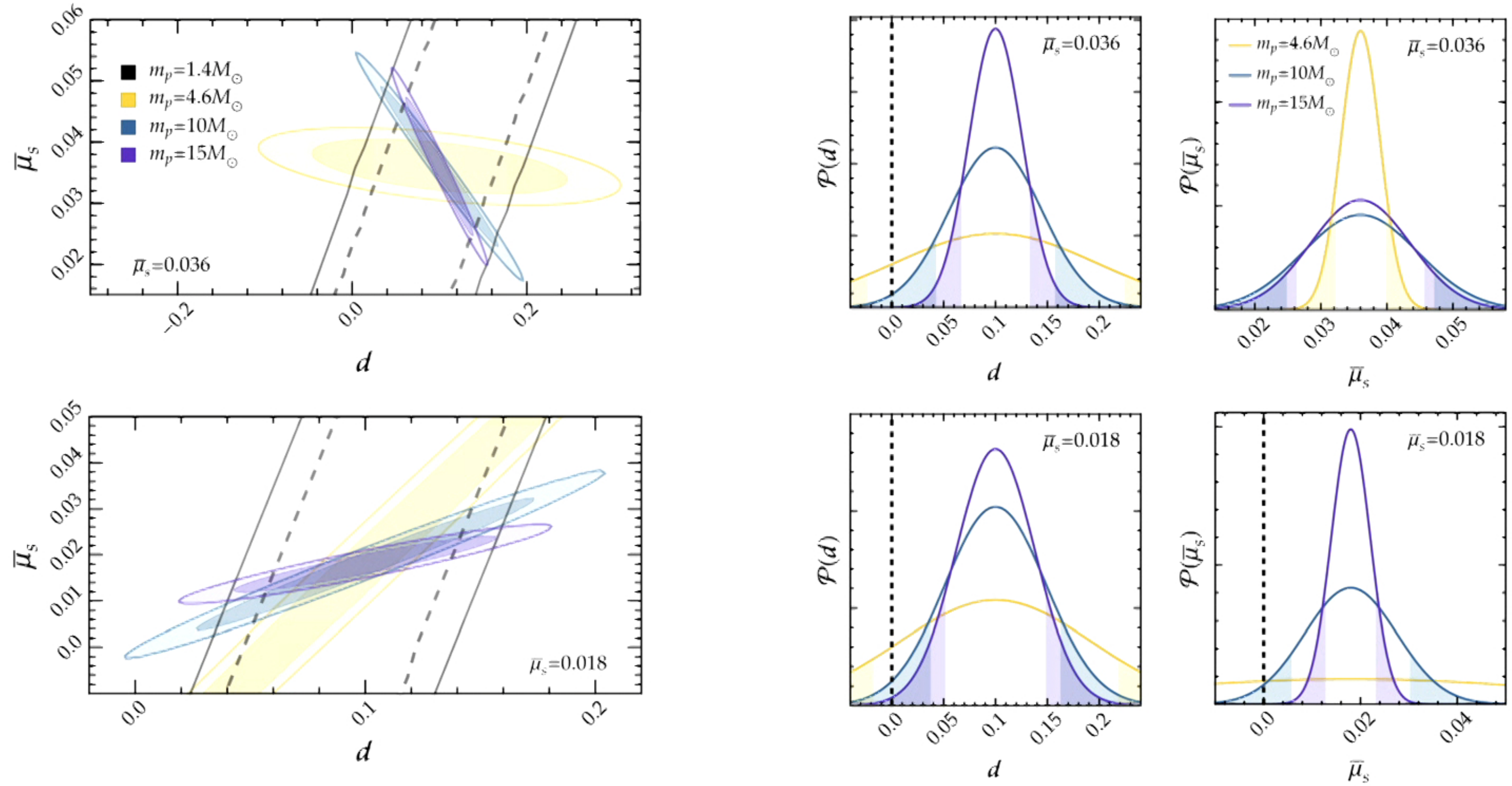


(Tagawa et al., arXiv 1912.08218)



(Baumann et al., arXiv 2206.01212)

Example: ultralight scalar



$d = 0.1\,,$

$M = 10^6 M_\odot\,,$

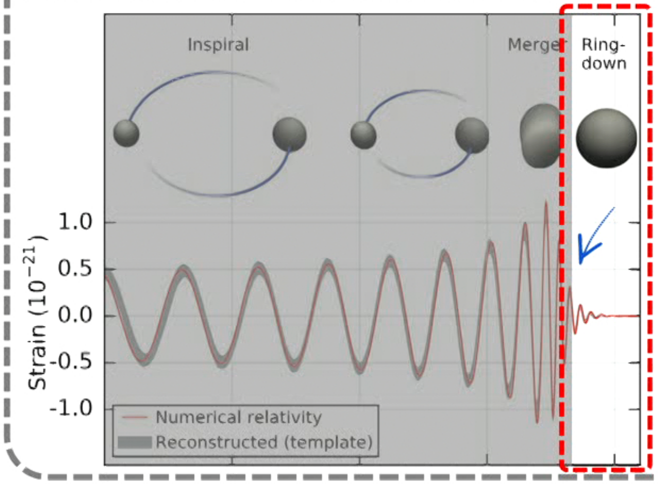
$\chi = 0.9$

Detect nonzero mass and charge

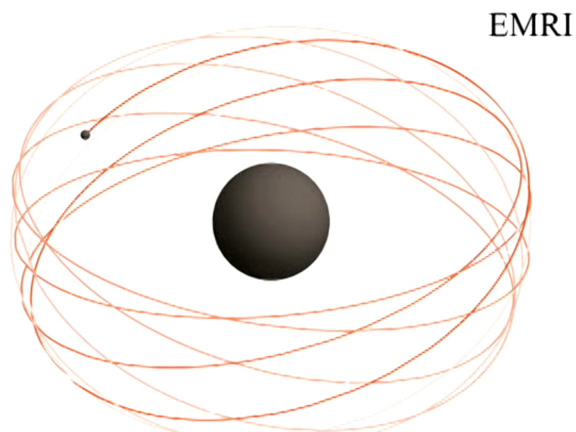
(Barsanti et al., arXiv 2212.03888)

Black Hole Perturbation Theory

Black Hole Perturbation Theory



(Image Credit: Leo C. Stein)



(Image Credit: N. Franchini)

Up to 10 highly coupled PDEs!

Non-rotating BHs

Regge-Wheeler and Zerilli-Moncrief eqns (perturb metric)
- ❑ 2 decoupled separable PDEs (Regge & Wheeler, 1957; Zerilli, 1970)

Also works for slowly rotating BHs

Rotating BHs

Teukolsky eqns (perturb curvature, based on NP formalism) **GR**
- ❑ > 50 PDEs → 2 decoupled separable PDEs for Ψ_0 & Ψ_4 (Teukolsky, 1973)

Metric can be reconstructed

Modified Teukolsky eqns (first-ever extension!) **bGR**
- ❑ Petrov type D → not necessarily for bGR BHs, e.g, dCS (Yagi et al., 2012)

Most remnant BHs are fast rotating!

- 65% of their maximum (Abbott et al., 2021; Buonanno et al., 2007)
- At least 5th **order** in the slow-rotation expansion (Pani, 2011)
  - → Coupling of many (l, m) modes!

(Abbott et al., arXiv 2111.03606)

