Title: What exactly does Bekenstein Bound?

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Abstract: The Bekenstein bound posits a maximum entropy for matter with finite energy confined to a spacetime region. It is often interpreted as a fundamental limit on the information that can be stored by physical objects. In this work, we test this interpretation by asking whether the Bekenstein bound imposes constraints on a channel's communication capacity, a context in which information can be given a mathematically rigorous and operationally meaningful definition. We first derive a bound on the accessible information and demonstrate that the Bekenstein bound constrains the decoding instead of the encoding. Then we study specifically the Unruh channel that describes a stationary Alice exciting different species of free scalar fields to send information to an accelerating Bob, who is therefore confined to a Rindler wedge and exposed to the noise of Unruh radiation. We show that the classical and quantum capacities of the Unruh channel obey the Bekenstein bound. In contrast, the entanglement-assisted capacity is as large as the input size even at arbitrarily high Unruh temperatures. This reflects that the Bekenstein bound can be violated if we do not properly constrain the decoding operation in accordance with the bound. We further find that the Unruh channel can transmit a significant number of zero-bits, which are communication resources that can be used as minimal substitutes for the classical/quantum bits needed for many primitive information processing protocols, such as dense coding and teleportation. We show that the Unruh channel has a large zero-bit capacity even at high temperatures, which underpins the capacity boost with entanglement assistance and allows Alice and Bob to perform quantum identification. Therefore, unlike classical bits and qubits, zero-bits and their associated information processing capability are not constrained by the Bekenstein bound. (This talk is based on the recent work (https://arxiv.org/abs/2309.07436) with Patrick Hayden.)

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Zoom link: https://pitp.zoom.us/j/98778081764?pwd=WktjNU84R3NWRXNyVmt1eDVMK2JnUT09

# WHAT EXACTLY DOES BEKENSTEIN BOUND?

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QI Seminar @ Perimeter

Based on 2309.07436 w/ Patrick Hayden

# BEKENSTEIN BOUND

- **Information is physical!**
- To make the black hole consistent with the second law of thermodynamics, [Bekenstein '81] argued that the **information** content contained in a box $O$ of size $R$ and energy $E$ is **bounded**. $S \leq \lambda RE$. It is conjectured to be **universally** true in any relativistic quantum field theory (QFT).
- However, it remained unclear how each term shall be defined precisely in QFT.
- [Casini '08] proposed a precise formulation in terms of the **relative entropy**. He observed that for any excited state $\rho$, $S(\rho_O || \Omega_O) = \Delta\langle K_O \rangle_\rho - \Delta S(\rho_O) \geq 0$, $K_O := -\log \Omega_O$ is the modular Hamiltonian. $\Delta$ means subtracting the vacuum entropy/energy $\langle K_W \rangle_\Omega \equiv S(\Omega_W)$. The Bekenstein bound thus follows from the **positivity of relative entropy**.
- E.g. for a Rindler wedge $W$, $K_W = \int dx^\perp \int_0^\infty dx\ x\ T_{00}(x) \sim RE$ .

# AN EXAMPLE

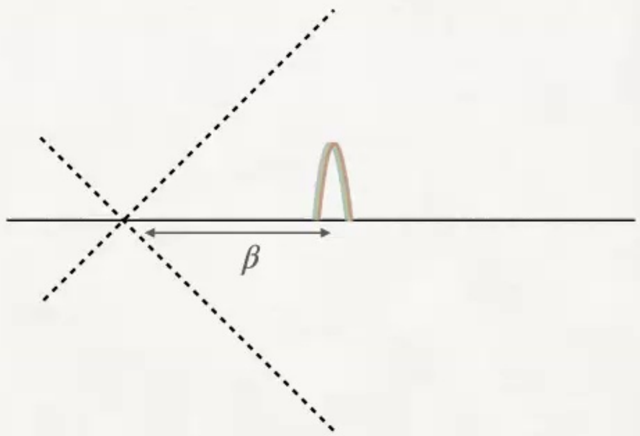
- Consider piling up entropy in a wave packet (of energy ~$E$) in Minkowski space.

