Title: A New Picture of Quantum Dynamics and A New Kind of Tensor Network

Speakers:

Series: Quantum Matter

Date: October 10, 2023 - 11:00 AM

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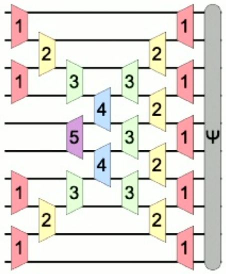
Abstract: I will introduce a new picture of quantum dynamics that might be thought of as "gauging" Schrodinger's picture that results in many "local" Hilbert spaces [1]. Truncating the dimensions of the local Hilbert spaces in this new picture yields an exciting new kind of tensor network whose computational cost does not increase with increasing spatial dimension (for fixed bond dimension) [2]. More detail: Although quantum dynamics are local for local Hamiltonians, the locality is not explicit in the Schrodinger picture since the wavefunction amplitudes do not obey a local equation of motion. In the first part of this talk, I will introduce a new picture of quantum dynamics--the gauge picture--which is similar to Schrodinger's picture, but with the feature that spatial locality is explicit in the equations of motion. In a sense, the gauge picture might be thought of as the result of "gauging" the global unitary symmetry of quantum dynamics into a local symmetry[1]. In the second part of the talk, I discuss a new kind of tensor network ansatz that is inspired from the gauge picture. In the gauge picture, different regions of space are associated with different Hilbert spaces, which are related by gauge connections. By relaxing the unitary constraint on the gauge connections, we can truncate the Hilbert space dimensions associated with different regions to obtain an approximate description of quantum dynamics. This truncated gauge picture, which we dub "quantum gauge network", is intriguingly similar to a classical lattice gauge theory coupled to a Higgs field (which are "local" wavefunctions in the gauge picture), but with non-unitary connections. In one spatial dimension, a quantum gauge network can be easily mapped to a matrix product density operator, and a matrix product state can be mapped to a quantum gauge network. Unlike tensor networks such as PEPS, quantum gauge networks boast the advantage that for fixed bond dimension, the computational cost does not increase with the number of spatial dimensions! Encoding fermionic wavefunctions is also remarkably straightforward. We provide a simple algorithm for approximately simulating quantum dynamics of bosonic or fermionic Hamiltonians in any spatial dimension. We compare the new quantum dynamics algorithm to exact methods for fermion systems in up to three spatial dimensions [2]. [1] The Gauge Picture of Quantum Dynamics. arXiv:2210.09314 [2] Quantum Gauge Networks: A New Kind of Tensor Network. arXiv:2210.12151

Zoom link: https://pitp.zoom.us/j/94596192271?pwd=MytzNUx4ZEZEemkvcEEzbllWM1J6QT09

A New Picture of Quantum Dynamics and A New Kind of Tensor Network

arXiv:2210.09314



Kevin Slagle
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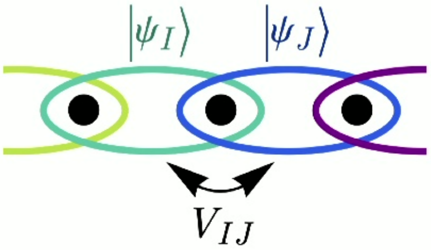
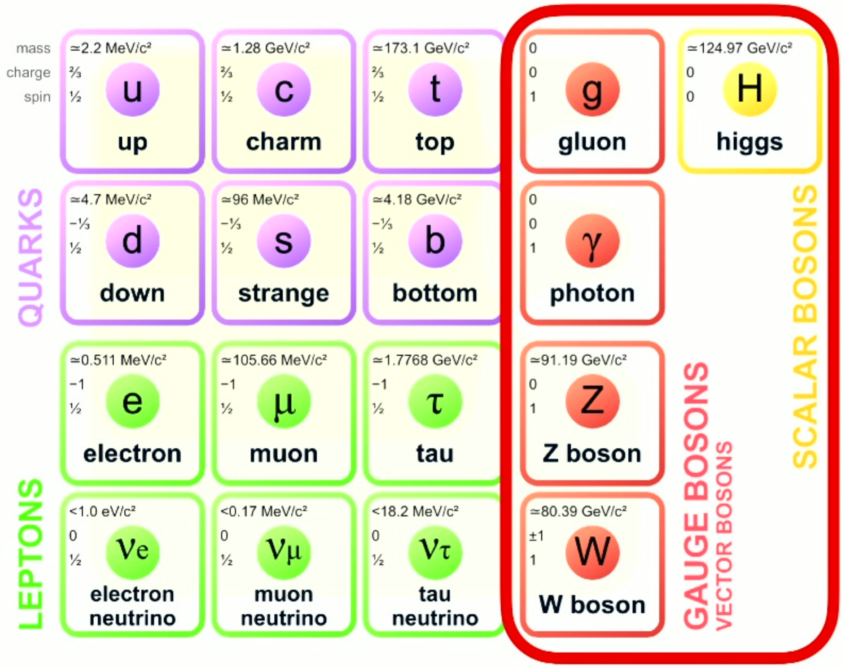
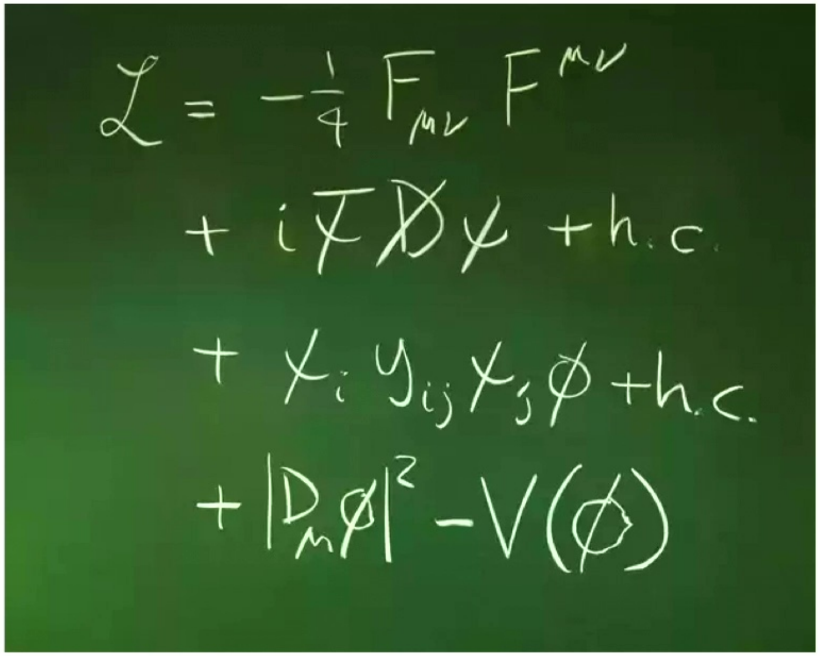
Quantum 7, 1113 (2023)



Gauge Theory

Describes particle interactions in the Standard Model of particle physics





$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \bar{\psi} \not{D} \psi + h.c. + \psi_i y_{ij} \psi_j \phi + h.c. + |D_\mu \phi|^2 - V(\phi)$$

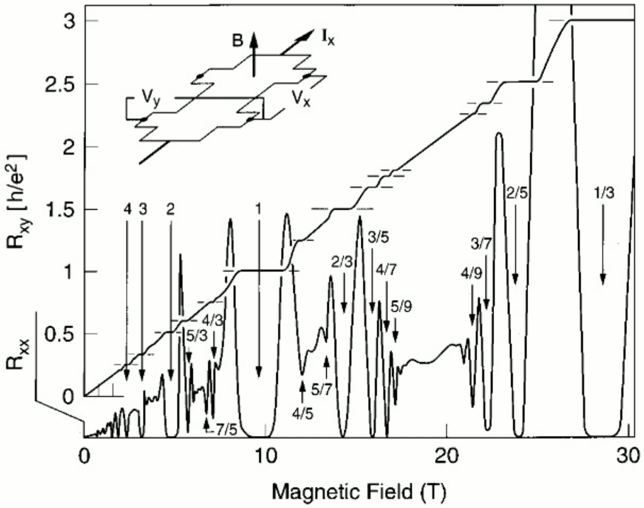
Gauge Theory

Describes exotic emergent physics

spin liquid

phys.org

fractional quantum Hall



Stormer, Tsui, Gossard 1999

Gauge Theory

We can “gauge” a global symmetry to make it a local gauge symmetry:

scalar field theory:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2} m^2 \phi^2$$

➡

coupled to gauge field:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi - A_\mu)^2 + \frac{1}{2} m^2 \phi^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

global symmetry:

$$\phi(x^\mu) \rightarrow \phi(x^\mu) + a$$

➡

local gauge symmetry:

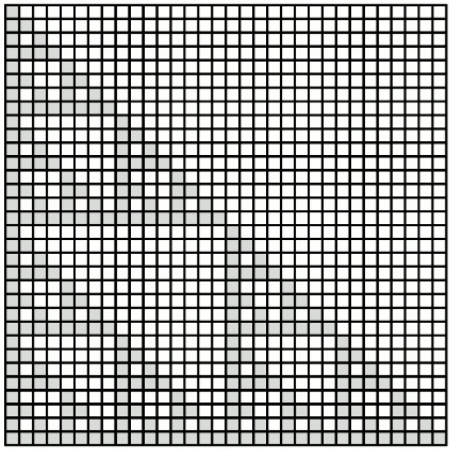
$$\phi(x^\mu) \rightarrow \phi(x^\mu) + \lambda(x^\mu)$$

$$A_\mu \rightarrow A_\mu + \partial_\mu \lambda$$

Gauge Theory

Gauging more exotic symmetries leads to more exotic physics

global fractal symmetry



Yoshida 2013

fracton topological order

Haah 2011
Vijay, Haah, Fu 2015
Shirley, KS, Chen 2019

What other crazy symmetries can one gauge?

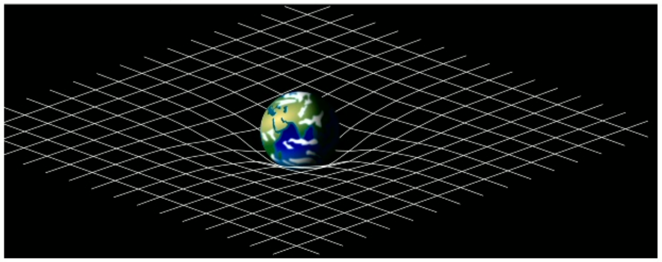
Gauging Lorentz/Poincaré invariance leads to general relativity

Lorentz transformation

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x' \\ y' \\ z' \end{pmatrix}$$

general relativity

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}$$



$$\mathcal{S}[e,\omega] = \int e \wedge e \wedge F[\omega]^\star$$

What other crazy symmetries can one gauge?

What about the global unitary invariance in quantum mechanics?

Schrodinger's equation and expectation values

$$i\,\partial_t \left|\psi(t)\right\rangle = \hat{H} \left|\psi(t)\right\rangle \qquad \left\langle\psi(t)\,\middle|\,\hat{A}(t)\,\middle|\,\psi(t)\right\rangle$$

are invariant under a global unitary transformation

$$\left|\psi(t)\right\rangle \to \hat{U} \left|\psi(t)\right\rangle \qquad \hat{A}(t) \to \hat{U}\,\hat{A}(t)\,\hat{U}^\dagger$$

$$\hat{H}(t) \to \hat{U}\,\hat{H}(t)\,\hat{U}^\dagger$$

Can we gauge this into a *local* invariance?

What about the global unitary invariance in quantum mechanics?

Schrodinger's equation and expectation values

$$i\,\partial_t \left|\psi(t)\right\rangle = \hat{H}\left|\psi(t)\right\rangle \qquad \left\langle \psi(t)\,\middle|\,\hat{A}(t)\,\middle|\,\psi(t)\right\rangle$$

are invariant under a global unitary transformation

$$\left|\psi(t)\right\rangle \to \hat{U}\left|\psi(t)\right\rangle \qquad \hat{A}(t) \to \hat{U}\,\hat{A}(t)\,\hat{U}^{\dagger}$$

$$\hat{H}(t) \to \hat{U}\,\hat{H}(t)\,\hat{U}^{\dagger}$$

Can we gauge this into a *local* invariance?

YES

The result is a new picture of quantum dynamics:

gauge picture

$$\partial_t \left|\psi_I\right\rangle = -iH^{\mathrm{G}}_{\langle I\rangle} \left|\psi_I\right\rangle$$

$$\partial_t U_{IJ} = -iH^{\mathrm{G}}_{\langle I\rangle} U_{IJ} + iU_{IJ} H^{\mathrm{G}}_{\langle J\rangle}$$

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Schrodinger picture

$$\partial_t \left|\Psi^{\mathrm{S}}\right\rangle = -iH \left|\Psi^{\mathrm{S}}\right\rangle$$

Heisenberg picture

$$\partial_t A^{\mathrm{H}} = -i[H, A^{\mathrm{H}}]$$

Gauge Picture

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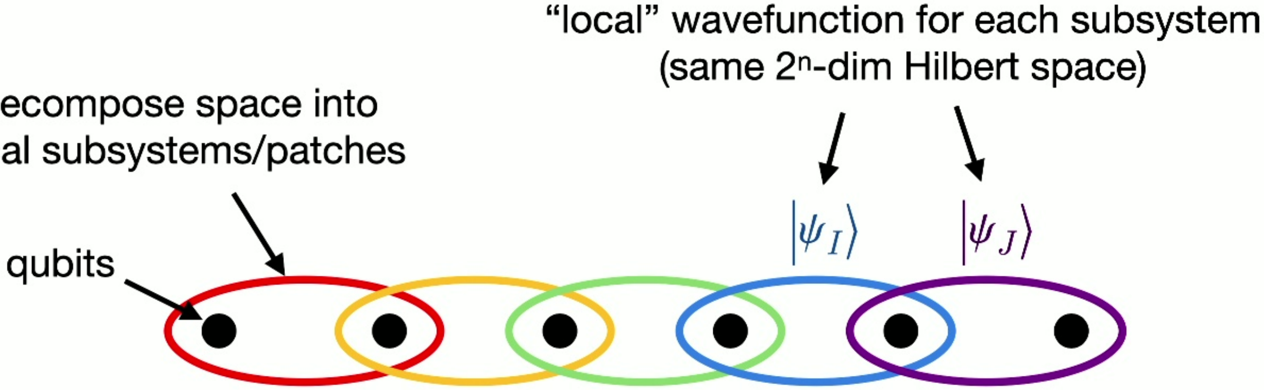

"local" wavefunction for each subsystem
(same 2^n-dim Hilbert space)

decompose space into
local subsystems/patches

qubits

Gauge Picture

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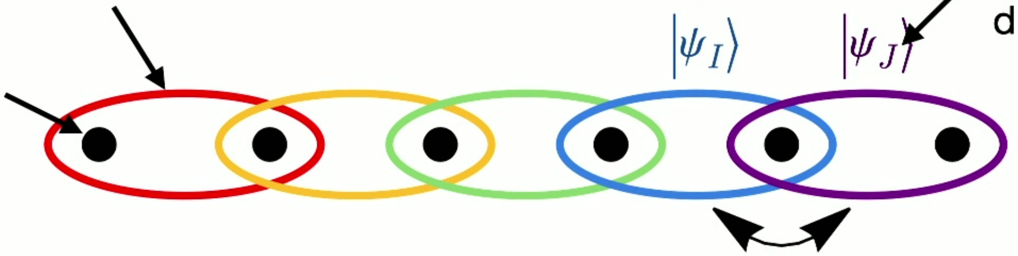