Event	M ($M_\odot$)	$\mathcal{M}$ ($M_\odot$)	m_1 ($M_\odot$)	m_2 ($M_\odot$)	$\chi_{\rm eff}$	$D_{\rm L}$ (Gpc)	z	$M_{\rm f}$ ($M_\odot$)	$\chi_{\rm f}$	$\Delta\Omega$ (deg^2)	SNR
GW191103_012549	$20.0^{+3.7}_{-1.8}$	$8.34^{+0.66}_{-0.57}$	$11.8^{+6.2}_{-2.2}$	$7.9^{+1.7}_{-2.4}$	$0.21^{+0.16}_{-0.10}$	$0.99^{+0.50}_{-0.47}$	$0.20^{+0.09}_{-0.09}$	$19.0^{+3.8}_{-1.7}$	$0.75^{+0.06}_{-0.05}$	2500	$8.9^{+0.3}_{-0.5}$
GW191105_143521	$18.5^{+2.1}_{-1.3}$	$7.82^{+0.61}_{-0.45}$	$10.7^{+3.7}_{-1.6}$	$7.7^{+1.4}_{-1.9}$	$-0.02^{+0.13}_{-0.09}$	$1.15^{+0.43}_{-0.48}$	$0.23^{+0.07}_{-0.09}$	$17.6^{+2.1}_{-1.2}$	$0.67^{+0.04}_{-0.05}$	640	$9.7^{+0.3}_{-0.5}$
GW191109_010717	112^{+20}_{-16}	$47.5^{+9.6}_{-7.5}$	65^{+11}_{-11}	47^{+15}_{-13}	$-0.29^{+0.42}_{-0.31}$	$1.29^{+1.13}_{-0.65}$	$0.25^{+0.18}_{-0.12}$	107^{+18}_{-15}	$0.61^{+0.18}_{-0.19}$	1600	$17.3^{+0.5}_{-0.5}$
GW191113_071753	$34.5^{+10.5}_{-9.8}$	$10.7^{+1.1}_{-1.0}$	29^{+12}_{-14}	$5.9^{+4.4}_{-1.3}$	$0.00^{+0.37}_{-0.29}$	$1.37^{+1.15}_{-0.62}$	$0.26^{+0.18}_{-0.11}$	34^{+11}_{-10}	$0.45^{+0.33}_{-0.11}$	3600	$7.9^{+0.5}_{-1.1}$
GW191126_115259	$20.7^{+3.4}_{-2.0}$	$8.65^{+0.95}_{-0.71}$	$12.1^{+5.5}_{-2.2}$	$8.3^{+1.9}_{-2.4}$	$0.21^{+0.15}_{-0.11}$	$1.62^{+0.74}_{-0.74}$	$0.30^{+0.12}_{-0.13}$	$19.6^{+3.5}_{-2.0}$	$0.75^{+0.06}_{-0.05}$	1400	$8.3^{+0.2}_{-0.5}$
GW191127_050227	80^{+39}_{-22}	$29.9^{+11.7}_{-9.1}$	53^{+47}_{-20}	24^{+17}_{-14}	$0.18^{+0.34}_{-0.36}$	$3.4^{+3.1}_{-1.9}$	$0.57^{+0.40}_{-0.29}$	76^{+39}_{-21}	$0.75^{+0.13}_{-0.29}$	980	$9.2^{+0.7}_{-0.6}$
GW191129_134029	$17.5^{+2.4}_{-1.2}$	$7.31^{+0.43}_{-0.28}$	$10.7^{+4.1}_{-2.1}$	$6.7^{+1.5}_{-1.7}$	$0.06^{+0.16}_{-0.08}$	$0.79^{+0.26}_{-0.33}$	$0.16^{+0.05}_{-0.06}$	$16.8^{+2.5}_{-1.2}$	$0.69^{+0.03}_{-0.05}$	850	$13.1^{+0.2}_{-0.3}$
GW191204_110529	$47.2^{+9.2}_{-8.0}$	$19.8^{+3.6}_{-3.3}$	$27.3^{+11.0}_{-6.0}$	$19.3^{+5.6}_{-6.0}$	$0.05^{+0.26}_{-0.27}$	$1.8^{+1.7}_{-1.1}$	$0.34^{+0.25}_{-0.18}$	$45.0^{+8.6}_{-7.6}$	$0.71^{+0.12}_{-0.11}$	3700	$8.8^{+0.4}_{-0.6}$
GW191204_171526	$20.21^{+1.70}_{-0.96}$	$8.55^{+0.38}_{-0.27}$	$11.9^{+3.3}_{-1.8}$	$8.2^{+1.4}_{-1.6}$	$0.16^{+0.08}_{-0.05}$	$0.65^{+0.19}_{-0.25}$	$0.13^{+0.04}_{-0.05}$	$19.21^{+1.79}_{-0.95}$	$0.73^{+0.03}_{-0.03}$	350	$17.5^{+0.2}_{-0.2}$
GW191215_223052	$43.3^{+5.3}_{-4.3}$	$18.4^{+2.2}_{-1.7}$	$24.9^{+7.1}_{-4.1}$	$18.1^{+3.8}_{-4.1}$	$-0.04^{+0.17}_{-0.21}$	$1.93^{+0.89}_{-0.86}$	$0.35^{+0.13}_{-0.14}$	$41.4^{+5.1}_{-4.1}$	$0.68^{+0.07}_{-0.07}$	530	$11.2^{+0.3}_{-0.4}$
GW191216_213338	$19.81^{+2.69}_{-0.94}$	$8.33^{+0.22}_{-0.19}$	$12.1^{+4.6}_{-2.3}$	$7.7^{+1.6}_{-1.9}$	$0.11^{+0.13}_{-0.06}$	$0.34^{+0.12}_{-0.13}$	$0.07^{+0.02}_{-0.03}$	$18.87^{+2.80}_{-0.94}$	$0.70^{+0.03}_{-0.04}$	490	$18.6^{+0.2}_{-0.2}$
GW191219_163120	$32.3^{+2.2}_{-2.7}$	$4.32^{+0.12}_{-0.17}$	$31.1^{+2.2}_{-2.8}$	$1.17^{+0.07}_{-0.06}$	$0.00^{+0.07}_{-0.09}$	$0.55^{+0.25}_{-0.16}$	$0.11^{+0.05}_{-0.03}$	$32.2^{+2.2}_{-2.7}$	$0.14^{+0.06}_{-0.06}$	1500	$9.1^{+0.5}_{-0.8}$
GW191222_033537	79^{+16}_{-11}	$33.8^{+7.1}_{-5.0}$	$45.1^{+10.9}_{-8.0}$	$34.7^{+9.3}_{-10.5}$	$-0.04^{+0.20}_{-0.25}$	$3.0^{+1.7}_{-1.7}$	$0.51^{+0.23}_{-0.26}$	$75.5^{+15.3}_{-9.9}$	$0.67^{+0.08}_{-0.11}$	2000	$12.5^{+0.2}_{-0.3}$
GW191230_180458	86^{+19}_{-12}	$36.5^{+8.2}_{-5.6}$	$49.4^{+14.0}_{-9.6}$	37^{+11}_{-12}	$-0.05^{+0.26}_{-0.31}$	$4.3^{+2.1}_{-1.9}$	$0.69^{+0.26}_{-0.27}$	82^{+17}_{-11}	$0.68^{+0.11}_{-0.13}$	1100	$10.4^{+0.3}_{-0.4}$
GW200105_162426	$11.0^{+1.5}_{-1.4}$	$3.42^{+0.08}_{-0.08}$	$9.0^{+1.7}_{-1.7}$	$1.91^{+0.33}_{-0.24}$	$0.00^{+0.13}_{-0.18}$	$0.27^{+0.12}_{-0.11}$	$0.06^{+0.02}_{-0.02}$	$10.7^{+1.5}_{-1.4}$	$0.43^{+0.05}_{-0.02}$	7900	$13.7^{+0.2}_{-0.4}$
GW200112_155838	$63.9^{+5.7}_{-4.6}$	$27.4^{+2.6}_{-2.1}$	$35.6^{+6.7}_{-4.5}$	$28.3^{+4.4}_{-5.9}$	$0.06^{+0.15}_{-0.15}$	$1.25^{+0.43}_{-0.46}$	$0.24^{+0.07}_{-0.08}$	$60.8^{+5.3}_{-4.3}$	$0.71^{+0.06}_{-0.06}$	4300	$19.8^{+0.1}_{-0.2}$
GW200115_042309	$7.4^{+1.8}_{-1.7}$	$2.43^{+0.05}_{-0.07}$	$5.9^{+2.0}_{-2.5}$	$1.44^{+0.85}_{-0.29}$	$-0.15^{+0.24}_{-0.42}$	$0.29^{+0.15}_{-0.10}$	$0.06^{+0.03}_{-0.02}$	$7.2^{+1.8}_{-1.7}$	$0.42^{+0.09}_{-0.05}$	370	$11.3^{+0.3}_{-0.5}$
GW200128_022011	75^{+17}_{-12}	$32.0^{+7.5}_{-5.5}$	$42.2^{+11.6}_{-8.1}$	$32.6^{+9.5}_{-9.2}$	$0.12^{+0.24}_{-0.25}$	$3.4^{+2.1}_{-1.8}$	$0.56^{+0.28}_{-0.28}$	71^{+16}_{-11}	$0.74^{+0.10}_{-0.10}$	2600	$10.6^{+0.3}_{-0.4}$
GW200129_065458	$63.4^{+4.3}_{-3.6}$	$27.2^{+2.1}_{-2.3}$	$34.5^{+9.9}_{-3.2}$	$28.9^{+3.4}_{-9.3}$	$0.11^{+0.11}_{-0.16}$	$0.90^{+0.29}_{-0.38}$	$0.18^{+0.05}_{-0.07}$	$60.3^{+4.0}_{-3.3}$	$0.73^{+0.06}_{-0.05}$	130	$26.8^{+0.2}_{-0.2}$
GW200202_154313	$17.58^{+1.78}_{-0.67}$	$7.49^{+0.24}_{-0.20}$	$10.1^{+3.5}_{-1.4}$	$7.3^{+1.1}_{-1.7}$	$0.04^{+0.13}_{-0.06}$	$0.41^{+0.15}_{-0.16}$	$0.09^{+0.03}_{-0.03}$	$16.76^{+1.87}_{-0.66}$	$0.69^{+0.03}_{-0.04}$	170	$10.8^{+0.2}_{-0.4}$
GW200208_130117	$65.4^{+7.8}_{-6.8}$	$27.7^{+3.6}_{-3.1}$	$37.8^{+9.2}_{-6.2}$	$27.4^{+6.1}_{-7.4}$	$-0.07^{+0.22}_{-0.27}$	$2.23^{+1.00}_{-0.85}$	$0.40^{+0.15}_{-0.14}$	$62.5^{+7.3}_{-6.4}$	$0.66^{+0.09}_{-0.13}$	30	$10.8^{+0.3}_{-0.4}$
GW200208_222617	63^{+100}_{-25}	$19.6^{+10.7}_{-5.1}$	51^{+104}_{-30}	$12.3^{+9.0}_{-5.7}$	$0.45^{+0.43}_{-0.44}$	$4.1^{+4.4}_{-1.9}$	$0.66^{+0.54}_{-0.28}$	61^{+100}_{-25}	$0.83^{+0.14}_{-0.27}$	2000	$7.4^{+1.4}_{-1.2}$
GW200209_085452	$62.6^{+13.9}_{-9.4}$	$26.7^{+6.0}_{-4.2}$	$35.6^{+10.5}_{-6.8}$	$27.1^{+7.8}_{-7.8}$	$-0.12^{+0.24}_{-0.30}$	$3.4^{+1.9}_{-1.8}$	$0.57^{+0.25}_{-0.26}$	$59.9^{+13.1}_{-8.9}$	$0.66^{+0.10}_{-0.12}$	730	$9.6^{+0.4}_{-0.5}$
GW200210_092254	$27.0^{+7.1}_{-4.3}$	$6.56^{+0.38}_{-0.40}$	$24.1^{+7.5}_{-4.6}$	$2.83^{+0.47}_{-0.42}$	$0.02^{+0.22}_{-0.21}$	$0.94^{+0.43}_{-0.34}$	$0.19^{+0.08}_{-0.06}$	$26.7^{+7.2}_{-4.3}$	$0.34^{+0.13}_{-0.08}$	1800	$8.4^{+0.5}_{-0.7}$

Most remnant BHs are fast rotating!

- ❑ 65% of their maximum (Abbott et al., 2021; Buonanno et al., 2007)
- ❑ At least **5th order** in the slow-rotation expansion (Pani, 2011)

  → Coupling of many (l, m) modes!

Need a formalism dealing with **arbitrarily spinning** BHs, which may **not be algebraically special!**

