Title: 3 short stories about the quantum <-> gravity interface

Speakers: Etera Livine

Series: Quantum Gravity

Date: October 12, 2023 - 2:30 PM

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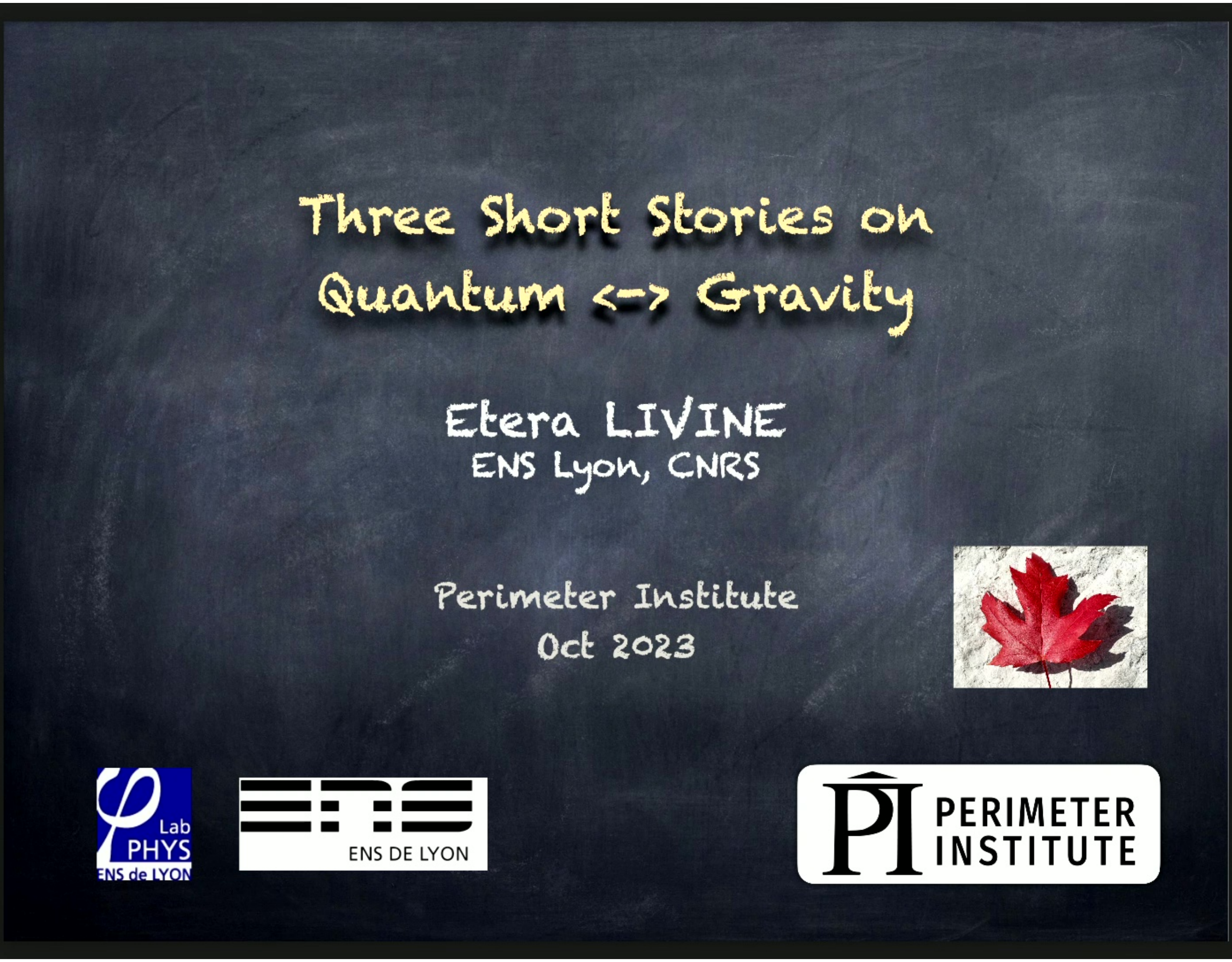
Abstract: Building on the fact that quantum uncertainty is a dynamical degree of freedom in itself in quantum mechanics, I will start with the remark that its evolution can provide a notion of intrinsic clock. To illustrate the non-triviality of this idea, we'll check the dynamics of the uncertainty if gravitizing quantum mechanics into a non-linear Schrödinger equation, and will see that it (surprisingly?) follows the same trajectories as planetary orbits. A third thread will come from the black hole mini-superspace in general relativity. We will see that, assuming that the quantization preserves all the symmetries of the theory, universal quantum gravity corrections can also be written as a non-linear Schrödinger equation, leading to a non-trivial evolution of the "classicality" of black hole wave-packets along the radial coordinate.

Zoom link: https://pitp.zoom.us/j/95808416532?pwd=cHJnVXpzdDd3eXdFbUlMamFsMFV3UT09



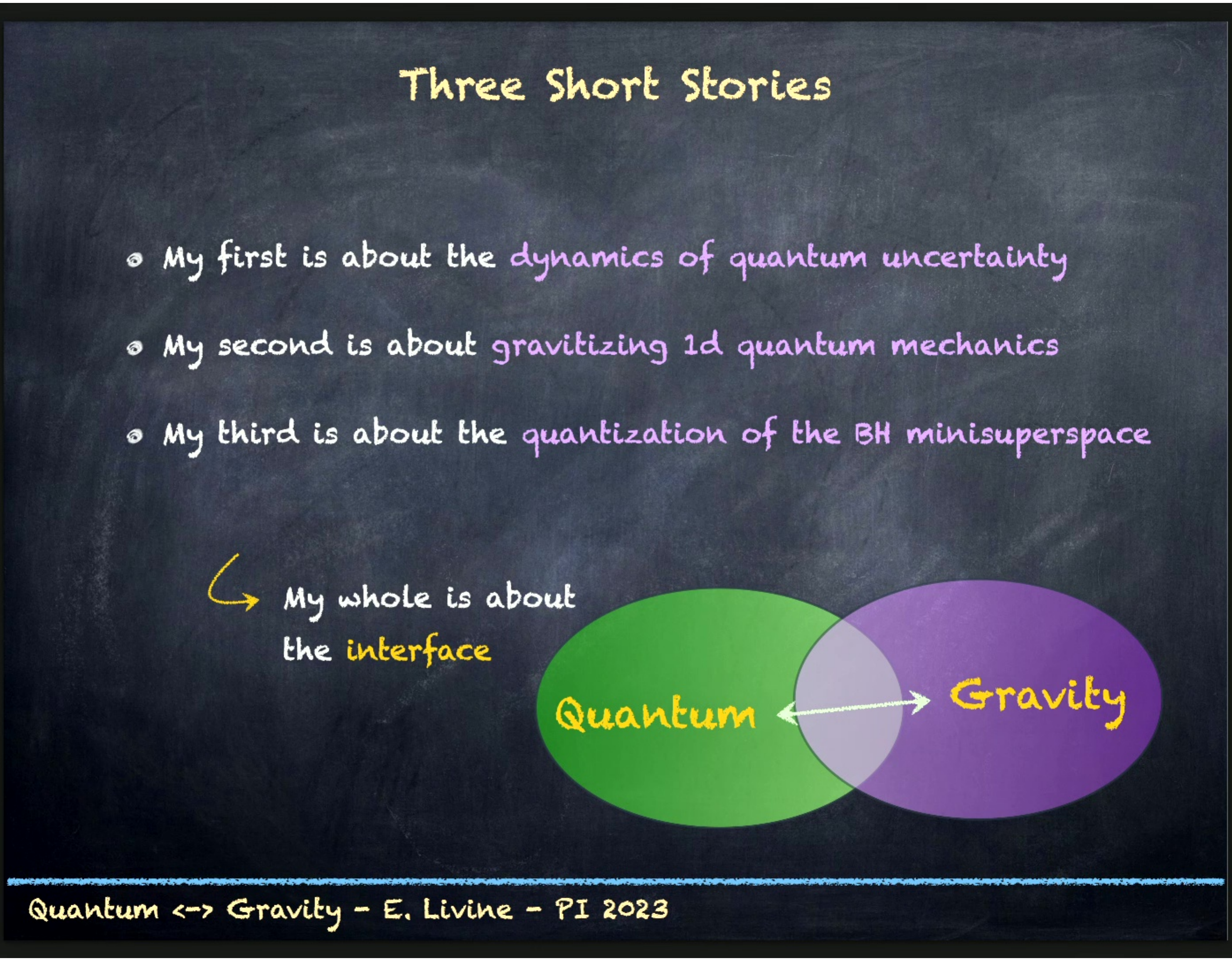
Three Short Stories on Quantum <-> Gravity

Etera LIVINE
ENS Lyon, CNRS

Perimeter Institute
Oct 2023

Three Short Stories

- My first is about the dynamics of quantum uncertainty
- My second is about gravitizing 1d quantum mechanics
- My third is about the quantization of the BH minisuperspace



Three Short Stories

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- My third is about the quantization of the BH minisuperspace

↳ My whole is about the interface

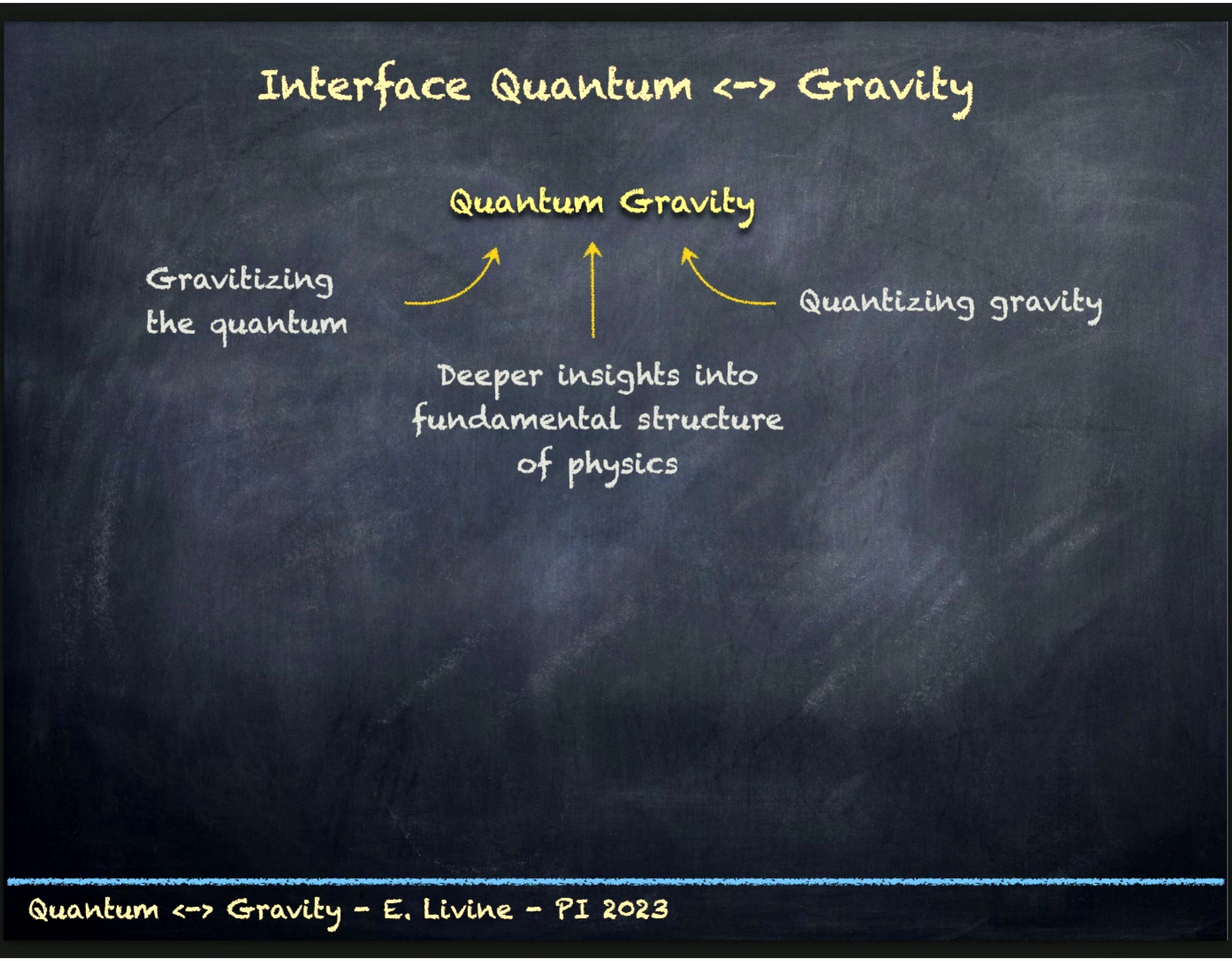
Quantum ⟷ Gravity

Interface Quantum <-> Gravity

Quantum Gravity

More than just
Quantizing gravity

QG Question has grown beyond the initial mission of taming the quantum fluctuation of the space-time geometry



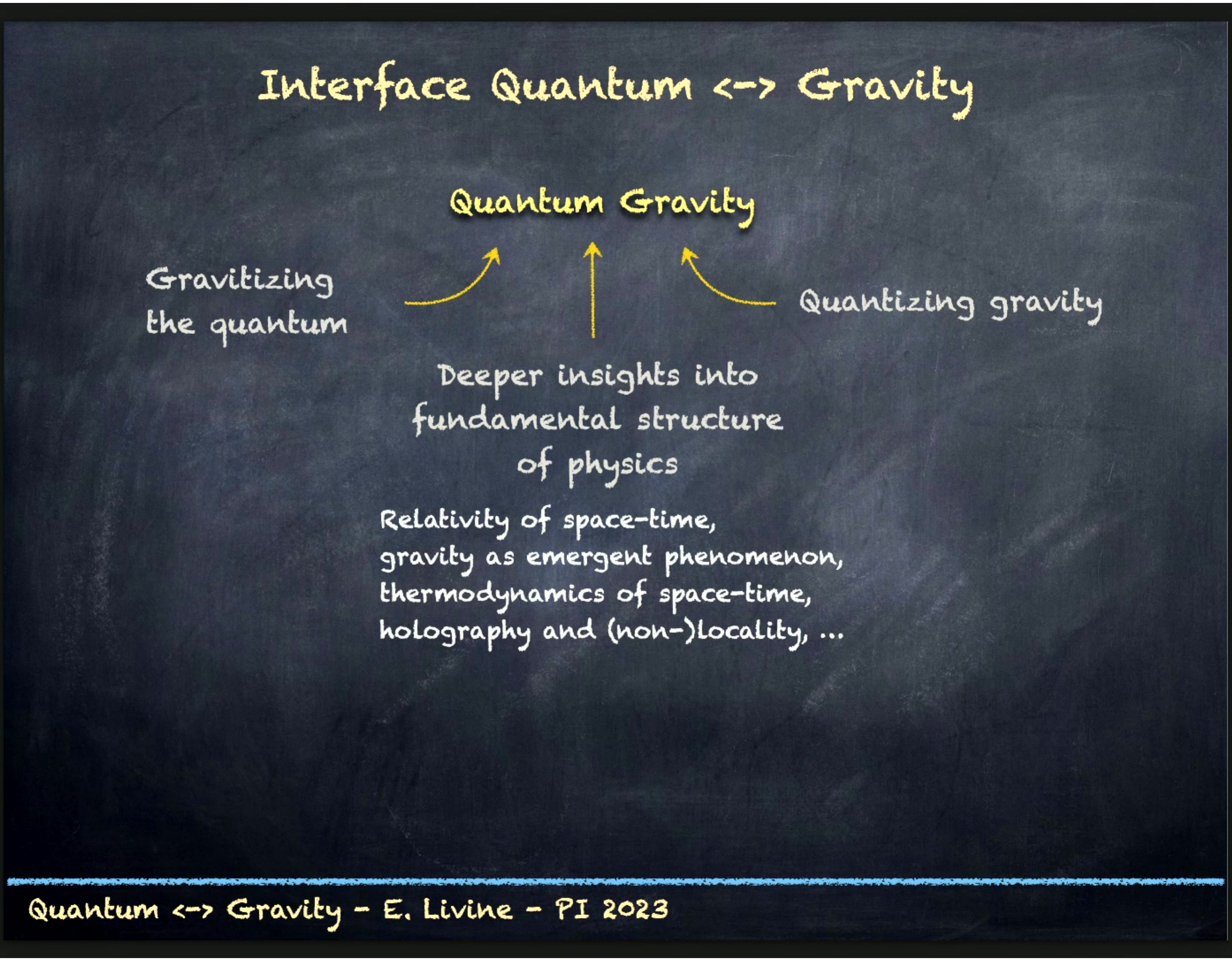
Interface Quantum <-> Gravity

Quantum Gravity

Gravitizing
the quantum

Quantizing gravity

Deeper insights into
fundamental structure
of physics



Interface Quantum <-> Gravity

Quantum Gravity

Gravitizing the quantum

Quantizing gravity

Deeper insights into fundamental structure of physics

Relativity of space-time,
gravity as emergent phenomenon,
thermodynamics of space-time,
holography and (non-)locality, ...

