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On the 4D SCFTs/VOAs correspondence

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$\mathcal{N}=2$ 4D SCFTs: algebra

We are interested in $\mathcal{N}=2$ 4D SCFTs from viewpoint of their OPE algebra,

$$\mathcal{O}_i(x)\mathcal{O}_j(y)=\sum_k \frac{c_{ij}^k}{|x-y|^{E_i+E_j-E_k}}\,\mathcal{O}_k(y)\,,\qquad x\,,y\in\mathbb{R}^4$$

$\{\mathcal{O}_i(x)\}$ can be organized in representation of the $\mathfrak{su}(2,2|2)$ superalgebra.

Bosonic subalgebra: $\mathfrak{su}(2,2)\times\mathfrak{su}(2)_R\times\mathfrak{u}(1)_r$.
$\mathfrak{su}(2,2)\cong\mathfrak{so}(4,2)$: conformal algebra in $D=4$,
$\mathfrak{su}(2)_R\times\mathfrak{u}(1)_r$ R-symmetry

Label operators by their Cartan quantum numbers, $\mathcal{O}_{E,R,r,(j_1,j_2)}$.

4d $\mathcal{N} = 2$ SCFTs: geometry

Two canonical varieties of supersymmetric vacua:

- Coulomb branch: $\mathcal{M}_C$ $SU(2)_R$ preserved, $\cancel{U(1)_r}$ broken.
- Higgs branch: $\mathcal{M}_H$ $\cancel{SU(2)_R}$ broken, $U(1)_r$ preserved.

Higgs geometry will be the one more relevant for this talk.

Geometry of the Higgs branch

In an $\mathcal{N} = 2$ SCFT, $\mathcal{M}_H$ is a hyperkähler cone.

In a Lagrangian theory $\mathcal{M}_H$ is given by the hyperkähler quotient:

$$\mathcal{M}_H = \{Q^i \tilde{Q}_j T_i^{A\,j} = 0\}/G_{\mathbb{C}}$$

More intrinsic characterization of $\mathcal{M}_H$ in terms of vevs of chiral operators.

Higgs chiral ring $\mathcal{R}_H$:

Scalar operators with $E = 2R$, $r = 0$, where E is the conformal dimension.
Their OPE is non-singular, so they form a commutative ring.

In all (superconformal) examples, $\mathcal{R}_H$ is a *reduced* commutative $\mathbb{C}$-algebra. (*Reduced* $\equiv$ no nilpotents).

One identifies $\mathcal{R}_H$ with the coordinate ring of $\mathcal{M}_H$,

$$\mathcal{R}_H \cong \mathbb{C}[\mathcal{M}_H] .$$

$\mathcal{M}_H$ is a holomorphic variety, equipped with a symplectic form Ω.

$\mathcal{R}_H = \mathbb{C}[\mathcal{M}_H]$ is the ring of holomorphic functions on $\mathcal{M}_H$, endowed with the Poisson bracket

$$\{f, g\} = (\Omega^{-1})^{ij} \partial_i f \partial_j g$$

R-grading defines a C^* action.

Generators with $R = 1$ correspond to global symmetries (moment maps).

Simple examples

- $\mathcal{M}_H = \mathbb{C}^2$
  $\mathcal{R}_H = \mathbb{C}[q, \tilde{q}]$
  Bracket: $\{q, \tilde{q}\} = 1$
  $R(q) = R(\tilde{q}) = \frac{1}{2}$

- $\mathcal{M}_H = \mathbb{C}^2/\mathbb{Z}_2$
  $\mathcal{R}_H = \mathbb{C}[\jmath^1, \jmath^2, \jmath^3]/\langle \jmath^A \jmath^A \rangle$
  Bracket: $\{\jmath^A, \jmath^B\} = \epsilon^{ABC} \jmath^C$
  $R(\jmath^A) = 1 \Rightarrow \mathfrak{sl}(2)$ symmetry

- $\mathcal{M}_H = \mathbb{C}^2/\mathbb{Z}_3$
  $\mathcal{R}_H = \mathbb{C}[\jmath, X, Y]/\langle XY - \jmath^3 \rangle$
  Bracket: $\{\jmath, X\} = X$, $\{\jmath, Y\} = -Y$, $\{Y, X\} = 3\jmath^2$
  $R(\jmath) = 1 \Rightarrow \mathfrak{u}(1)$ symmetry, $R(X) = R(Y) = \frac{3}{2}$

SCFT/VOA correspondence

For generic operators, the 4D OPE algebra is very complicated.