Event	M $(M_\odot)$	$\mathcal{M}$ $(M_\odot)$	m_1 $(M_\odot)$	m_2 $(M_\odot)$	$\chi_{\rm eff}$	$D_{\rm L}$ (Gpc)	z	$M_{\rm f}$ $(M_\odot)$	$\chi_{\rm f}$	$\Delta\Omega$ (deg^2)	SNR
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GW191105_143521	$18.5^{+2.1}_{-1.3}$	$7.82^{+0.61}_{-0.45}$	$10.7^{+3.7}_{-1.6}$	$7.7^{+1.4}_{-1.9}$	$-0.02^{+0.13}_{-0.09}$	$1.15^{+0.43}_{-0.48}$	$0.23^{+0.07}_{-0.09}$	$17.6^{+2.1}_{-1.2}$	$0.67^{+0.04}_{-0.05}$	640	$9.7^{+0.3}_{-0.5}$
GW191109_010717	112^{+20}_{-16}	$47.5^{+9.6}_{-7.5}$	65^{+11}_{-11}	47^{+15}_{-13}	$-0.29^{+0.42}_{-0.31}$	$1.29^{+1.13}_{-0.65}$	$0.25^{+0.18}_{-0.12}$	107^{+18}_{-15}	$0.61^{+0.18}_{-0.19}$	1600	$17.3^{+0.5}_{-0.5}$
GW191113_071753	$34.5^{+10.5}_{-9.8}$	$10.7^{+1.1}_{-1.0}$	29^{+12}_{-14}	$5.9^{+4.4}_{-1.3}$	$0.00^{+0.37}_{-0.29}$	$1.37^{+1.15}_{-0.62}$	$0.26^{+0.18}_{-0.11}$	34^{+11}_{-10}	$0.45^{+0.33}_{-0.11}$	3600	$7.9^{+0.5}_{-1.1}$
GW191126_115259	$20.7^{+3.4}_{-2.0}$	$8.65^{+0.95}_{-0.71}$	$12.1^{+5.5}_{-2.2}$	$8.3^{+1.9}_{-2.4}$	$0.21^{+0.15}_{-0.11}$	$1.62^{+0.74}_{-0.74}$	$0.30^{+0.12}_{-0.13}$	$19.6^{+3.5}_{-2.0}$	$0.75^{+0.06}_{-0.05}$	1400	$8.3^{+0.2}_{-0.5}$
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GW191129_134029	$17.5^{+2.4}_{-1.2}$	$7.31^{+0.43}_{-0.28}$	$10.7^{+4.1}_{-2.1}$	$6.7^{+1.5}_{-1.7}$	$0.06^{+0.16}_{-0.08}$	$0.79^{+0.26}_{-0.33}$	$0.16^{+0.05}_{-0.06}$	$16.8^{+2.5}_{-1.2}$	$0.69^{+0.03}_{-0.05}$	850	$13.1^{+0.2}_{-0.3}$
GW191204_110529	$47.2^{+9.2}_{-8.0}$	$19.8^{+3.6}_{-3.3}$	$27.3^{+11.0}_{-6.0}$	$19.3^{+5.6}_{-6.0}$	$0.05^{+0.26}_{-0.27}$	$1.8^{+1.7}_{-1.1}$	$0.34^{+0.25}_{-0.18}$	$45.0^{+8.6}_{-7.6}$	$0.71^{+0.12}_{-0.11}$	3700	$8.8^{+0.4}_{-0.6}$
GW191204_171526	$20.21^{+1.70}_{-0.96}$	$8.55^{+0.38}_{-0.27}$	$11.9^{+3.3}_{-1.8}$	$8.2^{+1.4}_{-1.6}$	$0.16^{+0.08}_{-0.05}$	$0.65^{+0.19}_{-0.25}$	$0.13^{+0.04}_{-0.05}$	$19.21^{+1.79}_{-0.95}$	$0.73^{+0.03}_{-0.03}$	350	$17.5^{+0.2}_{-0.2}$
GW191215_223052	$43.3^{+5.3}_{-4.3}$	$18.4^{+2.2}_{-1.7}$	$24.9^{+7.1}_{-4.1}$	$18.1^{+3.8}_{-4.1}$	$-0.04^{+0.17}_{-0.21}$	$1.93^{+0.89}_{-0.86}$	$0.35^{+0.13}_{-0.14}$	$41.4^{+5.1}_{-4.1}$	$0.68^{+0.07}_{-0.07}$	530	$11.2^{+0.3}_{-0.4}$
GW191216_213338	$19.81^{+2.69}_{-0.94}$	$8.33^{+0.22}_{-0.19}$	$12.1^{+4.6}_{-2.3}$	$7.7^{+1.6}_{-1.9}$	$0.11^{+0.13}_{-0.06}$	$0.34^{+0.12}_{-0.13}$	$0.07^{+0.02}_{-0.03}$	$18.87^{+2.80}_{-0.94}$	$0.70^{+0.03}_{-0.04}$	490	$18.6^{+0.2}_{-0.2}$
GW191219_163120	$32.3^{+2.2}_{-2.7}$	$4.32^{+0.12}_{-0.17}$	$31.1^{+2.2}_{-2.8}$	$1.17^{+0.07}_{-0.06}$	$0.00^{+0.07}_{-0.09}$	$0.55^{+0.25}_{-0.16}$	$0.11^{+0.05}_{-0.03}$	$32.2^{+2.2}_{-2.7}$	$0.14^{+0.06}_{-0.06}$	1500	$9.1^{+0.5}_{-0.8}$
GW191222_033537	79^{+16}_{-11}	$33.8^{+7.1}_{-5.0}$	$45.1^{+10.9}_{-8.0}$	$34.7^{+9.3}_{-10.5}$	$-0.04^{+0.20}_{-0.25}$	$3.0^{+1.7}_{-1.7}$	$0.51^{+0.23}_{-0.26}$	$75.5^{+15.3}_{-9.9}$	$0.67^{+0.08}_{-0.11}$	2000	$12.5^{+0.2}_{-0.3}$
GW191230_180458	86^{+19}_{-12}	$36.5^{+8.2}_{-5.6}$	$49.4^{+14.0}_{-9.6}$	37^{+11}_{-12}	$-0.05^{+0.26}_{-0.31}$	$4.3^{+2.1}_{-1.9}$	$0.69^{+0.26}_{-0.27}$	82^{+17}_{-11}	$0.68^{+0.11}_{-0.13}$	1100	$10.4^{+0.3}_{-0.4}$
GW200105_162426	$11.0^{+1.5}_{-1.4}$	$3.42^{+0.08}_{-0.08}$	$9.0^{+1.7}_{-1.7}$	$1.91^{+0.33}_{-0.24}$	$0.00^{+0.13}_{-0.18}$	$0.27^{+0.12}_{-0.11}$	$0.06^{+0.02}_{-0.02}$	$10.7^{+1.5}_{-1.4}$	$0.43^{+0.05}_{-0.02}$	7900	$13.7^{+0.2}_{-0.4}$
GW200112_155838	$63.9^{+5.7}_{-4.6}$	$27.4^{+2.6}_{-2.1}$	$35.6^{+6.7}_{-4.5}$	$28.3^{+4.4}_{-5.9}$	$0.06^{+0.15}_{-0.15}$	$1.25^{+0.43}_{-0.46}$	$0.24^{+0.07}_{-0.08}$	$60.8^{+5.3}_{-4.3}$	$0.71^{+0.06}_{-0.06}$	4300	$19.8^{+0.1}_{-0.2}$
GW200115_042309	$7.4^{+1.8}_{-1.7}$	$2.43^{+0.05}_{-0.07}$	$5.9^{+2.0}_{-2.5}$	$1.44^{+0.85}_{-0.29}$	$-0.15^{+0.24}_{-0.42}$	$0.29^{+0.15}_{-0.10}$	$0.06^{+0.03}_{-0.02}$	$7.2^{+1.8}_{-1.7}$	$0.42^{+0.09}_{-0.05}$	370	$11.3^{+0.3}_{-0.5}$
GW200128_022011	75^{+17}_{-12}	$32.0^{+7.5}_{-5.5}$	$42.2^{+11.6}_{-8.1}$	$32.6^{+9.5}_{-9.2}$	$0.12^{+0.24}_{-0.25}$	$3.4^{+2.1}_{-1.8}$	$0.56^{+0.28}_{-0.28}$	71^{+16}_{-11}	$0.74^{+0.10}_{-0.10}$	2600	$10.6^{+0.3}_{-0.4}$
GW200129_065458	$63.4^{+4.3}_{-3.6}$	$27.2^{+2.1}_{-2.3}$	$34.5^{+9.9}_{-3.2}$	$28.9^{+3.4}_{-9.3}$	$0.11^{+0.11}_{-0.16}$	$0.90^{+0.29}_{-0.38}$	$0.18^{+0.05}_{-0.07}$	$60.3^{+4.0}_{-3.3}$	$0.73^{+0.06}_{-0.05}$	130	$26.8^{+0.2}_{-0.2}$
GW200202_154313	$17.58^{+1.78}_{-0.67}$	$7.49^{+0.24}_{-0.20}$	$10.1^{+3.5}_{-1.4}$	$7.3^{+1.1}_{-1.7}$	$0.04^{+0.13}_{-0.06}$	$0.41^{+0.15}_{-0.16}$	$0.09^{+0.03}_{-0.03}$	$16.76^{+1.87}_{-0.66}$	$0.69^{+0.03}_{-0.04}$	170	$10.8^{+0.2}_{-0.4}$
GW200208_130117	$65.4^{+7.8}_{-6.8}$	$27.7^{+3.6}_{-3.1}$	$37.8^{+9.2}_{-6.2}$	$27.4^{+6.1}_{-7.4}$	$-0.07^{+0.22}_{-0.27}$	$2.23^{+1.00}_{-0.85}$	$0.40^{+0.15}_{-0.14}$	$62.5^{+7.3}_{-6.4}$	$0.66^{+0.09}_{-0.13}$	30	$10.8^{+0.3}_{-0.4}$
GW200208_222617	63^{+100}_{-25}	$19.6^{+10.7}_{-5.1}$	51^{+104}_{-30}	$12.3^{+9.0}_{-5.7}$	$0.45^{+0.43}_{-0.44}$	$4.1^{+4.4}_{-1.9}$	$0.66^{+0.54}_{-0.28}$	61^{+100}_{-25}	$0.83^{+0.14}_{-0.27}$	2000	$7.4^{+1.4}_{-1.2}$
GW200209_085452	$62.6^{+13.9}_{-9.4}$	$26.7^{+6.0}_{-4.2}$	$35.6^{+10.5}_{-6.8}$	$27.1^{+7.8}_{-7.8}$	$-0.12^{+0.24}_{-0.30}$	$3.4^{+1.9}_{-1.8}$	$0.57^{+0.25}_{-0.26}$	$59.9^{+13.1}_{-8.9}$	$0.66^{+0.10}_{-0.12}$	730	$9.6^{+0.4}_{-0.5}$
GW200210_092254	$27.0^{+7.1}_{-4.3}$	$6.56^{+0.38}_{-0.40}$	$24.1^{+7.5}_{-4.6}$	$2.83^{+0.47}_{-0.42}$	$0.02^{+0.22}_{-0.21}$	$0.94^{+0.43}_{-0.34}$	$0.19^{+0.08}_{-0.06}$	$26.7^{+7.2}_{-4.3}$	$0.34^{+0.13}_{-0.08}$	1800	$8.4^{+0.5}_{-0.7}$

(Abbott et al., arXiv 2111.03606)

Black Hole Perturbation Theory beyond GR

I. Basics in Teukolsky formalism (GR)

II. Modified Teukolsky formalism

PERTURBATIONS OF A ROTATING BLACK HOLE. I. FUNDAMENTAL EQUATIONS FOR GRAVITATIONAL, ELECTROMAGNETIC, AND NEUTRINO-FIELD PERTURBATIONS*

SAUL A. TEUKOLSKY†
California Institute of Technology, Pasadena
Received 1973 April 12

ABSTRACT

This paper derives linear equations that describe dynamical gravitational, electromagnetic, and neutrino-field perturbations of a rotating black hole. The equations decouple into a single gravitational equation, a single electromagnetic equation, and a single neutrino equation. Each of these equations is completely separable into ordinary differential equations. The paper lays the mathematical groundwork for later papers in this series, which will deal with astrophysical applications: stability of the hole, tidal friction effects, superradiant scattering of electromagnetic waves, and gravitational-wave processes.

Perturbations of Spinning Black Holes beyond General Relativity: Modified Teukolsky Equation

Dongjun Li[1,*] Pratik Wagle[2,†] Yanbei Chen[1] and Nicolás Yunes[2]
[1]*Theoretical Astrophysics 350-17, California Institute of Technology, Pasadena, California 91125, USA*
[2]*Illinois Center for Advanced Studies of the Universe and Department of Physics, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801, USA*

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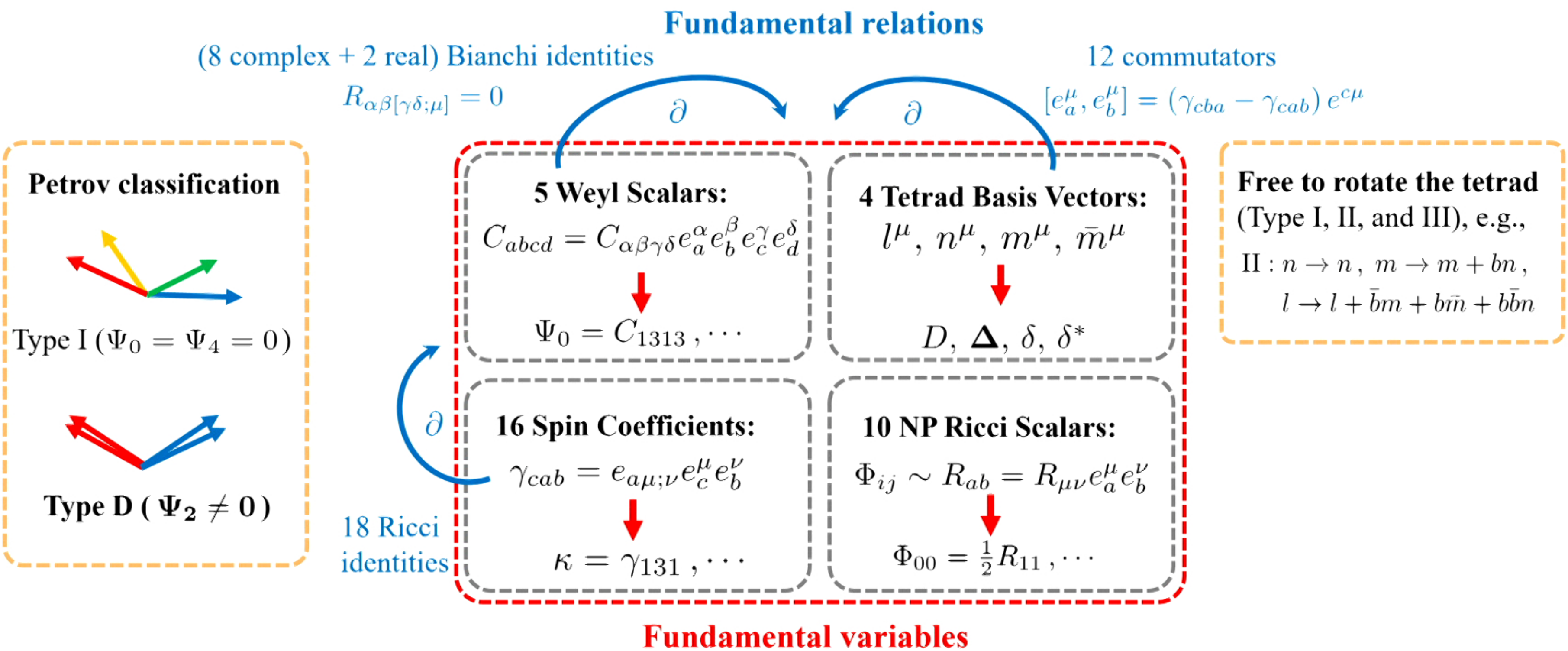
The detection of gravitational waves from compact binary mergers by the LIGO/Virgo Collaboration has, for the first time, allowed for tests of relativistic gravity in the strong, dynamical, and nonlinear regime. Outside Einstein's relativity, spinning black holes may be different from their general relativistic counterparts, and their merger may then lead to a modified ringdown. We study the latter and, for the first time, derive a modified Teukolsky equation, i.e., a set of linear, decoupled differential equations that describe dynamical perturbations of non-Kerr black holes for the radiative Newman-Penrose scalars Ψ_0 and

NP Formalism

- A special **tetrad formalism** adaptable to BH perturbations
- Four null tetrad basis vectors satisfying certain orthogonality conditions



Teukolsky equation in GR (vacuum, type D)

2 Bianchi identities & 1 Ricci identity

$$F_1\Psi_0 - J_1\Psi_1 - 3\kappa\Psi_2 = 0\,,$$
$$F_2\Psi_0 - J_2\Psi_1 - 3\sigma\Psi_2 = 0\,,$$
$$E_2\sigma - E_1\kappa - \Psi_0 = 0$$