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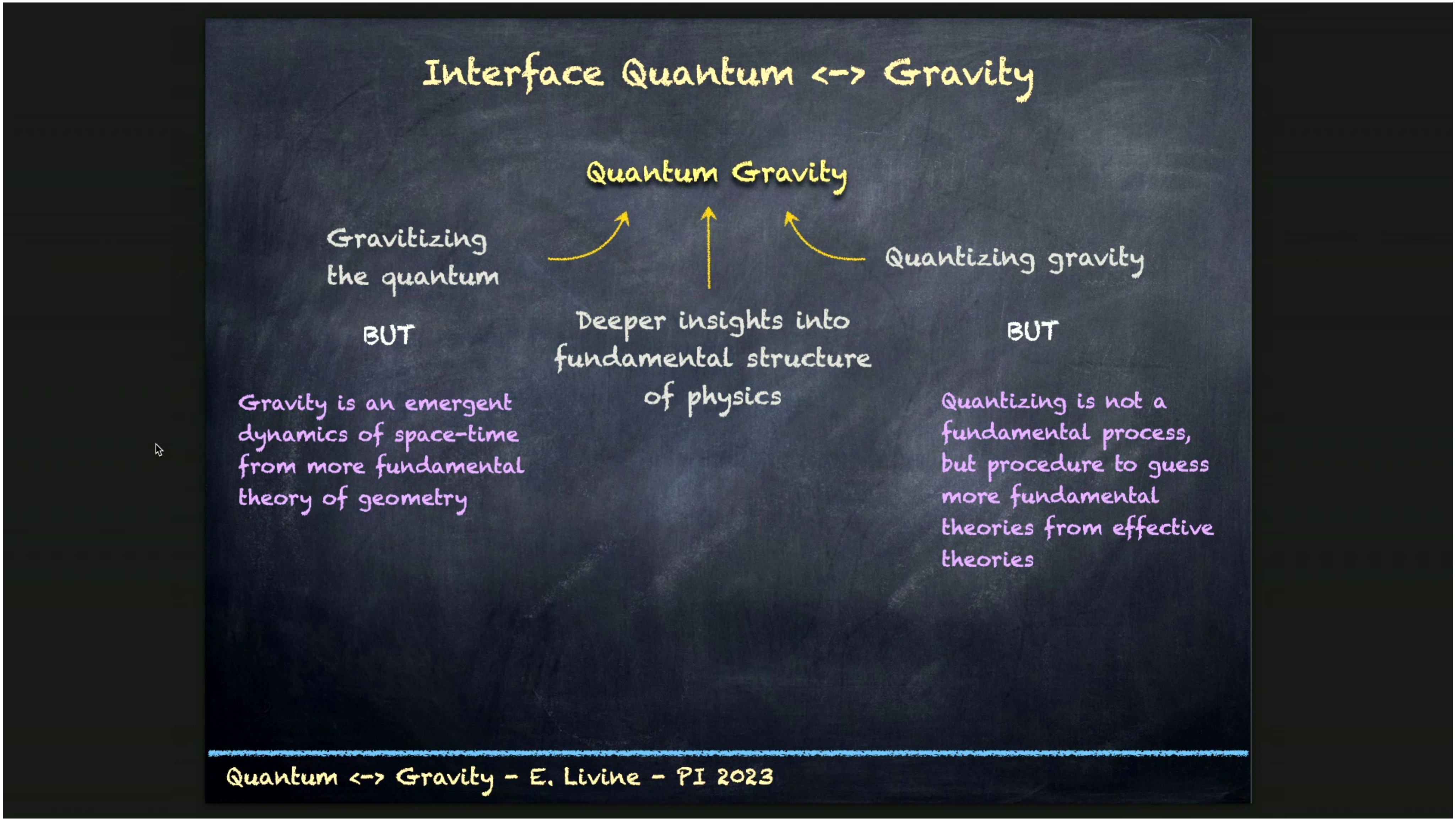

Interface Quantum <-> Gravity

Quantum Gravity

Gravitizing
the quantum

BUT

Gravity is an emergent dynamics of space-time from more fundamental theory of geometry

Deeper insights into
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Quantizing gravity

BUT

Quantizing is not a fundamental process, but procedure to guess more fundamental theories from effective theories

Interface Quantum <-> Gravity

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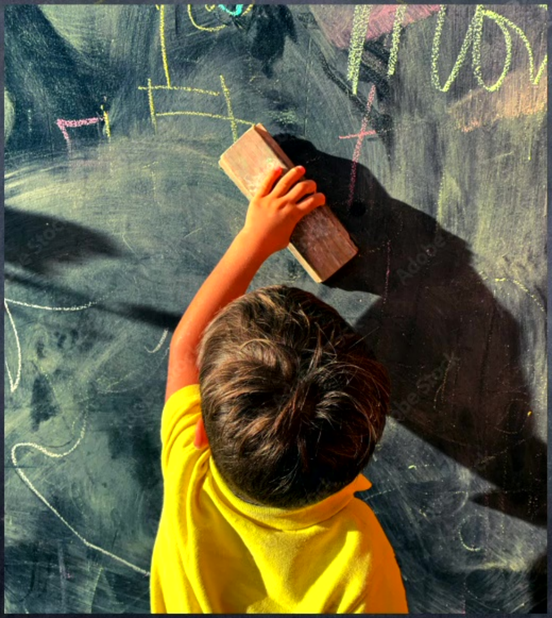


antizing gravity

BUT

Quantizing is not a fundamental process, but procedure to guess more fundamental theories from effective theories

Interface Quantum <-> Gravity

Quantum Gravity

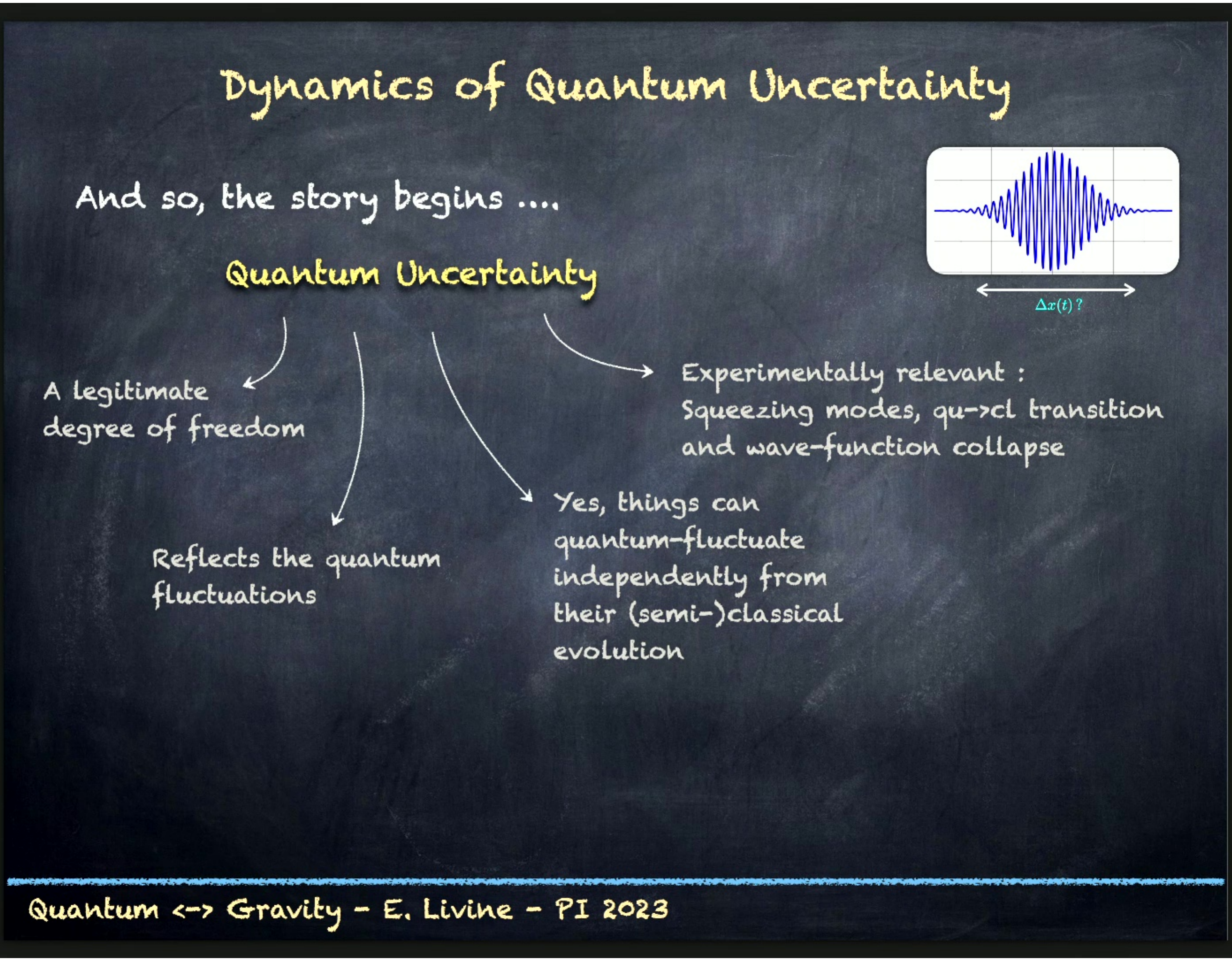
A deepening of BOTH concepts

1. Dynamics of quantum uncertainty

2. Gravitizing 1d quantum mechanics

3. Quantization of the BH minisuperspace

"Our love keeps getting stronger and stronger... like a piece of old cheese."



Dynamics of Quantum Uncertainty

And so, the story begins ....

Quantum Uncertainty

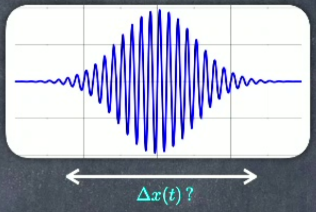
A legitimate degree of freedom

Reflects the quantum fluctuations

Yes, things can quantum-fluctuate independently from their (semi-)classical evolution

Experimentally relevant : Squeezing modes, qu->cl transition and wave-function collapse

Dynamics of Quantum Uncertainty



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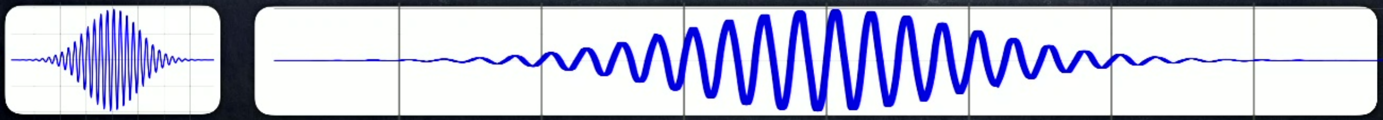
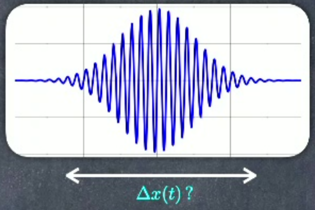


Stretched Quantum uncertainty = Quantum Superposition

Dynamics of Quantum Uncertainty



And so, the story begins ....

Quantum Uncertainty

$x(t)$ versus $\Psi(x,t)$

Classical particle Mechanics

Quantum particle Mechanics

as field Mechanics

Wave-function carries an infinity of d.o.f.s → Fourier modes $\Psi_p(t)$

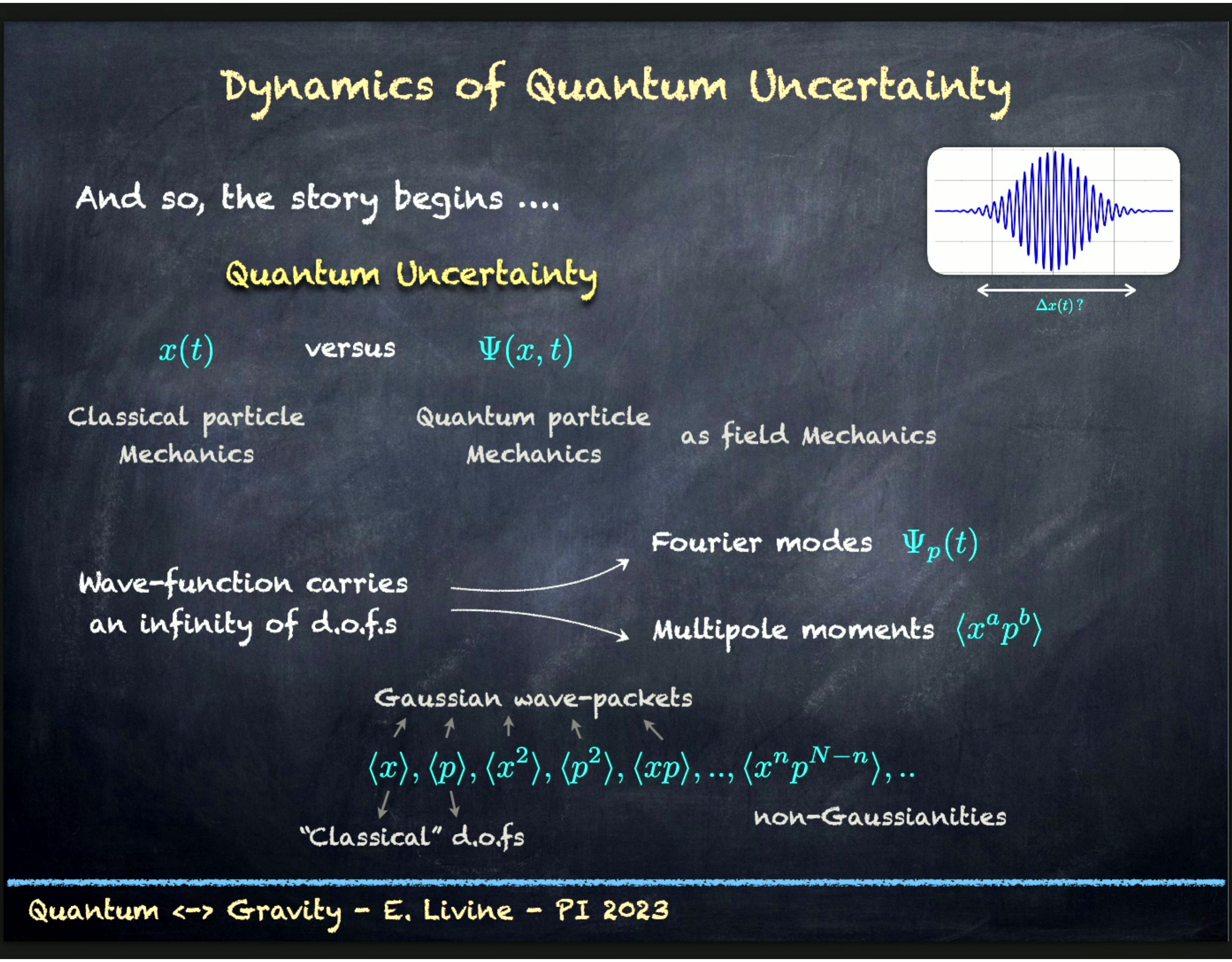
Wave-function carries an infinity of d.o.f.s → Multipole moments $\langle x^a p^b \rangle$

Gaussian wave-packets

$\langle x\rangle, \langle p\rangle, \langle x^2\rangle, \langle p^2\rangle, \langle xp\rangle, .., \langle x^n p^{N-n}\rangle, ..$