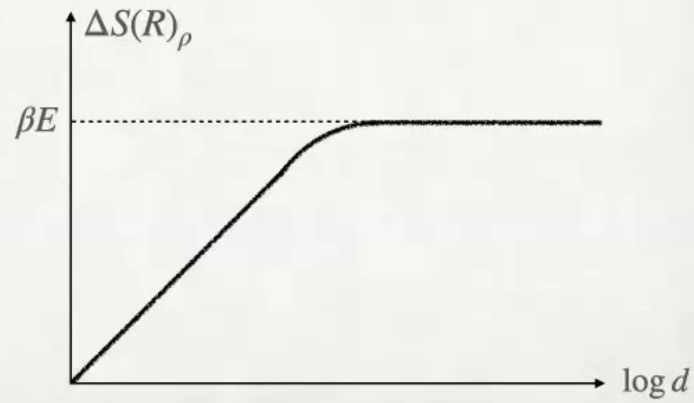
- How? We can add entropy without increasing energy by introducing more **particle species**, (i.e. say by making $d$ copies of free scalar field theory), and then the entropy of an unknown particle excitation, described by $\rho$, grows like $\log d$.

- What if we put this wave packet in a Rindler wedge, $\rho \to \rho_R$?

- [Marolf-Minic-Ross '03] showed that the von Neumann entropy of such a mixed state keeps increasing with $\log d$ until gets **capped off** by the Bekenstein bound $\beta E$ . **No species problem!**

- Large d: Signals carried by particle excitations are **no longer distinguishable** from the background thermal excitations. $S(\rho_R || \Omega_R) \approx 0$. Overlapping 'codewords' leads to entropy saturation.



# A BOUND ON INFORMATION?

- Casini's bound reformulates the Bekenstein Bound as a statement about states **distinguishability**.
- However, it is unclear what kind of **information** is bounded in Casini's version.
- The vN entropy describes the ultimate **compression** rate of an information source, but this operational meaning becomes obscure after **vacuum subtraction**.
- Could there be other versions of the Bekenstein bound that's operationally meaningful and UV finite? Since von Neumann entropy isn't generally well-defined in QFT, we shall resort to other information-theoretic quantities.

# LET'S EXAMINE AN OPERATIONAL TASK!

# REVISITING A BOUND ON THE ACCESSIBLE INFO

- Consider Alice encoding a classical message $X$ by local unitaries $U_x|\Omega\rangle$, that are supported over a spatial region $O$. How much of this information can Bob access from $O$?
- The figure of merit is the **accessible information**, defined as the optimal $I(X:Y)$ over Bob's measurements that yield the outcome $Y$. It is upper bounded by the **Holevo Information**:
- $\chi(\{p_x, \rho_x\}) := \sum_x p_x S(\rho_x || \bar{\rho})$, where $\rho_x$ denotes the reduced state of $U_x|\Omega\rangle$ in region $O$, $\bar{\rho} := \sum_x p_x \rho_x$ .
- [Bousso '16] derived a Bekenstein bound on the Holevo information, which in this scenario reads: $\chi(\{p_x, \rho_x\}) \leq \sum_x p_x S(\rho_x || \Omega_O) = \Delta \langle K_O \rangle_{\bar{\rho}}$ . This bound follows from Casini's bound. One changes the vacuum subtraction $-S(\Omega_O)$ to $-\sum_x p_x S(\rho_x)$. (The Holevo information is **UV-finite**, so no regularization is needed.)
- It's fruitful to continue Bousso's study of the Bekenstein Bound on communication, using the tools from **quantum Shannon theory**. Because **channel capacities**, such as the Holevo Information, are operationally meaningful UV-finite information measures, and thus are potentially better alternatives to the von Neumann entropy in the standard Bekenstein bound.