$\Psi = \Psi^{(0)} + \epsilon\Psi^{(1)}$

e.g., $F_1 \equiv \bar{\delta} - 4\alpha + \pi$

Teukolsky's original approach

Additional 2 Bianchi ids and 1 commutator to decouple

$\Psi_1^{(1)}, \kappa^{(1)}, \sigma^{(1)}$

$$\left(E_2^{\rm GR}F_2 - E_1^{\rm GR}F_1 - 3\Psi_2\right)\Psi_0^{(1)} = 0$$

$D\Psi_2 = 3\rho\Psi_2\,,$

$\delta\Psi_2 = 3\tau\Psi_2$

Chandrasekhar's approach

Type II rotation with $b \sim \mathcal{O}(\epsilon)$ to remove $\Psi_1^{(1)}$ directly

$$\Psi_{0,2,3,4}^{(1)} \to \Psi_{0,2,3,4}^{(1)}\,,\ \Psi_1^{(1)} \to \Psi_1^{(1)} + 3b\Psi_2^{(0)}$$

$$\left(\mathcal{E}_2F_2 - \mathcal{E}_1F_1 - 3\Psi_2\right)\Psi_0^{(1)} = 0$$

Solutions

$$\Psi_0^{(1)} = e^{-i(\omega t - m\phi)}{}_2R_{lm\omega}(r){}_2S_{lm\omega}(\theta)$$

Leaver's method (Leaver, 1985)

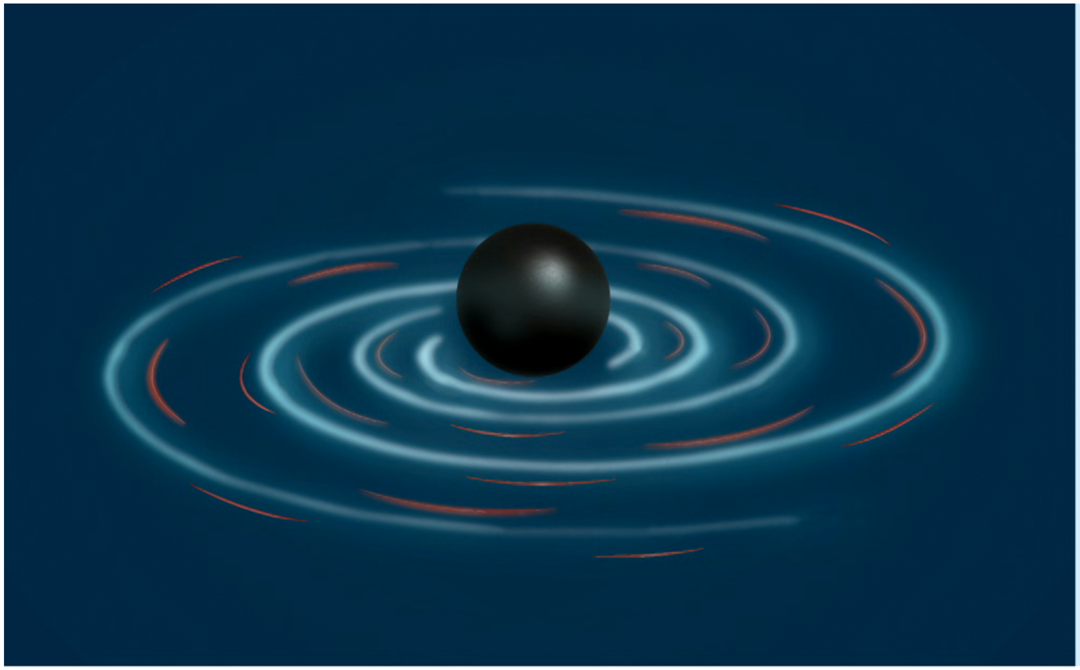
Spin-weighted spheroidal harmonics

From Petrov type D to non-Petrov-type-D

- Teukolsky's approach versus Chandrasekhar's approach (Petrov type D)
  - **Equivalent** approaches ← Ψ_0 is gauge-invariant at linear order
  - More subtleties for non-Ricci-flat backgrounds (e.g., dCS, EdGB, …)

$$(D-\rho)\Psi_2 = -(\boldsymbol{\Delta} + \bar{\mu} - 2\gamma - 2\bar{\gamma})\Phi_{00} + \cdots$$

  - No need for additional Bianchi identities and commutation relations
- To non-Petrov-type-D
  - Many Weyl scalars are nonzero, may not be able to decouple NP quantities, e.g., $\left(\bar{\boldsymbol{\Delta}} - 4\gamma + \mu\right)\Psi_1$
  - **Really non-Petrov-type-D? bGR theories cannot deviate too much!**



An EFT approach:

$\zeta \to$ bGR expansion

$\epsilon \to$ GW expansion

DL, Wagle, Chen, & Yunes,
PhysRevX.13.021029 (arXiv 2206.10652)

Background spacetime in GR, type D always!

GWs in GR

$$\Psi = \Psi^{(0,0)}(\mathbf{x}) + \zeta \Psi^{(1,0)}(\mathbf{x}) + \epsilon \Psi^{(0,1)}(\mathbf{x}, t) + \zeta \epsilon \Psi^{(1,1)}(\mathbf{x}, t)$$

The stationary part of spacetime in bGR, can be algebraically general

Leading correction to GWs due to bGR

- **Modified Teukolsky equation**

$$H_0^{(0,0)}\Psi_0^{(1,1)} = \mathcal{S}_{0,\mathrm{D}}^{(1,1)} + \mathcal{S}_{0,\text{non-D}}^{(1,1)} + \mathcal{S}_{1,\text{non-D}}^{(1,1)} + \mathcal{S}^{(1,1)}$$

$$H_0 = (D - \rho + \cdots)(\boldsymbol{\Delta} - 4\gamma + \cdots) + \cdots$$

$$H_1 = (D - \rho + \cdots)(\delta - 4\tau - 2\beta) + \cdots$$

Follow Chandrasekhar: rotate away $\Psi_{1,3}^{(1,1)}$

Works for any linear deviation of a type D spacetime!

- **Theory-independent** source terms (purely geometrical, $\mathcal{S}_{\text{geo}}$)

$$\mathcal{S}_{0,\mathrm{D}}^{(1,1)} \sim H_0^{(1,0)}\Psi_0^{(0,1)} \sim h^{(1,0)}\Psi_0^{(0,1)}$$ Non-GR type D

$$\mathcal{S}_{i,\text{non-D}}^{(1,1)} \sim H_i^{(0,1)}\Psi_i^{(1,0)} \sim h^{(0,1)}\Psi_i^{(1,0)}$$ Non-type-D

- **Theory-dependent** source terms (effective $T^{\mu\nu}$)

Extra fields

$$\mathcal{S} = \mathcal{E}_2 S_2 - \mathcal{E}_1 S_1 \sim h^{(0,1)}\vartheta^{(1,0)} + g^{(0,0)}\vartheta^{(1,1)}$$

$$\mathcal{E}_2 = \Psi_2(D - \rho + \cdots)\Psi_2^{-1}\cdots,\ S_2 = (\delta - 2\beta + 2\bar{\pi})\Phi_{01} + \cdots$$

$h^{(0,1)}$ needs metric reconstruction, but only for GR GWs

$$h_{\mu\nu}^{(0,1)} = \left(\mathcal{O}_{\mu\nu} + \bar{\mathcal{O}}_{\mu\nu}\mathcal{C}\right)\Psi_0^{(0,1)}$$

(**DL**, Wagle, Chen, & Yunes, arXiv 2206.10652)

An alternative tensorial approach

- **Wald's formalism in GR** (Wald, 1978)

$$H\mathcal{T}\left[h_{cd}\right]=H\left[\psi_s\right]=\mathcal{S}^{ab}\mathcal{E}_{ab}\left[h_{cd}\right]$$

H : Teukolsky operator $\qquad \mathcal{E}_{ab}$: Einstein tensor

$\mathcal{T}$: convert metric to Weyl scalars

$\mathcal{S}^{ab}$: convert EE to the Teukolsky equation

$\Rightarrow$ $h_{ab}=2\,\mathrm{Re}\left(\mathcal{S}^{\dagger}_{ab}\Psi_H\right),\ \mathrm{P}^4\bar{\Psi}_H=-2\Psi_0$

Metric reconstruction!

- **An extension to bGR theories** (Hussain & Zimmerman, 2022)
  - The same $\mathcal{S}^{ab}$ operator can be applied to the EE in bGR theories
  - $\mathcal{S}^{ab(0,0)}$ converts EE at $\mathcal{O}(\zeta^1,\epsilon^1)$ to Teukolsky equation at $\mathcal{O}(\zeta^1,\epsilon^1)$ $\Rightarrow$ $\Psi_0^{(1,1)}$ naturally decouples
  - $\mathcal{S}^{ab(0,1)}$ converts EE at $\mathcal{O}(\zeta^1,\epsilon^0)$ to source terms (similar to $\mathcal{S}^{(1,1)}$, $\mathcal{S}^{(1,1)}_{\mathrm{geo}}$), similarly for $\mathcal{S}^{ab(1,0)}$ Needs metric reconstruction!

(Hussain & Zimmerman, arXiv 2206.10653)

Metric Reconstruction (vacuum)

- **Direct reconstruction** (Chandrasekhar, 1983; Loutrel et al. in 2020)
- **CCK-Ori (**Cohen & Kegeles, 1975; Kegeles & Cohen, 1978; Chrzanowski, 1975; Ori, 2003**)**
  - An inversion of Wald's relation: $h_{ab} = \mathrm{Re}\left(\mathcal{S}^{\dagger}\Phi\right)_{ab},\ \mathrm{P}^4\bar{\Phi} = -\Psi_0$
  - Relies on IRG ($l^\mu h_{\mu\nu} = 0,\ h = 0$) or ORG ($n^\mu h_{\mu\nu} = 0,\ h = 0$)
  - Reconstruct NP quantities (Campanelli & Lousto, 1999)

I. Tetrad

$$e_a^{\mu(0,1)} = A_a{}^{b(0,1)} e_b^{\mu(0,0)}$$

Orthogonality

$$n^{\mu(0,1)} = \frac{1}{2} h_{nn}^{(0,1)} l^\mu$$

→

II. Spin coefficients

Commutators

$$\nu^{(0,1)} = -\tfrac{1}{2}\bar{\delta}_{[2,2,-1,-1]} h_{nn}^{(0,1)} + \cdots$$

III. Weyl scalars

Ricci identities

$$\Psi_0^{(0,1)} = \tfrac{1}{2} D_{[-3,1,0,-1]} D_{[-2,2,0,-1]} h_{mm}^{(0,1)}$$

e.g., $\bar{\delta}_{[a,b,c,d]} = \bar{\delta} + a\alpha + b\bar{\beta} + c\pi + d\bar{\tau}$

→

III. Rotate the tetrad

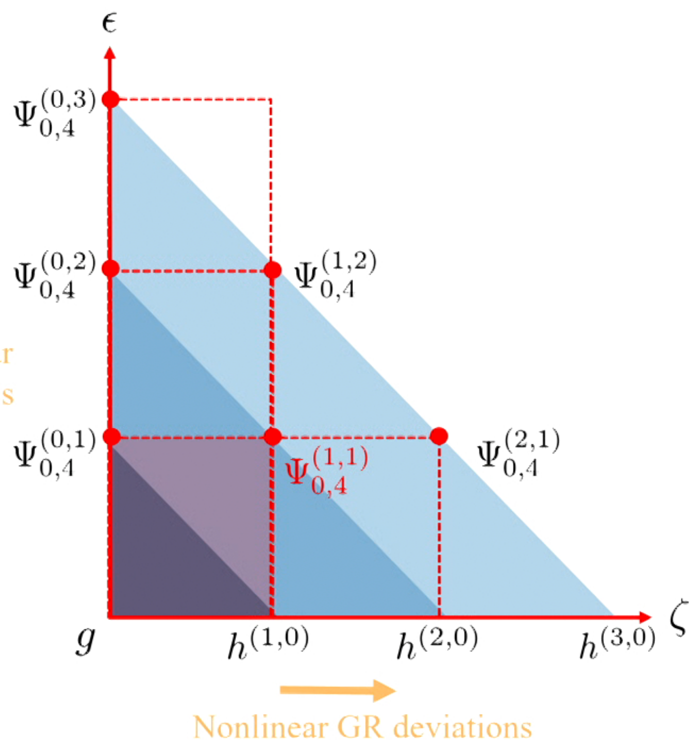
$$\Psi_{1,3}^{(0,1)} = 0$$

$$a^{(0,1)} = -\bar{\Psi}_3^{(0,1)}/(3\Psi_2)$$

$$b^{(0,1)} = -\Psi_1^{(0,1)}/(3\Psi_2)$$

Higher-order (modified) Teukolsky formalism

- 2nd order in GR (Campanelli & Lousto ,1999): $h^{(0,1)} \times h^{(1,0)} \longrightarrow \left[h^{(0,1)}\right]^2$
- Nonlinear in bGR: iterate the steps at $\mathcal{O}(\zeta^1, \epsilon^1)$ to $\mathcal{O}(\zeta^m, \epsilon^n)$



$$H_0^{(0,0)} \Psi_0^{(m,n)} = \mathcal{S}_{\rm geo}^{(m,n)} + \mathcal{S}^{(m,n)}$$