"Classical" d.o.fs

non-Gaussianities



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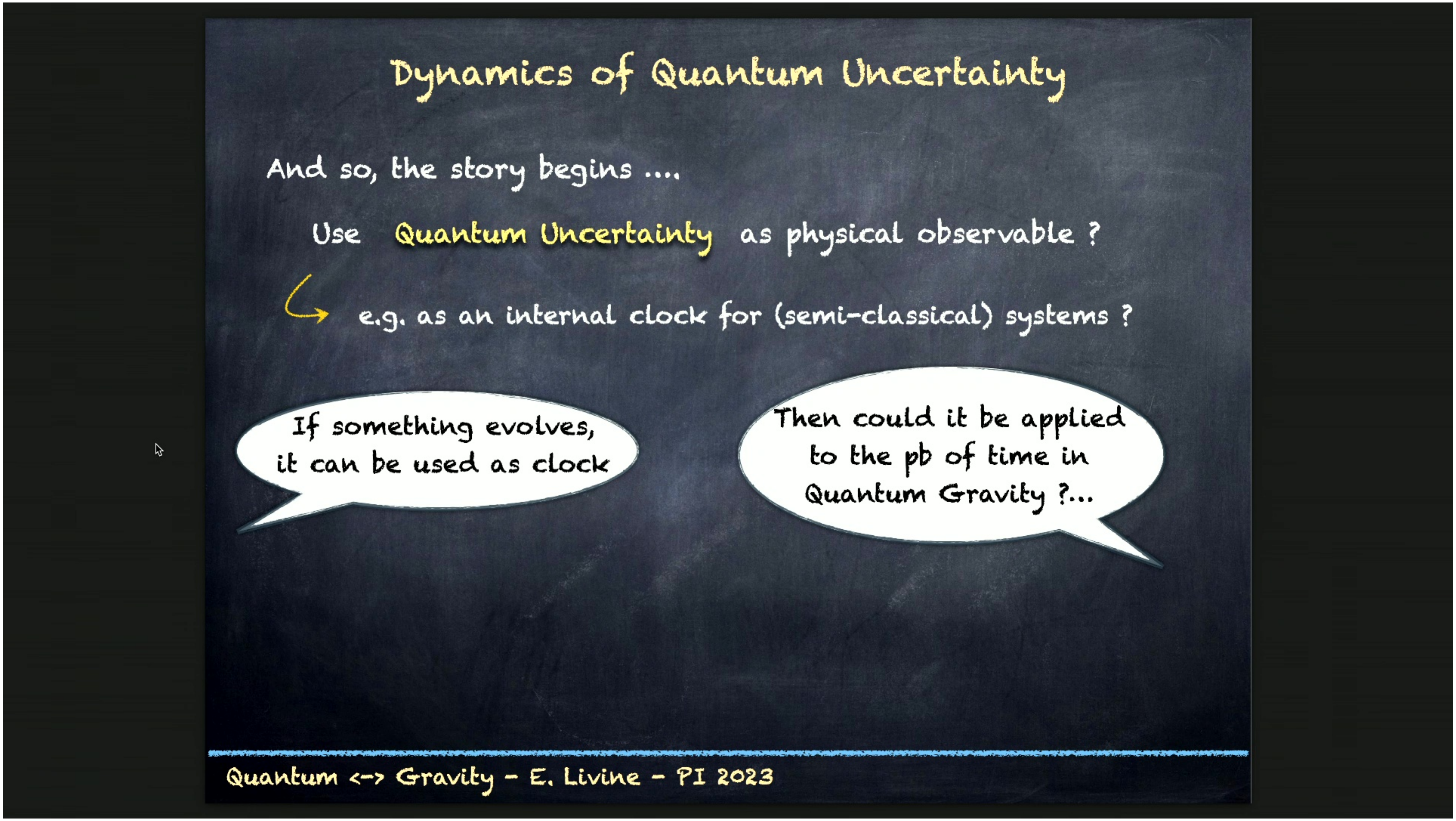


Dynamics of Quantum Uncertainty

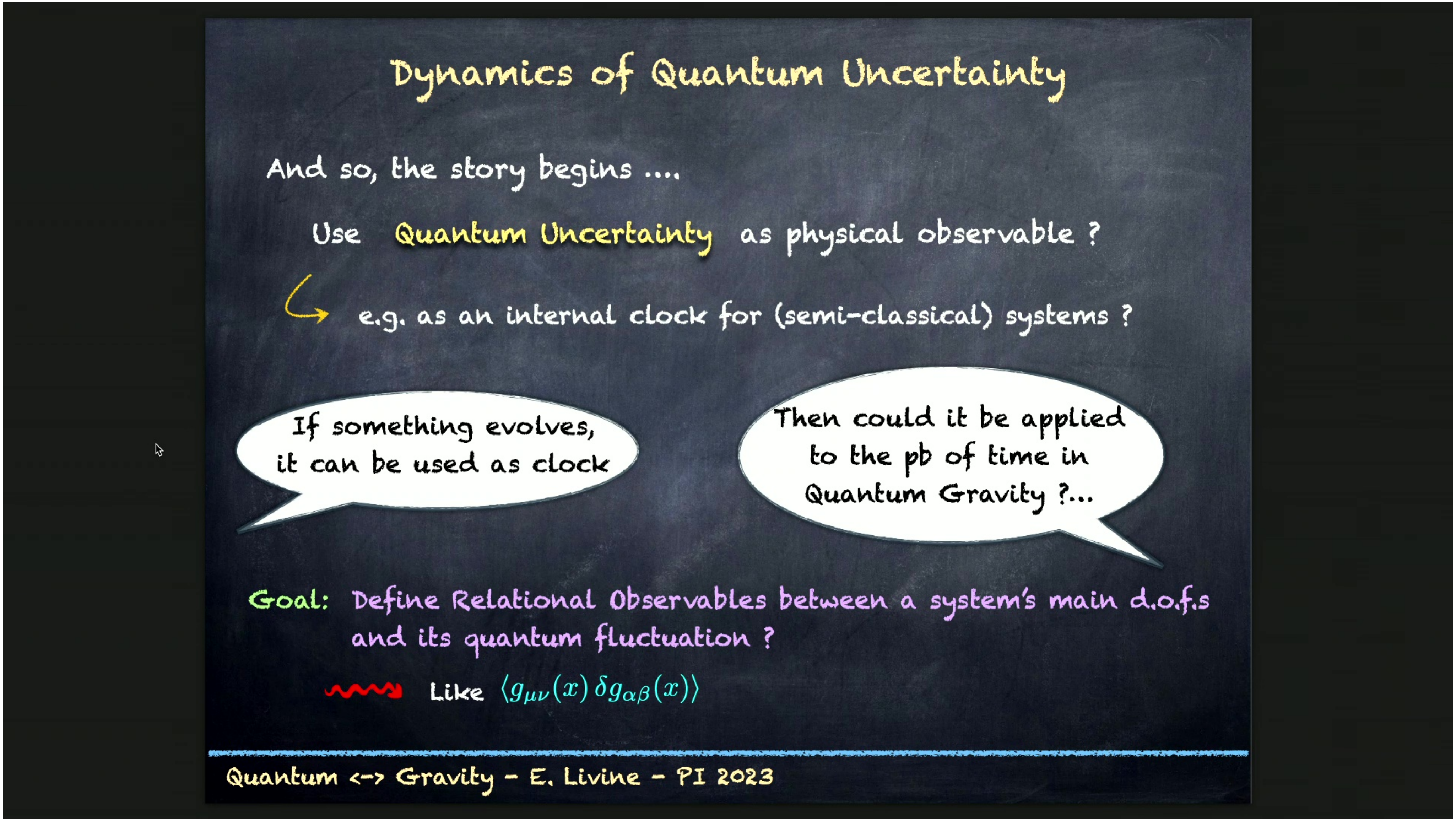
And so, the story begins ....

Use **Quantum Uncertainty** as physical observable ?

↳ e.g. as an internal clock for (semi-classical) systems ?

If something evolves,
it can be used as clock

Then could it be applied
to the pb of time in
Quantum Gravity ?...



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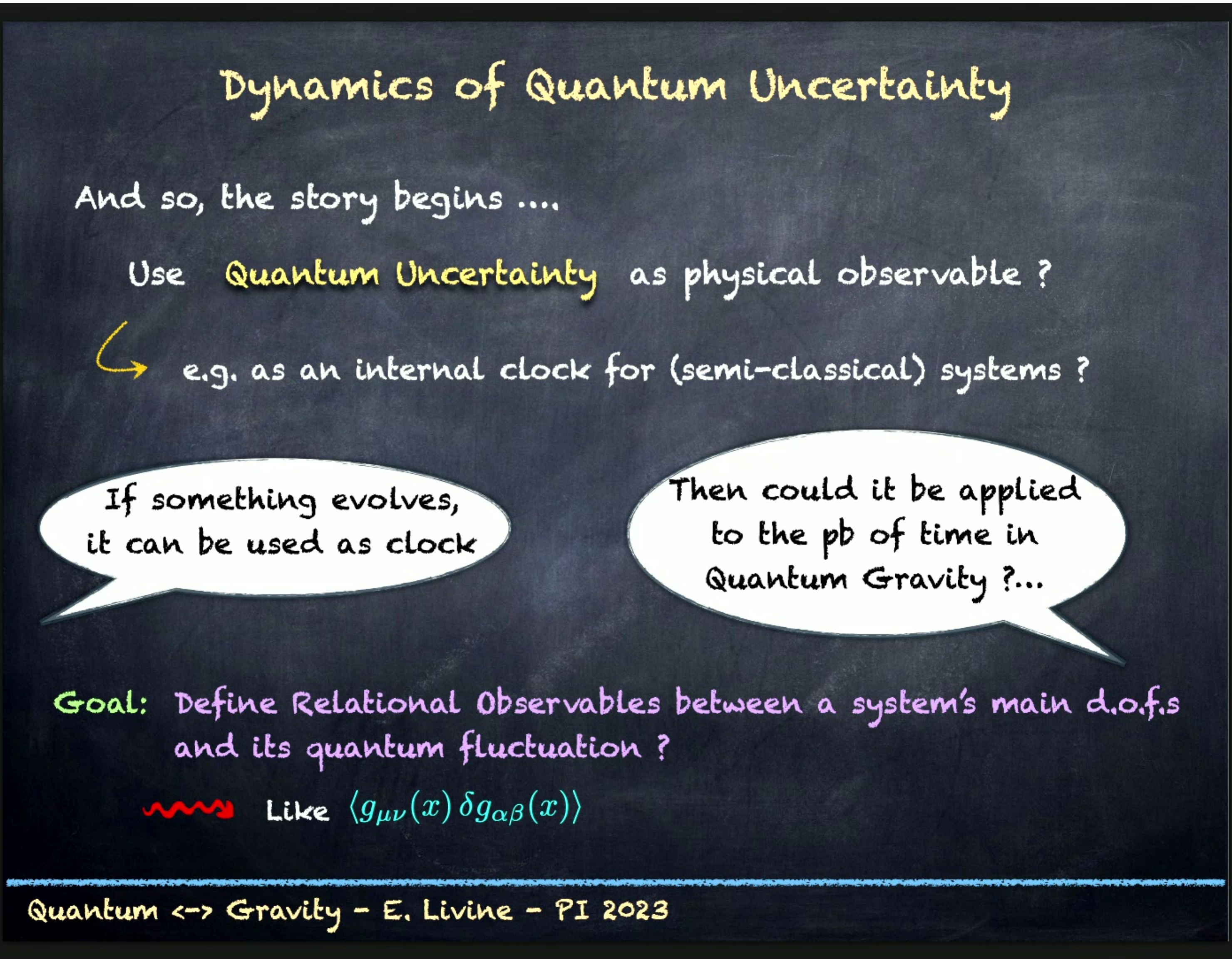
Goal: Define Relational Observables between a system's main d.o.f.s and its quantum fluctuation ?

⇝ Like $\langle g_{\mu\nu}(x)\,\delta g_{\alpha\beta}(x)\rangle$



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But here: Illustrate the idea in a much Much MUCH simpler context !

⟿ Back to the simplest model in classical mechanics

Solo adventures [2212.09442, 2305.03847, 2307.01061]

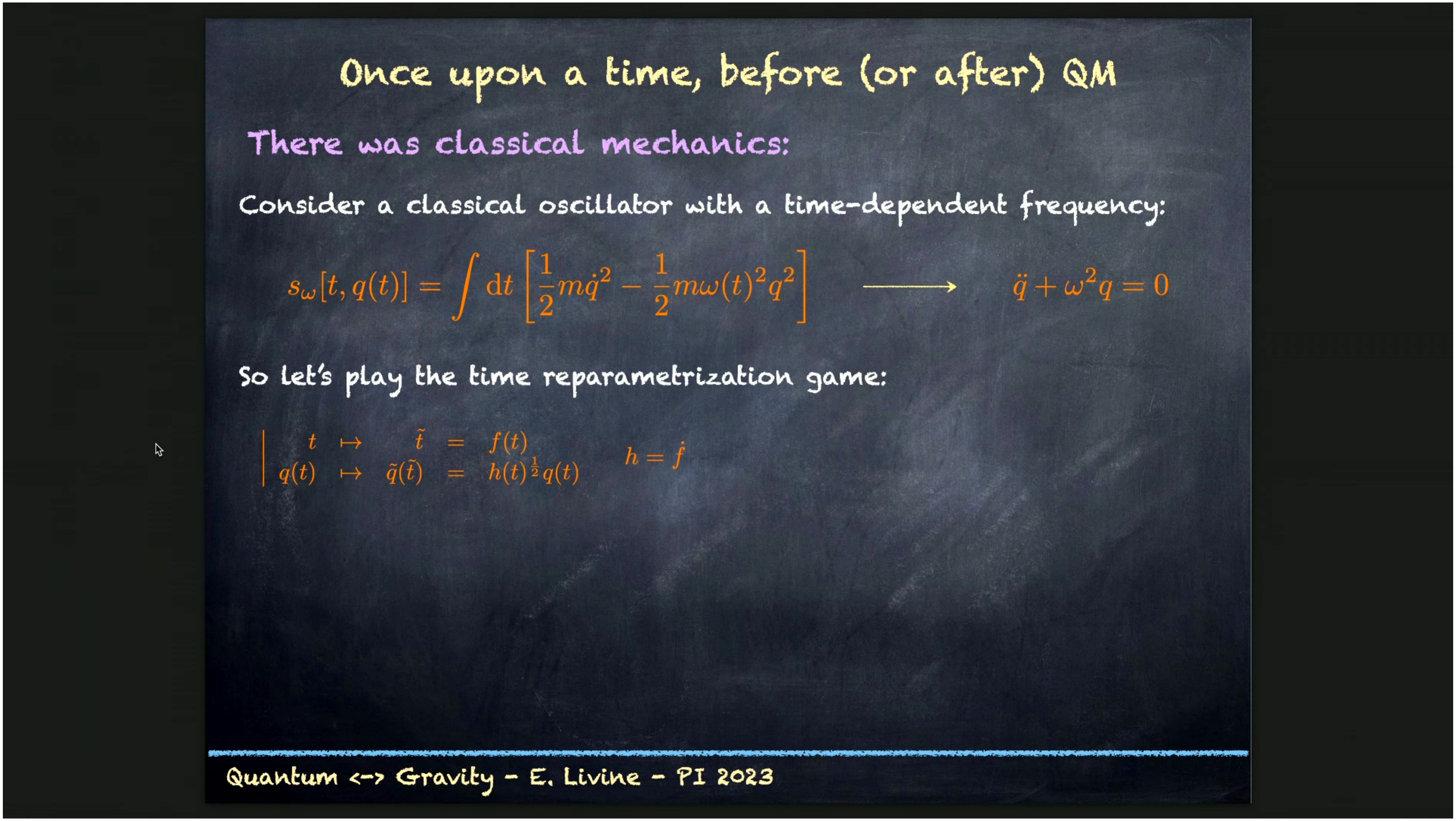
Once upon a time, before (or after) QM

There was classical mechanics:

Consider a classical oscillator with a time-dependent frequency:

$$s_\omega[t,q(t)] = \int \mathrm{d}t \left[\frac{1}{2}m\dot{q}^2 - \frac{1}{2}m\omega(t)^2 q^2\right] \qquad \longrightarrow \qquad \ddot{q} + \omega^2 q = 0$$

Explicit time dependence opens the door to
explore different choices of clock
and investigate role of time reparametrization