"local" wavefunction for each subsystem
(same 2^n-dim Hilbert space)

decompose space into
local subsystems/patches

I, J, K index
different subsystems

qubits

U_{IJ} Hilbert spaces of neighboring
subsystems related by
time-dependent unitary
transformations/connections

Gauge Picture

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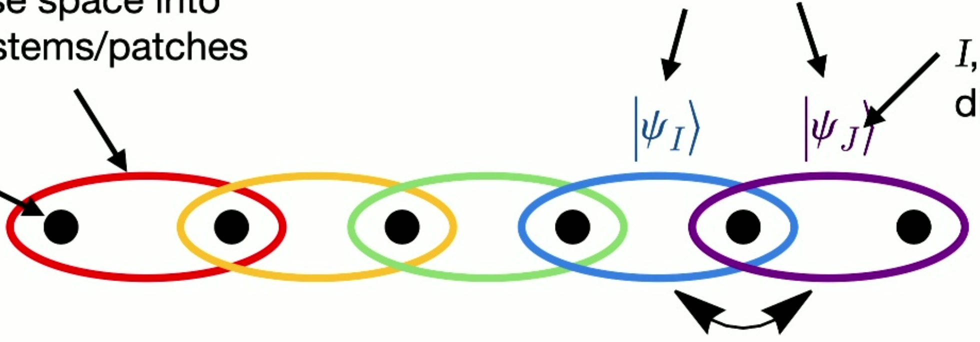

“local” wavefunction for each subsystem (same 2^n-dim Hilbert space)

decompose space into local subsystems/patches

I, J, K index different subsystems

qubits

$|\psi_I\rangle$ $|\psi_J\rangle$

U_{IJ} Hilbert spaces of neighboring subsystems related by time-dependent unitary transformations/connections

Gauge Picture

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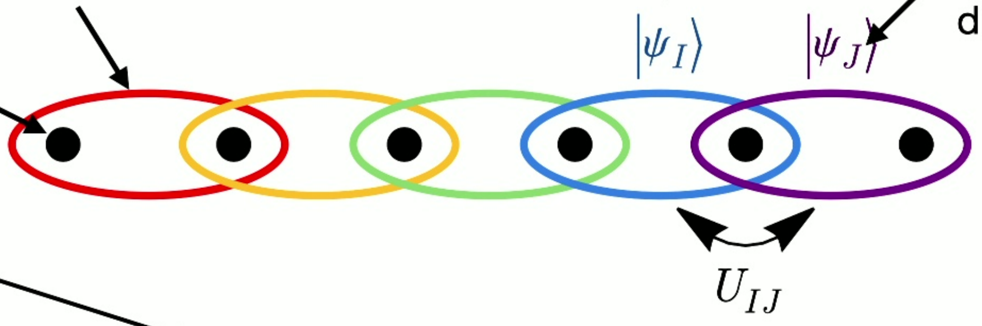


"local" wavefunction for each subsystem
(same 2^n-dim Hilbert space)

decompose space into
local subsystems/patches

I, J, K index
different subsystems

qubits

"G" for gauge
picture

$$\partial_t |\psi_I\rangle = -iH^{\mathrm{G}}_{\langle I\rangle} |\psi_I\rangle$$

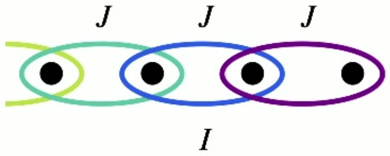
$$\partial_t U_{IJ} = -iH^{\mathrm{G}}_{\langle I\rangle} U_{IJ} + iU_{IJ} H^{\mathrm{G}}_{\langle J\rangle}$$

(same time-independent operators as
Shrodinger's picture)

sum of nearby Hamiltonian terms

$$H^{\mathrm{G}}_{\langle I\rangle} = \sum_{J}^{J\cap I\neq\emptyset} U_{IJ} H^{\mathrm{G}}_J U_{JI}$$

$$H = \sum_J H_J$$



$$\partial_t |\psi_I\rangle = -iH^{\mathrm{G}}_{\langle I\rangle} |\psi_I\rangle$$
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$$H^{\mathrm{G}}_{\langle I\rangle} = \sum_{J}^{J\cap I\neq\emptyset} U_{IJ} H^{\mathrm{G}}_J U_{JI}$$

Gauge Picture

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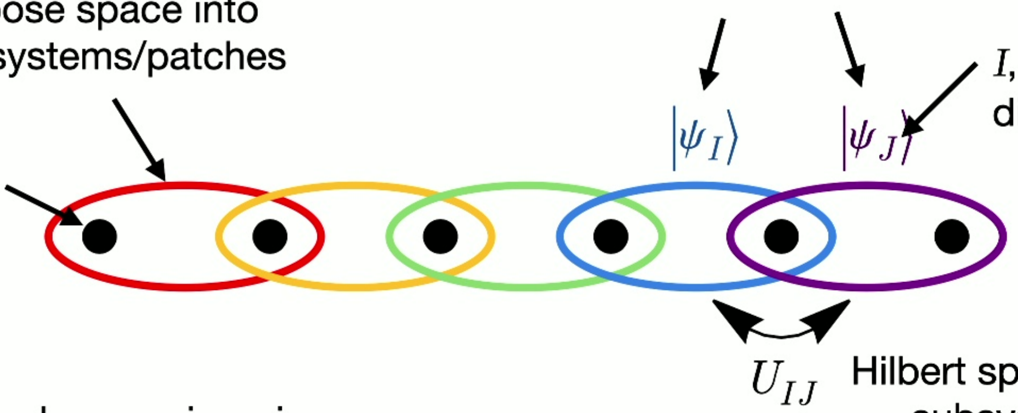

“local” wavefunction for each subsystem
(same 2^n-dim Hilbert space)

decompose space into
local subsystems/patches

qubits

$|\psi_I\rangle$ $|\psi_J\rangle$

I, J, K index
different subsystems

U_{IJ} Hilbert spaces of neighboring
subsystems related by
time-dependent unitary
transformations/connections

local gauge invariance:

$$|\psi_I\rangle \rightarrow \Lambda_I |\psi_I\rangle \quad U_{IJ} \rightarrow \Lambda_I U_{IJ} \Lambda_J^\dagger$$
$$A_I^{\mathrm{G}} \rightarrow \Lambda_I A_I^{\mathrm{G}} \Lambda_I^\dagger$$

arbitrary
unitary

Gauge Picture

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$$\partial_t |\psi_I\rangle = -iH^{\mathrm{G}}_{\langle I\rangle}|\psi_I\rangle$$

$$\partial_t U_{IJ} = -iH^{\mathrm{G}}_{\langle I\rangle}U_{IJ} + iU_{IJ}H^{\mathrm{G}}_{\langle J\rangle}$$

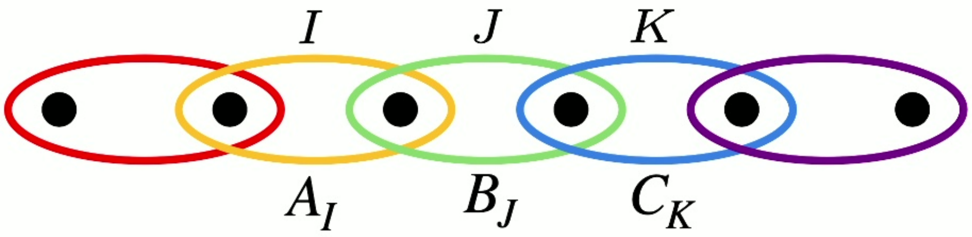
$$H^{\mathrm{G}}_{\langle I\rangle} = \sum_{J}^{J\cap I\neq\emptyset} U_{IJ}H^{\mathrm{G}}_J U_{JI}$$

local gauge invariance:

$$|\psi_I\rangle \to \Lambda_I |\psi_I\rangle$$

$$A^{\mathrm{G}}_I \to \Lambda_I A^{\mathrm{G}}_I \Lambda^\dagger_I$$

$$U_{IJ} \to \Lambda_I U_{IJ} \Lambda^\dagger_J$$



expectation values between patches require a connection:

$$\langle \psi^{\mathrm{H}} | A^{\mathrm{H}}_I \quad B^{\mathrm{H}}_J \quad C^{\mathrm{H}}_K | \psi^{\mathrm{H}} \rangle$$ Heisenberg picture

$$= \langle \psi_I | A^{\mathrm{G}}_I U_{IJ} B^{\mathrm{G}}_J U_{JK} C^{\mathrm{G}}_K | \psi_K \rangle$$ gauge picture

$$\neq \langle \psi_I | A^{\mathrm{G}}_I \, B^{\mathrm{G}}_J \, C^{\mathrm{G}}_K | \psi_K \rangle$$

missing connections U_{IJ} and U_{JK}

Pictures of Quantum Dynamics

Heisenberg picture

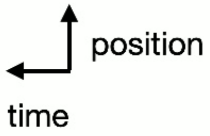
$$\partial_t A^{\mathrm{H}} = i[H, A^{\mathrm{H}}]$$

Schrödinger picture

$$\partial_t \left|\psi^{\mathrm{S}}\right\rangle = -iH \left|\psi^{\mathrm{S}}\right\rangle$$

same expectation values:

$$\langle\psi|A|\psi\rangle$$



Pictures of Quantum Dynamics

Heisenberg picture

$$\partial_t A^{\mathrm{H}} = i[H, A^{\mathrm{H}}]$$

$$A^{\mathrm{H}}(t) = e^{+iHt} A^{\mathrm{H}}(0) e^{-iHt}$$

Schrödinger picture

$$\partial_t \left|\psi^{\mathrm{S}}\right\rangle = -iH \left|\psi^{\mathrm{S}}\right\rangle$$

$$\left|\psi^{\mathrm{S}}(t)\right\rangle = e^{-iHt} \left|\psi^{\mathrm{S}}(0)\right\rangle$$

same expectation values:

$$\langle\psi|A|\psi\rangle$$

Pictures of Quantum Dynamics

Heisenberg picture

$$\partial_t A^{\mathrm{H}} = i[H, A^{\mathrm{H}}]$$

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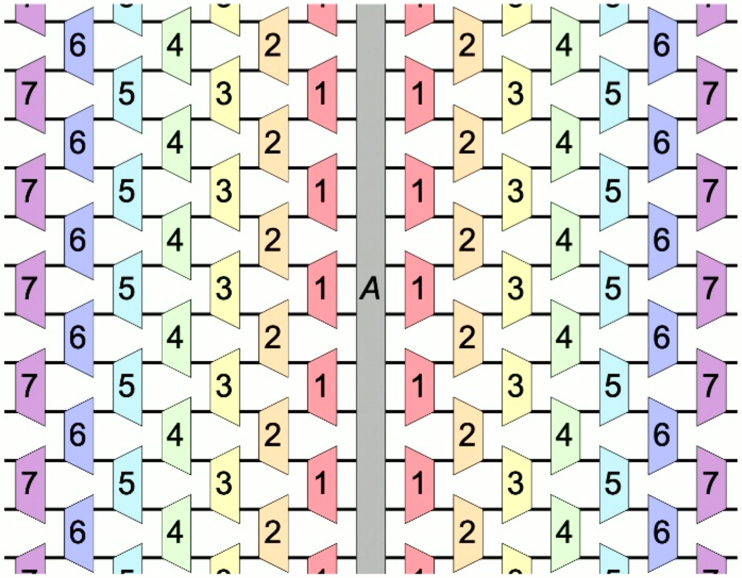
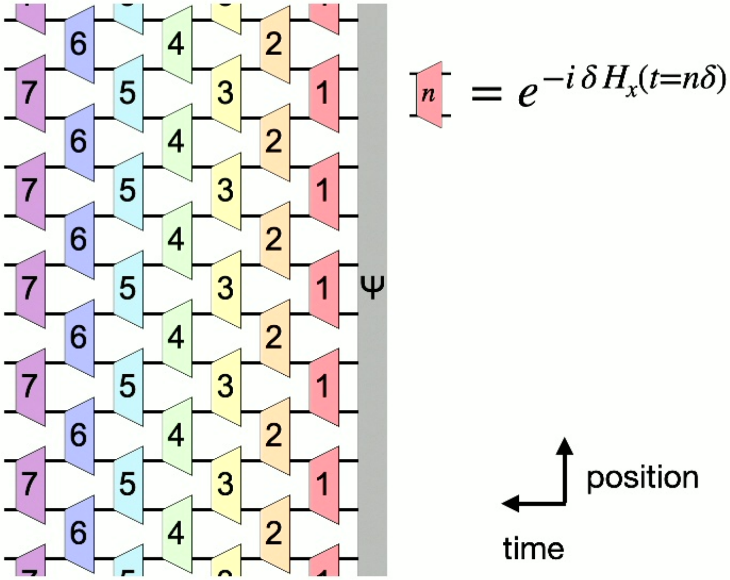


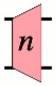
same

$$\langle\psi|A|\psi\rangle$$



$$= e^{-i\,\delta\,H_x(t=n\delta)}$$

Locality

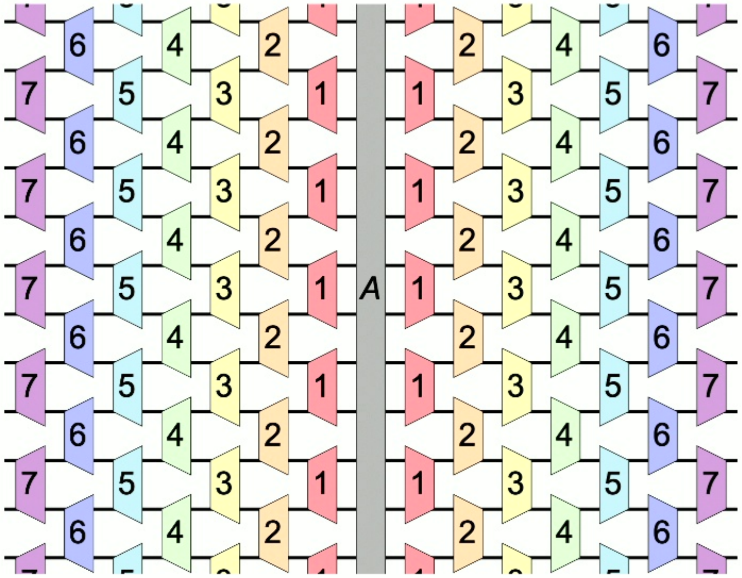


$$= e^{-i\,\delta\, H_x(t=n\delta)}$$

Heisenberg picture

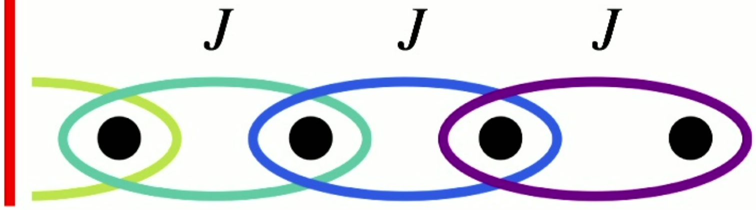
$$\partial_t A^{\mathrm{H}} = i[H, A^{\mathrm{H}}]$$

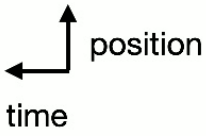
$$A^{\mathrm{H}}(t) = e^{+iHt} A^{\mathrm{H}}(0) e^{-iHt}$$