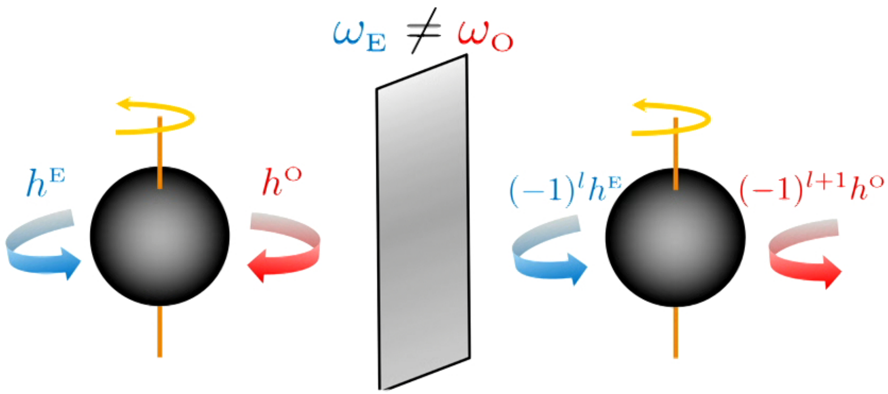
$\zeta = 0$ ↓

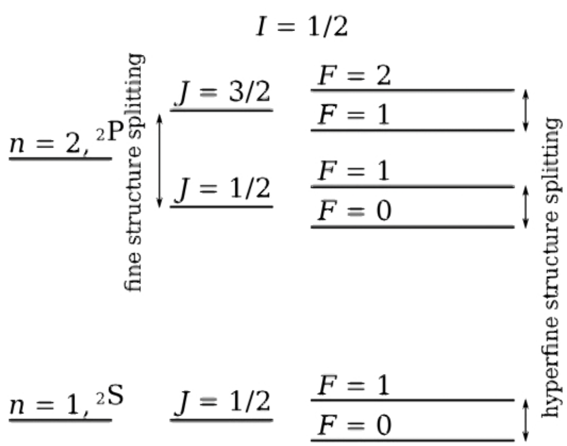
Campanelli & Lousto, 1999

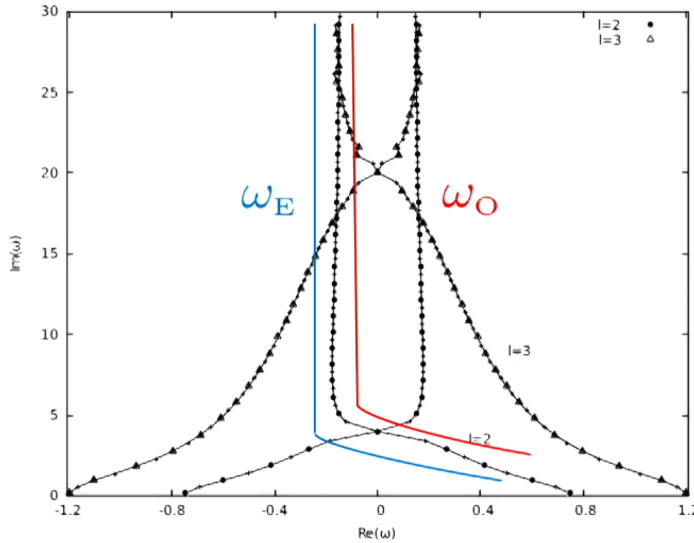
(**DL**, Wagle, Chen, & Yunes, arXiv 2206.10652)

Isospectrality Breaking

- GR (isospectrality): Both $h_{\mu\nu}^{\mathrm{E,O}}$ have the same frequency
- bGR: QNMs of $h_{\mu\nu}^{\mathrm{E,O}}$ get shifted differently
- Example: for non-rotating BHs in dCS, only odd modes are modified (Cardoso et al., 2010)







(Leaver, 1985)

Isospectrality Breaking

- **Definite-parity modes**
  - Metric perturbations (RW & ZM): $\hat{P} h_{\mu\nu}^{\mathrm{E,O}} = \pm(-1)^l h_{\mu\nu}^{\mathrm{E,O}}$ under $\hat{P} : (\theta, \phi) \to (\pi - \theta, \phi + \pi)$
  - **GR:** definite-parity modes of curvature perturbations (Chrzanowski, 1975; Nichols et al., 2012):

$$\Psi_{lm\omega}^{\mathrm{E,O}} = \Psi_{lm\omega} \pm (-1)^l \hat{\mathcal{P}} \Psi_{lm\omega} \,, \quad \hat{\mathcal{P}} f(\theta, \phi) = \hat{P} \bar{f} = \bar{f}(\pi - \theta, \phi + \pi)$$

  - **bGR:**
    - **Type D:** the same definition still applies
    - **Non-type-D:** $\Psi_{0,4}$ is not invariant under $\mathcal{O}(\zeta^0, \epsilon^1)$ transformations

      Similar to 2^{nd} order Teukolsky equation in GR (Campanelli & Lousto, 1999)

(**DL**, et al., arXiv 2310.06033)

Corrections to QNMs

- **Eigenvalue perturbation (EVP)** (e.g., Mark, et al., 2015; Hussain & Zimmerman, 2022)
  - $H^{(0)}\Psi^{(0)} = 0\,, \quad H^{(0)}\Psi^{(1)} + H^{(1)}\Psi^{(0)} = 0$
  - Motivated by QM and Poincaré-Lindstedt method

    $$\Psi^{(0)} + \zeta\Psi^{(1)} \approx \exp\left[-i\left(\omega_{lm}^{(0)} + \zeta\omega_{lm}^{(1)}\right)t + im\phi\right]\left[\psi_{lm}^{(0)}(r,\theta) + \zeta\psi_{lm}^{(1)}(r,\theta)\right]$$
  - Make $\tilde{H}^{(0)}$ self-adjoint

    $$\langle\psi_{lm}^{(0)}|\tilde{H}^{(0)}\psi_{lm}^{(1)}\rangle = \langle\tilde{H}^{(0)}\psi_{lm\omega}^{(0)}|\psi_{lm}^{(1)}\rangle$$
  - Contract $\psi_{lm}^{(0)}$ with the equation

    $$\tilde{H}^{(0)}\psi_{lm}^{(1)} + \omega^{(1)}\partial_\omega\tilde{H}^{(0)}\psi_{lm}^{(0)} + \tilde{H}^{(1)}\psi_{lm}^{(0)} = 0$$

Corrections to QNMs

- **Eigenvalue perturbation (EVP)** (e.g., Mark, et al., 2015; Hussain & Zimmerman, 2022)
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  - ❑ Make $\tilde{H}^{(0)}$ self-adjoint

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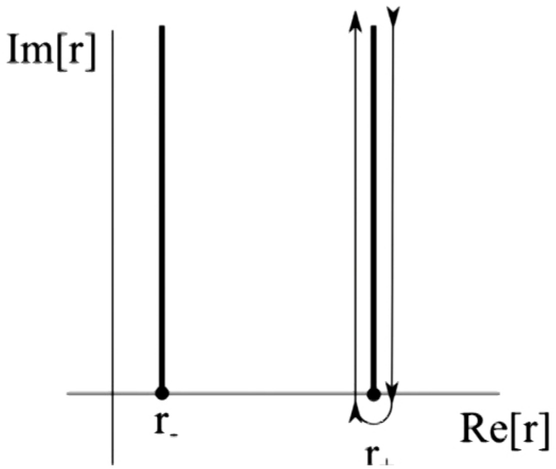
  - ❑ Contract $\psi_{lm}^{(0)}$ with the equation

    $$\tilde{H}^{(0)}\psi_{lm}^{(0)} = 0$$

    $$\tilde{H}^{(0)}\psi_{lm}^{(1)} + \omega^{(1)}\partial_\omega\tilde{H}^{(0)}\psi_{lm}^{(0)} + \tilde{H}^{(1)}\psi_{lm}^{(0)} = 0$$

$$\omega^{(1)} = -\langle\tilde{H}^{(1)}\rangle/\langle\partial_\omega\tilde{H}^{(0)}\rangle\,,\ \langle\mathcal{O}\rangle = \langle\psi_{lm\omega}^{(0)}|\mathcal{O}\psi_{lm\omega}^{(0)}\rangle$$



(arXiv 1409.5800)

Isospectrality Breaking

- Schemetically: $H_0\Psi_0^{(1,1)} = \mathcal{S}^{\mu\nu}\left(\mathcal{O}_{\mu\nu} + \bar{\mathcal{O}}_{\mu\nu}\hat{\mathcal{C}}\right)\Psi_0^{(0,1)}$
- Ansatz: $\Psi_0^{\eta(1,1)} = \Psi_0^{(1,1)} + \eta\hat{\mathcal{P}}\Psi_0^{(1,1)}$

$$\Psi_0^{(0,1)}+\zeta\Psi_0^{(1,1)} \approx \exp\left[-i\left(\omega_{lm}^{(0)} + \zeta\omega_{lm}^{(1)}\right)t + im\phi\right]\left[\psi_{lm}^{(0,1)}(r,\theta) + \zeta\psi_{lm}^{(1,1)}(r,\theta)\right]$$

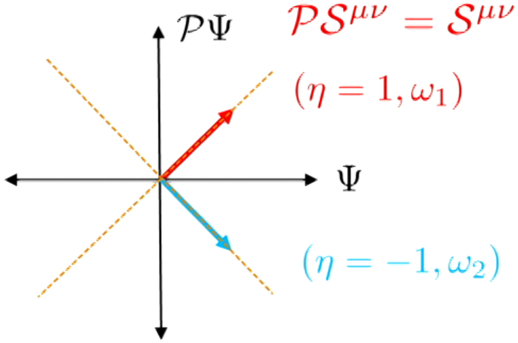
- Evaluation of QNMs:

$$\frac{1}{\langle\partial_\omega \tilde{H}_0\rangle}\begin{pmatrix} \langle\mathcal{S}^{\mu\nu}\mathcal{O}_{\mu\nu}\rangle & \langle\mathcal{S}^{\mu\nu}\bar{\mathcal{O}}_{\mu\nu}\hat{\mathcal{C}}\hat{\mathcal{P}}\rangle \\ \langle(\hat{\mathcal{P}}\mathcal{S}^{\mu\nu})\bar{\mathcal{O}}_{\mu\nu}\hat{\mathcal{C}}\hat{\mathcal{P}}\rangle & \langle(\hat{\mathcal{P}}\mathcal{S}^{\mu\nu})\mathcal{O}_{\mu\nu}\rangle \end{pmatrix}\begin{pmatrix}1\\ \bar{\eta}\end{pmatrix} = \omega_{lm}^{(1)}\begin{pmatrix}1\\ \bar{\eta}\end{pmatrix}$$

- Solutions:

$$\bar{\eta}^2\left\langle\mathcal{S}^{\mu\nu}\bar{\mathcal{O}}_{\mu\nu}\hat{\mathcal{C}}\hat{\mathcal{P}}\right\rangle + \bar{\eta}\left\langle\left[\mathcal{S}^{\mu\nu} - (\hat{\mathcal{P}}\mathcal{S}^{\mu\nu})\right]\mathcal{O}_{\mu\nu}\right\rangle - \left\langle(\hat{\mathcal{P}}\mathcal{S}^{\mu\nu})\bar{\mathcal{O}}_{\mu\nu}\hat{\mathcal{C}}\hat{\mathcal{P}}\right\rangle = 0$$

$$\omega_{lm}^{(1)} = \frac{\langle\mathcal{S}^{\mu\nu}\left(\mathcal{O}_{\mu\nu}+\bar{\eta}_i\bar{\mathcal{O}}_{\mu\nu}\hat{\mathcal{C}}\hat{\mathcal{P}}\right)\rangle}{\langle\partial_\omega\tilde{H}_0\rangle}$$



Even and Odd modes

(**DL**, et al., arXiv 2310.06033)