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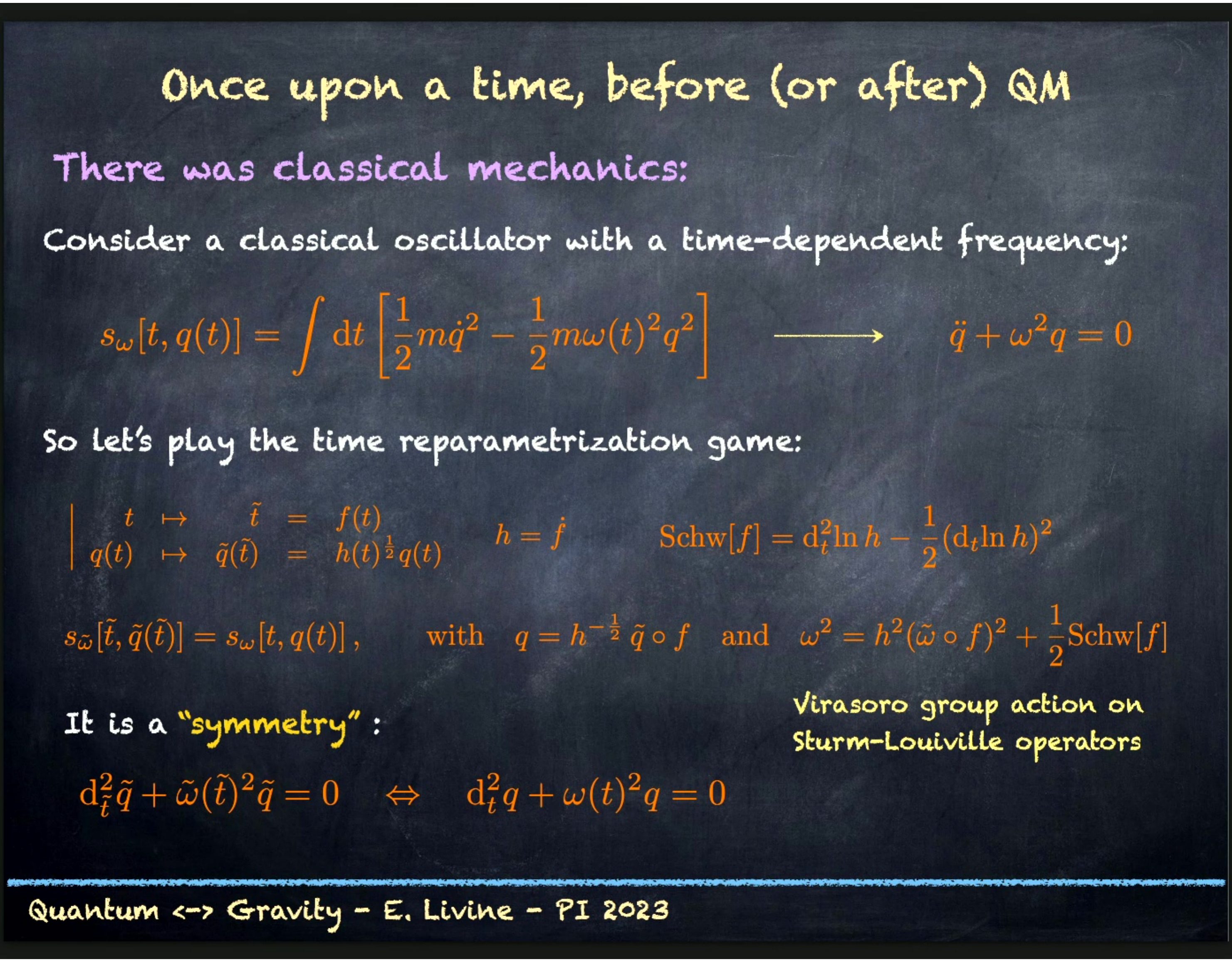
So let's play the time reparametrization game:

$$\left|\begin{array}{rcrcl} t & \mapsto & \tilde{t} & = & f(t) \\ q(t) & \mapsto & \tilde{q}(\tilde{t}) & = & h(t)^{\frac{1}{2}}q(t) \end{array}\right. \qquad h = \dot{f}$$



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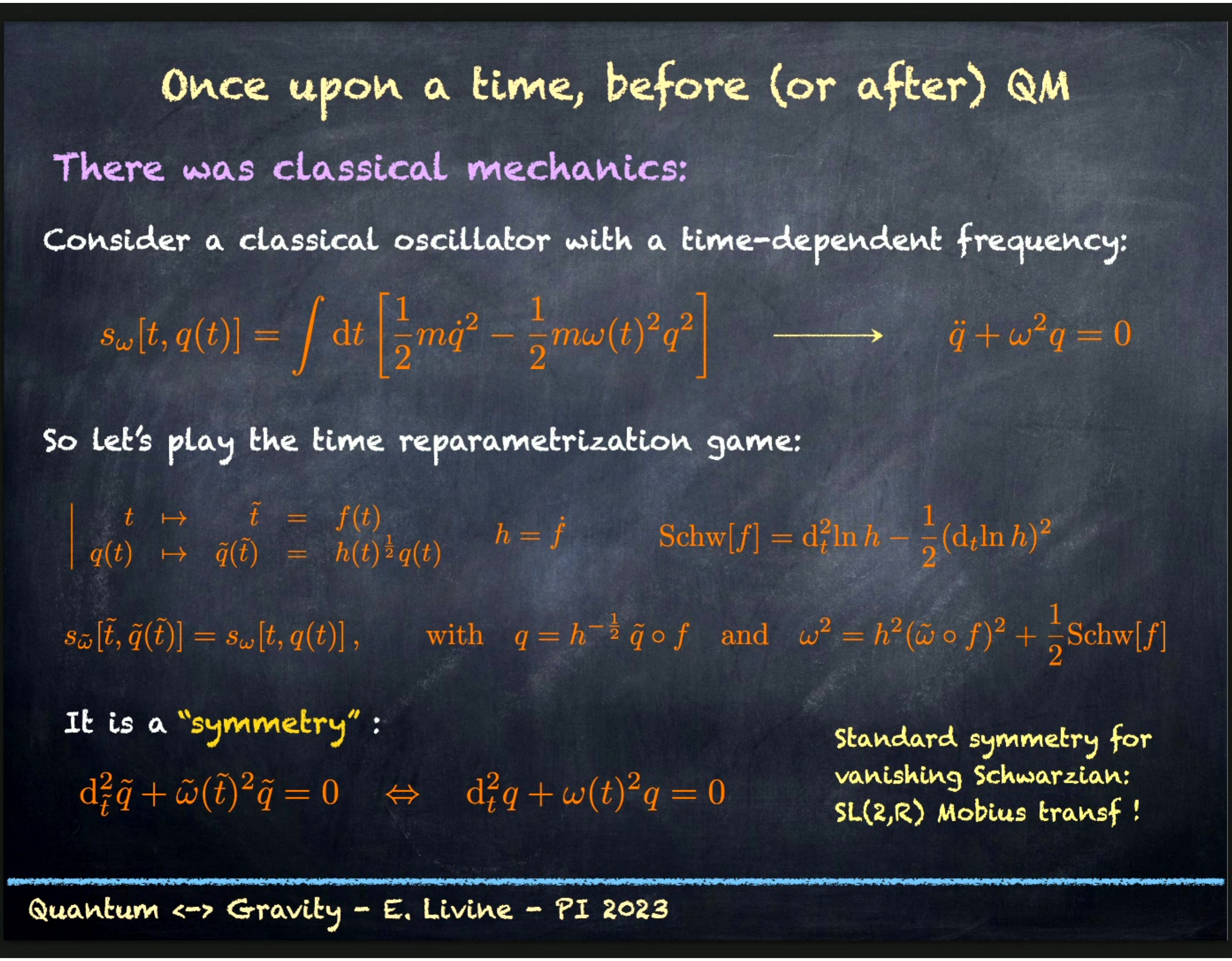
$$\left| \begin{array}{ccccc} t & \mapsto & \tilde t & = & f(t) \\ q(t) & \mapsto & \tilde q(\tilde t) & = & h(t)^{\frac12} q(t) \end{array}\right. \qquad h=\dot f \qquad \mathrm{Schw}[f] = \mathrm{d}_t^2\ln h - \frac12(\mathrm{d}_t\ln h)^2$$

$$s_{\tilde\omega}[\tilde t,\tilde q(\tilde t)] = s_\omega[t,q(t)]\,, \qquad \text{with} \quad q = h^{-\frac12}\,\tilde q\circ f \quad \text{and} \quad \omega^2 = h^2(\tilde\omega\circ f)^2 + \frac12\mathrm{Schw}[f]$$

It is a "symmetry" :

Virasoro group action on Sturm-Louiville operators

$$\mathrm{d}_{\tilde t}^2\tilde q + \tilde\omega(\tilde t)^2\tilde q = 0 \quad\Leftrightarrow\quad \mathrm{d}_t^2 q+\omega(t)^2 q = 0$$



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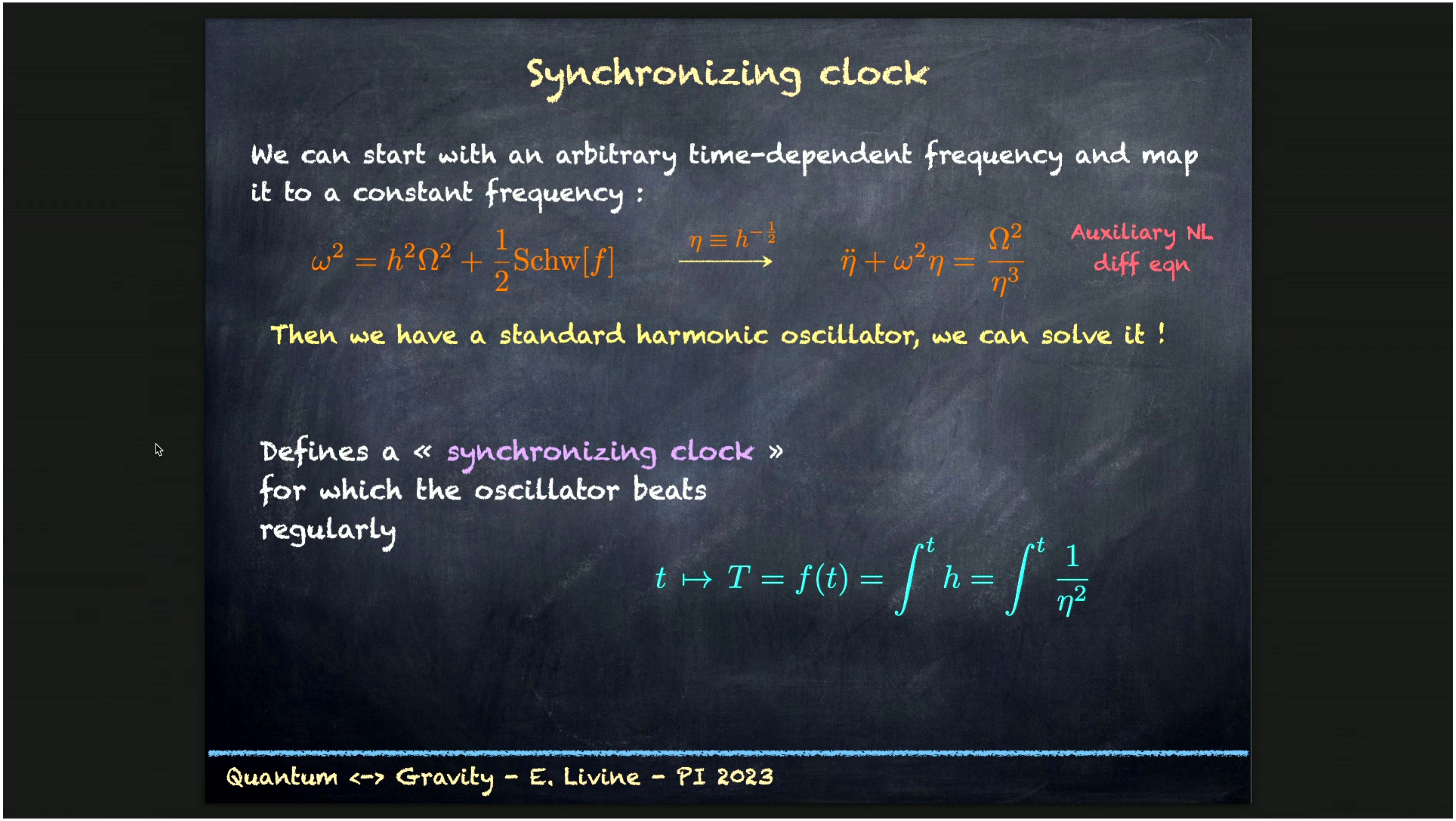
Standard symmetry for
vanishing Schwarzian:
SL(2,R) Mobius transf !



Synchronizing clock

We can start with an arbitrary time-dependent frequency and map it to a constant frequency :

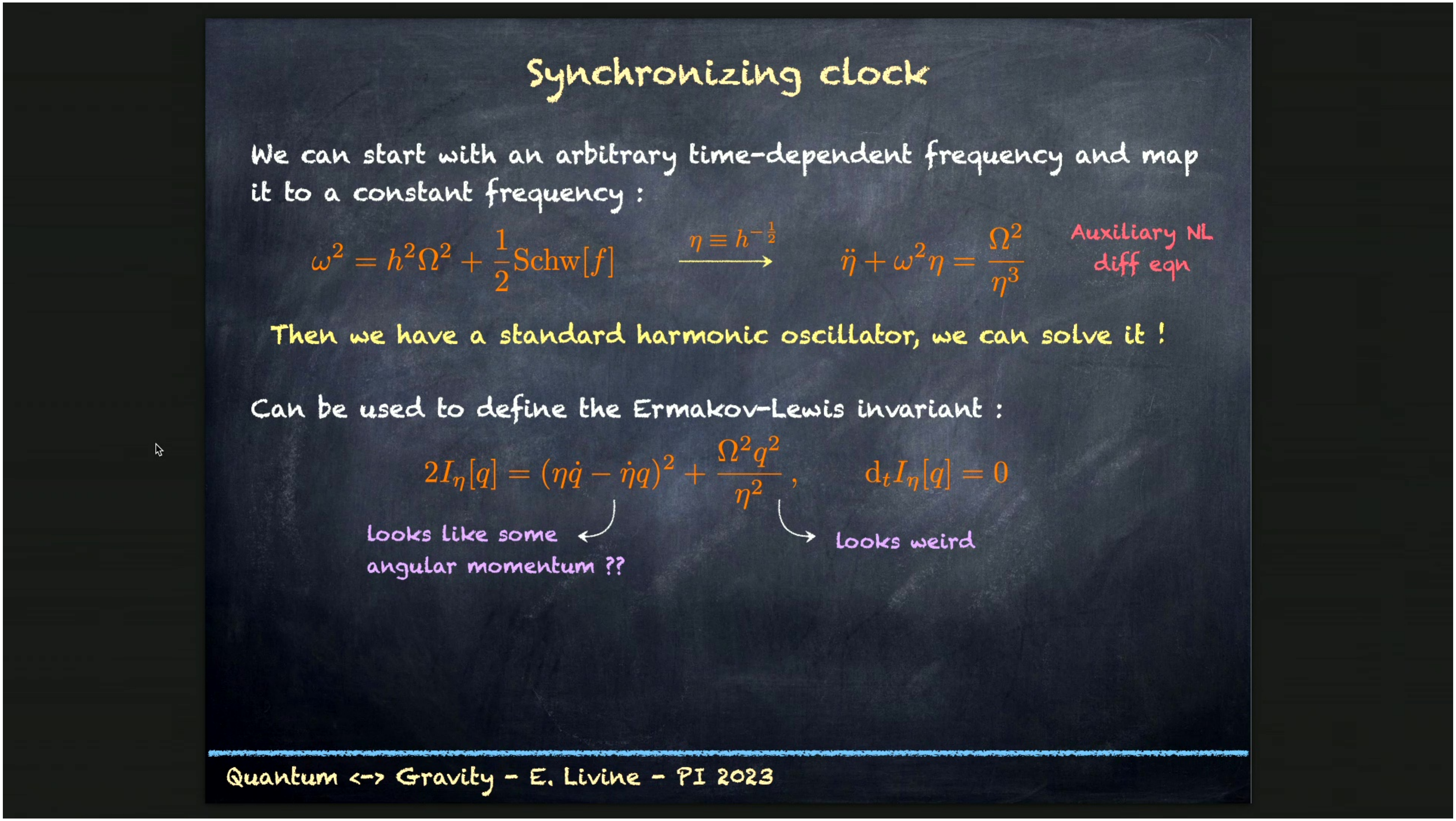
$$\omega^2 = h^2\Omega^2 + \frac{1}{2}\mathrm{Schw}[f] \quad \xrightarrow{\eta \equiv h^{-\frac{1}{2}}} \quad \ddot{\eta} + \omega^2\eta = \frac{\Omega^2}{\eta^3}$$

Auxiliary NL diff eqn

Then we have a standard harmonic oscillator, we can solve it !