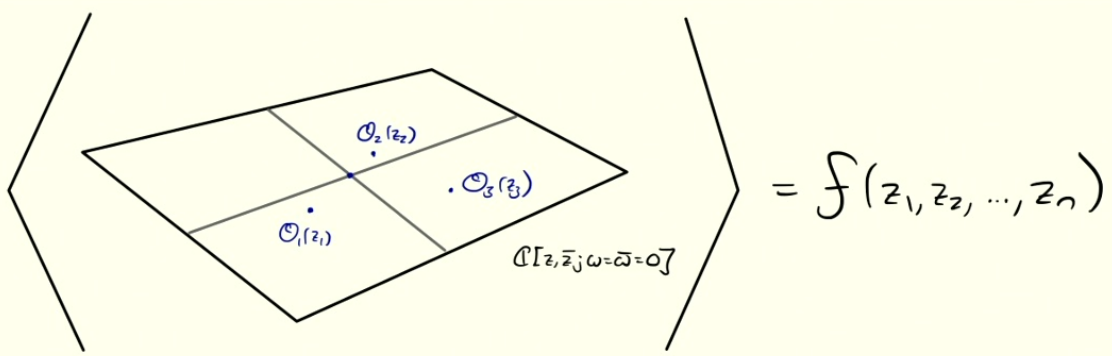
One can however carve out *closed subalgebras* of operators in shortened representations. I will describe one of the richest.

To any 4D $\mathcal{N} = 2$ SCFT, one canonically associates a Vertex Operator Algebra,

$$\chi : \ \ \text{4D} \ \ \mathcal{N} = 2 \ \ \text{SCFT} \longrightarrow \text{VOA}$$

Beem Lemos Liendo Peelaers LR van Rees

VOA obtained by passing to the cohomology of a certain nilpotent $\mathbb{Q} \sim \mathcal{Q} + \mathcal{S}$



$\mathcal{V}$ = vector space of Schur operators

As a vector space, $\mathcal{V} \cong$ "Schur operators" of the SCFT, defined by

$$E = 2R + \ell \quad \text{with} \;\; \ell = j_1 + j_2$$

$$(\Rightarrow \quad r = j_2 - j_1 \quad \text{for unitary multiplets})$$

Schur operators are thus labelled by three quantum numbers (R, ℓ, r).

4D Schur operator $\mathcal{O}_{R,\ell,r} \longrightarrow$ VOA operator $\varphi_{h(R,r)}$, where

$$h = R + \ell\,.$$

is the holomorphic dimension of φ.

- r *is* conserved in OPE. Today, mostly focus on theories where $r \equiv 0$.
- R *not* conserved in OPE, but can at most decrease, so we have a *filtration*.
  Important fact: VOAs that descend from 4D SCFTs inherit R-filtration.
  [But if one is handed an abstract presentation of $\mathcal{V}$ without the χ map from 4D, only h is manifest and not obvious how to assign R-filtration].

Structural properties

Let $\mathcal{V} = \chi[\mathcal{A}]$ for some 4D $\mathcal{N} = 2$ SCFT $\mathcal{A}$

- Both Bose and Fermi statistics are allowed, $\mathcal{V}$ *super* VOA.
- $\mathcal{V}$ always simple quotient.
- $\mathcal{V}$ always a conformal vertex algebra, i.e. there is a stress-tensor T with $c_{2d} = -12\, c_{4d}$, where c_{4d} is the Weyl^2 central charge of $\mathcal{T}$.
- $\mathcal{V}$ is either $\mathbb{Z}$ graded or $\mathbb{Z}/2$ graded by L_0, but no spin-statistics relation.
- Vacuum character $\mathrm{STr}_{\mathcal{V}}(q^{L_0 - \frac{c_{2d}}{24}})$ = Schur index of $\mathcal{A}$

- $\mathcal{V}$ (believed to be) finitely strongly generated.
- Generators of $\mathcal{R}_H[\mathcal{A}] \to$ strong generators of $\mathcal{V}$.
- In general, $\mathcal{V}$ has additional strong generators, notably stress tensor T (conformal vector), and often more.