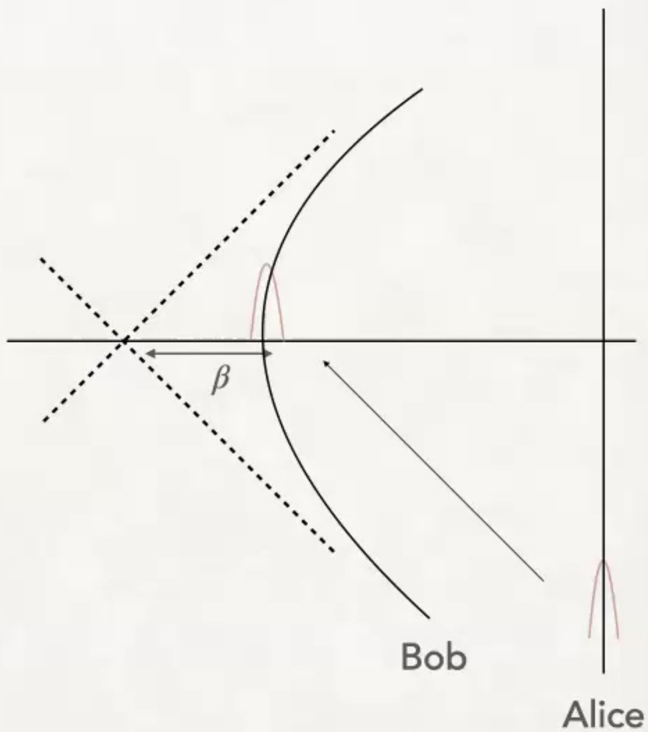
LET'S DO A CASE STUDY IN RINDLER SPACE.

# COMMUNICATION IN RINDLER SPACE

- A stationary Alice sends a wave pulse to an accelerating Bob. How much information can they communicate in this way?

- We model the communication scenario as the ''**Unruh channel**'', which encodes the message as single-particle excitation in an **Unruh mode** of distinct particle species, followed by tracing out the left Rindler wedge, which gives rise to the **noise of Unruh radiation**.

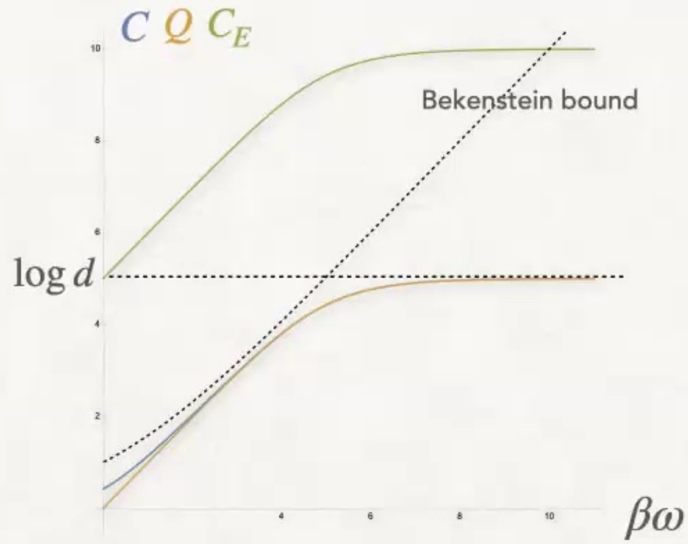
- The Unruh channel is characterized by the input dimension (species number that Alice can access) $d$, the energy/frequency of the mode $\omega$, and Bob's local Unruh temperature $\beta$.