Defines a « synchronizing clock » for which the oscillator beats regularly

$$t \mapsto T = f(t) = \int^t h = \int^t \frac{1}{\eta^2}$$



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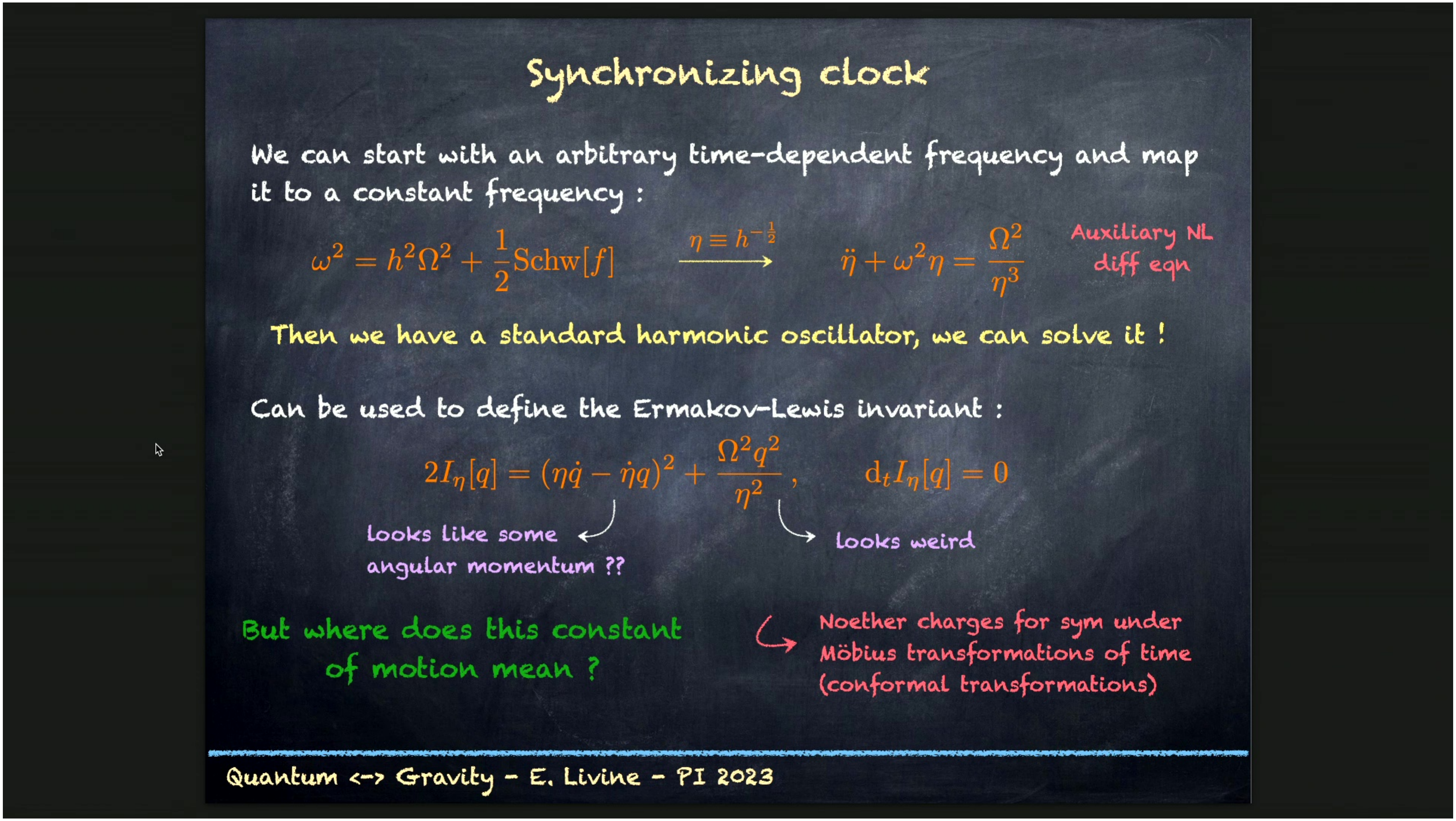
Can be used to define the Ermakov-Lewis invariant :

$$2I_\eta[q] = (\eta\dot{q} - \dot{\eta}q)^2 + \frac{\Omega^2 q^2}{\eta^2}\,, \qquad \mathrm{d}_t I_\eta[q] = 0$$

looks like some angular momentum ??

looks weird



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But where does this constant of motion mean ?

Noether charges for sym under Möbius transformations of time (conformal transformations)

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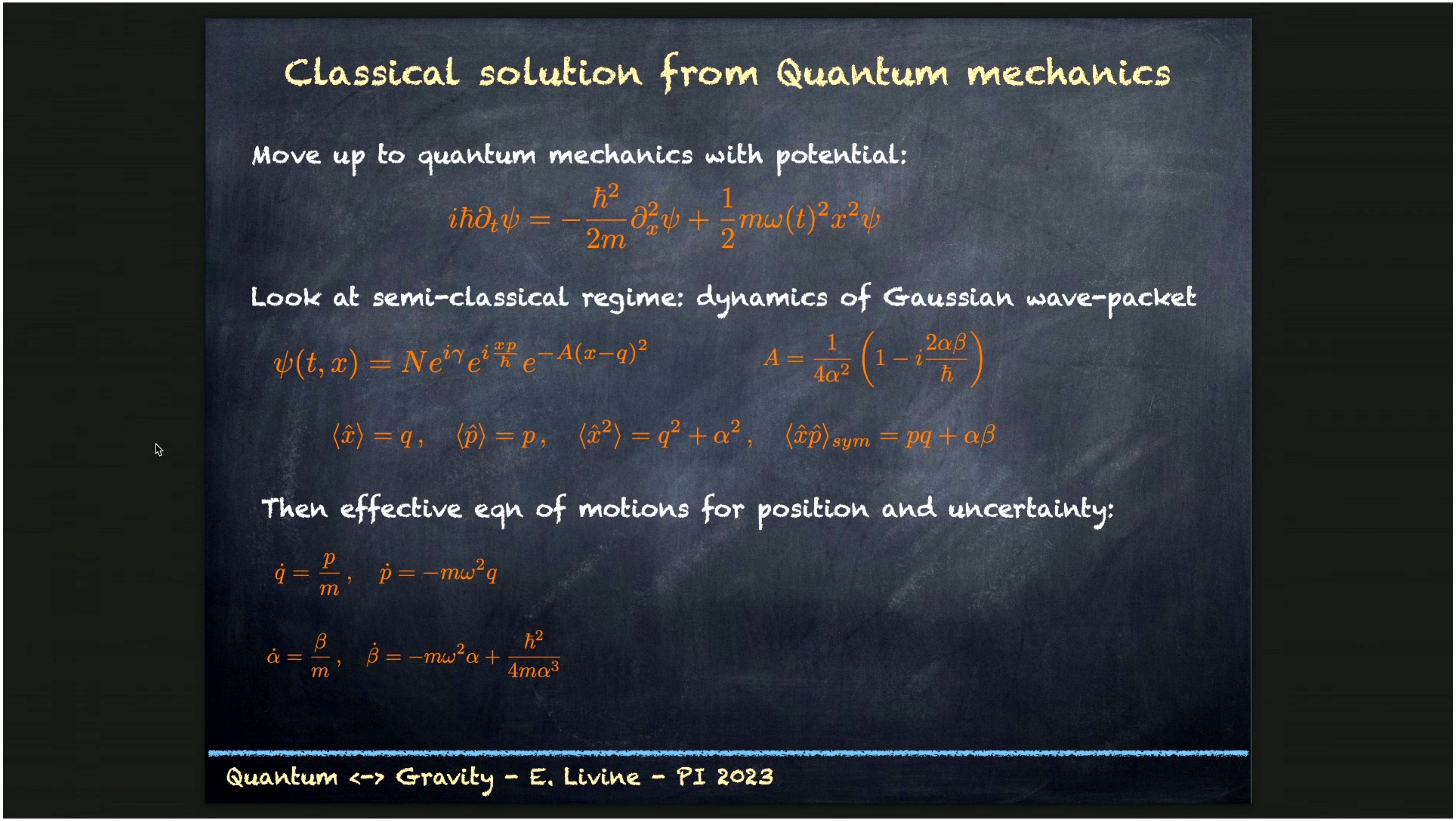


Classical solution from Quantum mechanics

Move up to quantum mechanics with potential:

$$i\hbar\partial_t\psi = -\frac{\hbar^2}{2m}\partial_x^2\psi + \frac{1}{2}m\omega(t)^2x^2\psi$$

Look at semi-classical regime: dynamics of Gaussian wave-packet

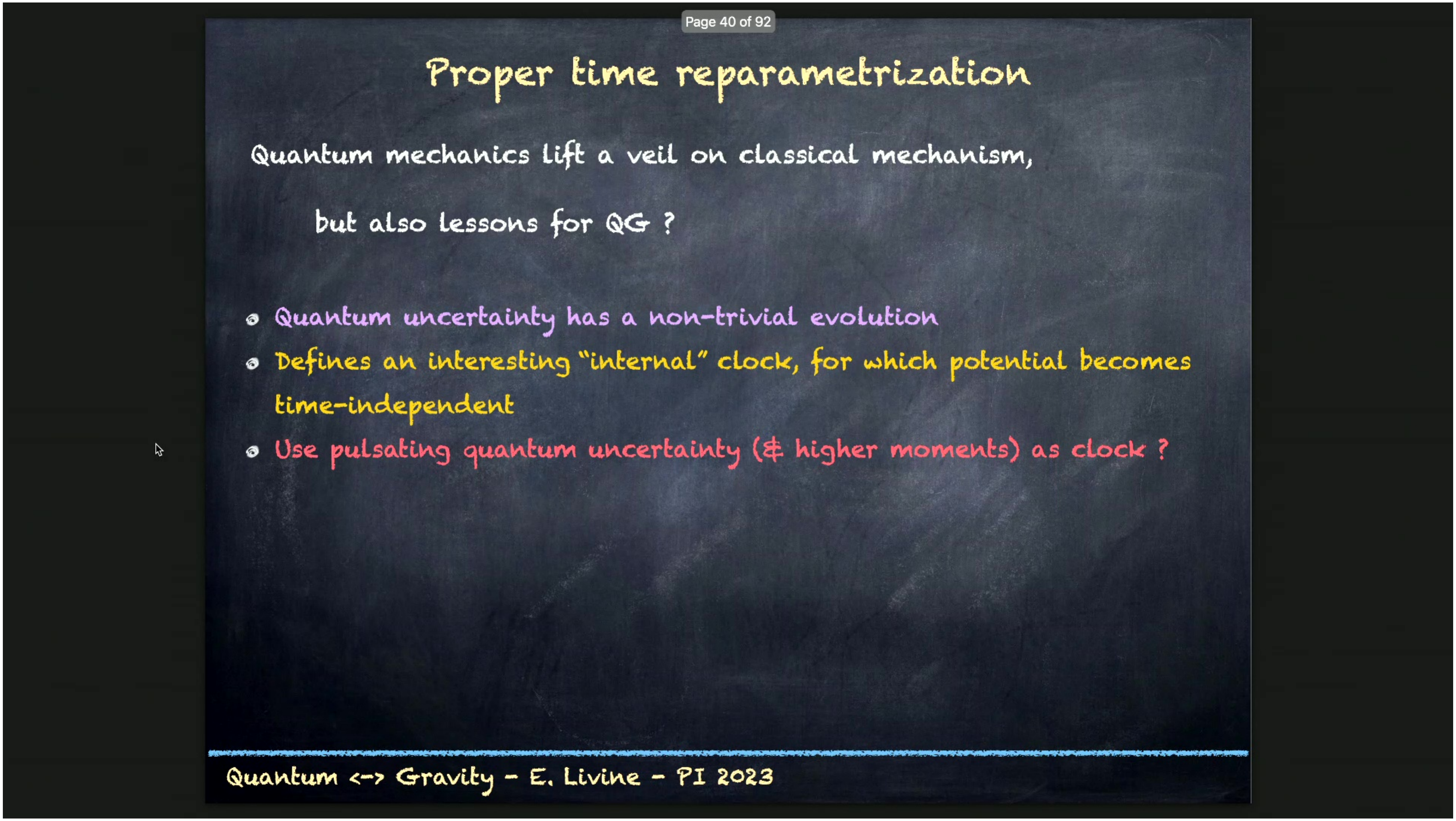
$$\psi(t,x) = Ne^{i\gamma}e^{i\frac{xp}{\hbar}}e^{-A(x-q)^2} \qquad A = \frac{1}{4\alpha^2}\left(1 - i\frac{2\alpha\beta}{\hbar}\right)$$

$$\langle\hat{x}\rangle = q\,, \quad \langle\hat{p}\rangle = p\,, \quad \langle\hat{x}^2\rangle = q^2+\alpha^2\,, \quad \langle\hat{x}\hat{p}\rangle_{sym} = pq+\alpha\beta$$

Then effective eqn of motions for position and uncertainty:

$$\dot{q} = \frac{p}{m}\,, \quad \dot{p} = -m\omega^2 q$$

$$\dot{\alpha} = \frac{\beta}{m}\,, \quad \dot{\beta} = -m\omega^2\alpha + \frac{\hbar^2}{4m\alpha^3}$$

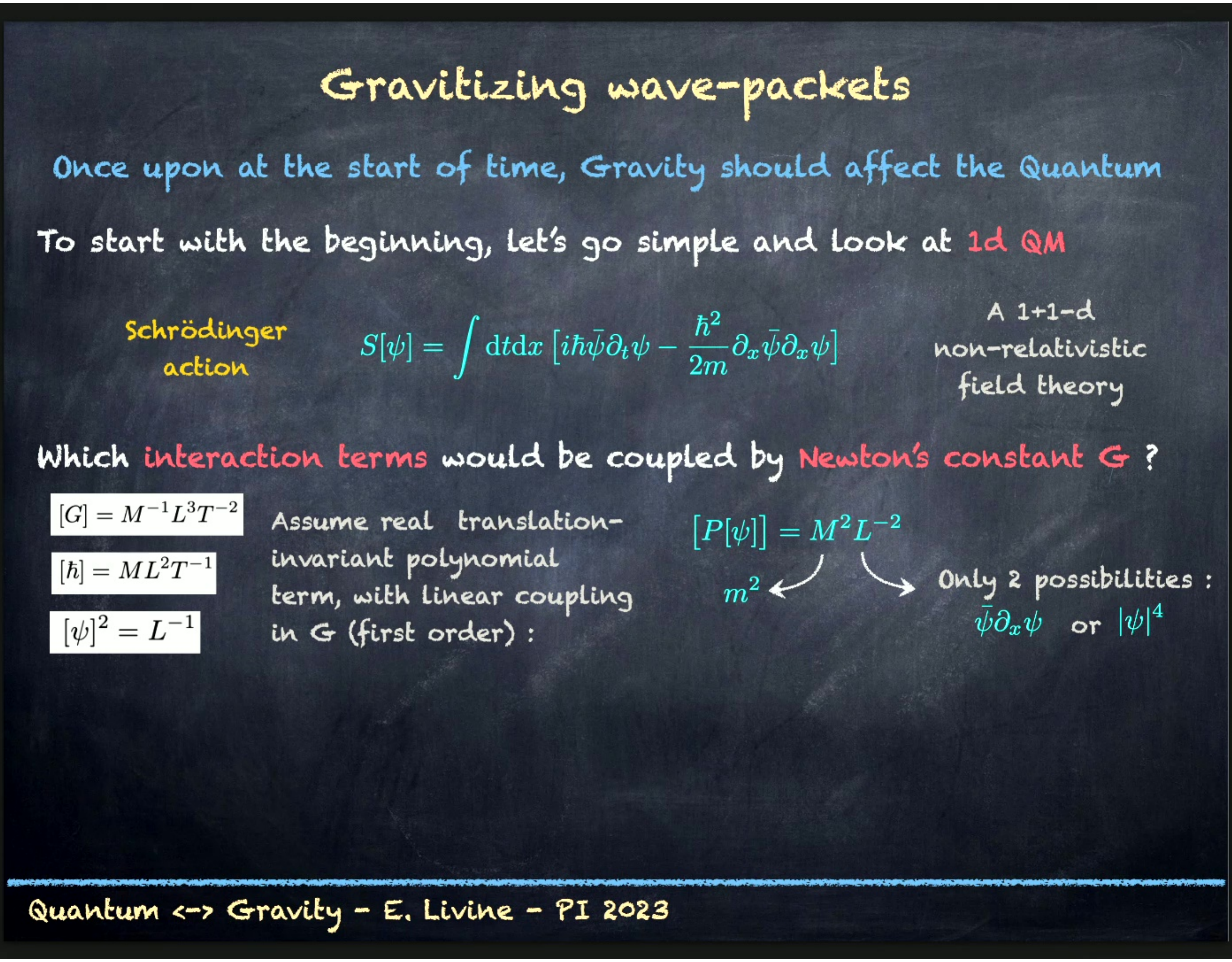


Proper time reparametrization

Quantum mechanics lift a veil on classical mechanism,

but also lessons for QG ?