Example: dCS Gravity

- **dCS Gravity** (convention in Yagi, Yunes, & Tanaka, 2012)
  - **Action:** $S \equiv \int d^4x\sqrt{-g}\left(\frac{1}{16\pi G}R + \frac{\alpha}{4}\vartheta R_{\nu\mu\rho\sigma}\,{}^*R^{\mu\nu\rho\sigma} - \frac{1}{2}\nabla_\mu\vartheta\nabla^\mu\vartheta\right),\ {}^*R^{\mu\nu\rho\sigma} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}R_{\alpha\beta}{}^{\rho\sigma}$
  - **EOM:** $R_{\mu\nu} = -\frac{\alpha}{\kappa_g}C_{\mu\nu} + \frac{1}{2\kappa_g}\bar{T}^{\vartheta}_{\mu\nu}$ (Alexander & Yunes, 2009)
    $\Box\vartheta = -\frac{\alpha}{4}R_{\nu\mu\rho\sigma}{}^*R^{\mu\nu\rho\sigma}$
  - **Expansion parameter:** $\zeta \equiv \frac{\alpha^2}{\kappa_g M^4}\,(\mathbf{dCS})\,,\ \chi \equiv \frac{a}{M}\,(\mathbf{slow\ rotation})\,,\ \epsilon\,(\mathbf{GW})$

    $$g_{\mu\nu} = g^{(0,0,0)}_{\mu\nu} + \sum_{l>0,m,n}^{\infty}\zeta^l\chi^m\epsilon^n h^{(l,m,n)}_{\mu\nu},\ \vartheta = \sum_{l>0,m,n}^{\infty}\zeta^l\chi^m\epsilon^n\vartheta^{(l,m,n)}$$
  - **Stationary solutions** (Yunes & Pretorius, 2009; Yagi, Yunes, & Tanaka, 2012)

    $$\vartheta^{(0,1,0)} = \frac{5}{8}\kappa_g^{\frac{1}{2}}M^2\frac{\cos\theta}{r^2}\left(1 + \frac{2M}{r} + \frac{18M^2}{5r^2}\right)$$

    $$h^{(1,1,0)}_{t\phi} = \frac{5M^5}{8r^4}\left(1 + \frac{12M}{7r} + \frac{27M^2}{10r^2}\right)\sin^2\theta\,,\quad h^{(1,1,0)}_{\mu\nu} = 0\ \text{otherwise}$$

    Petrov type D up to $\mathcal{O}(\chi)$, but Petrov type I beyond $\mathcal{O}(\chi)$

Isospectrality Breaking

- Schemetically: $H_0\Psi_0^{(1,1)} = \mathcal{S}^{\mu\nu}\left(\mathcal{O}_{\mu\nu} + \bar{\mathcal{O}}_{\mu\nu}\hat{\mathcal{C}}\right)\Psi_0^{(0,1)}$
- Ansatz: $\Psi_0^{\eta(1,1)} = \Psi_0^{(1,1)} + \eta\hat{\mathcal{P}}\Psi_0^{(1,1)}$

$$\Psi_0^{(0,1)} + \zeta\Psi_0^{(1,1)} \approx \exp\left[-i\left(\omega_{lm}^{(0)} + \zeta\omega_{lm}^{(1)}\right)t + im\phi\right]\left[\psi_{lm}^{(0,1)}(r,\theta) + \zeta\psi_{lm}^{(1,1)}(r,\theta)\right]$$

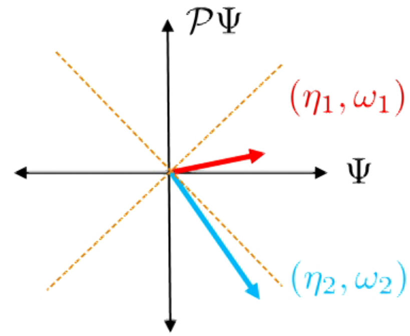
- Evaluation of QNMs:

$$\frac{1}{\langle\partial_\omega \tilde{H}_0\rangle}\begin{pmatrix}\langle\mathcal{S}^{\mu\nu}\mathcal{O}_{\mu\nu}\rangle & \langle\mathcal{S}^{\mu\nu}\bar{\mathcal{O}}_{\mu\nu}\hat{\mathcal{C}}\hat{\mathcal{P}}\rangle \\ \langle(\hat{\mathcal{P}}\mathcal{S}^{\mu\nu})\bar{\mathcal{O}}_{\mu\nu}\hat{\mathcal{C}}\hat{\mathcal{P}}\rangle & \langle(\hat{\mathcal{P}}\mathcal{S}^{\mu\nu})\mathcal{O}_{\mu\nu}\rangle\end{pmatrix}\begin{pmatrix}1\\ \bar{\eta}\end{pmatrix} = \omega_{lm}^{(1)}\begin{pmatrix}1\\ \bar{\eta}\end{pmatrix}$$

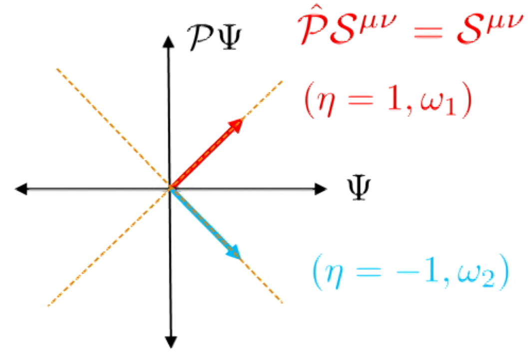
- Solutions:

$$\bar{\eta}^2\left\langle\mathcal{S}^{\mu\nu}\bar{\mathcal{O}}_{\mu\nu}\hat{\mathcal{C}}\hat{\mathcal{P}}\right\rangle + \bar{\eta}\left\langle\left[\mathcal{S}^{\mu\nu} - (\hat{\mathcal{P}}\mathcal{S}^{\mu\nu})\right]\mathcal{O}_{\mu\nu}\right\rangle - \left\langle(\hat{\mathcal{P}}\mathcal{S}^{\mu\nu})\bar{\mathcal{O}}_{\mu\nu}\hat{\mathcal{C}}\hat{\mathcal{P}}\right\rangle = 0$$

$$\omega_{lm}^{(1)} = \frac{\langle\mathcal{S}^{\mu\nu}(\mathcal{O}_{\mu\nu} + \bar{\eta}_i\bar{\mathcal{O}}_{\mu\nu}\hat{\mathcal{C}}\hat{\mathcal{P}})\rangle}{\langle\partial_\omega\tilde{H}_0\rangle}$$



Isospectrality is broken



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(**DL**, et al., arXiv 2310.06033)

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    Petrov type D up to $\mathcal{O}(\chi)$, but Petrov type I beyond $\mathcal{O}(\chi)$

Example: dCS Gravity (Metric)

- **Step 1: Decompose metric δg into $\delta g^{\text{odd,even}}$**
  - ❑ Regge-Wheeler Gauge (10+1 → 6+1):

$$\delta g_{\mu\nu}^{\text{odd}} : h_0 \to h_{t\theta}\,, h_{t\phi};\ h_1 \to h_{r\theta}\,, h_{r\phi}$$

$$\delta g_{\mu\nu}^{\text{even}} : H_0 \to h_{tt};\ H_1 \to h_{tr};\ H_2 \to h_{rr};\ K \to h_{\theta\theta}\,, h_{\phi\phi}$$

$$\vartheta : R$$

All decomposed into harmonics (ℓ, m)

- **Step 2: 10+1 equations to 2+1**

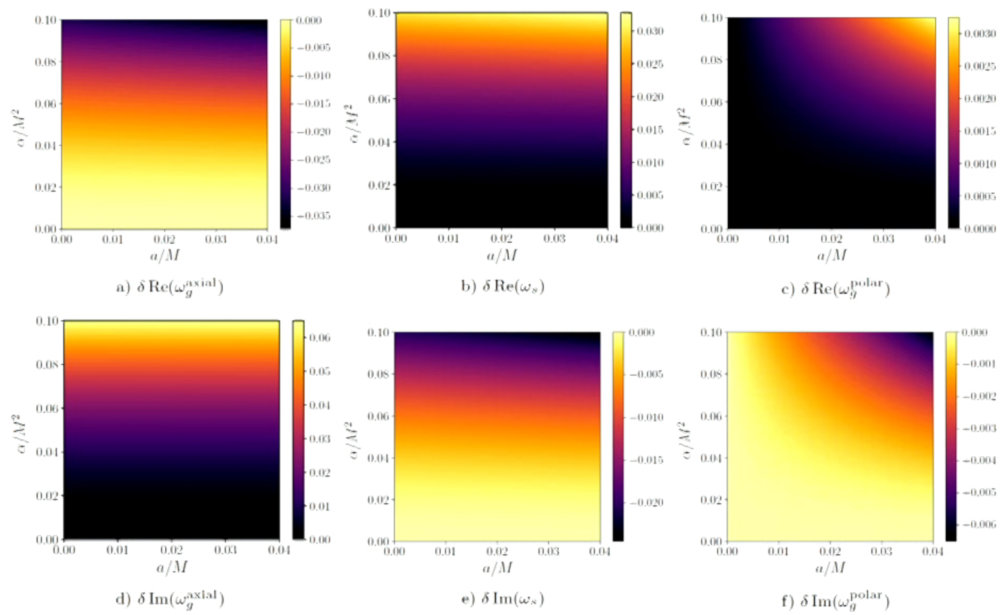
$$\mathcal{D}\Psi_j + V_j\Psi_j = S_j\left[\Psi_k, \partial_r\Psi_k\right]$$

$$\delta g_{\mu\nu}^{\text{odd}} : \Psi_{\text{RW}}\,, \quad \delta g_{\mu\nu}^{\text{even}} : \Psi_{\text{ZM}}$$

$$j \in \{\text{R}, \text{RW}, \text{ZM}\}, k \in \{\{\text{R}, \text{RW}, \text{ZM}\} - j\}$$

Only Ψ_{RW} and R are coupled

- **Step 3: Evaluate QNM shifts**
  - ❑ Direct integration



	a/M	α/M^2
$\mathrm{Re}(\omega_g^{\mathrm{axial}})$	↑	↑
$\mathrm{Im}(\omega_g^{\mathrm{axial}})$	↓	∼
$\mathrm{Re}(\omega_s)$	↓	↓
$\mathrm{Im}(\omega_s)$	↑	∼
$\mathrm{Re}(\omega_g^{\mathrm{polar}})$	↑	∼
$\mathrm{Im}(\omega_g^{\mathrm{polar}})$	↓	∼

(Wagle et al., arXiv 2103.09913)

Example: dCS Gravity (NP)

$$H_0^{(0,0)}\Psi_0^{(1,1)} = \mathcal{S}^{(1,1)}_{\rm geo}(\Psi_0^{(0,1)}) + \mathcal{S}^{(1,1)}(\vartheta^{(1,1)}, h^{(0,1)})$$

Metric Reconstruction

Scalar equation (IRG):

$$\left[r(r-2M)\partial_r^2 + 2(r-M)\partial_r - \frac{4\chi m M^2\omega}{r-2M} - {}_0A_{\ell m}\right]\Theta_{\ell m}(r)$$
$$= -\pi^{-\frac{1}{2}}M^2\left(V^R_{\ell m}(r) + \bar{\eta}_{\ell m}V^{\dagger R}_{\ell\, -m}(r)\right) - \left(V^{\square}_{\ell m}(r) + \bar{\eta}_{\ell m}V^{\dagger\square}_{\ell\, -m}(r)\right)$$

Driven by $\vartheta^{(1,1)}$

Driven by $h^{(0,1)}$

Teukolsky equation (IRG):

$$\left[r(r-r_s)\partial_r^2 + 6(r-M)\partial_r + \frac{C(r)}{r-r_s} + \frac{4m\chi M(i(r-M) - M\omega r)}{r(r-r_s)} - {}_{+2}A_{\ell m}\right]{}_2R^{(1,1)}_{\ell m}(r)$$
$$= -2r^2\left[\mathcal{S}^{\rm geo}_{\ell m}(r) + \left(\mathcal{S}^A_{\ell m}(r) + \bar{\eta}_{\ell m}\mathcal{S}^{\dagger A}_{\ell\, -m}(r)\right) + \left(\mathcal{S}^B_{\ell m}(r) + \bar{\eta}_{\ell m}\mathcal{S}^{\dagger B}_{\ell\, -m}(r)\right)\right]$$

QNMs can now be computed!