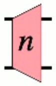
local Hamiltonian

$$H = \sum_J H_J$$





Locality

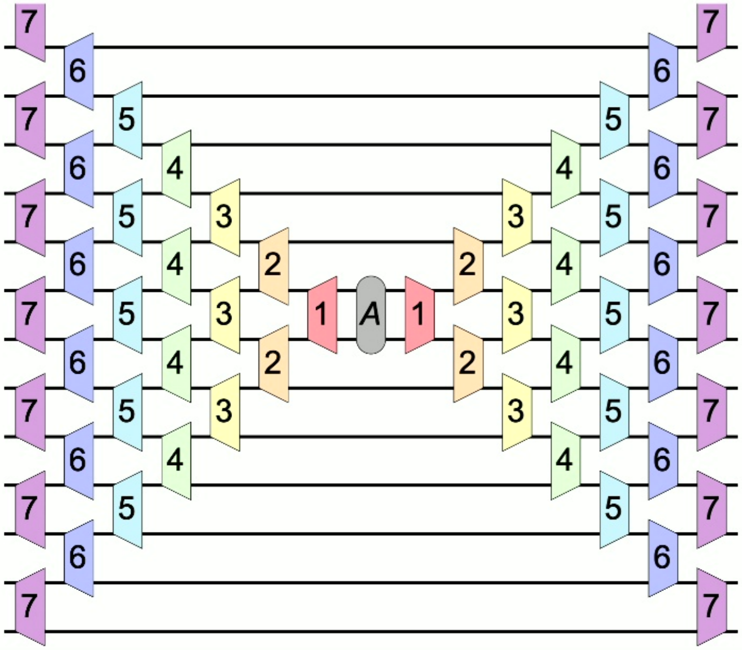


$$= e^{-i\,\delta\,H_x(t=n\delta)}$$

Heisenberg picture

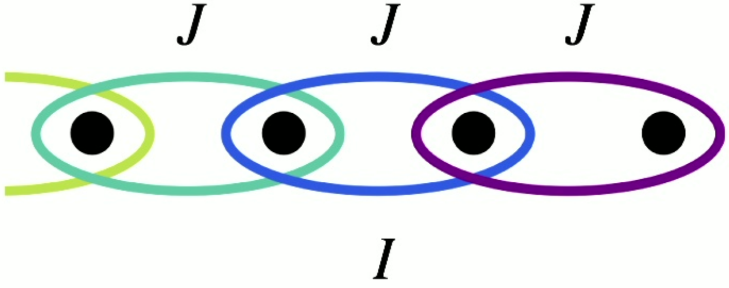
$$\partial_t A_I^{\mathrm{H}} = i[H,\, A_I^{\mathrm{H}}] = i[\underline{H_{\langle I\rangle}^{\mathrm{H}}},\, A_I^{\mathrm{H}}]$$

$$A^{\mathrm{H}}(t) = e^{+iHt}A^{\mathrm{H}}(0)e^{-iHt}$$



local Hamiltonian

$$H = \sum_J H_J$$



patch I only affected by overlapping patches J:

$$\underline{H_{\langle I\rangle} = \sum_{J}^{J\cap I\neq\emptyset} H_J}$$

$$H_{\langle I \rangle} = \sum_{J}^{J \cap I \neq \emptyset} H_J$$

Locality

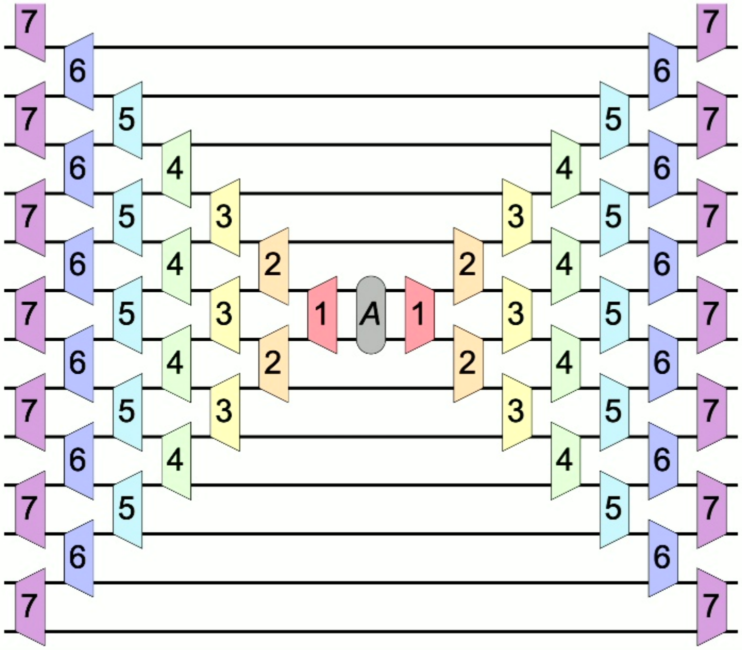
Heisenberg picture

$$\partial_t A_I^{\mathrm{H}} = i[H, A_I^{\mathrm{H}}] = i[\underline{H_{\langle I \rangle}^{\mathrm{H}}}, A_I^{\mathrm{H}}]$$

← explicitly local equation of motion

$$A^{\mathrm{H}}(t) = e^{+iHt} A^{\mathrm{H}}(0) e^{-iHt}$$

results in "light cone" operator growth



$$H_{\langle I \rangle} = \sum_{J}^{J \cap I \neq \emptyset} H_J$$

Locality

Heisenberg picture

$$\partial_t A_I^{\mathrm{H}} = i[H, A_I^{\mathrm{H}}] = i[\underline{H_{\langle I \rangle}^{\mathrm{H}}}, A_I^{\mathrm{H}}]$$

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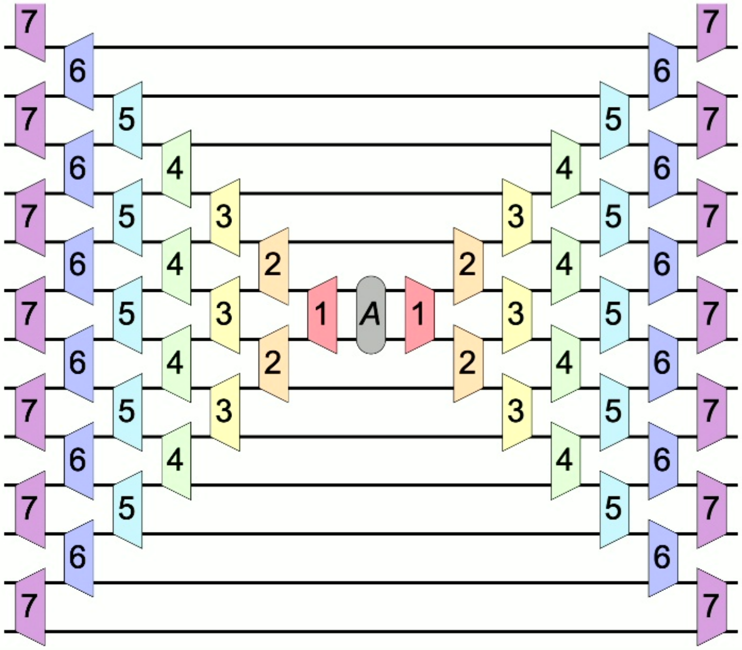


results in "light cone" operator growth

Schrodinger's equation is not explicitly local

$$\partial_t |\psi^{\mathrm{S}}\rangle = -iH |\psi^{\mathrm{S}}\rangle$$

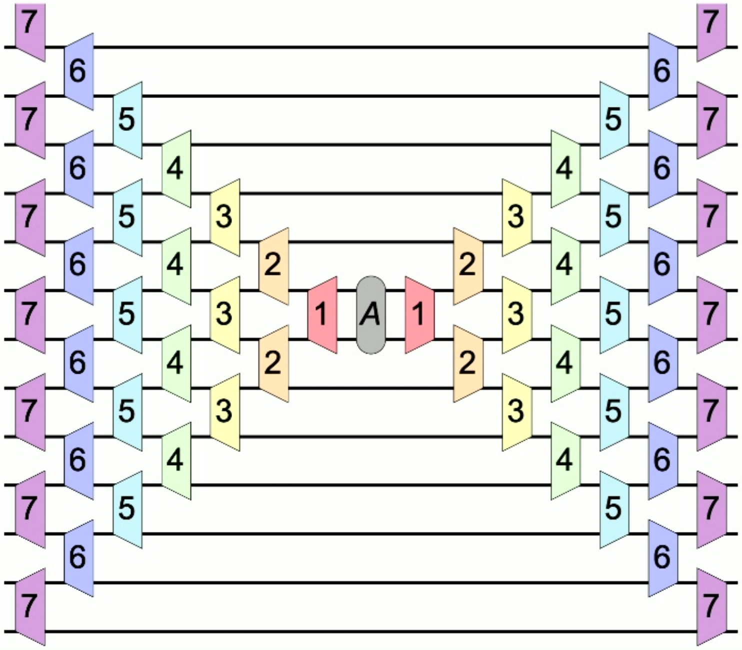
$$H_{\langle I\rangle} = \sum_{J}^{J\cap I\neq\emptyset} H_J$$

Locality

Heisenberg picture

$$\partial_t A_I^{\mathrm{H}} = i[H,\, A_I^{\mathrm{H}}] = i[\underline{H_{\langle I\rangle}^{\mathrm{H}}},\, A_I^{\mathrm{H}}]$$

$$A^{\mathrm{H}}(t) = e^{+iHt} A^{\mathrm{H}}(0) e^{-iHt}$$



← explicitly local equation of motion

results in "light cone" operator growth

The gauge picture extends this sense of locality to wavefunctions!

Local Wavefunction $\partial_t |\psi_I\rangle = -iH^{\mathrm{G}}_{\langle I\rangle} |\psi_I\rangle$

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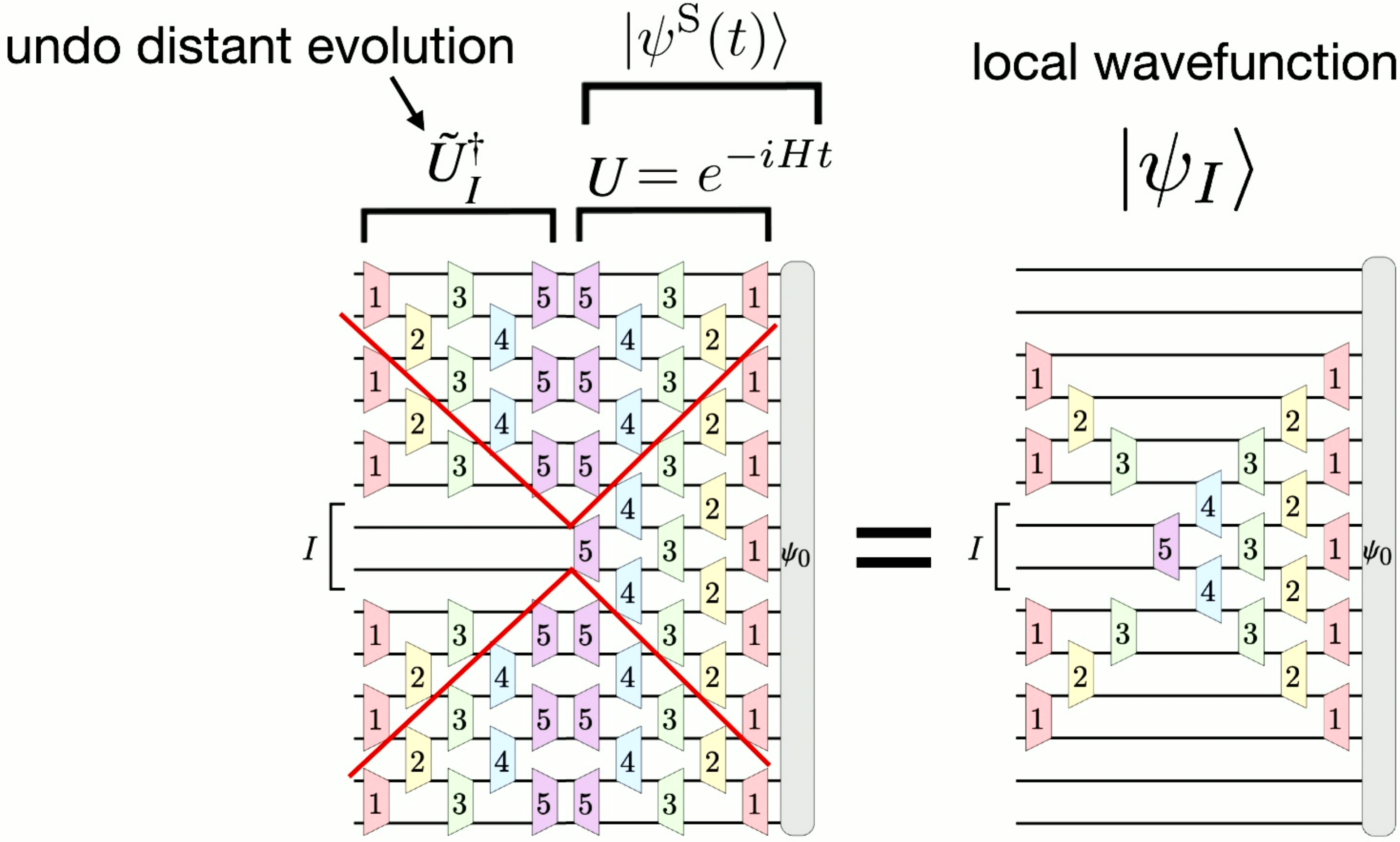

undo distant evolution

$|\psi^{\mathrm{S}}(t)\rangle$

local wavefunction

$\tilde{U}_I^\dagger$

$U = e^{-iHt}$

$|\psi_I\rangle$

Local Wavefunction $\partial_t |\psi_I\rangle = -iH^{\mathrm{G}}_{\langle I\rangle} |\psi_I\rangle$