- Quantum uncertainty has a non-trivial evolution
- Defines an interesting "internal" clock, for which potential becomes time-independent
- Use pulsating quantum uncertainty (& higher moments) as clock ?



Gravitizing wave-packets

Once upon at the start of time, Gravity should affect the Quantum

To start with the beginning, let's go simple and look at 1d QM

Schrödinger action

$$S[\psi] = \int \mathrm{d}t\mathrm{d}x\, [i\hbar\bar{\psi}\partial_t\psi - \frac{\hbar^2}{2m}\partial_x\bar{\psi}\partial_x\psi]$$

A 1+1-d non-relativistic field theory

Which interaction terms would be coupled by Newton's constant G ?

$[G] = M^{-1}L^3T^{-2}$

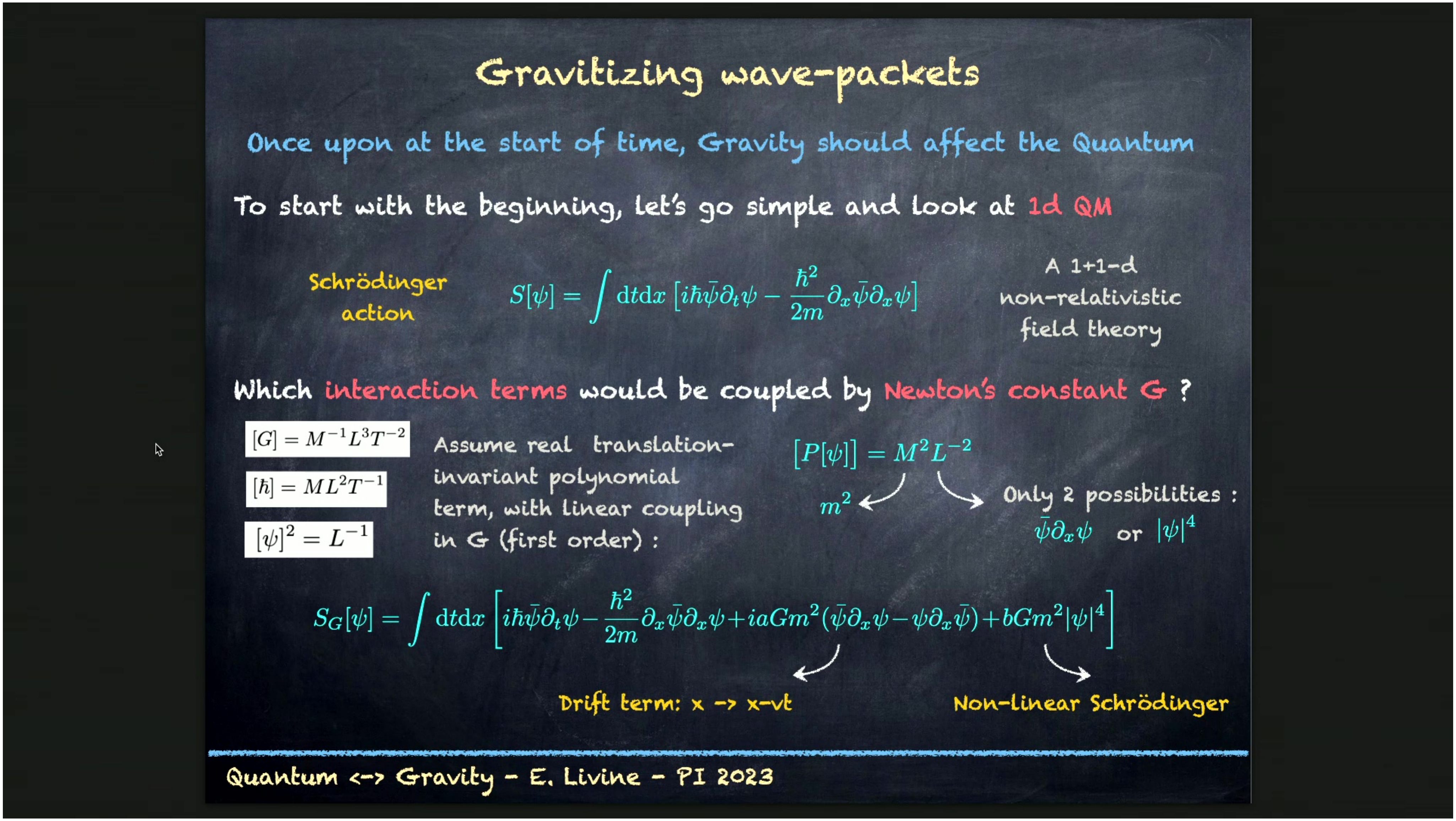
$[\hbar] = ML^2T^{-1}$

$[\psi]^2 = L^{-1}$

Assume real translation-invariant polynomial term, with linear coupling in G (first order) :

$$\begin{tikzcd} & {[P[\psi]] = M^2L^{-2}} \arrow[dl] \arrow[dr] & \\ m^2 & & \text{Only 2 possibilities :} \end{tikzcd}$$

$\bar{\psi}\partial_x\psi$ or $|\psi|^4$



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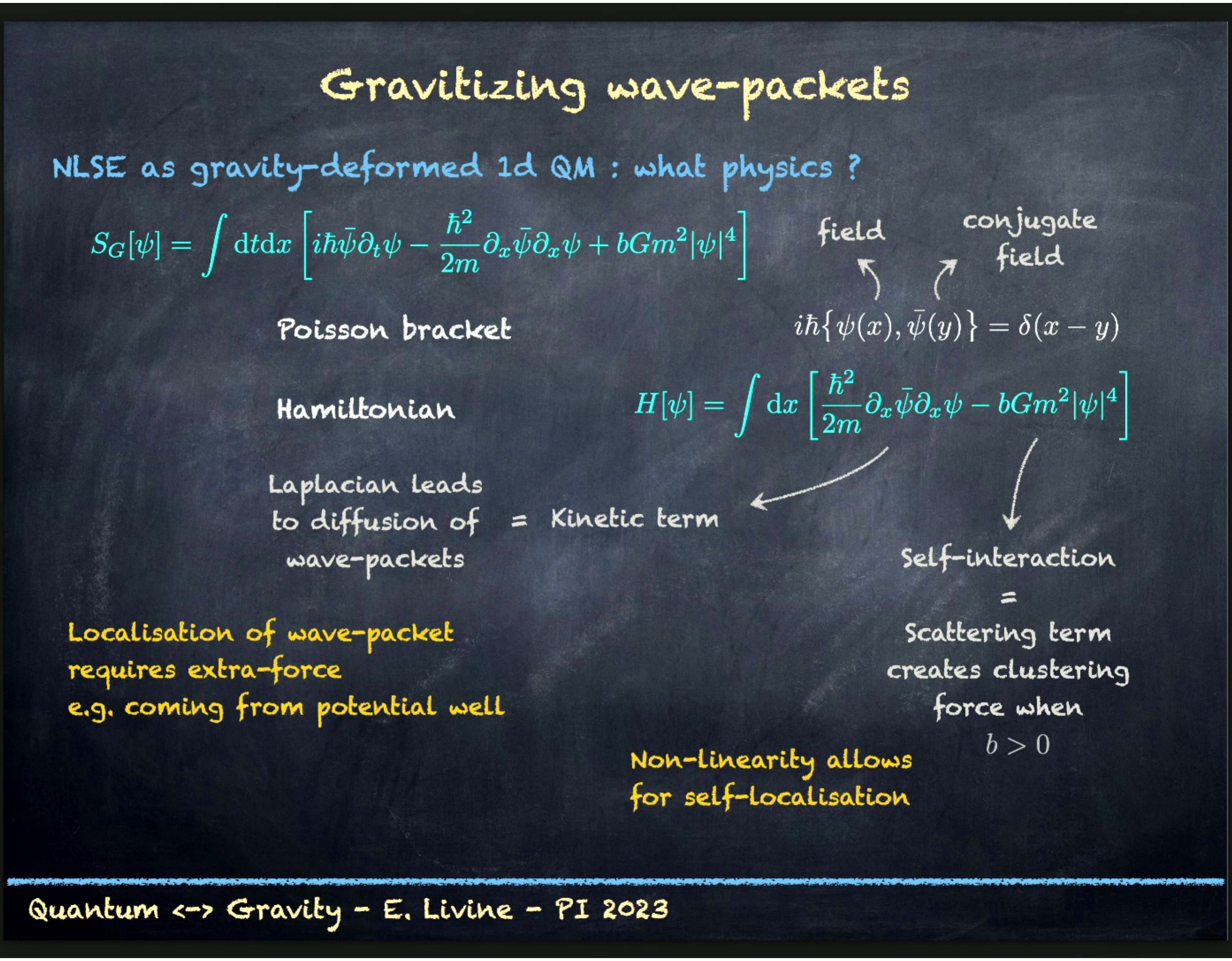
m^2

Only 2 possibilities : $\bar{\psi}\partial_x\psi$ or $|\psi|^4$

$$S_G[\psi] = \int \mathrm{d}t\mathrm{d}x \left[i\hbar\bar{\psi}\partial_t\psi - \frac{\hbar^2}{2m}\partial_x\bar{\psi}\partial_x\psi + iaGm^2(\bar{\psi}\partial_x\psi - \psi\partial_x\bar{\psi}) + bGm^2|\psi|^4\right]$$

Drift term: x -> x-vt

Non-linear Schrödinger



Gravitizing wave-packets

NLSE as gravity-deformed 1d QM : what physics ?

$$S_G[\psi] = \int \mathrm{d}t\mathrm{d}x \left[i\hbar\bar{\psi}\partial_t\psi - \frac{\hbar^2}{2m}\partial_x\bar{\psi}\partial_x\psi + bGm^2|\psi|^4 \right]$$

field

conjugate field

Poisson bracket

$$i\hbar\{\psi(x),\bar{\psi}(y)\} = \delta(x-y)$$

Hamiltonian

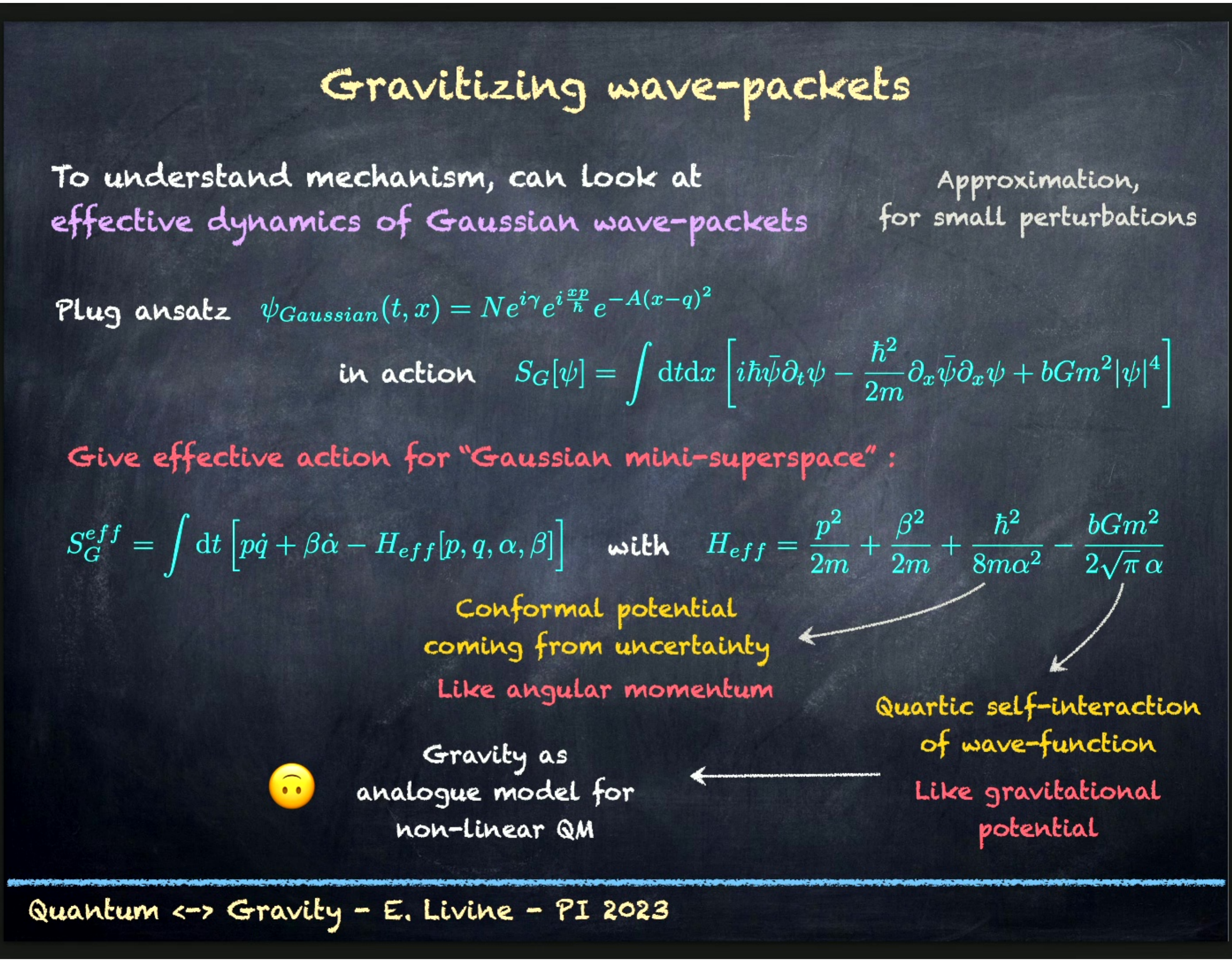
$$H[\psi] = \int \mathrm{d}x \left[\frac{\hbar^2}{2m}\partial_x\bar{\psi}\partial_x\psi - bGm^2|\psi|^4 \right]$$

Laplacian leads to diffusion of wave-packets = Kinetic term

Self-interaction = Scattering term creates clustering force when $b > 0$

Localisation of wave-packet requires extra-force e.g. coming from potential well

Non-linearity allows for self-localisation



Gravitizing wave-packets

To understand mechanism, can look at
effective dynamics of Gaussian wave-packets

Approximation,
for small perturbations

Plug ansatz $\psi_{Gaussian}(t,x) = Ne^{i\gamma}e^{i\frac{xp}{\hbar}}e^{-A(x-q)^2}$

in action $S_G[\psi] = \int \mathrm{d}t\mathrm{d}x \left[i\hbar\bar{\psi}\partial_t\psi - \frac{\hbar^2}{2m}\partial_x\bar{\psi}\partial_x\psi + bGm^2|\psi|^4 \right]$

Give effective action for "Gaussian mini-superspace" :

$$S_G^{eff} = \int \mathrm{d}t \left[p\dot{q} + \beta\dot{\alpha} - H_{eff}[p,q,\alpha,\beta] \right] \quad \text{with} \quad H_{eff} = \frac{p^2}{2m} + \frac{\beta^2}{2m} + \frac{\hbar^2}{8m\alpha^2} - \frac{bGm^2}{2\sqrt{\pi}\,\alpha}$$

Conformal potential
coming from uncertainty

Like angular momentum

Quartic self-interaction
of wave-function

Like gravitational
potential

🙃 Gravity as
analogue model for
non-linear QM

Gravitizing wave-packets

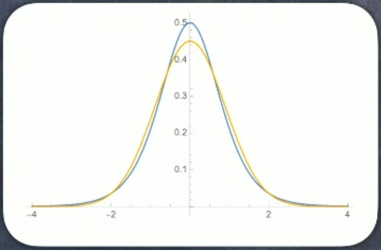
Compare exact solitons and approx Gaussian wave-packet:

$$\psi = \mathcal{A}\frac{e^{-i\omega t}e^{\frac{i}{\hbar}xp}}{\cosh\gamma(x-vt)}$$

$$\frac{1}{\gamma} = \frac{\hbar^2}{bGm^3}\frac{1}{N}$$

$$\Delta x = \sqrt{\langle x^2\rangle - \langle x\rangle^2} = \frac{\pi}{\gamma\sqrt{12}} = \frac{\pi}{\sqrt{12}}\frac{1}{N}\frac{\hbar^2}{bGm^3}$$

$$\frac{\pi}{\sqrt{12}} \simeq 0.907$$



$$\psi(x,t) = Ke^{i\gamma}\,e^{\frac{i}{\hbar}p(x-q)}\,e^{-A(x-q)^2}$$

$$\alpha_0 = \frac{\sqrt{\pi}}{2}\frac{\hbar^2}{bGm^3}$$

$$\frac{\sqrt{\pi}}{2} \simeq 0.886$$

Elliptic trajectory should approx exact pulsating solitons

↳ Oscillating quantum uncertainty,
like beating heart of wave-packet,
a.k.a. internal clock