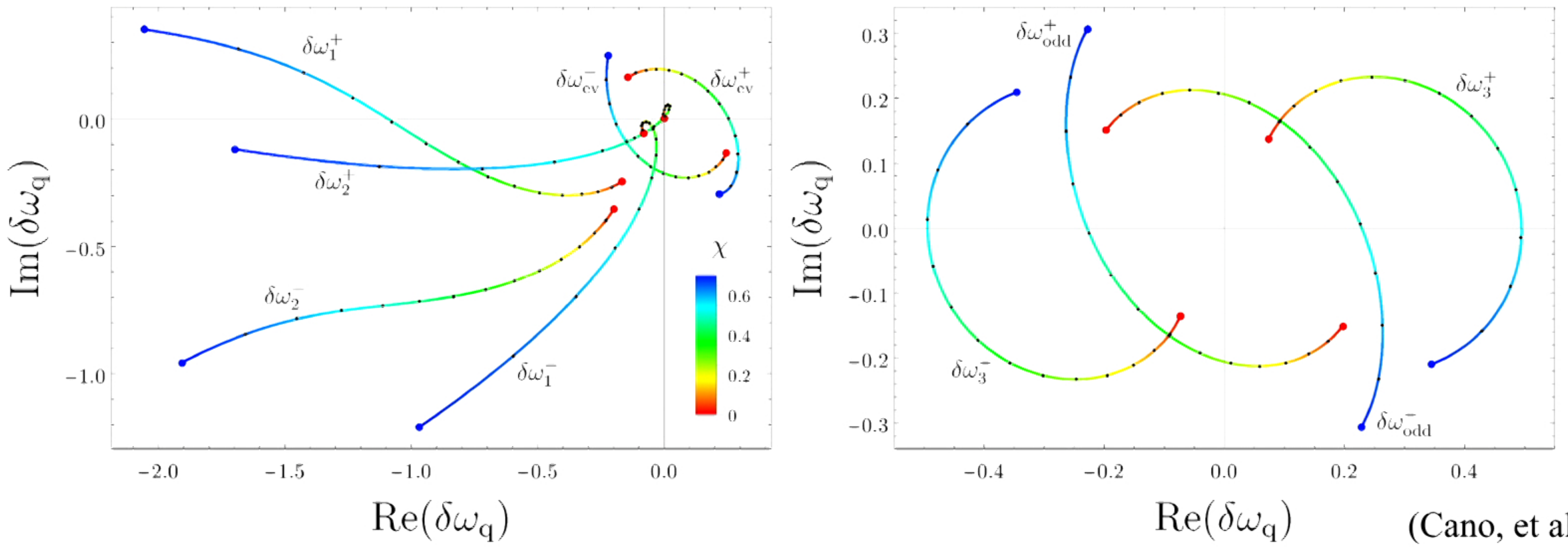
(Wagle, **DL**, Chen, & Yunes, to submit)

Example: Higher-derivative Gravity

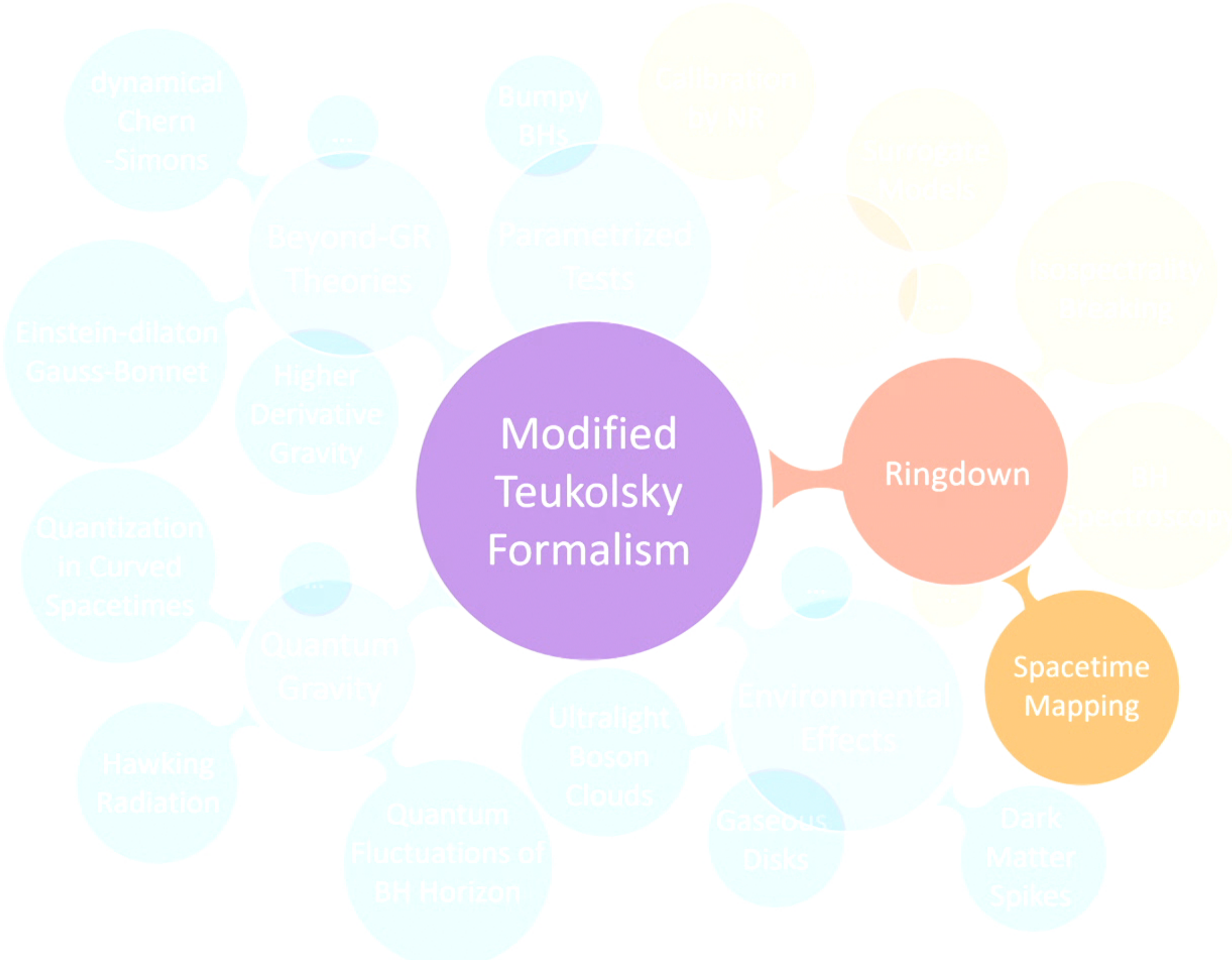
- Action: $S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left\{R + \ell^4\mathcal{L}_{(6)} + \ell^6\mathcal{L}_{(8)} + \ldots\right\}$

$$\mathcal{L}_{(6)} = \lambda_{\rm e} R_{\mu\nu}{}^{\rho\sigma} R_{\rho\sigma}{}^{\delta\gamma} R_{\delta\gamma}{}^{\mu\nu} + \lambda_{\rm o} R_{\mu\nu}{}^{\rho\sigma} R_{\rho\sigma}{}^{\delta\gamma}\, {}^{*}R_{\delta\gamma}{}^{\mu\nu}$$

- Up to $\mathcal{O}(\chi^{14})$, the results are valid for $\chi \lesssim 0.7$
- Matches well with metric perturbations (mismatch $\lesssim 5\%$)!



(Cano, et al., 2023)



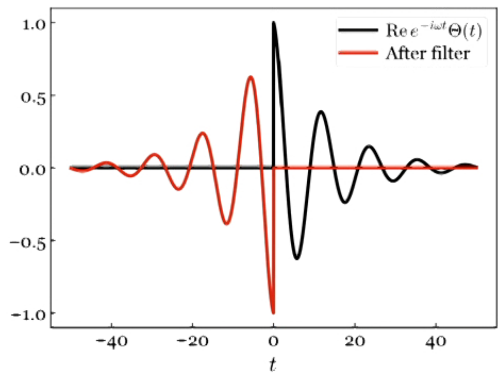
Modified Teukolsky Formalism
Ringdown
Spacetime Mapping
dynamical Chern -Simons
Bumpy BHs
Beyond-GR Theories
Parametrized Tests
Einstein-dilaton Gauss-Bonnet
Higher Derivative Gravity
Quantization in Curved Spacetimes
Quantum Gravity
Hawking Radiation
Quantum Fluctuations of BH Horizon
Ultralight Boson Clouds
Environmental Effects
Gaseous Disks
Dark Matter Spikes

QNM Filter

- Invented by Ma et al. (Ma et al., 2022; Ma et al., 2023)
- Rational Filter:

$$\left(\frac{d}{dt} + i\omega_{lmn}\right) e^{-i\omega_{lmn}t} = 0$$

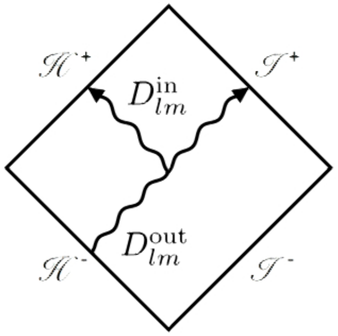
$$\mathcal{F}_{lmn} = \frac{\omega - \omega_{lmn}}{\omega - \omega^*_{lmn}}, \quad \mathcal{F}_{\rm tot} = \prod_{lmn} \mathcal{F}_{lmn}$$



(Ma et al., arXiv 2207.10870)

- Useful for revealing nonlinearities, overtones, etc.
- Full Filter:

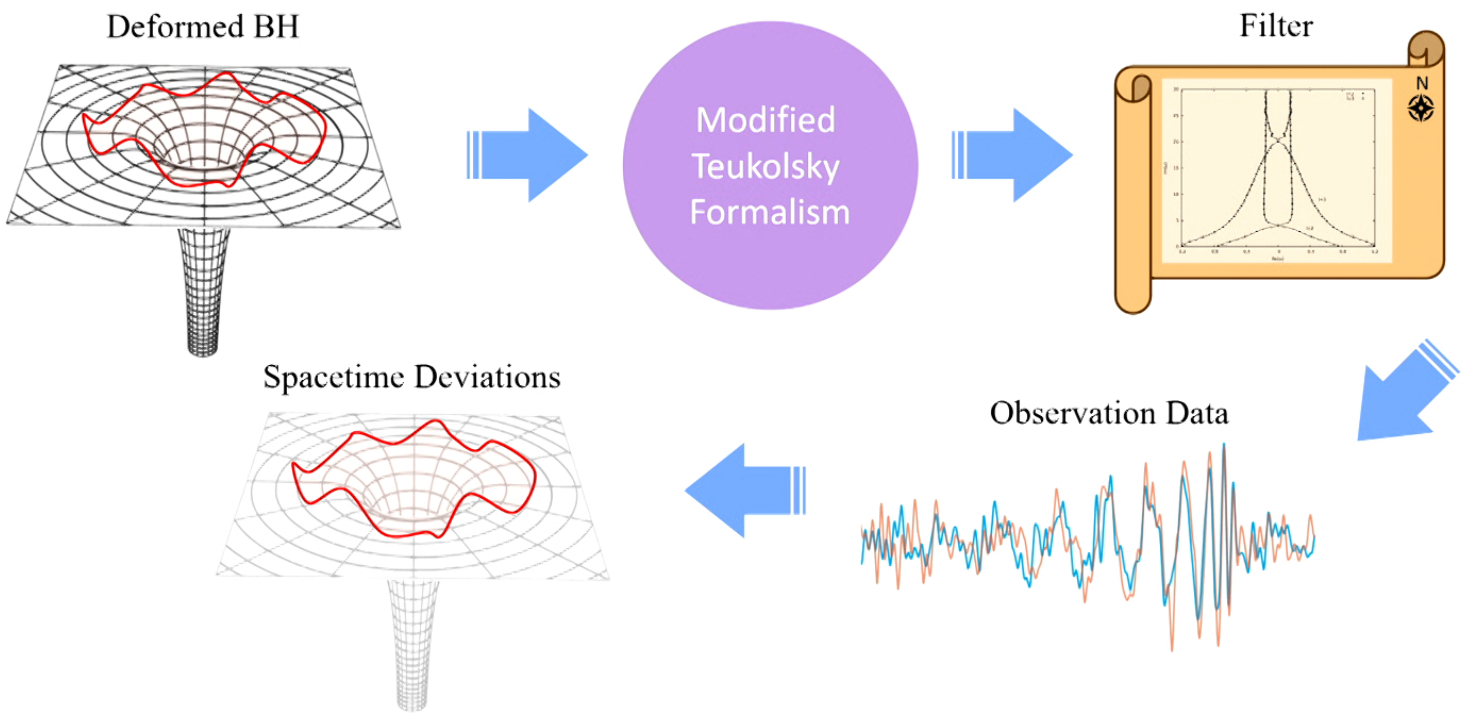
$$D^{\rm out}_{lm}\left(\omega_{lmn}\right) = 0 \rightarrow \mathcal{F}^D_{lm} = \frac{D^{\rm out}_{lm}}{D^{\rm out *}_{lm}}$$