  [Exception: $\mathcal{V}$ is just affine Kac-Moody and $T =$ Segal-Sugawara.]
- Nilpotent Higgsing of $\mathcal{A} =$ DS reduction of $\mathcal{V}$.
- Conformal gauging of a global symmetry of $\mathcal{A} =$ chiral analog of the holomorphic symplectic quotient.
- "Hidden" R-filtration. More later.
- 4D unitarity. More later.

Two general questions

Remarkably, it appears that $\chi[\mathcal{A}]$ is a *perfect invariant* of $\mathcal{A}$!

In other terms the map $\chi : 4D\ \mathcal{N} = 2$ SCFTs $\rightarrow$ VOAs is injective.

It is clearly not surjective, VOAs that descend from 4D are very special.

1. Can one fully axiomatize the subcategory of VOAs in the image of χ?
2. Are there *even simpler* algebraic invariants that fully characterize $\mathcal{A}$?

The goal is to reduce the classification problem of 4D $\mathcal{N} = 2$ SCFTs to the classification of much simpler and well-defined mathematical structures.

The associated scheme $\widetilde{X}_{\mathcal{V}}$ and the associated variety $X_{\mathcal{V}}$

There is a natural commutative algebra associated to $\mathcal{V}$, called $\mathcal{R}_{\mathcal{V}}$.

$$\mathcal{R}_{\mathcal{V}} := \mathcal{V}/C_2[\mathcal{V}]\,, \quad C_2[\mathcal{V}] := \mathrm{Span}\{a_{-h_a-1}b\,,\ a\,,b \in \mathcal{V}\}\,.$$

As a vector space, $\mathcal{R}_{\mathcal{V}}$ is obtained from $\mathcal{V}$ by just *removing* all operators containing holomorphic derivatives. So

$$\mathcal{R}_{\mathcal{V}} = \mathbb{C}[s_1, \ldots, s_n]/\langle \mathrm{nulls} \rangle\,,$$

with s_i a formal variable, image of the strong generator $S_i(z)$.

Normal ordering descends to a commutative product on $\mathcal{R}_{\mathcal{V}}$.
$\mathcal{R}_{\mathcal{V}}$ also endowed with a Poisson structure $\{,\}$ induced by the VOA $\{,\}$.

In general, $\mathcal{R}_{\mathcal{V}}$ is not *reduced*
One defines [Arakawa]

$$\widetilde{X}_{\mathcal{V}} := \mathrm{Spec}\,\mathcal{R}_{\mathcal{V}}\,, \quad X_{\mathcal{V}} := \mathrm{Spec}\,(\mathcal{R}_{\mathcal{V}})_{\mathrm{red}}\,.$$

4D $\mathcal{N}=2$ SCFT $\mathcal{T} \Longrightarrow$ VOA $\mathcal{V} \Longrightarrow$ scheme $\widetilde{X}_{\mathcal{V}}$.

If we *remove* nilpotents, we find the ordinary variety $X_{\mathcal{V}}$.

Our central conjecture: Higgs branch reconstruction [Beem LR 2017]

$$\mathcal{M}_H = X_{\mathcal{V}} := \mathrm{Spec}\,(\mathcal{R}_{\mathcal{V}})_{\mathrm{red}}$$

Higgs branch $=$ Associated variety

[Equivalently, $\mathcal{R}_H = (\mathcal{R}_{\mathcal{V}})_{\mathrm{red}}$]

In our context, we call $\widetilde{X}_{\mathcal{V}}$ the Higgs scheme:
natural extension of the standard Higgs variety $X_{\mathcal{V}}$ by nilpotents.

Many examples of different SCFTs with the same $\mathcal{M}_H \cong X_{\mathcal{V}}$.
E.g., infinitely many SCFTs with *trivial* $\mathcal{M}_H$. But experimentally, no two different SCFTs have the same Higgs scheme $\widetilde{X}_{\mathcal{V}}$.

Conjecture: [Beem Martone Mogol LR, in progress]
The Higgs scheme is a perfect invariant of a 4D $\mathcal{N}=2$ SCFT.

Lisse and quasi-lisse VOAs and 4d SCFTs

If $X_{\mathcal{V}} = \mathrm{pt.}$, $\mathcal{V}$ is called lisse. Believed to be necessary condition for rationality.