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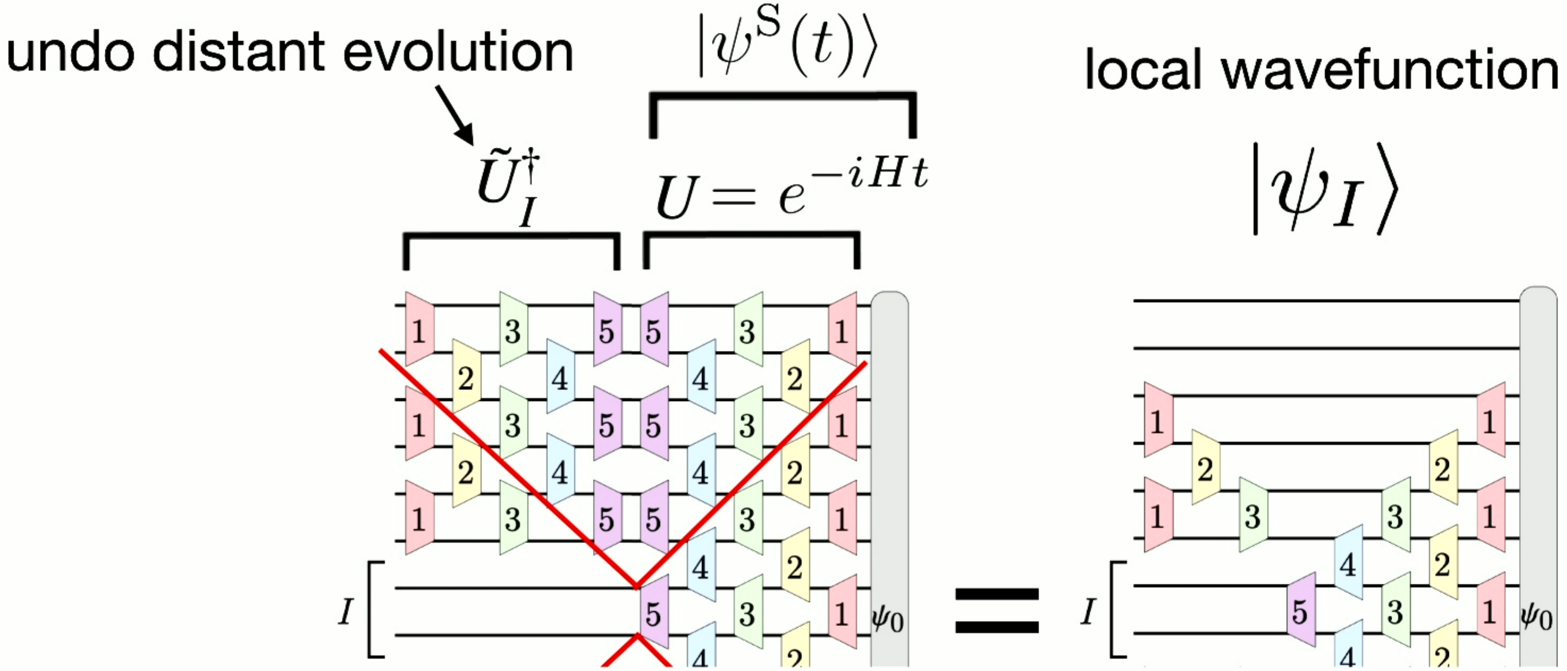


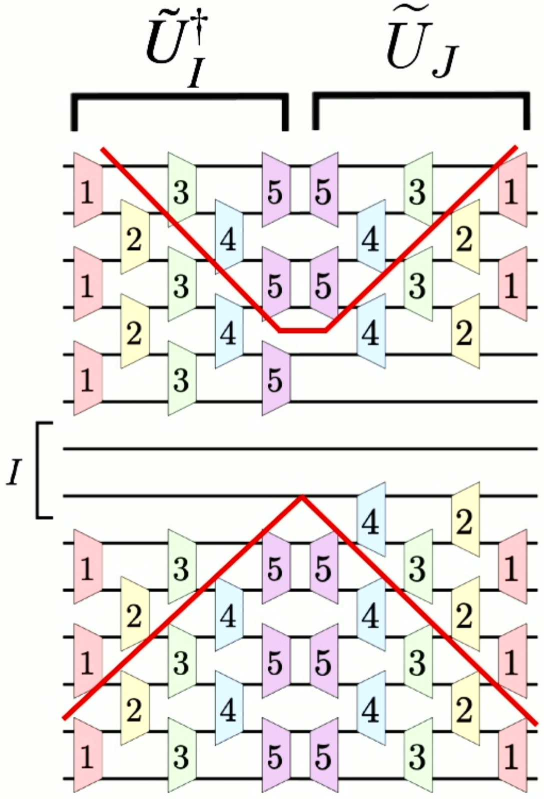
undo distant evolution

$|\psi^{\mathrm{S}}(t)\rangle$

local wavefunction

Same wavefunction as in the interaction picture with interaction Hamiltonian

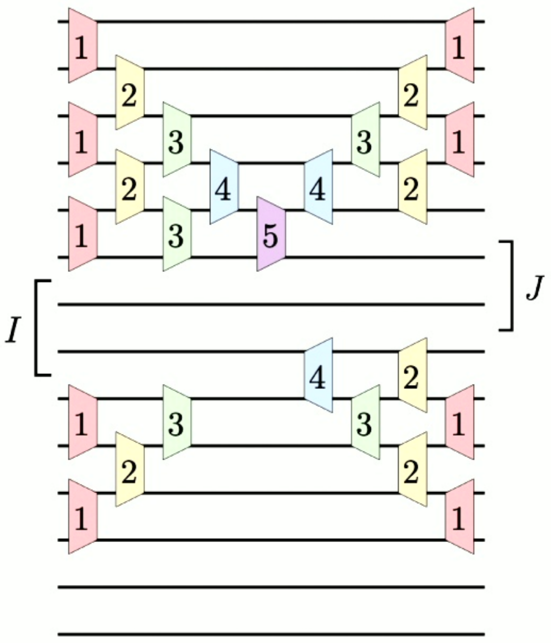
$$H_{\langle I\rangle} = \sum_{J}^{J\cap I\neq\emptyset} H_J$$



$=$

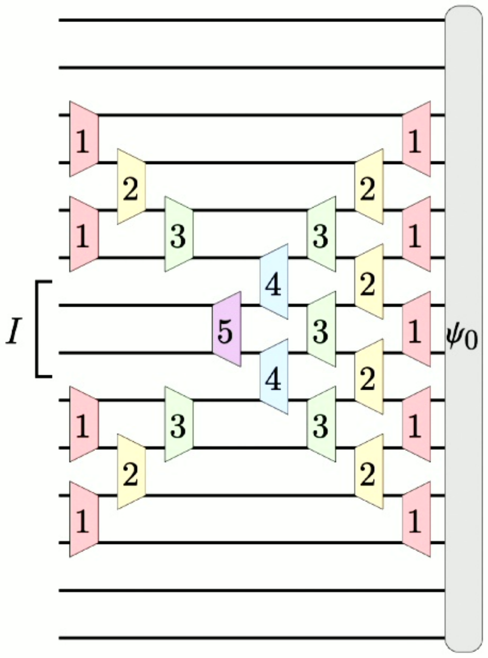
connection

$$U_{IJ}$$



local wavefunction

$$|\psi_I\rangle$$



Local Equations of Motion

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Schrödinger picture

$$\partial_t \left|\psi^{\mathrm{S}}\right\rangle = -iH \left|\psi^{\mathrm{S}}\right\rangle$$

- not explicitly local 😔
- global wavefunction derivative depends on all Hamiltonian terms

---

Heisenberg picture

$$\partial_t A_I^{\mathrm{H}} = i[H_{\langle I\rangle}^{\mathrm{H}}, A_I^{\mathrm{H}}]$$

gauge picture

$$\partial_t \left|\psi_I\right\rangle = -iH_{\langle I\rangle}^{\mathrm{G}} \left|\psi_I\right\rangle$$

$$\partial_t U_{IJ} = -iH_{\langle I\rangle}^{\mathrm{G}} U_{IJ} + iU_{IJ} H_{\langle J\rangle}^{\mathrm{G}}$$

- explicitly local 😎
- local operator/wavefunction derivative only depends on local Hamiltonian terms in these pictures

$$H_{\langle I\rangle} = \sum_J^{J\cap I\neq\emptyset} H_J$$

Interpolating between Pictures

Schrödinger picture

$$\partial_t |\psi^{\mathrm{S}}\rangle = -iH\,|\psi^{\mathrm{S}}\rangle$$

$$\gamma \gg 1$$

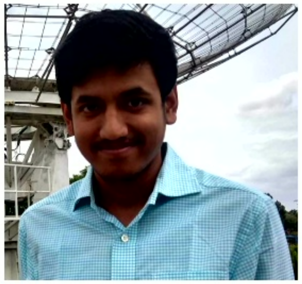
gauge picture

$$\partial_t |\psi_I\rangle = -iH^{\mathrm{G}}_{\langle I\rangle}\,|\psi_I\rangle$$

$$\begin{aligned}\partial_t U_{IJ} &= -iH^{\mathrm{G}}_{\langle I\rangle}U_{IJ}\\ &\quad + iU_{IJ}H^{\mathrm{G}}_{\langle J\rangle}\end{aligned}$$

$$H_{\langle I\rangle} \to H_{\langle I\rangle} + \gamma X_I$$

$$X_I = \mathrm{Tr}_I \sum_{J}^{I\cap J\neq\emptyset} (-i)\left(U_{IJ} - U^{\dagger}_{IJ}\right)$$



Sayak Guha Roy

Guha Roy, KS 2023

Schrodinger → Heisenberg

KS 2022

- Apply time-dependent unitary:

$$\left|\psi^{\mathrm{H}}(t)\right\rangle = U^{\dagger}(t)\left|\psi^{\mathrm{S}}(t)\right\rangle$$
$$A^{\mathrm{H}}(t) = U^{\dagger}(t)A^{\mathrm{S}}(t)U(t)$$

- Let $\partial_t U(t) = -iG^{\mathrm{S}}(t)U(t)$
- Want constant wavefunction

$$\partial_t\left|\psi^{\mathrm{H}}\right\rangle = -i(H^{\mathrm{H}} - G^{\mathrm{H}})\left|\psi^{\mathrm{H}}\right\rangle$$

- Need $G^{\mathrm{H}}(t) = H^{\mathrm{H}}(t)$

Heisenberg → Gauge Picture

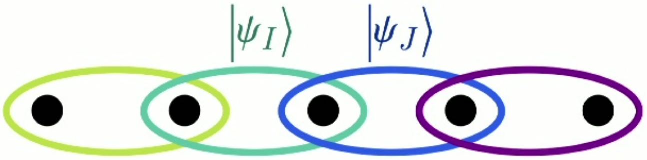
KS 2022

- Apply time and **space**-dependent unitary:

$$|\psi_I\rangle = U_I|\psi^{\mathrm{H}}\rangle$$

gauge picture →

$$A_I^{\mathrm{G}} = U_I A_I^{\mathrm{H}} U_I^\dagger$$



- Let $\partial_t U_I = -iG_I U_I$
- Want constant operators

$$\partial_t A_I^{\mathrm{G}} = i[H_{\langle I\rangle}^{\mathrm{G}} - G_I, A_I^{\mathrm{G}}]$$

$$H_{\langle I\rangle} = \sum_J^{J\cap I\neq\emptyset} H_J$$

- Need $G_I = H_{\langle I\rangle}^{\mathrm{G}}$

Gauge Picture
of Quantum Dynamics

- Spatial locality is not explicit in Schrodinger's equation:

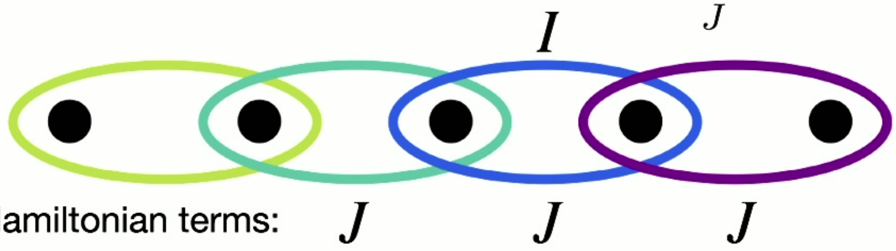
$$\partial_t \left|\Psi^{\mathrm{S}}\right\rangle = -iH \left|\Psi^{\mathrm{S}}\right\rangle$$

- Gauge picture is explicitly local:

$$\partial_t \left|\psi_I\right\rangle = -iH^{\mathrm{G}}_{\langle I\rangle} \left|\psi_I\right\rangle$$

$$H_{\langle I\rangle} = \sum_J^{J\cap I\neq\emptyset} H_J$$

SO WHAT?



only local Hamiltonian terms:

Gauge Picture of Quantum Dynamics

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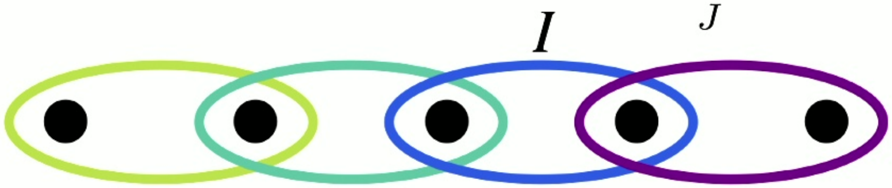
- **Can we use this locality to improve computer algorithms?**

$$\xleftarrow{\qquad} H_{\langle I\rangle} = \sum_{J}^{J\cap I\neq\emptyset} H_J$$



only local Hamiltonian terms: J J J

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Gauge Picture of Quantum Dynamics

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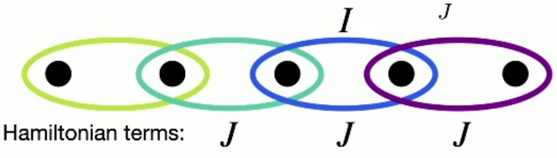
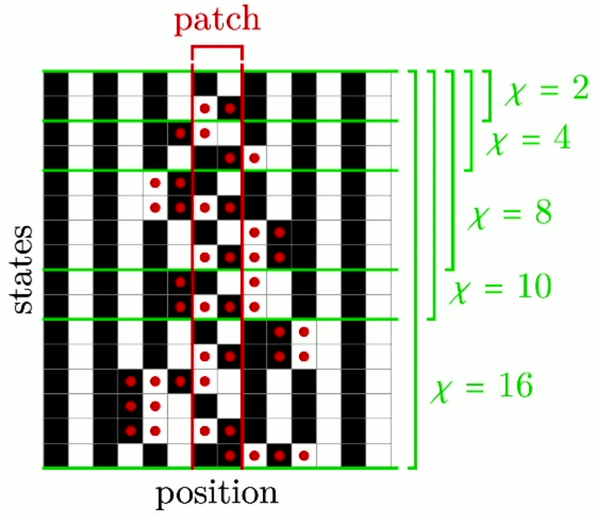
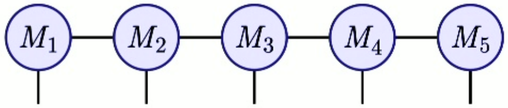


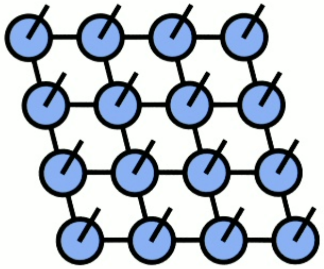
- **Can we use this locality to improve computer algorithms?**
- **Yes, truncate the Hilbert space!**
  - different truncation for each patch



Tensor Networks

- Accurate simulations of quantum physics with low entanglement
  - Very efficient in 1D
  - Less efficient in 2D
  - Rarely used in 3D



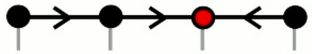


Tensor Networks in 1D

- **Matrix Product State (MPS)**
  - Very efficient in 1D
  - DMRG, TEBD, TDVP, …
  - CPU ~ χ^3
  - χ ~ 1000 for laptops

$\chi \times \chi$ matrix

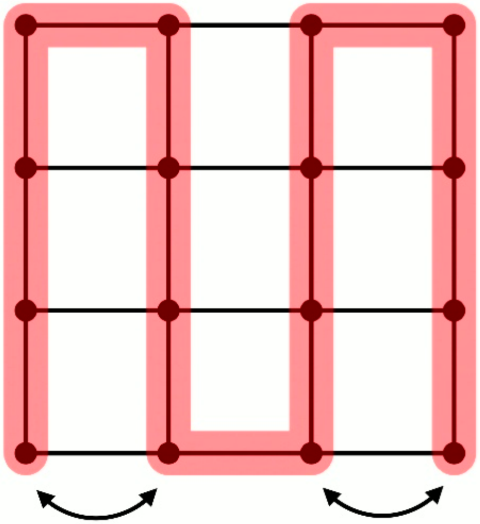
$$\left| \psi \right\rangle = \Sigma_{s_1 s_2 s_3 s_4} \mathrm{tr}\left(A_1^{s_1} A_2^{s_2} A_3^{s_3} A_4^{s_4}\right) \left| s_1 s_2 s_3 s_4 \right\rangle$$