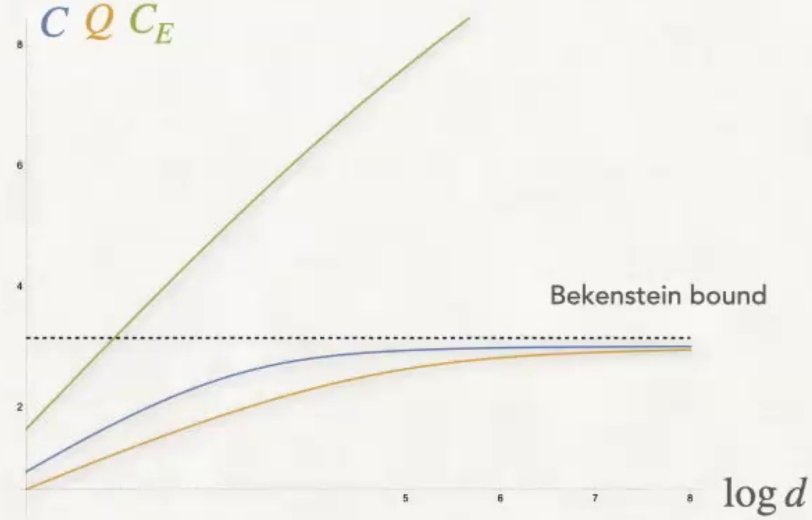


# CHANNEL CAPACITIES

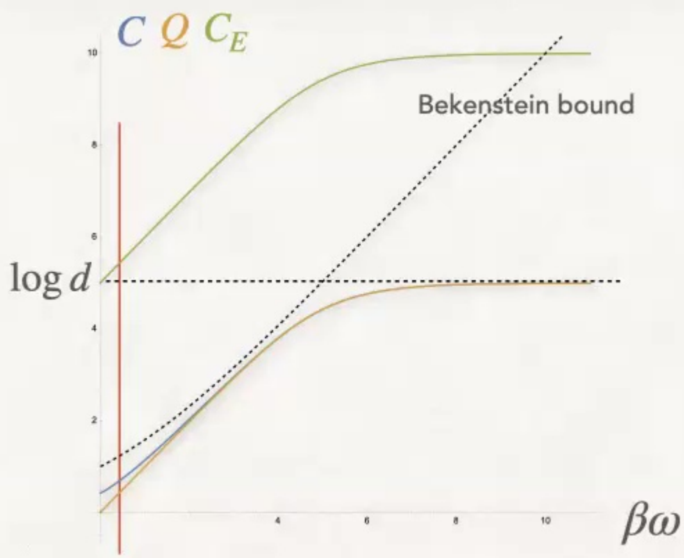
- We are interested in the **classical capacity** $C$, **quantum capacity** $Q$, and **entanglement-assisted classical capacity** $C_E$ .
- They are defined as the optimal rates to transmit classical bits (cbits), qubits, and cbits with the help of free entanglement over i.i.d. uses of the channel.
- The Unruh channel conveniently has **additive** capacities, i.e., all of these capacities admit **single-letter formulas**. They are given by the Holevo information, coherent information, and mutual information respectively.
- The maximization is also straightforward to work out in the evaluating them.

# RESULTS

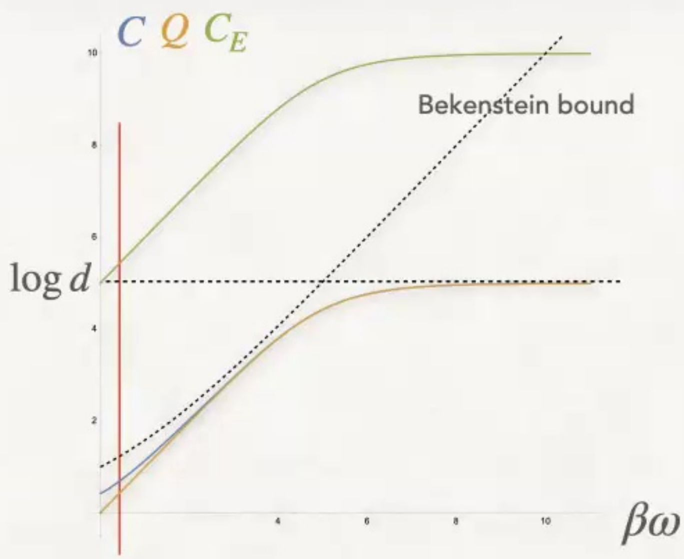



- Qubits and cbits transmitted are constrained by the Bekenstein bound.
- The entanglement-assisted capacity, which is at least $\log d$, doesn't obey the bound. This is because it uses free Bell pairs stored in an auxiliary system, which the bound doesn't allow Bob to use in his decoding.




- At high temperature, however, the fact they can communicate with a seemingly useless noisy channel is still surprising, even if there is entanglement assistance! (Extensive separation: $0 < C, Q < 1, C_E > \log d$). What's the reason for such a **capacity boost**? Since Bell pairs alone cannot be used for communication, the Unruh channel still transmits quantum information even at high temperatures.