Gravitizing wave-packets

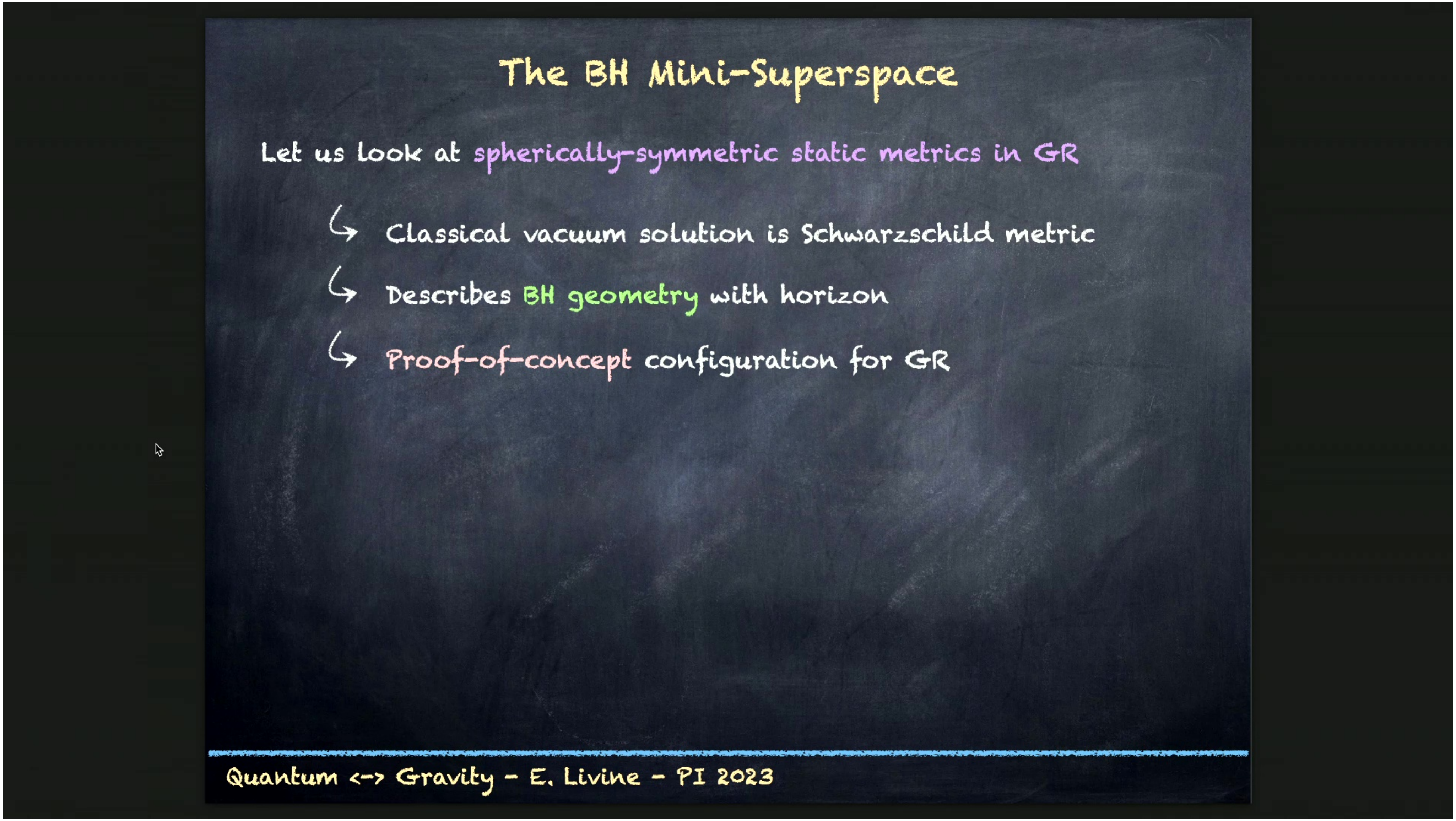
Moral of the story ?

Gravity effects on QM

Self-interaction of wave-function

Non-linear Schrödinger

Approximated by gravity in field space ?!



The BH Mini-Superspace

Let us look at spherically-symmetric static metrics in GR

- ↳ Classical vacuum solution is Schwarzschild metric
- ↳ Describes BH geometry with horizon
- ↳ Proof-of-concept configuration for GR

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$$S[g] = \frac{1}{\ell_P^2} \int_{\mathcal{M}} \mathrm{d}^4x \left[\mathcal{R} - 2\Lambda\right]$$

$$\mathrm{d}s^2 = \epsilon \left(-N^2(r)\mathrm{d}r^2 + \gamma_{tt}(r)\mathrm{d}t^2\right) + \gamma_{\theta\theta}(r)\mathrm{d}\Omega^2$$

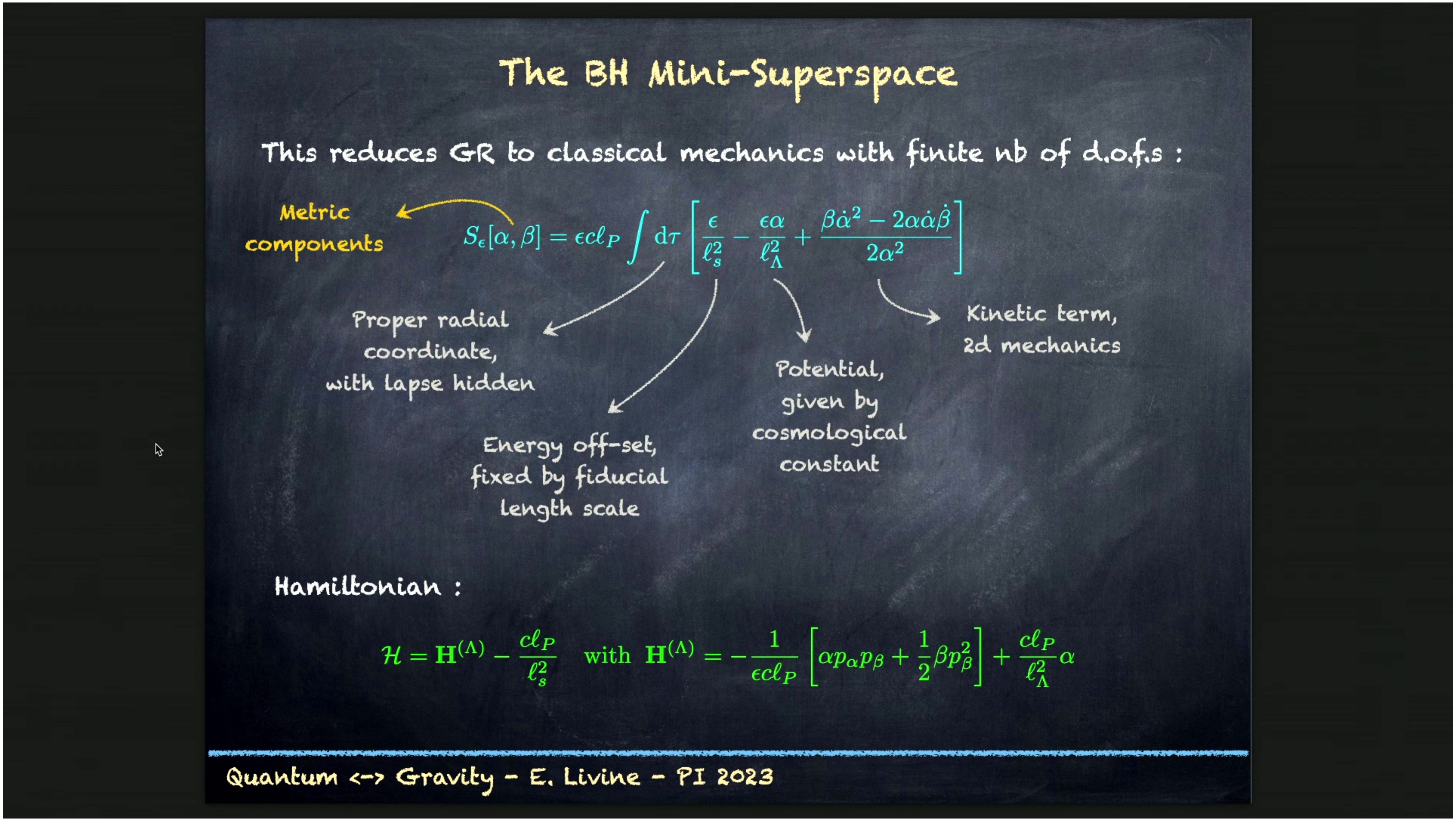
$$\gamma_{tt} := 2\beta(r)/\alpha(r), \quad \gamma_{\theta\theta} := \ell_s^2\alpha(r) \qquad c = \frac{\ell_0\ell_s^2}{\ell_p^3} = \frac{\mathcal{V}_{IR}}{\mathcal{V}_{UV}}$$

$\epsilon = -$ BH exterior

$\epsilon = +$ BH interior

$$S_\epsilon[\alpha, \beta] = \epsilon c \ell_P \int \mathrm{d}\tau \left[\frac{\epsilon}{\ell_s^2} - \frac{\epsilon\alpha}{\ell_\Lambda^2} + \frac{\beta\dot{\alpha}^2 - 2\alpha\dot{\alpha}\dot{\beta}}{2\alpha^2}\right]$$

Evolution along r : $\mathrm{d}\tau = \sqrt{\frac{2\beta}{\alpha}}N(r)\mathrm{d}r$



The BH Mini-Superspace

This reduces GR to classical mechanics with finite nb of d.o.f.s :

$$S_\epsilon[\alpha,\beta] = \epsilon c \ell_P \int \mathrm{d}\tau \left[\frac{\epsilon}{\ell_s^2} - \frac{\epsilon\alpha}{\ell_\Lambda^2} + \frac{\beta\dot{\alpha}^2 - 2\alpha\dot{\alpha}\dot{\beta}}{2\alpha^2} \right]$$

Metric components

Proper radial coordinate, with lapse hidden

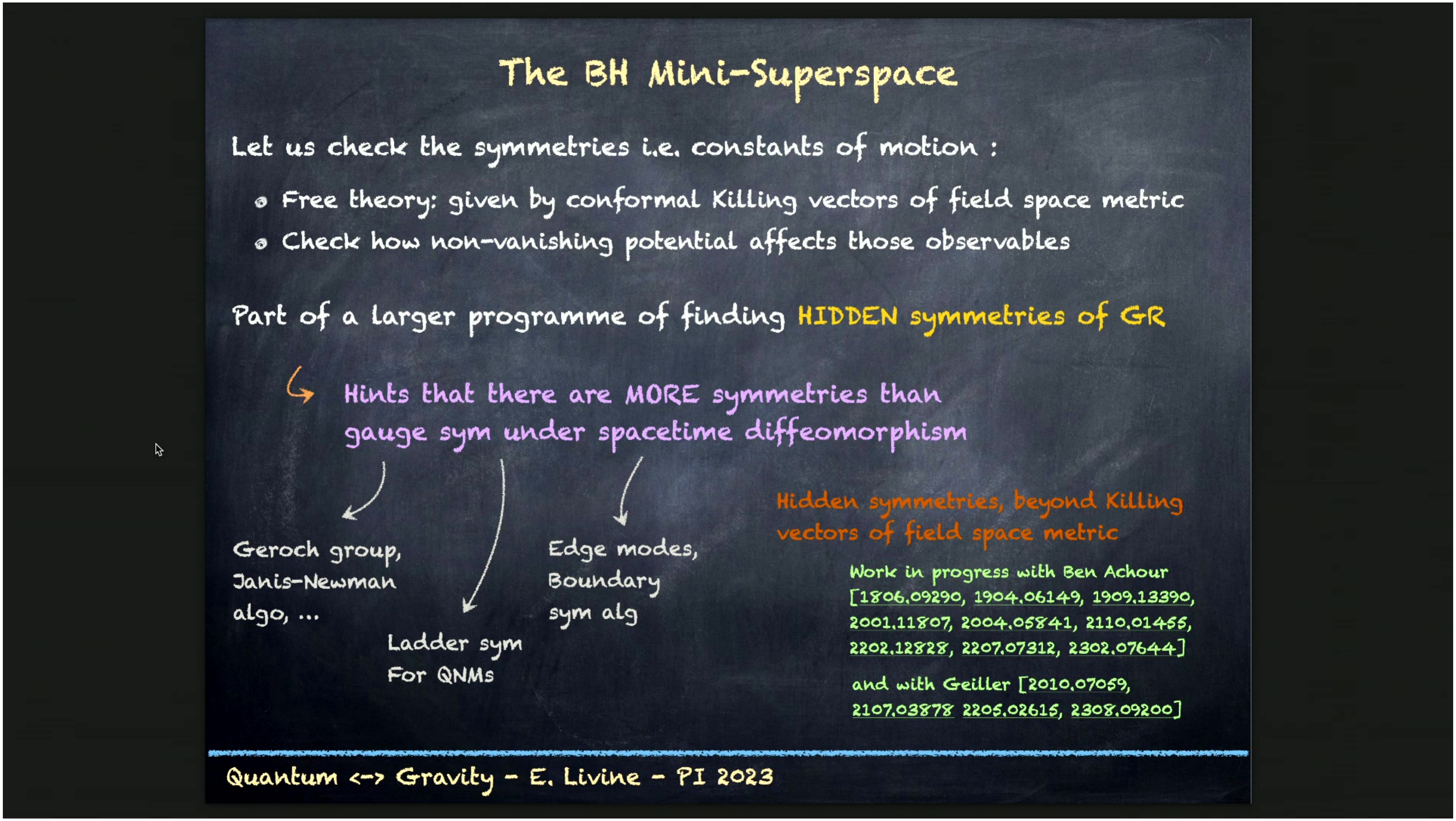
Energy off-set, fixed by fiducial length scale

Potential, given by cosmological constant

Kinetic term, 2d mechanics

Hamiltonian :

$$\mathcal{H} = \mathbf{H}^{(\Lambda)} - \frac{c\ell_P}{\ell_s^2} \quad \text{with} \quad \mathbf{H}^{(\Lambda)} = -\frac{1}{\epsilon c \ell_P}\left[\alpha p_\alpha p_\beta + \frac{1}{2}\beta p_\beta^2\right] + \frac{c\ell_P}{\ell_\Lambda^2}\alpha$$



The BH Mini-Superspace

Let us check the symmetries i.e. constants of motion :

- Free theory: given by conformal Killing vectors of field space metric
- Check how non-vanishing potential affects those observables

Part of a larger programme of finding HIDDEN symmetries of GR

↳ Hints that there are MORE symmetries than gauge sym under spacetime diffeomorphism

Geroch group,
Janis-Newman
algo, ...