DMRG: White 1992
TEDB: Vidal 2003
TDVP: Haegeman, Cirac, Osborne,
Pizorn, Verschelde, Verstraete 2011

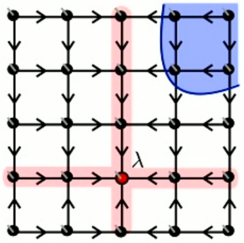
Tensor Networks from 1D to 3D

- **Matrix Product State (MPS)**
  - CPU ~ χ^3, e.g. χ ~ 1000
  - snake MPS for 2D

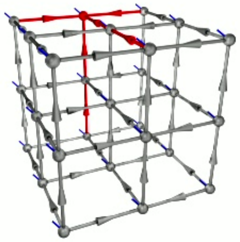


limited entanglement

- **PEPS, canonical PEPS, 2D DMRG, ...**
- **Isometric Tensor Networks**
  - max entanglement limited by:
  - CPU ~ χ^7 (2D)
  - χ ~ **12** & **6**
  - CPU ~ χ^{12} (3D)
  - χ ~ **36** & **6**



Zaletel, Pollmann 2019
Lin, Zaletel, Pollmann 2021
Hyatt, Stoudenmire 2019
Haghshenas, O'Rourke, Chan 2019
…



Tepaske, Luitz 2021

Tensor Networks from 1D to 3D

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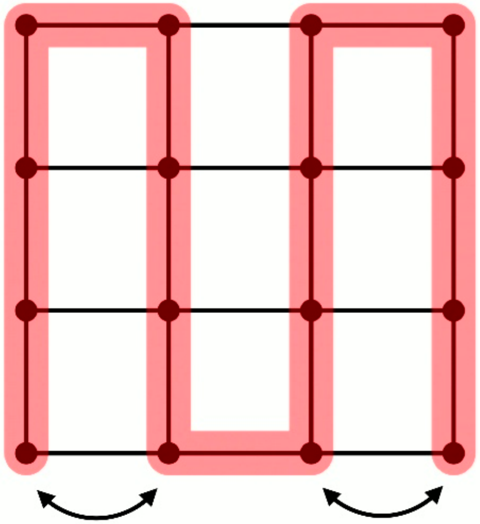
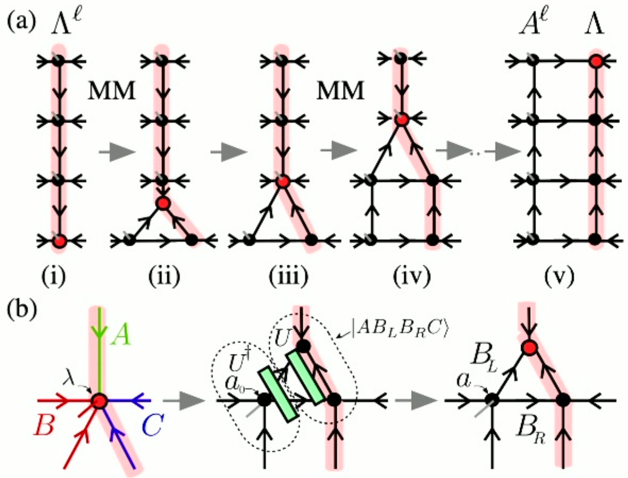


limited entanglement

- **PEPS, canonical PEPS, 2D DMRG, …**
- **Isometric Tensor Networks**
  - “Moses Move” required



Zaletel, Pollmann 2020

Quantum Gauge Network:
A New Kind of Tensor Network

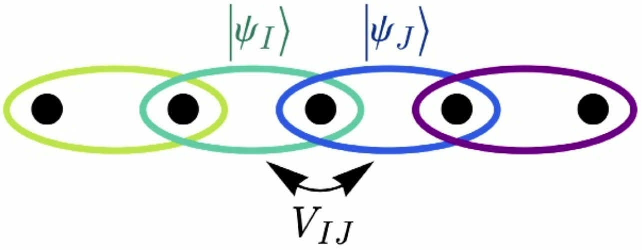
KS 2022

truncate gauge picture Hilbert space:

$2^n \times 2^n$ unitary matrix $U_{IJ} \rightarrow \chi \times \chi$ matrix V_{IJ}

2^n-vector $|\Psi_I\rangle \rightarrow \chi$-vector $|\psi_I\rangle$

$\chi \ll 2^n$, CPU ~ χ^3



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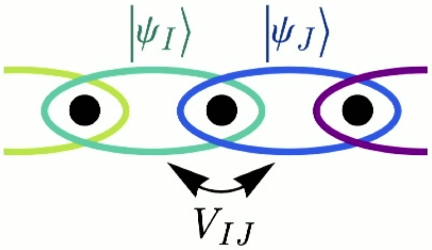

simple equations of motion for quantum dynamics:

$$\partial_t |\psi_I\rangle = -iH'_I |\psi_I\rangle$$

$$\partial_t V_{IJ} = -iH'_I V_{IJ} + iV_{IJ} H'_J$$

$$H'_I = \sum_J^{J\cap I \neq \emptyset} V_{IJ} H_J V_{JI}$$

Quantum Gauge Network:

Simple and (potentially) Efficient

KS 2022

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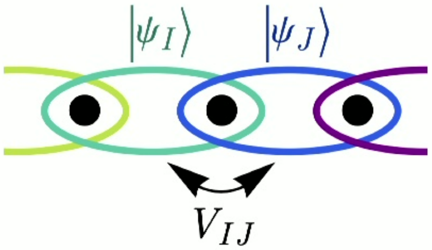

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Mapping QGN to MPDO

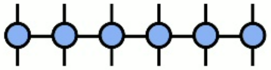
matrix product density operator (MPDO):

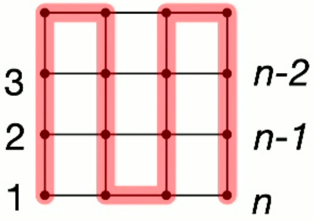
matrices

$$\hat{\rho}_{\text{MPDO}} = 2^{-n} \Sigma_{\mu_1\ldots\mu_n} \hat{\sigma}^{\mu_1\cdots\mu_n} \underbrace{A_1^{\mu_1} A_2^{\mu_2} \cdots A_{n-1}^{\mu_{n-1}} A_n^{\mu_n}}_{\text{matrix product coefficient}}$$

row vector

column vector





Mapping QGN to MPDO

quantum gauge network (QGN) density matrix:

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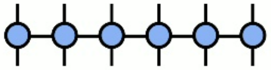

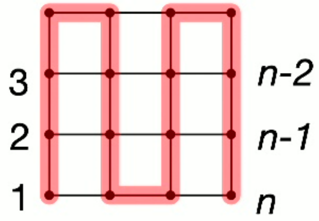

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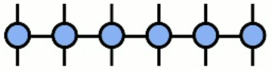

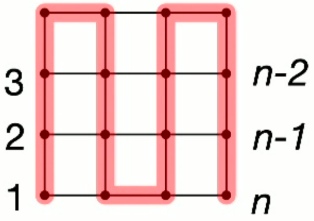

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simple mapping

harder due to constraints:
(in progress)

$$V_{i,i+1}\, \big|\psi_{i+1}\big\rangle = \big|\psi_i\big\rangle$$

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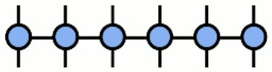

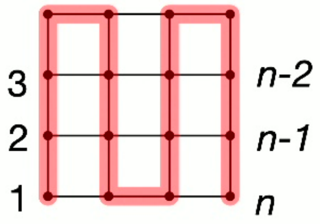

QGN generalize(?) MPDO to higher dimensions while retaining χ^3 scaling!

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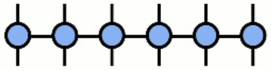
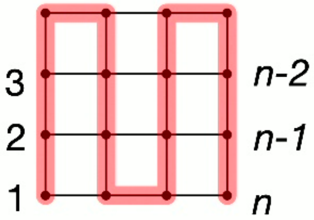
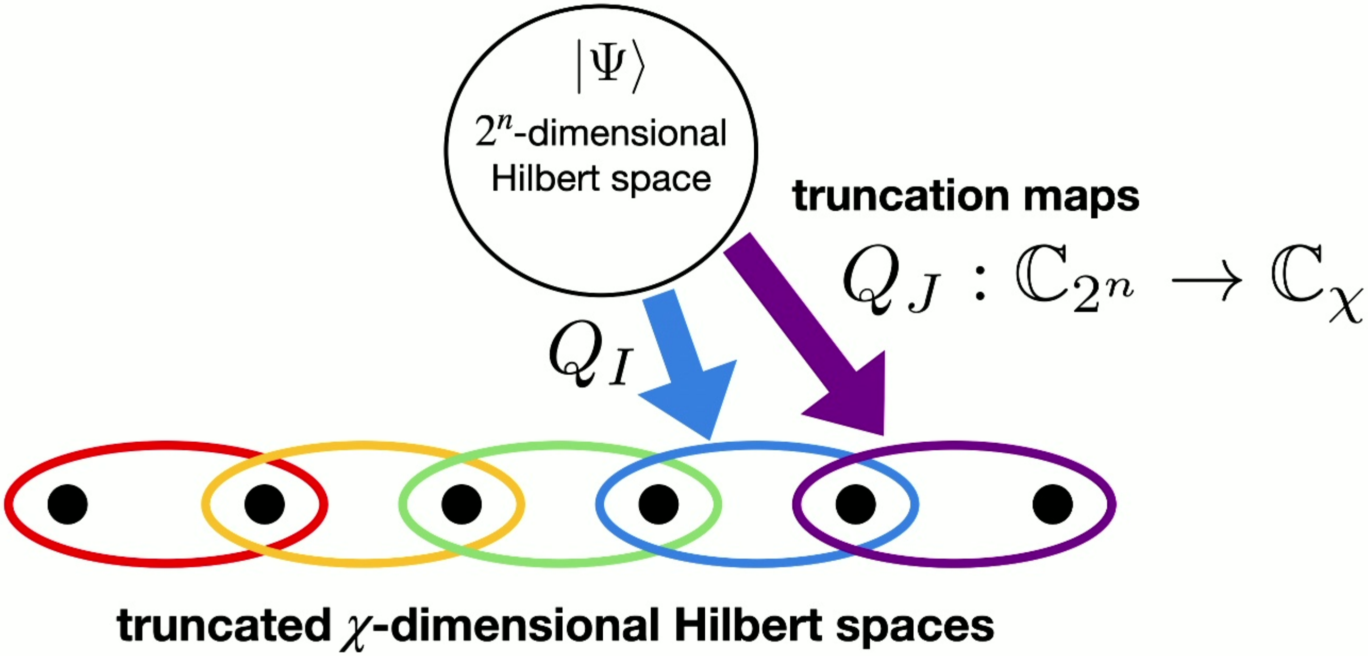




A Quantum Gauge Network Construction

(useful for theory and initialization)

KS 2022



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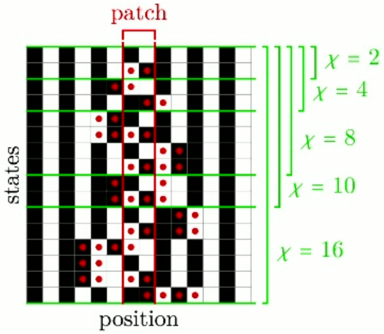
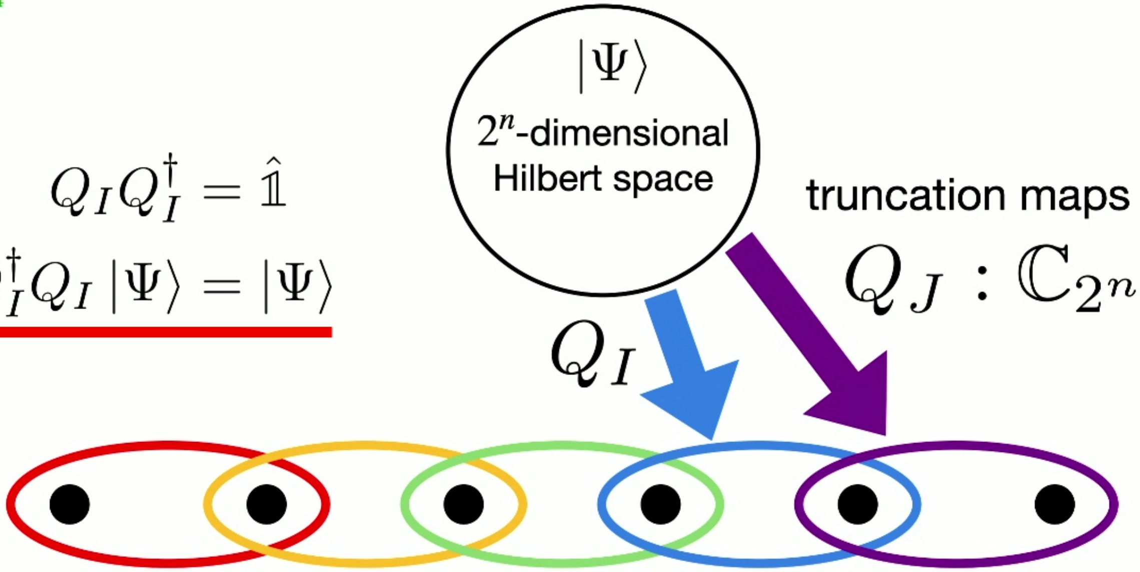


$$Q_I Q_I^\dagger = \hat{\mathbb{1}}$$

$$\underline{Q_I^\dagger Q_I \left|\Psi\right\rangle = \left|\Psi\right\rangle}$$



$|\Psi\rangle$

2^n-dimensional Hilbert space

Q_I

truncation maps

$$Q_J : \mathbb{C}_{2^n} \rightarrow \mathbb{C}_\chi$$

truncated χ-dimensional Hilbert spaces

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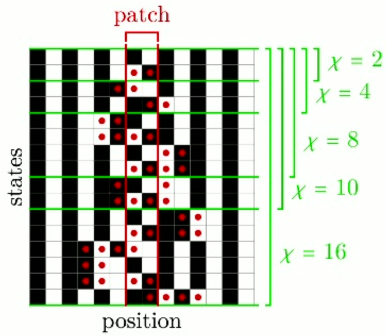