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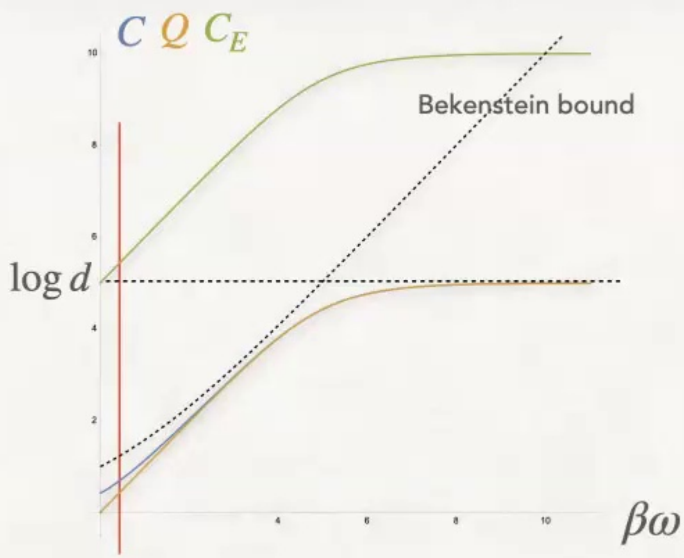
- The standard dense coding, **1 qubit+1 ebit > 2 cbits,** is not helpful here for the lack of qubits.

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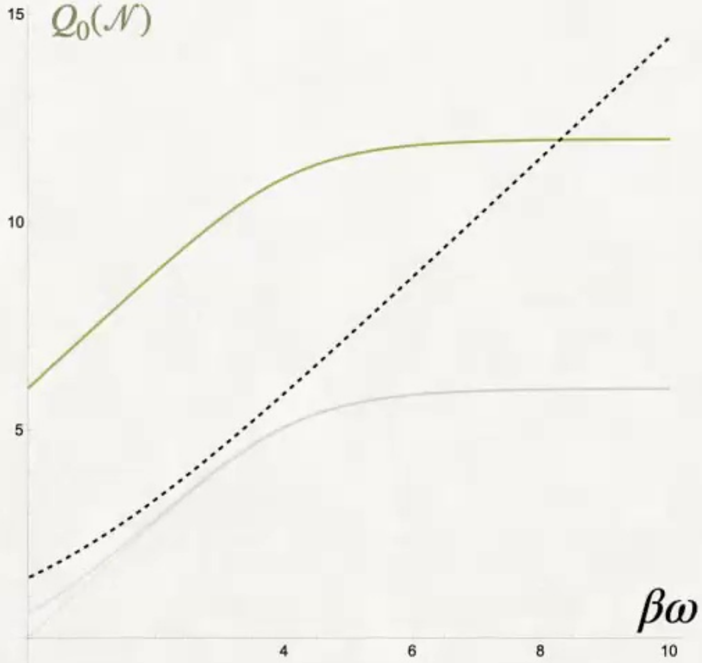
- The standard dense coding, **1 qubit+1 ebit > 2 cbits,** is not helpful here for the lack of qubits.
- Unless perhaps one can use a weaker version: **1 zero-bit+1 ebit > 1 cbit ?**