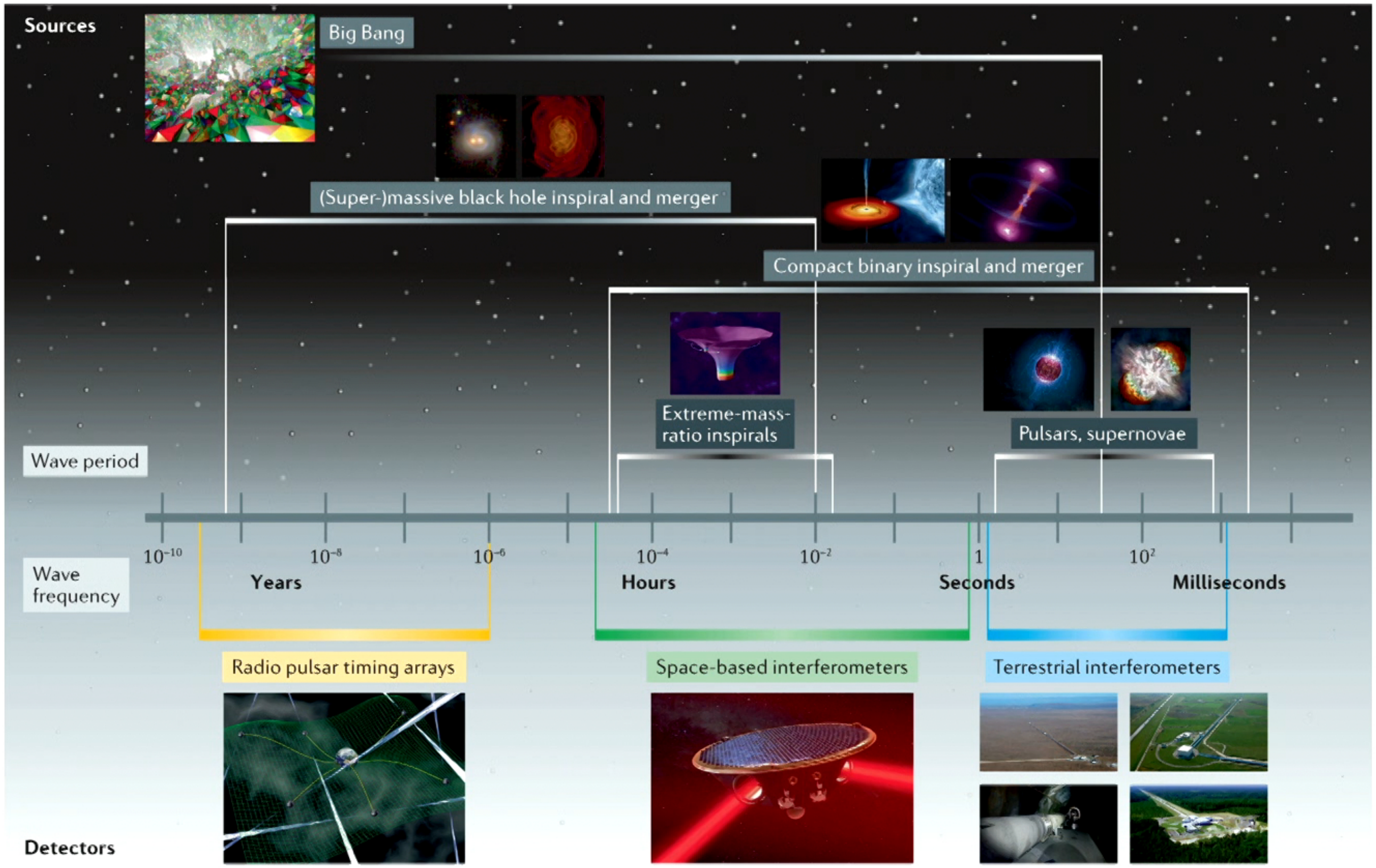


(Pound & Wardell, arXiv 2101.04592)

- Stable against perturbations to BH potential

Spacetime Matching via Filter

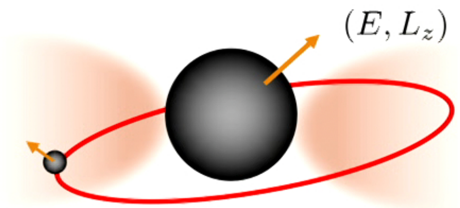




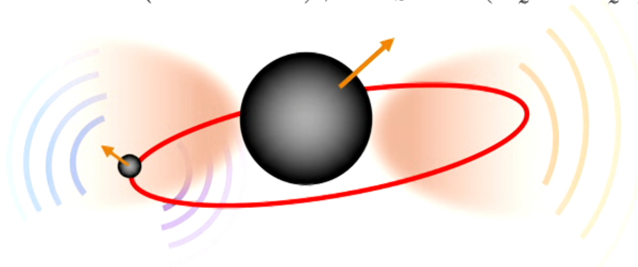
(Image credit: Bailes et al.)

EMRIs beyond GR

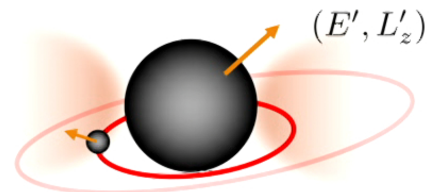
$$\dot{E} = -(\dot{E}^H + \dot{E}^\infty)\,, \quad \dot{L}_z = -(\dot{L}_z^H + \dot{L}_z^\infty)$$



Geodesics in a deformed BH



Radiation Reaction



Corrected geodesics

$$\dot{E}^{H,\infty} \propto \frac{|\Psi_0^{H,\infty}|^2}{\omega^2}\,, \quad \dot{L}_z^{H,\infty} \propto \frac{\dot{E}^{H,\infty}}{\omega}$$

$$H_0 \Psi_0^{(1,1)} = \mathcal{S}^{\mu\nu} h_{\mu\nu}^{(0,1)}$$

$$H_0 \Psi_0^{(0,1)} = \mathcal{S}_p^{(0,1)} \longleftarrow T^{\mu\nu}(x) = \mu \int d\tau u^\mu u^\nu \delta^{(4)}[x - z(\tau)]$$

Vacuum: $h_{\mu\nu}^{(0,1)} = (\mathcal{O}_{\mu\nu} + \bar{\mathcal{O}}_{\mu\nu}\hat{\mathcal{C}})\Psi_0^{(0,1)}$

Non-vacuum: $h_{\mu\nu}^{(0,1)} \overset{?}{\longleftrightarrow} \Psi_0^{(0,1)}$

(LaHaye, Weller, **DL**, Yang, in prepare)

Metric Reconstruction (non-vacuum)

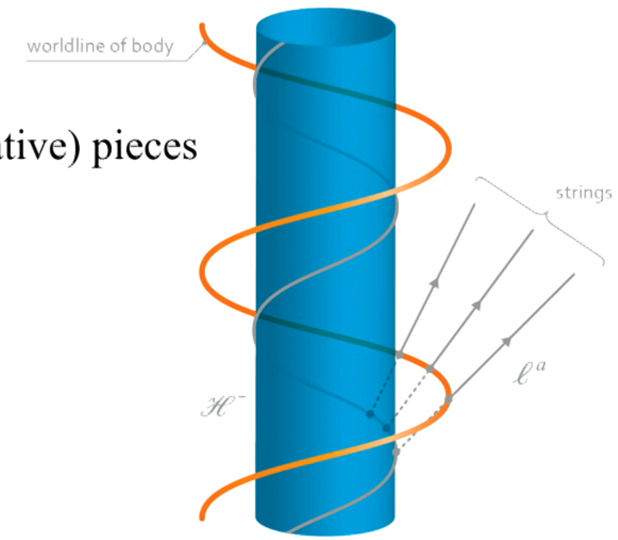
- Necessary for nonlinear effects in both GR (e.g., 2nd order self-force) and bGR
- **Lorenz gauge** (Dolan et al., 2022; Dolan et al., 2023)
  - CCK with an additional gauge transformation

$$\xi^\mu = \zeta^2 \nabla_\nu H^{\mu\nu} - g^{\mu\nu}\nabla_\nu \chi$$

  - (Spin 0 + Spin 1) [gauge in vacuum] + Spin 2 + completion (non-radiative) pieces
- **GHZ** (Green, Holland, & Zimmerman, 2020; Toomani, 2022)
  - CCK with an additional correction tensor

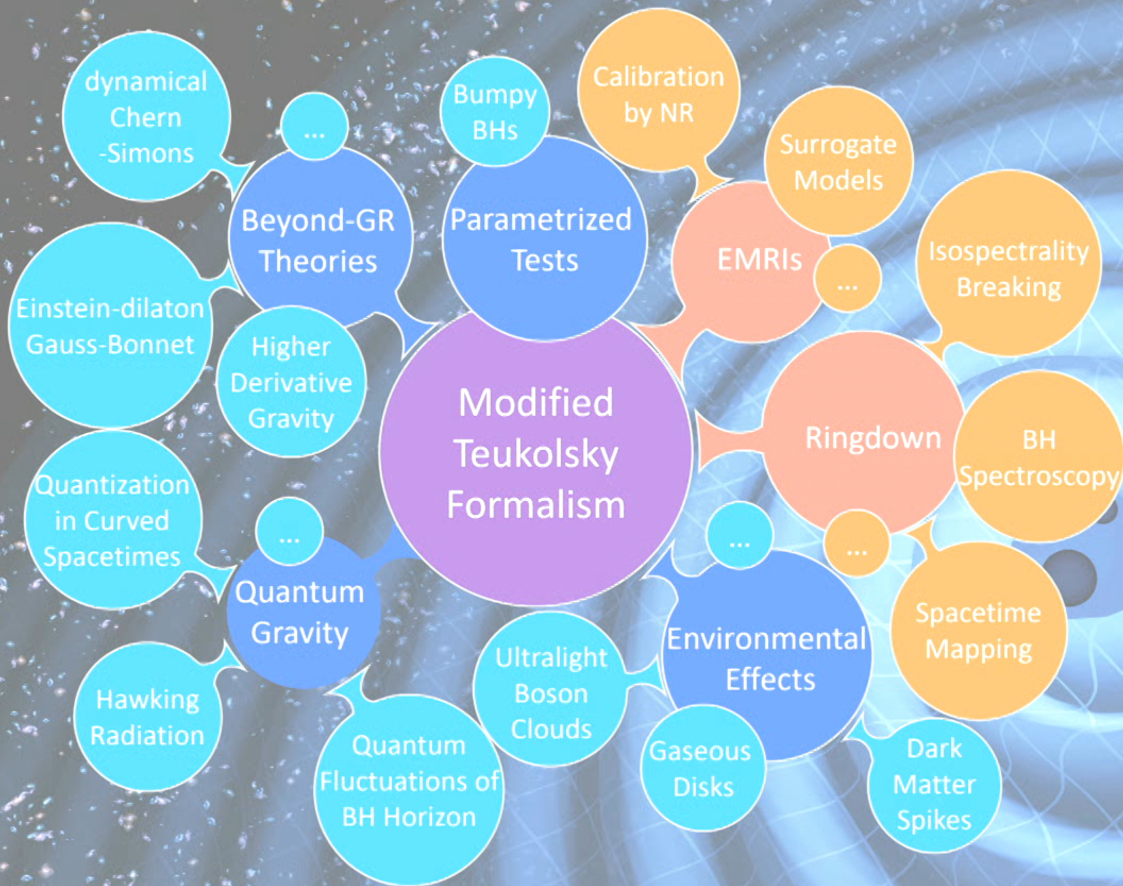
$$h_{ab} = (\mathcal{L}_\xi g)_{ab} + \dot{g}_{ab} + x_{ab} + \mathrm{Re}\left(\mathcal{S}^\dagger \Phi\right)_{ab}$$

  - ξ : transform to the shadowless gauge



(Green et al., arXiv 1908.09095)

Applying these procedures to bGR EMRIs
(LaHaye, **DL,** Weller, Chen, Yang, in prepare)



Full IMR waveform:
- PN and BHP
- Calibration to comparable mass

Fast waveform generation:
- Surrogate models
- Phenom models

Environmental vs bGR

A pipeline:
- Joint efforts with established frameworks
- Effective data analysis framework