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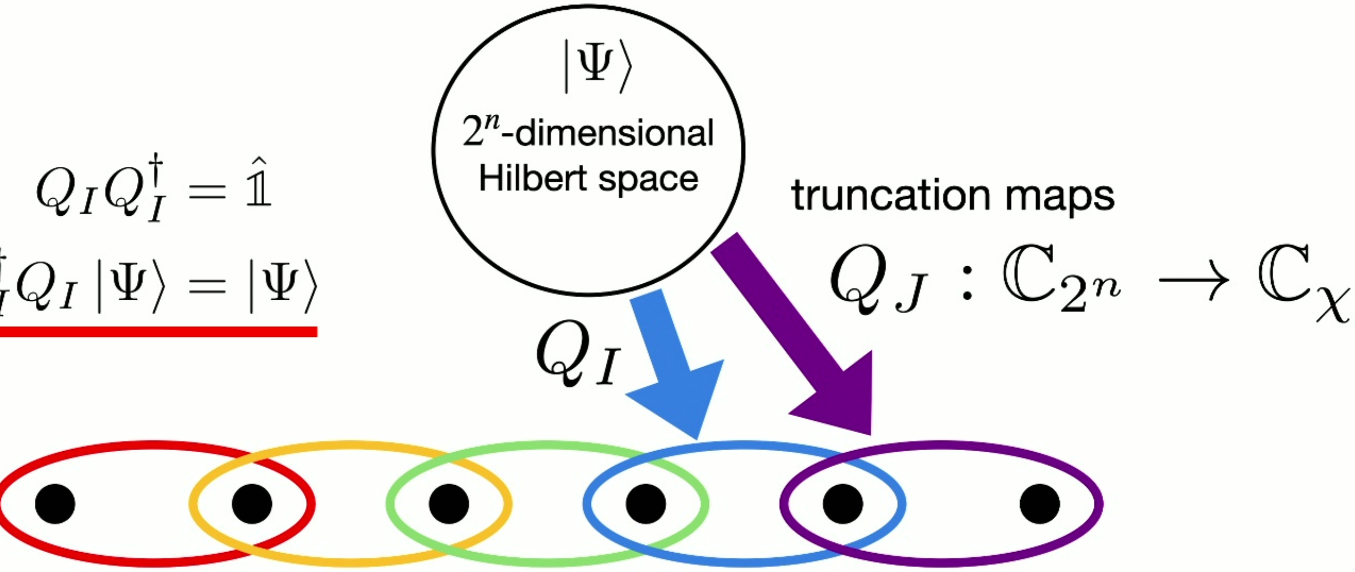

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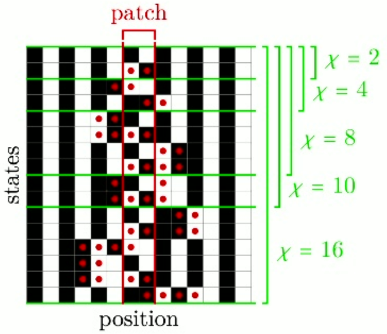

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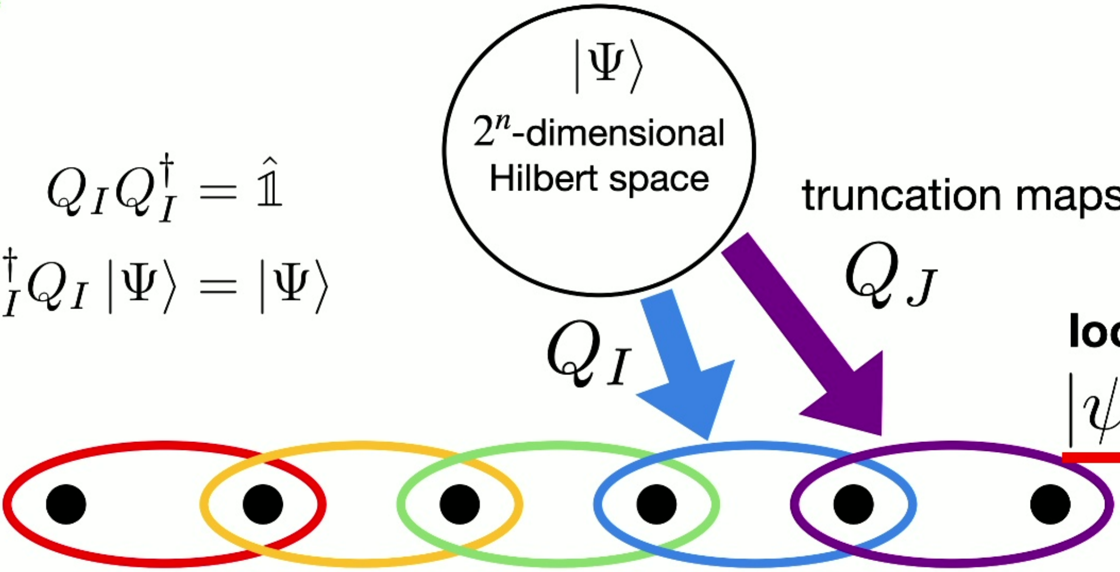

truncation maps

local wavefunction

$$|\psi_J\rangle = Q_J |\Psi\rangle$$

truncated χ-dimensional Hilbert spaces

A Quantum Gauge Network Construction
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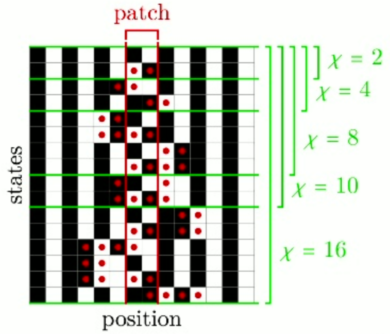
KS 2022



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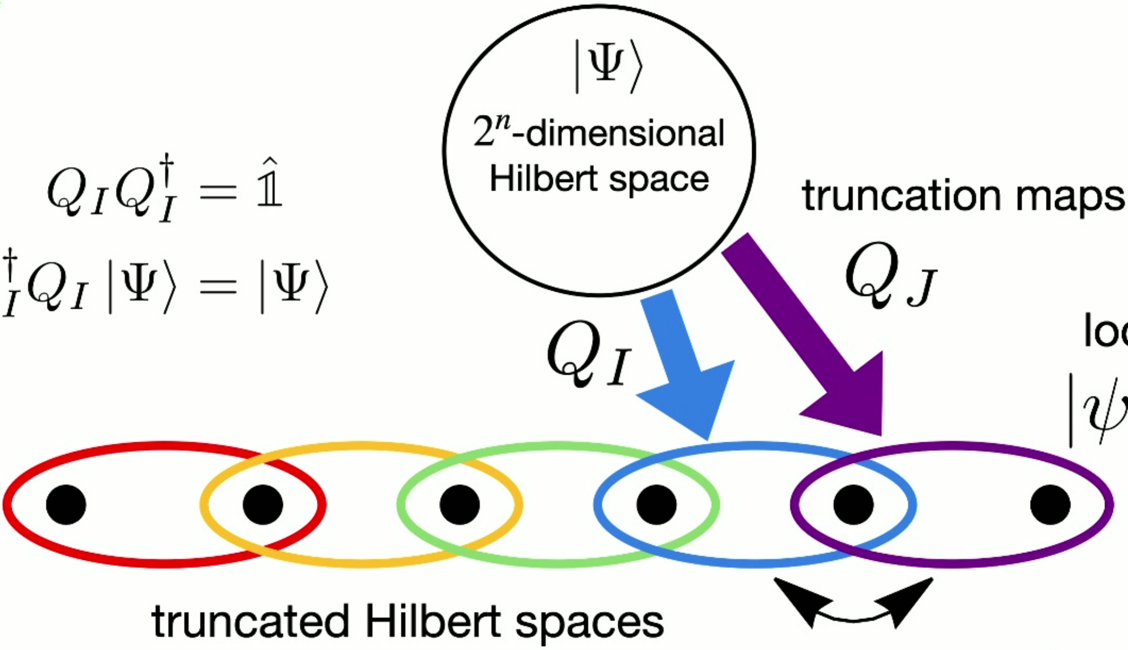


truncation maps

local wavefunction

$$|\psi_J\rangle = Q_J |\Psi\rangle$$

$$V_{IJ} |\psi_J\rangle = |\psi_I\rangle$$

truncated Hilbert spaces

singular values ≤1

$$V_{IJ} = Q_I Q_J^\dagger$$

connection

A Quantum Gauge Network Construction

(useful for theory and initialization)

KS 2022

$$Q_I Q_I^\dagger = \hat{\mathbb{1}}$$

$$Q_I^\dagger Q_I \left|\Psi\right\rangle = \left|\Psi\right\rangle$$

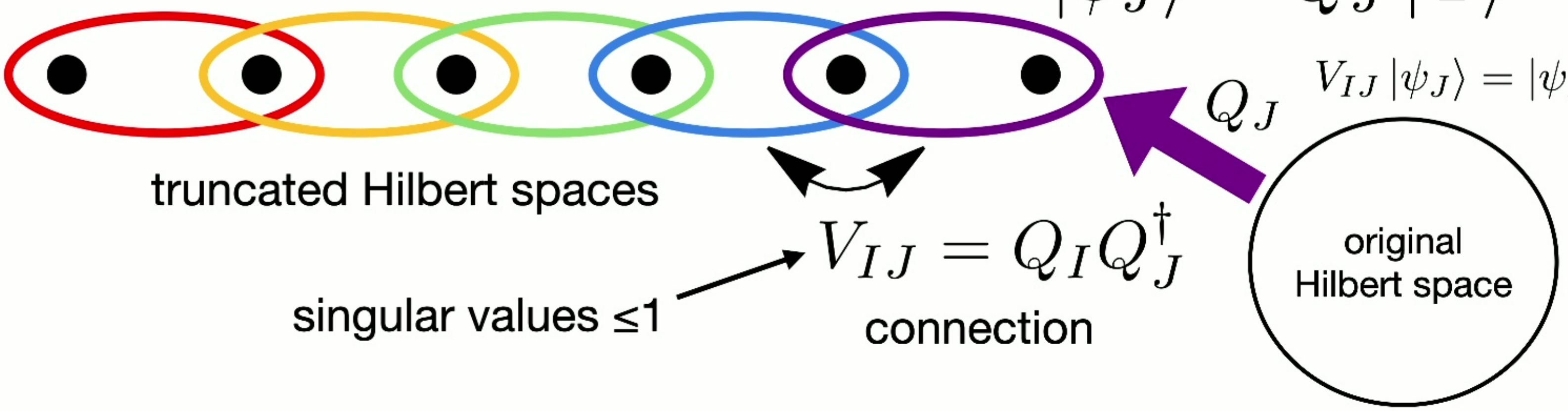
truncated local operator

$$A_I = Q_I \hat{A}_I Q_I^\dagger$$

original operators have hats

local wavefunction

$$\left|\psi_J\right\rangle = Q_J \left|\Psi\right\rangle$$



Q_J

$V_{IJ}\left|\psi_J\right\rangle = \left|\psi_I\right\rangle$

original Hilbert space

truncated Hilbert spaces

$$V_{IJ} = Q_I Q_J^\dagger$$

singular values ≤1

connection

A Quantum Gauge Network Construction

(useful for theory and initialization)

KS 2022

$$Q_I Q_I^\dagger = \hat{\mathbb{1}}$$

$$Q_I^\dagger Q_I |\Psi\rangle = |\Psi\rangle$$

connections required for string operators:

$$\langle \psi_I | A_I V_{IJ} B_J V_{JK} C_K | \psi_K \rangle$$

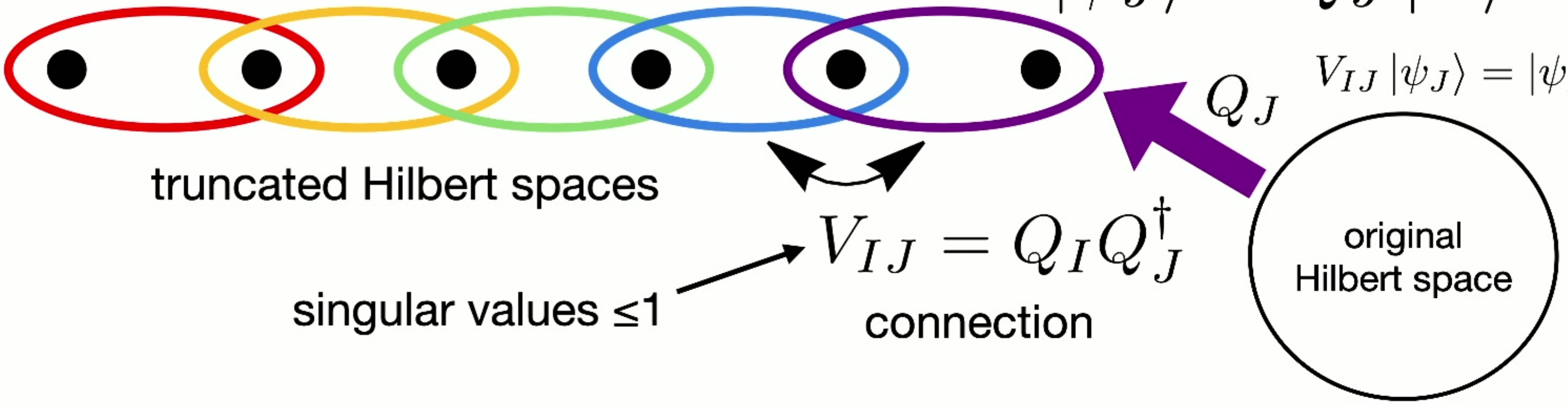
$$\approx \langle \Psi | \hat{A}_I \hat{B}_J \hat{C}_K | \Psi \rangle$$

truncated local operator

$$A_I = Q_I \hat{A}_I Q_I^\dagger$$

local wavefunction

$$|\psi_J\rangle = Q_J |\Psi\rangle$$



Q_J

$V_{IJ} |\psi_J\rangle = |\psi_I\rangle$

truncated Hilbert spaces

$V_{IJ} = Q_I Q_J^\dagger$

connection

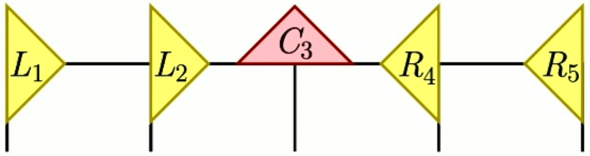
singular values ≤1

original Hilbert space

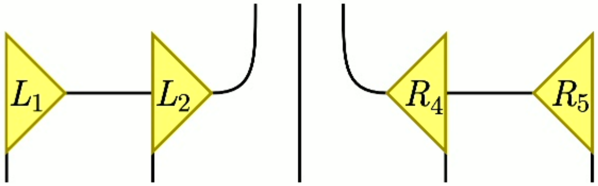
MPS to QGN

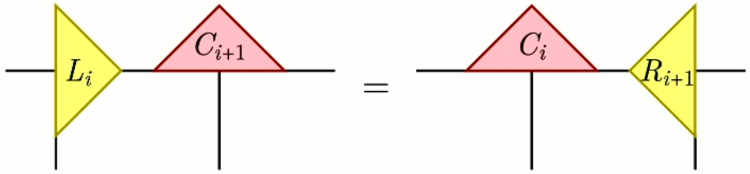
KS 2022

$|\Psi_{\text{MPS}}\rangle =$

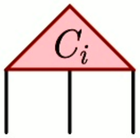


$Q_i^\dagger \overset{(i=3)}{\underset{(n=5)}{=}}$

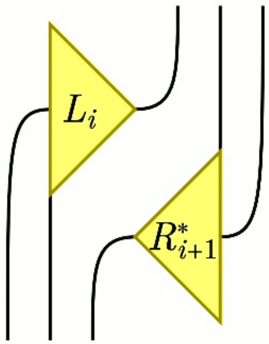




$|\psi_i\rangle =$



$V_{i,i+1} =$



Expectation Value Encoding

KS 2022

- QGN: $|\psi_I\rangle = Q_I\,|\Psi\rangle \quad V_{IJ} = Q_I Q_J^\dagger \qquad A_I = Q_I \hat{A}_I Q_I^\dagger$
- Suppose we want to perfectly encode:

$$\langle\psi_I|A_I V_{IJ} B_J V_{JK} C_K|\psi_K\rangle = \langle\Psi|\hat{A}_I \hat{B}_J \hat{C}_K|\Psi\rangle$$