If $X_{\mathcal{V}}$ has a finite number of symplectic leaves, $\mathcal{V}$ is called quasi-lisse.

Quasi-lisse VOAs are very interesting generalizations of rational VOAs.
In particular, they enjoy modular properties. [Arakawa Kawasetsu]

Physics translation:
Assuming the Higgs branch reconstruction conjecture, *all* VOAs arising from 4d SCFTs are quasi-lisse (as the Higgs branch is a symplectic singularity).
The subset of SCFTs with *no* Higgs branch give rise to lisse VOAs.

Striking consequence: the Schur index (= vacuum character) satisfies a finite-order monic modular differential equation, which morally arises from nilpotency of T in $\mathcal{R}_{\mathcal{V}}$. [Beem LR]

Any quasi-lisse VOAs has rational central charge. [LR Rayhaun]
It follows that $c_{4d} = -12 c_{2d}$ is rational for any $\mathcal{N} = 2$ SCFT.
With a bit more work, one can also argue that a_{4d} is rational

Examples: trivial Higgs branch, I

$\mathrm{AD}_{(A_1,A_{2n-2})}$ theories: $\mathcal{M}_H = \mathrm{pt}.$

$\chi[\mathrm{AD}_{(A_1,A_{2n-2})}] = \mathrm{Vir}_{2,2n+1}.$

Single generator $T(z)$ of $\mathcal{V} \to$ single generator t of $\mathcal{R}_{\mathcal{V}}$.

Primitive null $T^n + \partial T(\dots)$ induces relation $t^n = 0$ in $\mathcal{R}_{\mathcal{V}}$.

$\mathcal{R}_{\mathcal{V}} = \mathbb{C}[t]/\langle t^n \rangle$, hence $(\mathcal{R}_{\mathcal{V}})_{\mathrm{red}} = \mathbb{C}$ and $X_{\mathcal{V}} = \mathrm{pt}.$

Examples: trivial Higgs branch, II

More generally, special sequences of W MMs are related to 4d SCFTs,

$\chi[\mathrm{AD}_{(A_{p-1},A_{N-1})}] = \mathcal{W}_p(p, p+N)$, with $(p, N) = 1$ [Cordova Shao]

For all these theories, $\mathcal{M}_H = \mathrm{pt}$.

- $\chi[\mathrm{AD}_{(A_2,A_3)}] = \mathcal{W}_3(3,7)$
  $\mathcal{R}_\mathcal{V} = \mathbb{C}[t,w]/\langle tw, t^3 + w^2\rangle$, $(\mathcal{R}_\mathcal{V})_{\mathrm{red}} = \mathbb{C}$ and $X_\mathcal{V} = \mathrm{pt}$.

- $\chi[\mathrm{AD}_{(A_2,A_4)}] = \mathcal{W}_3(3,8)$
  $\mathcal{R}_\mathcal{V} = \mathbb{C}[t,w]/\langle t^2w, t^3 + w^2\rangle$, $(\mathcal{R}_\mathcal{V})_{\mathrm{red}} = \mathbb{C}$ and $X_\mathcal{V} = \mathrm{pt}$.

- $\dots$

Note that $\mathcal{W}_p(p, p+N)$ arises from DS reduction of $\mathfrak{sl}(p)_{-p+\frac{p}{p+N}}$, which are the boundary admissible levels.

Examples: $\mathcal{M}_H = \mathbb{C}^2/\mathbb{Z}_2$

$\mathrm{AD}_{(A_1,D_{2n+1})}$ theories: $\mathcal{M}_H = \mathbb{C}^2/\mathbb{Z}_2$.

$\chi[\mathrm{AD}_{(A_1,D_{2n+1})}] = \mathfrak{sl}(2)_{\frac{-4n}{2n+1}}$.

$\mathcal{R}_{\mathcal{V}} = \mathbb{C}[\jmath_1, \jmath_2, \jmath_3]/\langle \mathrm{nulls} \rangle = \mathbb{C}[\jmath^1, \jmath^2, \jmath^3]/\langle \jmath^A(\jmath_1^2 + \jmath_2^2 + \jmath_3^2)^n \rangle$.

Note that $(\jmath_1^2 + \jmath_2^2 + \jmath_3^2)^{n+1} = 0$, so $\mathcal{R}_{\mathcal{V}}$ is *not* reduced.