Ladder sym
For QNMs

Edge modes,
Boundary
sym alg

Hidden symmetries, beyond Killing vectors of field space metric

Work in progress with Ben Achour [1806.09290, 1904.06149, 1909.13390, 2001.11807, 2004.05841, 2110.01455, 2202.12828, 2207.07312, 2302.07644]

and with Geiller [2010.07059, 2107.03878 2205.02615, 2308.09200]

The BH Mini-Superspace

Let us check the symmetries i.e. constants of motion :

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Find maximal symmetry algebra :

Schrödinger alg $\mathfrak{sh}(2) = (\mathfrak{sl}(2,\mathbb{R}) \oplus \mathfrak{so}(1,1)) \oplus_s (\mathbb{R}^2 \oplus \mathbb{R}^2)$

Translations

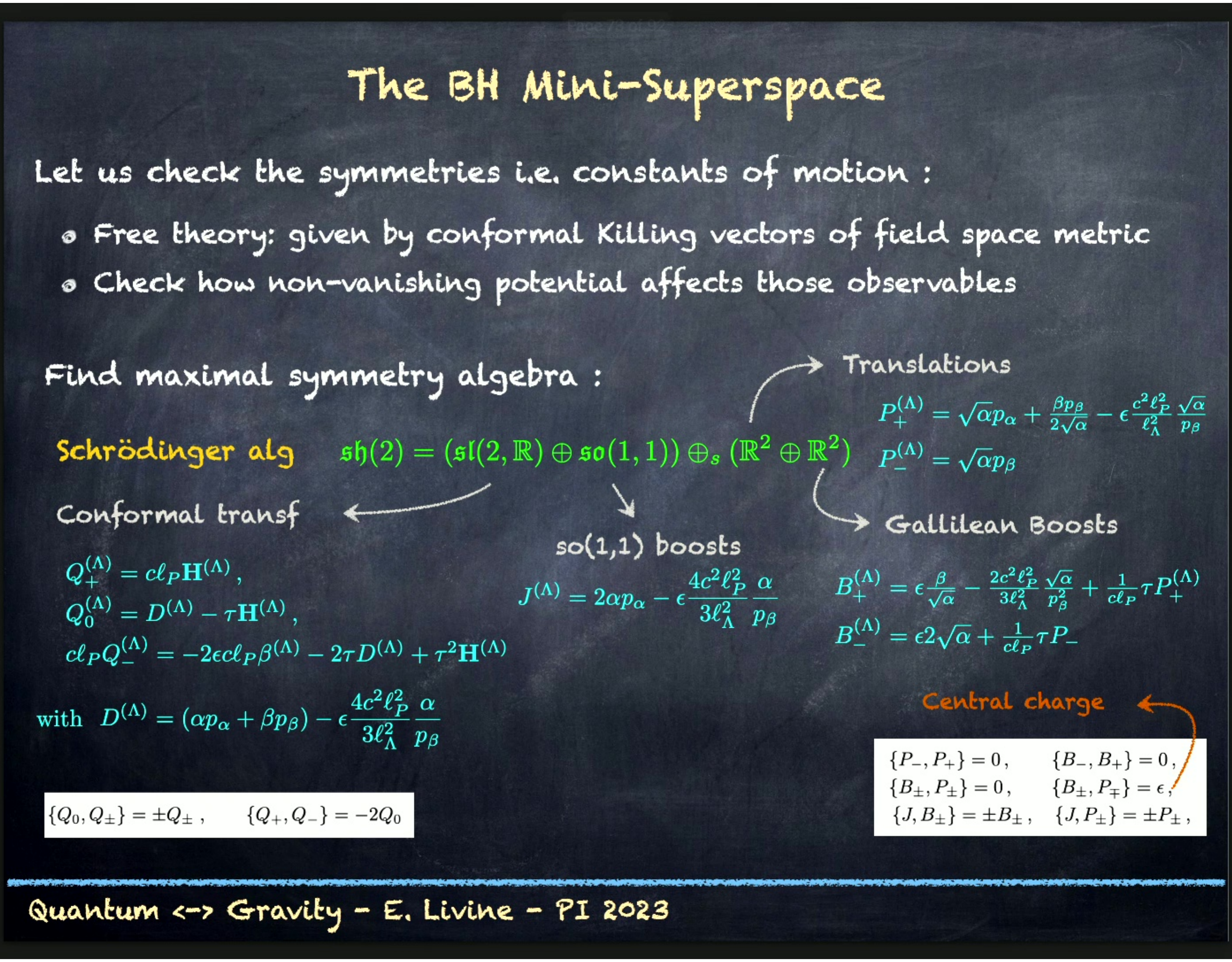
$$P_+^{(\Lambda)} = \sqrt{\alpha}p_\alpha + \frac{\beta p_\beta}{2\sqrt{\alpha}} - \epsilon\frac{c^2\ell_P^2}{\ell_\Lambda^2}\frac{\sqrt{\alpha}}{p_\beta}$$

$$P_-^{(\Lambda)} = \sqrt{\alpha}p_\beta$$

Gallilean Boosts

$$B_+^{(\Lambda)} = \epsilon\frac{\beta}{\sqrt{\alpha}} - \frac{2c^2\ell_P^2}{3\ell_\Lambda^2}\frac{\sqrt{\alpha}}{p_\beta^2} + \frac{1}{c\ell_P}\tau P_+^{(\Lambda)}$$

$$B_-^{(\Lambda)} = \epsilon 2\sqrt{\alpha} + \frac{1}{c\ell_P}\tau P_-$$



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Conformal transf

$$Q_+^{(\Lambda)} = c\ell_P \mathbf{H}^{(\Lambda)},$$

$$Q_0^{(\Lambda)} = D^{(\Lambda)} - \tau\mathbf{H}^{(\Lambda)},$$

$$c\ell_P Q_-^{(\Lambda)} = -2\epsilon c\ell_P \beta^{(\Lambda)} - 2\tau D^{(\Lambda)} + \tau^2\mathbf{H}^{(\Lambda)}$$

with $D^{(\Lambda)} = (\alpha p_\alpha + \beta p_\beta) - \epsilon \dfrac{4c^2\ell_P^2}{3\ell_\Lambda^2}\dfrac{\alpha}{p_\beta}$

$$\{Q_0, Q_\pm\} = \pm Q_\pm\,, \qquad \{Q_+, Q_-\} = -2Q_0$$

so(1,1) boosts

$$J^{(\Lambda)} = 2\alpha p_\alpha - \epsilon \frac{4c^2\ell_P^2}{3\ell_\Lambda^2}\frac{\alpha}{p_\beta}$$

Gallilean Boosts

$$B_+^{(\Lambda)} = \epsilon \tfrac{\beta}{\sqrt{\alpha}} - \tfrac{2c^2\ell_P^2}{3\ell_\Lambda^2}\tfrac{\sqrt{\alpha}}{p_\beta^2} + \tfrac{1}{c\ell_P}\tau P_+^{(\Lambda)}$$

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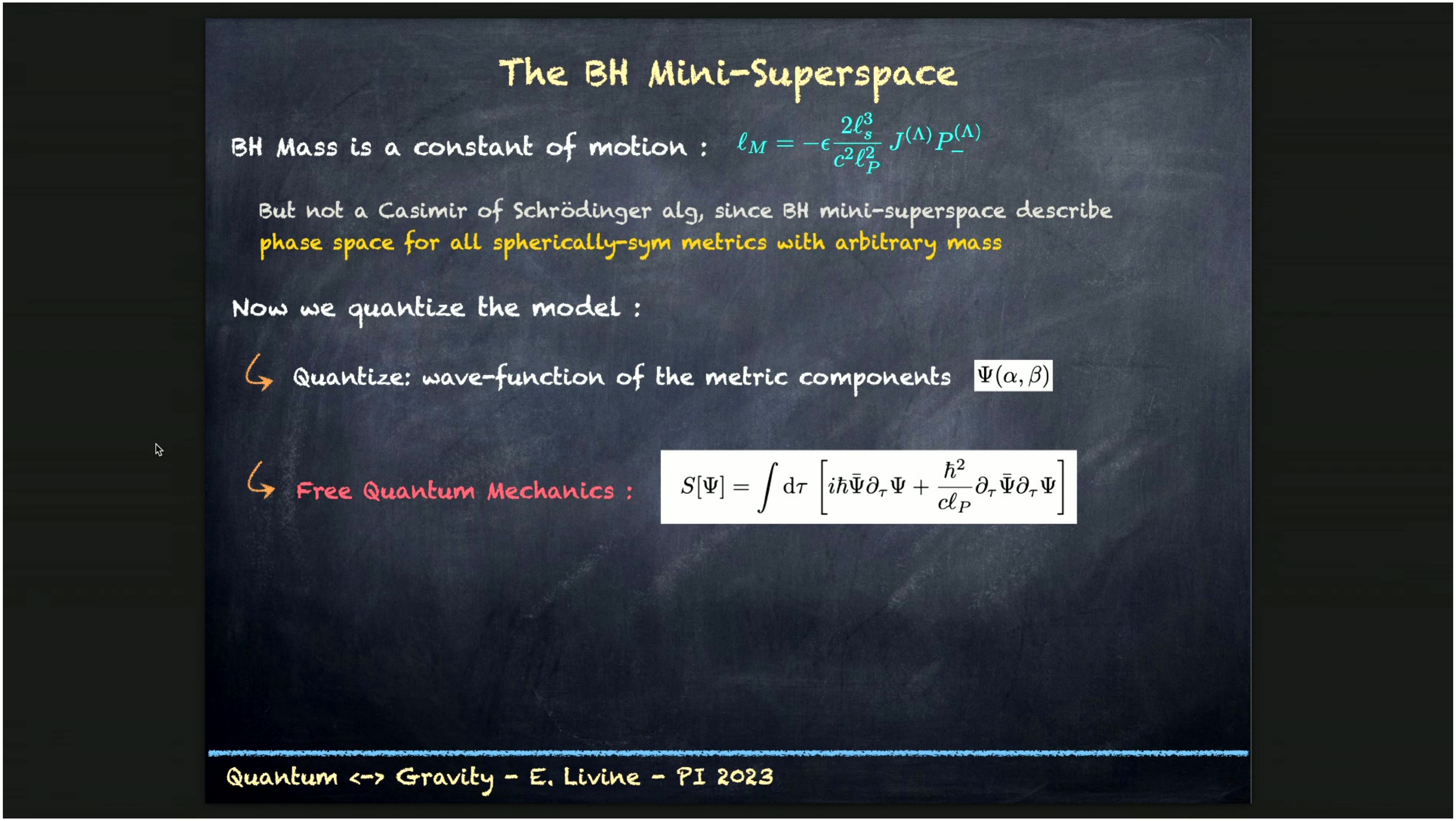
Central charge

$$\begin{aligned} &\{P_-, P_+\} = 0\,, && \{B_-, B_+\} = 0\,, \\ &\{B_\pm, P_\pm\} = 0\,, && \{B_\pm, P_\mp\} = \epsilon\,, \\ &\{J, B_\pm\} = \pm B_\pm\,, && \{J, P_\pm\} = \pm P_\pm\,, \end{aligned}$$

Quantum <-> Gravity - E. Livine - PI 2023



The BH Mini-Superspace

BH Mass is a constant of motion : $\ell_M = -\epsilon \frac{2\ell_s^3}{c^2\ell_P^2} J^{(\Lambda)} P_-^{(\Lambda)}$

But not a Casimir of Schrödinger alg, since BH mini-superspace describe phase space for all spherically-sym metrics with arbitrary mass

Now we quantize the model :

↳ Quantize: wave-function of the metric components $\boxed{\Psi(\alpha,\beta)}$

↳ Free Quantum Mechanics : $$\boxed{S[\Psi] = \int \mathrm{d}\tau \left[i\hbar \bar{\Psi} \partial_\tau \Psi + \frac{\hbar^2}{c\ell_P} \partial_\tau \bar{\Psi} \partial_\tau \Psi \right]}$$

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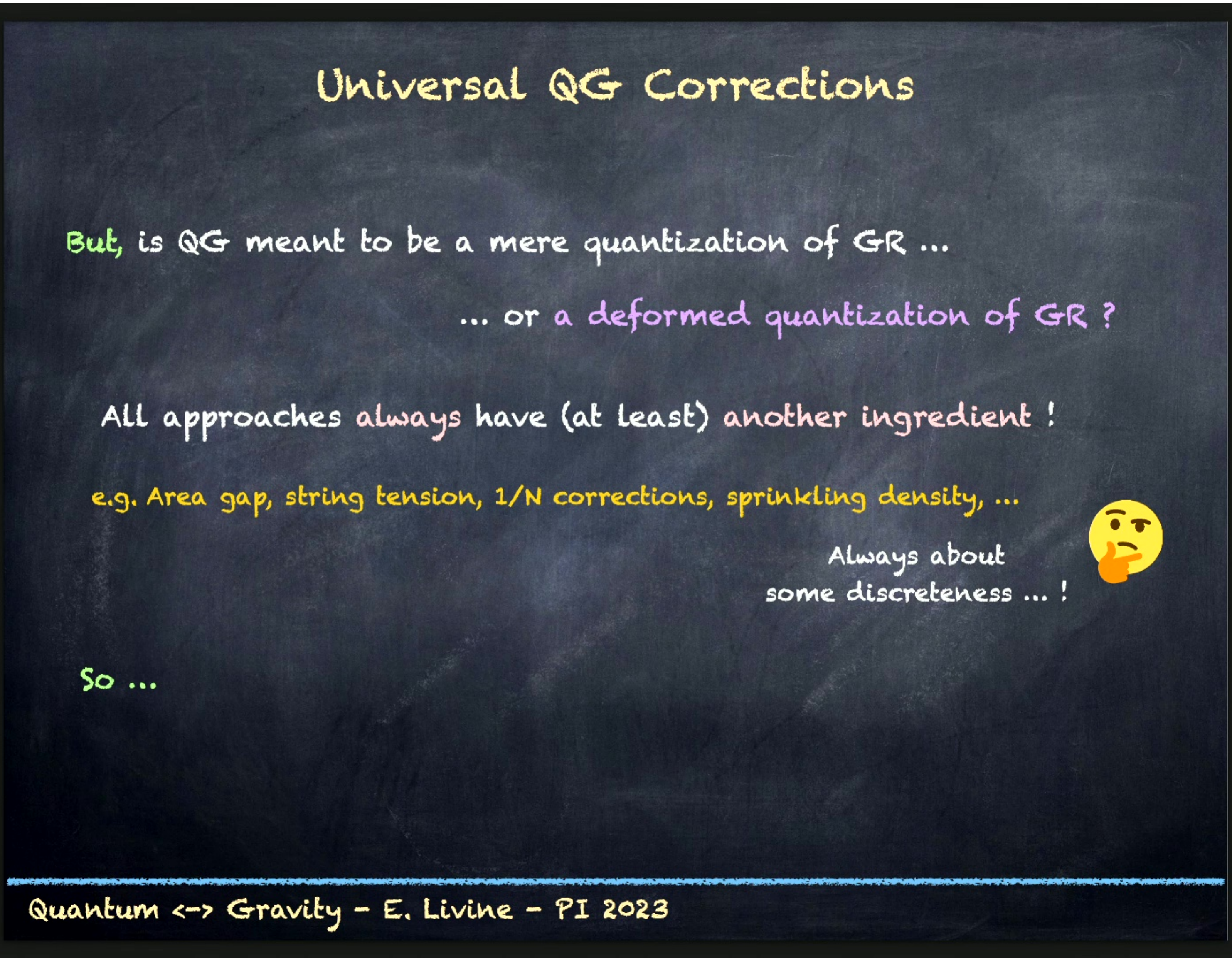

Universal QG Corrections

But, is QG meant to be a mere quantization of GR …

… or a deformed quantization of GR ?

All approaches always have (at least) another ingredient !

e.g. Area gap, string tension, 1/N corrections, sprinkling density, …

Always about
some discreteness … !

So …

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Universal QG correction preserving Schrodinger symmetry $$S[\Psi] = \int \mathrm{d}\tau \left[i\hbar \bar{\Psi} \partial_\tau \Psi + \frac{\hbar^2}{c\ell_P} \partial_\tau \bar{\Psi} \partial_\tau \Psi + \kappa^2 |\Psi|^4 \right]$$

The BH Mini-Superspace

Universal QG correction preserving Schrodinger symmetry

$$S[\Psi] = \int \mathrm{d}\tau \left[i\hbar\bar{\Psi}\partial_\tau\Psi + \frac{\hbar^2}{c\ell_P}\partial_\tau\bar{\Psi}\partial_\tau\Psi + \kappa^2|\Psi|^4 \right]$$

What kind of phenomenology ?

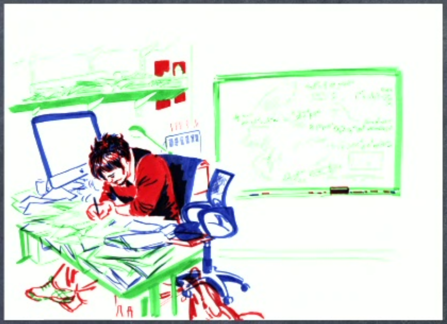
- ↳ Self-interaction of BH wave-function
- ↳ New parameter controls attraction/repulsion of probability peaks !
- ↳ Modulates "evolution" of metric superpositions along radial direction
- ↳ Explores qu -> cl transition in QG
- ↳ New type of analogue QG models for BH

Ben Achour, L, Oriti, [2302.07644]

At the interface Quantum <-> Gravity

What's next ?

More work !



The Quantum
of
Gravity ?

BH-WH
oscillation
accelerant

Non-linear corner
inducer of soft
graviton EPR pairs

Holographic
dissolvant of
spacetime bulk

LQG spin oil
to reduce
information
viscosity

At the interface Quantum <-> Gravity

What's next ?   Questions !