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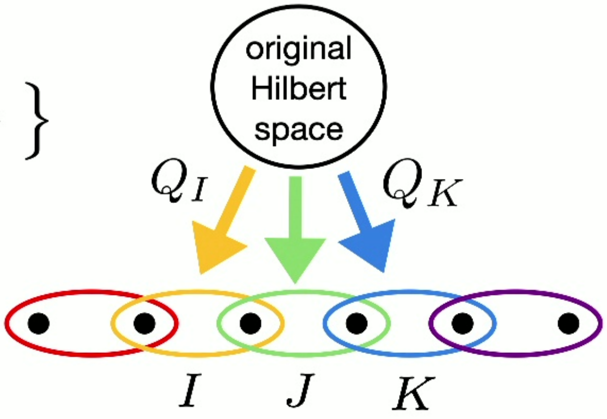
- Guaranteed if truncated Hilbert spaces are large enough:

$$\mathrm{im}(Q_I^\dagger) \supseteq \mathrm{span}\{\,|\Psi\rangle\,,\ \hat{A}_I^\dagger\,|\Psi\rangle\,\}$$

$$\mathrm{im}(Q_J^\dagger) \supseteq \mathrm{span}\{\,|\Psi\rangle\,,\ \hat{A}_I^\dagger\,|\Psi\rangle\,,\ \hat{C}_K\,|\Psi\rangle\,\}$$

$$\mathrm{im}(Q_K^\dagger) \supseteq \mathrm{span}\{\,|\Psi\rangle\,,\ \hat{C}_K\,|\Psi\rangle\,\}$$



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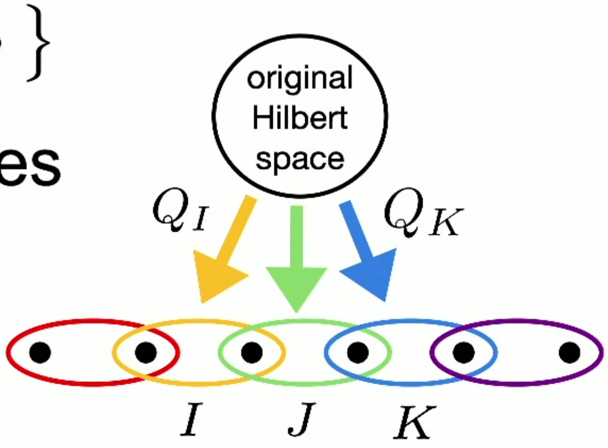
- Guaranteed if truncated Hilbert spaces are large enough:
  $\mathrm{im}(Q_J^\dagger) \supseteq \mathrm{span}\{\,|\Psi\rangle\,,\ \hat{A}_I^\dagger\,|\Psi\rangle\,,\ \hat{C}_K\,|\Psi\rangle\,\}$
- Each expectation value only increases dimension χ by at most two



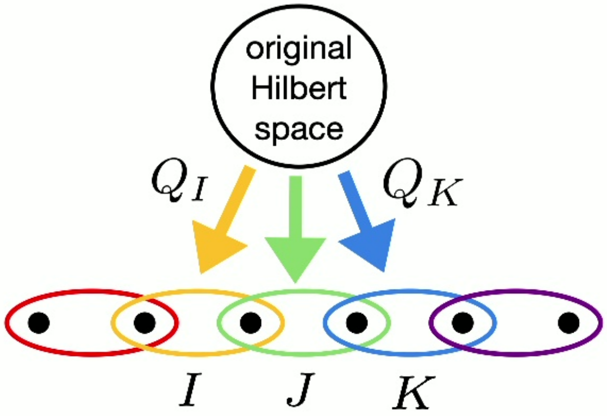
Long Range Correlations

KS 2022

- QGN: $|\psi_I\rangle = Q_I\,|\Psi\rangle \quad V_{IJ} = Q_I Q_J^\dagger \quad A_I = Q_I \hat{A}_I Q_I^\dagger$
- Suppose we want to perfectly encode all 2*k*-point correlation functions for *M* different operators $\hat{\tau}_i^\mu$

$$\left\langle \Psi \middle| \hat{\tau}_{i_1}^{\mu_1 \dagger} \ldots \hat{\tau}_{i_k}^{\mu_k \dagger} \hat{\tau}_{j_1}^{\nu_1} \ldots \hat{\tau}_{j_k}^{\nu_k} \middle| \Psi \right\rangle$$

- Requires dimension O(M^k)



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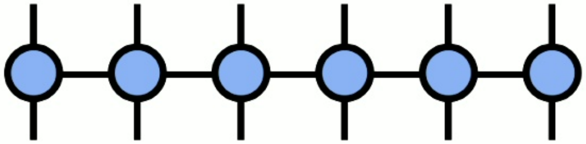
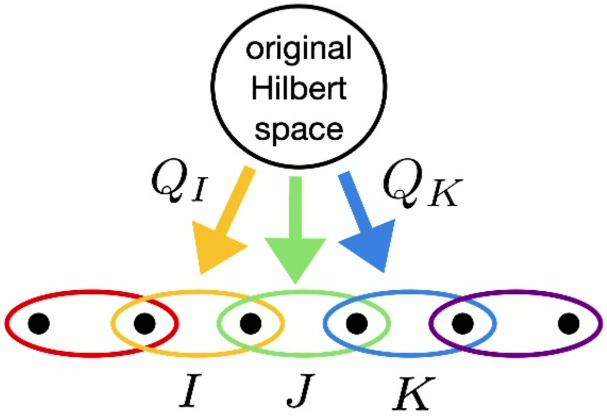
- Requires dimension O(M^k)
  - same as MPDO





Long Range Correlations

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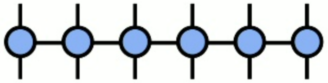
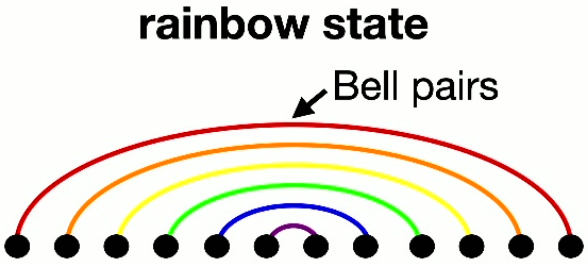




- Exponentially better than MPS, which can require $\chi \sim 2^{M/6}$ for $k = 1$!

Time Evolution Algorithm

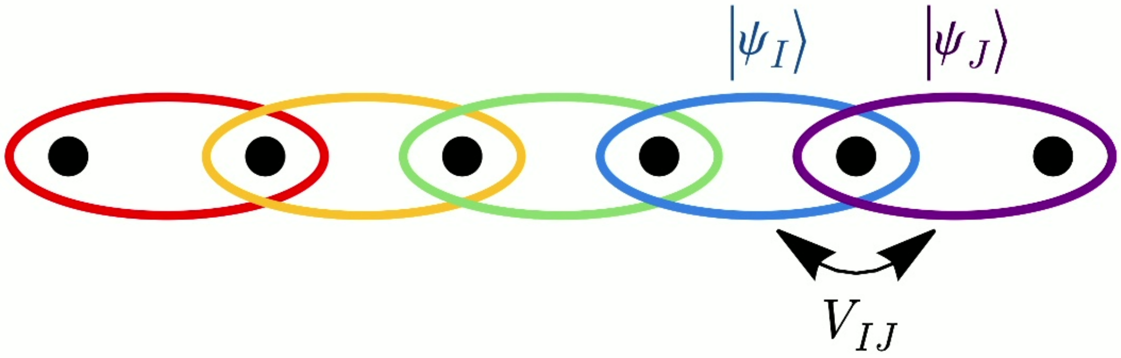
KS 2022

- Simple time evolution algorithm via gauge picture:

$$\partial_t \left|\psi_I\right\rangle = -iH'_I \left|\psi_I\right\rangle$$

$$\partial_t V_{IJ} = -iH'_I V_{IJ} + iV_{IJ} H'_J$$

$$H'_I = \sum_J^{J\cap I\neq\emptyset} V_{IJ}\, H_J\, V_{JI}$$

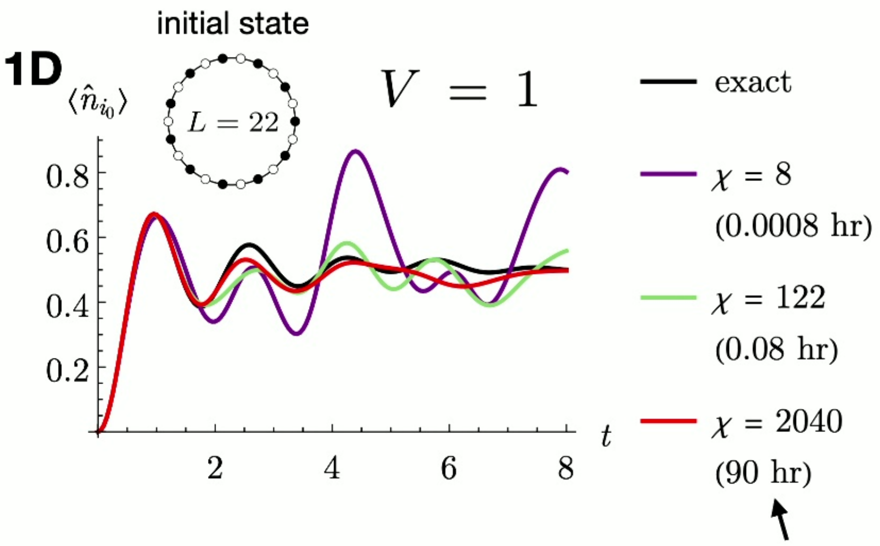


Spinless Fermion Quench

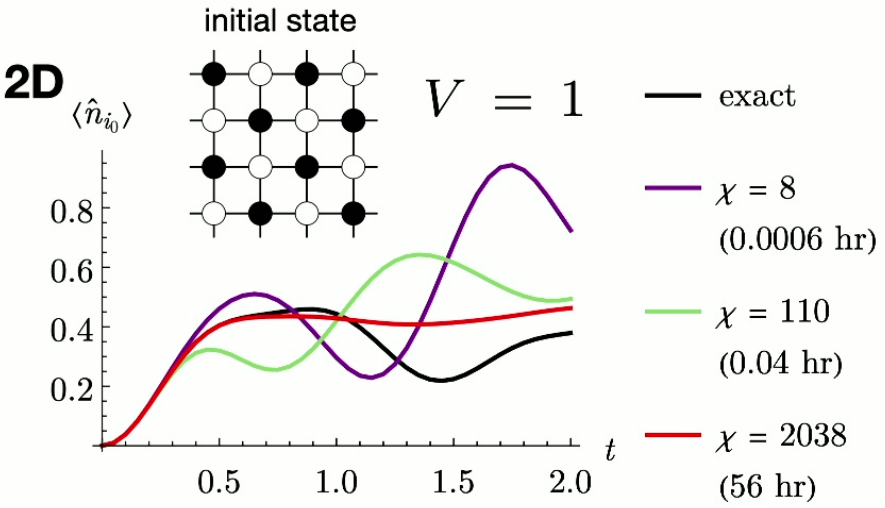
KS 2022

$$\hat{H}^{\text{Fermi}} = \sum_{\langle ij\rangle} \hat{H}^{\text{Fermi}}_{\langle ij\rangle}$$

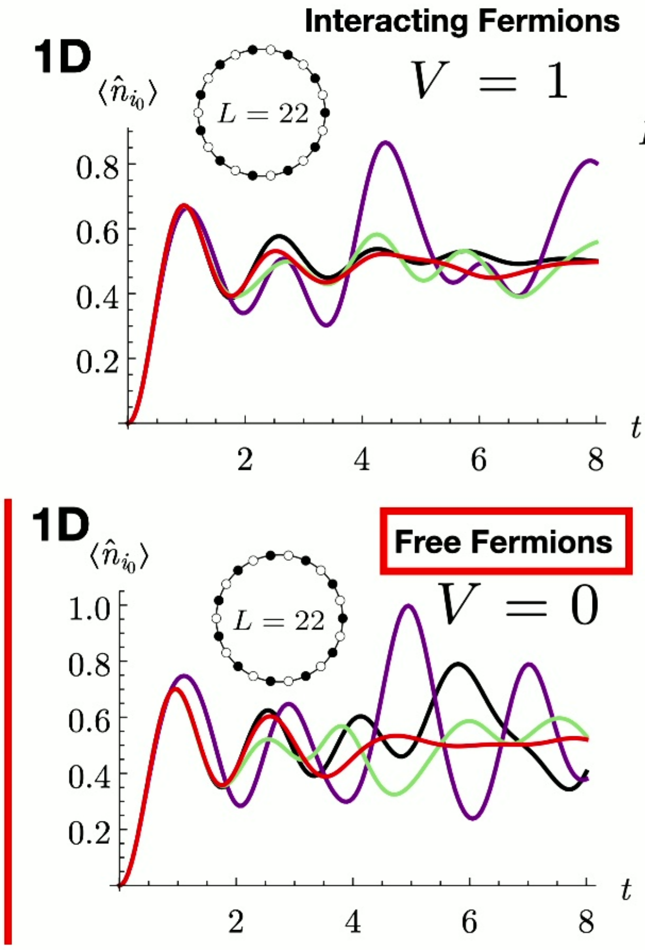
$$\hat{H}^{\text{Fermi}}_{I=\langle ij\rangle} = -\hat{c}_i^{\dagger}\hat{c}_j - \hat{c}_j^{\dagger}\hat{c}_i + V\hat{n}_i\hat{n}_j$$



CPU time



KS 2022



$$\hat{H}^{\text{Fermi}} = \sum_{\langle ij \rangle} \hat{H}^{\text{Fermi}}_{\langle ij \rangle}$$

$$\hat{H}^{\text{Fermi}}_{I=\langle ij \rangle} = -\hat{c}_i^\dagger \hat{c}_j - \hat{c}_j^\dagger \hat{c}_i + V \hat{n}_i \hat{n}_j$$