Clearly $(\mathcal{R}_{\mathcal{V}})_{\mathrm{red}} = \mathbb{C}[\jmath_1, \jmath_2, \jmath_3]/\langle \jmath_1^2 + \jmath_2^2 + \jmath_3^2 \rangle \Rightarrow X_{\mathcal{V}} = \mathbb{C}^2/\mathbb{Z}_2$.

Can one reconstruct $\mathcal{V}$ from $\mathcal{R}_{\mathcal{V}}$?

In general, no. The map $\mathcal{V} \to \mathcal{R}_{\mathcal{V}}$ is many-to-one.
E.g., both $\mathrm{Vir}_{(2,7)}$ and $\mathrm{Vir}_{(3,4)}$ have a null of the form $T^3 + \partial(\dots)$.
For both, $\mathcal{R}_{\mathcal{V}} = \mathbb{C}[t]/\langle t^3 \rangle$.

In some special cases however, $\mathcal{R}_{\mathcal{V}}$ suffices to reconstruct not quite $\mathcal{V}$, but its commutative limit.

Every VOA is endowed with a canonical filtration (Li filtration).
Define $F^n(\mathcal{V}) \subset \mathcal{V}$ as the subspace spanned by monomials with *at least* n holomorphic derivatives. Clearly $F^n(\mathcal{V}) \supset F^{n+1}(\mathcal{V})$. The associated graded

$$\mathrm{gr}_{\partial}\mathcal{V} := \oplus_n F^n(\mathcal{V})/F^{n+1}(\mathcal{V})$$

is an ∞-dim. commutative algebra (in fact a vertex Poisson algebra (VPA)).

A simple way to construct a vertex Poisson algebra is to start with a finite dimensional Poisson algebra A and add generators corresponding to formal derivatives, subject to the original relations and their derivatives.

The resulting VPA is called $J_\infty(A)$. E.g.

$$A = \mathbb{C}[t]/\langle t^n \rangle, \quad J_\infty(A) = \mathbb{C}[t, \partial t, \partial^2 t, \dots]/\langle t^n, \partial(t^n), \partial^2(t^n), \dots \rangle .$$

It is a general theorem that there is a *surjective* morphism

$$J_\infty(\mathcal{R}_\mathcal{V}) \twoheadrightarrow \mathrm{gr}_\partial \mathcal{V} .$$

If the kernel is zero, i.e. $J_\infty(\mathcal{R}_\mathcal{V}) \cong \mathrm{gr}_\partial \mathcal{V}$, the VOA is called *classically free*.
[van Ekeren Heluani]

Classically free VOAs are rather special.

Remarkably, in all cases that we have checked so far,
VOAs that descend from 4D SCFTs appear to be classically free!

- $\mathrm{Vir}_{(p,q)}$ is classically free *if and only if* $(p,q) = (2, 2n+1)$.
  [van Ekeren Heluani]

  Recall $\chi[\mathrm{AD}_{(A_1,A_{2n-2})}] = \mathrm{Vir}_{2,2n+1}$

- $\mathfrak{sl}(2)_k$ at the *boundary* admissible levels $k = -\frac{4n}{2n+1}$ are expected (by mathematicians) to be classically free.
  Recall $\chi[\mathrm{AD}_{(A_1,D_{2n+1})}] = \mathfrak{sl}(2)_{\frac{-4n}{2n+1}}$.
  Remark: $\mathrm{DS}[\mathfrak{sl}(2)_{-4n/(2n+1)}] = \mathrm{Vir}_{2,2n+1}$.

- We have also checked in examples (by level-by-level evaluation of characters) that the W_3 MMs related to 4d SCFTs are classically free. (They correspond to DS reductions of $\mathfrak{sl}(3)$ at boundary admissible levels.)

- Also, "near failures" where this could have been falsified but is not. E.g., $H_0^{(2)}$ theory.

Tempting to speculate that classical freeness is a general property.

If so, we predict a huge list of non-trivial examples of classically free VOAs. Some notable cases:

- Deligne series: affine Kac-Moody $\hat{g}_k$ with $\mathfrak{g} \in \{A_1, A_2, G_2, D_4, F_4, E_6, E_7, E_8\}$ and $k = -\frac{h^\vee}{6} - 1$.
- Class $\mathcal{S}$ VOAs
- A plethora of Argyres-Douglas theories.