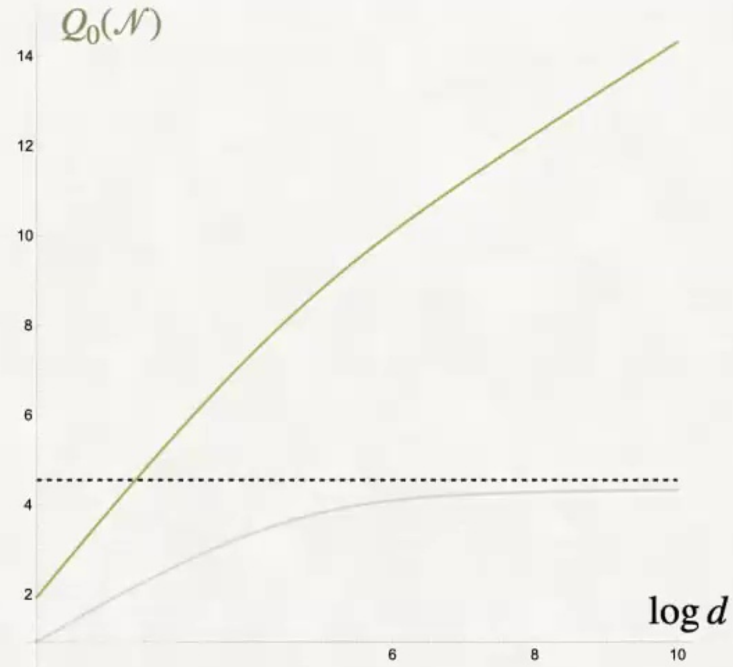
# ZERO-BITS

- Roughly, we say a channel transmits $d$ zero-bits if any **two-dimensional subspace** of a $2^d$ dimensional message space can be decoded.
- [Hayden-Winter '10] proposed the following QI task: Alice wants Bob to simulate any binary measurement on her $d$-dim input state at the output end. An $(d, \epsilon)$-QID code is $(\mathscr{E}, D)$ such that: $\forall \psi \in \mathscr{H}_A, \forall \{ |\phi\rangle\langle\phi|, 1 - |\phi\rangle\langle\phi| \}, \exists \{D_\phi, 1 - D_\phi\}, st. |\mathrm{Tr} D_\phi \mathscr{N}(\psi) - \langle\phi|\psi|\phi\rangle| < \epsilon.$
- **Operational definition of zero-bits**: we say a channel $\mathscr{N}$ transmits $\log d$ zero-bits if for any $\epsilon > 0$, there exists a $(d^n, \epsilon)$-QID code for Alice and Bob to perform **quantum identification** with $\mathscr{N}^{\otimes n}$ at large enough $n$.
- This task doesn't involve other communication resources, and it is certainly quantum!
- A zero-bit is like a tiny fraction of a qubit, less powerful but more robust against noise. Operationally, they can do quantum identification, dense coding, teleportation, and more.

# ZERO-BIT CAPACITY

It turns out that the Unruh channel always transmits a considerable ( $> \log d$) amount of **zero-bits!**



NO BEKENSTEIN BOUND FOR ZERO-BITS!

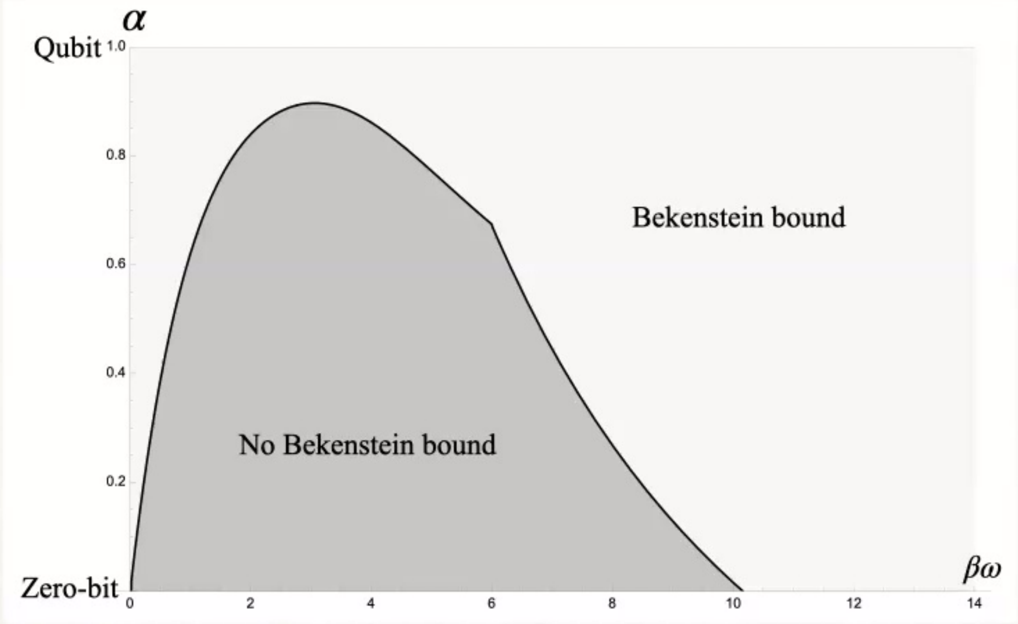
# NO BEKENSTEIN BOUND FOR $\alpha$-BITS EITHER

- [Hayden-Penington '17] generalized zero-bits to $\alpha$-bits ($0 \leq \alpha \leq 1$) that interpolate between zero-bits ($\alpha = 0$) and qubits ($\alpha = 1$).

- We say a channel transmits $d$ $\alpha$-bits if any $2^{\alpha d}$-dim **subspace** of a $2^d$ dimensional message space can be decoded.

- Sending an $\alpha$-bit means to make good use of an $\alpha$-fraction of a noisy qubit.

- It turns out they are not constrained by the Bekenstein bound either.



# WHAT EXACTLY DOES BEKENSTEIN BOUND?

CBITS & QUBITS (PERHAPS)