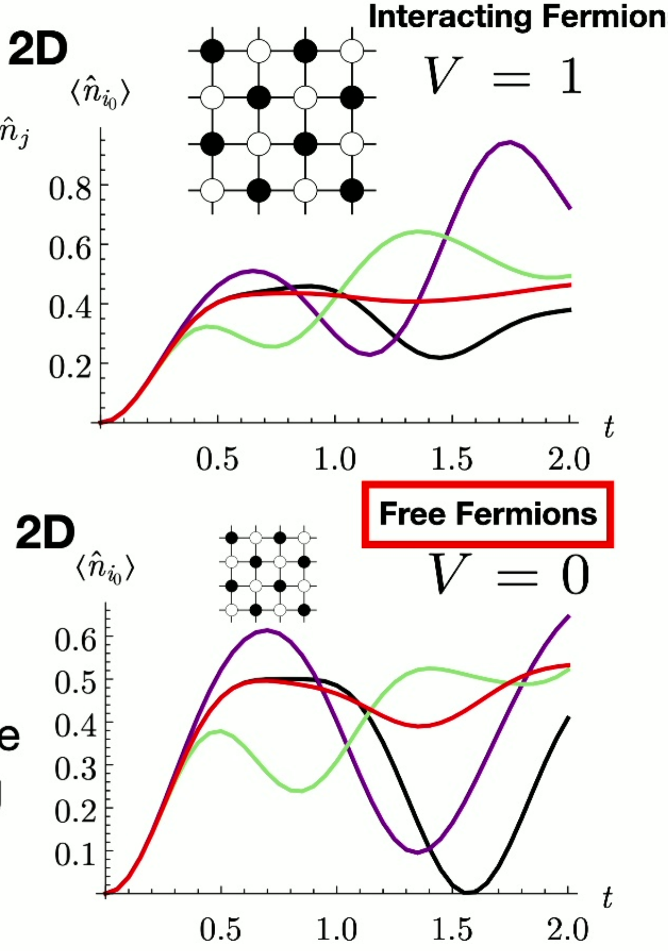
— exact

— χ = 8 (0.0008 hr)

— χ = 122 (0.08 hr)

— χ = 2040 (90 hr)

Similar performance for non-interacting fermions



— exact

— χ = 8 (0.0006 hr

— χ = 110 (0.04 hr)

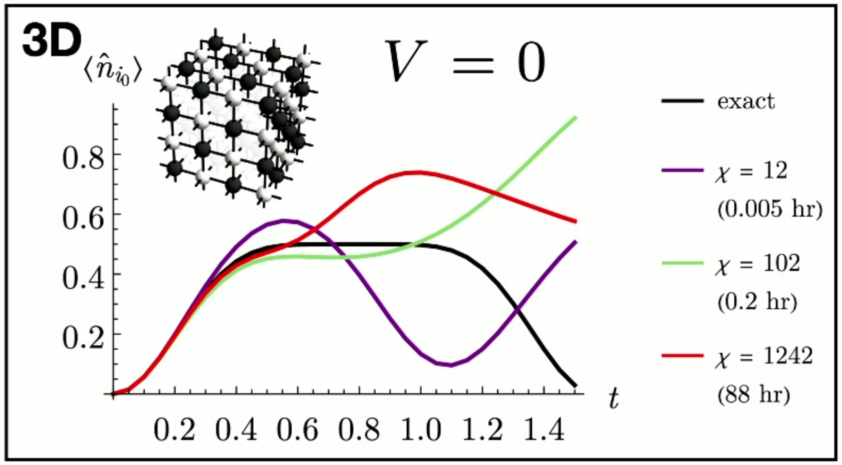
— χ = 2038 (56 hr)

Free Fermion Quench

KS 2022

Accurate for short time that increases with bond dimension and decreases with spatial dimension

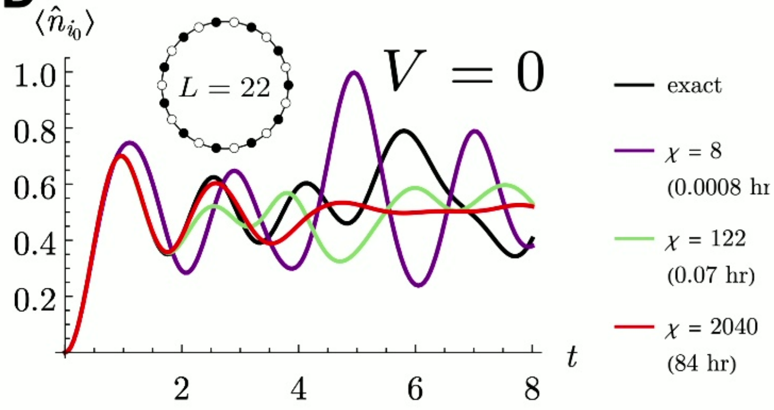
3D



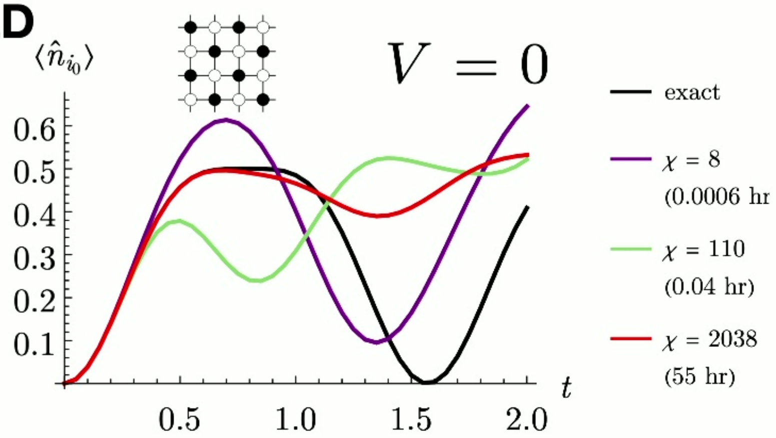
$$\hat{H}^{\mathrm{Fermi}} = \sum_{\langle ij \rangle} \hat{H}^{\mathrm{Fermi}}_{\langle ij \rangle}$$

$$\hat{H}^{\mathrm{Fermi}}_{I=\langle ij \rangle} = -\hat{c}_i^\dagger \hat{c}_j - \hat{c}_j^\dagger \hat{c}_i$$

1D



2D



Time Evolution Algorithm

KS 2022

- Accurate for short time that increases with bond dimension and decreases with spatial dimension

- Efficient and simple
  - new way to generalize MPDO to higher dimensions
  - only vectors and matrices (no tensors)
  - CPU ~ χ^3 in any dimension
  - many dimensions and fermions are easy to code
  - energy conserved exactly

Time Evolution Algorithm

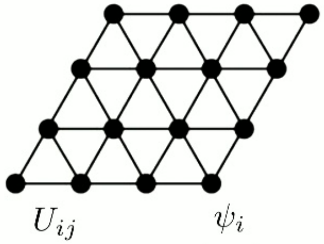
- Accurate for short time that increases with bond dimension and decreases with spatial dimension

- Significant room for future improvement
  - Dynamically increase truncation dimension
  - Adjust truncated Hilbert space
  - Conserve charge (in addition to energy)

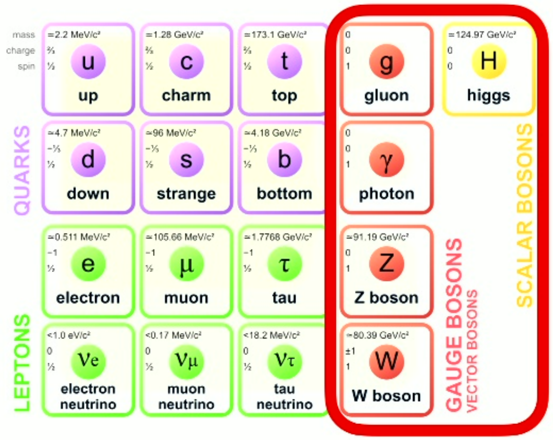
Lattice Gauge Theory

classical U(*N*) lattice gauge theory coupled to Higgs field ψ_i

$$E = - \sum_{ijk}^{\text{plaquettes}} \operatorname{tr} U_{ij} U_{jk} U_{ki} - \sum_{\langle ij \rangle} \psi_i^* \cdot U_{ij} \cdot \psi_j$$



unitary $|\psi_i| = 1$



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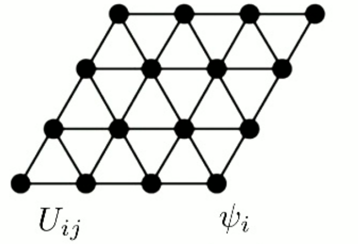


unitary $|\psi_i| = 1$

ground states:

$$U_{ij} \cdot \psi_j = \psi_i$$

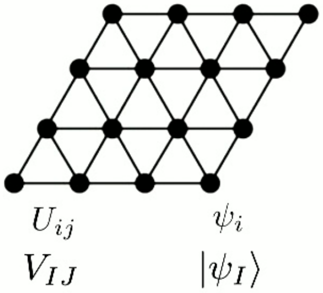
$$U_{ij} U_{jk} = U_{ik}$$

$$U_{ij}^\dagger = U_{ji}$$

QGN ~ Non-unitary Gauge Theory

classical U(*N*) lattice gauge theory coupled to Higgs field ψ_i

$$E = -\sum_{ijk}^{\text{plaquettes}} \operatorname{tr} U_{ij} U_{jk} U_{ki} - \sum_{\langle ij \rangle} \psi_i^* \cdot U_{ij} \cdot \psi_j$$



ground states:	similar to quantum gauge networks:
$U_{ij} \cdot \psi_j = \psi_i$	$V_{IJ} \, \lvert\psi_J\rangle = \lvert\psi_I\rangle$
$U_{ij} U_{jk} = U_{ik}$	$V_{IJ} V_{JK} \approx V_{IK}$
$U_{ij}^\dagger = U_{ji}$	$V_{IJ}^\dagger = V_{JI}$

New Kind of Gauge Theory

Higgs'd classical lattice gauge theory

$$U_{ij} \cdot \psi_j = \psi_i$$

$$U_{ij} U_{jk} = U_{ik}$$

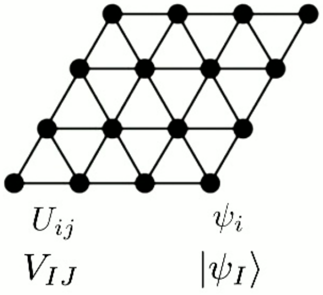
unitary U_{ij}

quantum gauge networks:

$$V_{IJ} \left|\psi_J\right\rangle = \left|\psi_I\right\rangle$$

$$V_{IJ} V_{JK} \approx V_{IK}$$

singularValues$(V_{IJ}) \leq 1$

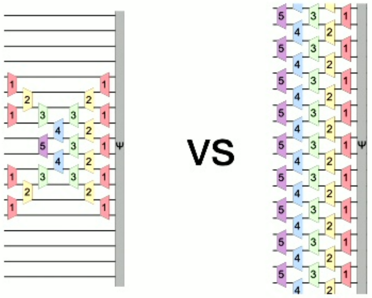


- By relaxing unitary constraint, classical lattice gauge theory locally encodes quantum wavefunctions!
- Emergent (approximate) quantum mechanics from classical gauge theory with non-unitary connections?

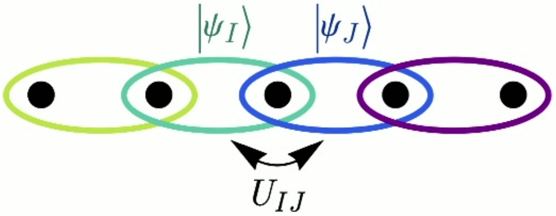
Outlook

(kevin.slagle@rice.edu)

arXiv:2210.09314



- **Gauge Picture** of quantum dynamics
  - spatial locality is explicit
  - many future directions!
    - quantum channels? Lindblad?
    - non-trivial connections? $U_{IJ}U_{JK} \neq U_{IK}$
    - applications for OTOC, MBL, or information speed?



$$\partial_t |\psi_I\rangle = -iH^{\mathrm{G}}_{\langle I\rangle} |\psi_I\rangle$$

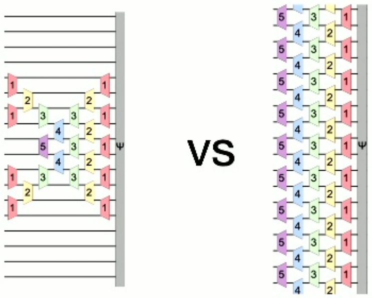
$$\partial_t U_{IJ} = -iH^{\mathrm{G}}_{\langle I\rangle} U_{IJ} + iU_{IJ} H^{\mathrm{G}}_{\langle J\rangle}$$

Outlook

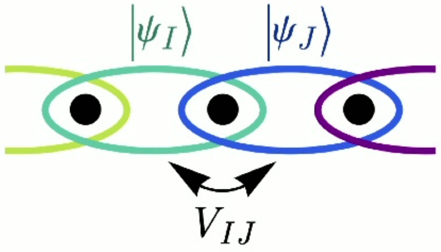
(kevin.slagle@rice.edu)

- **Gauge Picture** of quantum dynamics
  - spatial locality is explicit
- **Quantum Gauge Networks**
  - new algorithm for simulating quantum dynamics
  - many dimensions, fermions, & long-range interactions are easy
  - higher-dimensional MPDO with χ^3 scaling!
  - many future directions!
    - better theory
    - algorithm improvements
    - ground state optimization

arXiv:2210.09314

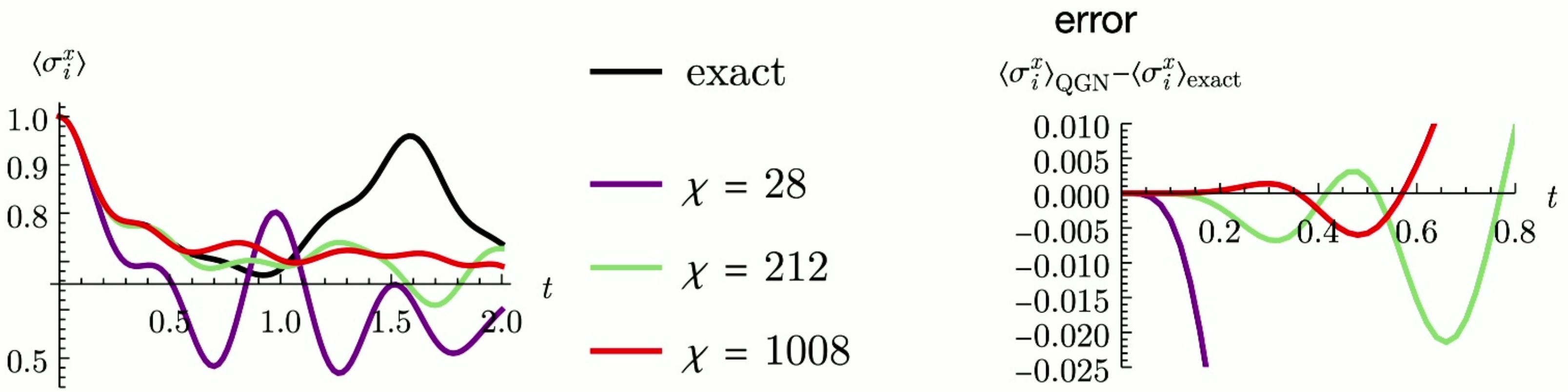


arXiv:2210.12151
Quantum 7, 1113 (2023)



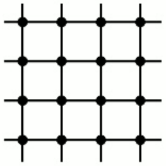
Ising Quench Benchmark

KS 2022



periodic 4x4 square lattice quantum quench

$$\hat{H}^{\mathrm{Ising}} = -\sum_{\langle ij\rangle} \hat{\sigma}_i^z \hat{\sigma}_j^z - h \sum_i \hat{\sigma}_i^x$$



$$h = 3$$

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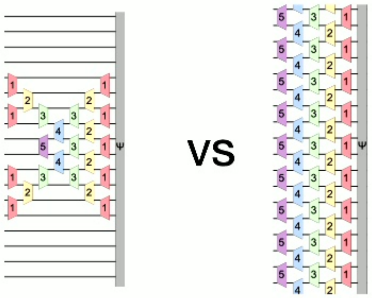

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Quantum 7, 1113 (2023)