Free field constructions from Higgs branch EFT

Experimentally, VOAs associated to 4d SCFTs admit free field realizations the mimic the 4d EFT on the Higgs branch. [Beem Meneghelli LR]

The VOA is recovered from the Higgs geometry, decorated by the data of the EFT $\mathcal{T}_{\mathrm{IR}}$ leaving at a generic point.

The simplest examples are again the Argyres-Douglas theories $\mathrm{AD}_{(A_1,D_{2n+1})}$.

$$\chi[\mathrm{AD}_{(A_1,D_{2n+1})}] = \mathfrak{sl}(2)_{\frac{-4n}{2n+1}}$$

For each n,

$$\mathcal{M}_{\mathrm{Higgs}} = X_{\mathcal{V}} = \mathbb{C}^2/\mathbb{Z}_2\,.$$

The EFT comprises one free hyper $\oplus$ the interacting SCFT $\mathrm{AD}_{(A_1,A_{2n-2})}$. Recall that $\chi[\mathrm{AD}_{(A_1,A_{2n-2})}] = \mathrm{Vir}_{2,2n+1}$.

There is a free field construction of $\mathfrak{sl}(2)_{\frac{-4n}{2n+1}}$ in terms of free bosons δ, φ $\oplus$ an additional "abstract" $\mathrm{Vir}_{2,2n+1}$.

Many generalizations, including to all rank-one SCFTs.
[Beem Martone Meneghelli LR, in progress]

Summary so far

The image of χ obeys a set rigorous structural axioms and some conjectural experimental properties.

Three striking experimental facts:

- Quasi-lisse property (thoroughly checked)
- Free-field realizations that mimic the Higgs branch EFT (many examples)
- Classical freeness (more tentative).

Can one deduce these conjectural properties from the axioms?

4D unitarity must be crucial.

R-filtration

[Focus for simplicity on theories where $r \equiv 0$]

$\mathcal{V} = \bigoplus_{h=0}^{\infty} \bigoplus_{R=0}^{h} \mathcal{V}_{h,R}$

$\mathcal{V}_{0,0} = \mathbb{C}$ (vacuum) and $\mathcal{V}_{k,0} = \varnothing$ for $k > 0$
$\mathcal{V}_{1,1} = \{J^A\}$ (affine currents)
$\tilde{\mathcal{V}}_{2,1} = \{T\}$ (stress tensor)
$\tilde{\mathcal{V}}_{n,1} = \varnothing$ (absence of 4D higher-spin currents)
$\bigoplus_{R=1}^{\infty} \tilde{\mathcal{V}}_{R,R} = \mathcal{R}_H = \mathbb{C}[\mathcal{M}]_H$ (Higgs chiral ring)

($\tilde{\mathcal{V}}_{h,R} \equiv$ restriction of $\mathcal{V}_{h,R}$ to quasi-primaries)

The OPE is *short*:

$a \in \mathcal{V}_{h_a,R_a}$, $b \in \mathcal{V}_{h_b,R_b} \Longrightarrow a_{-h_a} b \in \bigoplus_{k=0}^{|R_a-R_b|} \mathcal{V}_{h_a+h_b,R_a+R_b-k}$
Reflection of the fact that R is the Cartan of $SU(2)_R$.

Experimentally, R-filtration naturally encoded in the free-field constructions! Free fields keeps track of what counts as "geometry" (the Higgs chiral ring operators, $E = 2R$) and what doesn't.

4D unitarity

Beem, Ardehali Beem Lemos LR (to appear)

4D CTP $\Rightarrow$ an *antilinear* automorphism σ of $\mathcal{V}$,
$\sigma : \mathcal{V}_{h,R} \to \mathcal{V}_{h,R}$
$\sigma^2 = (-1)^{2R}$.

Positivity $(-1)^{h-R} z^{2h} \langle a(z)\, \sigma(a(0)) \rangle \geqslant 0$ for $a \in \mathcal{V}_{h,R}$.

For stress tensor $\sigma(T) = T$, $h = 2$, $R = 1$ so $\langle TT \rangle \sim c_{2d} < 0$

Our expectation is that unitarity (possibly supplemented by 4D Lorentzian analytic properties) is extremely constraining.

E.g., if $\mathcal{V} = \mathrm{Vir}_c$, we believe that $c = c_{2,2n+1}$ are the only allowed possibilities. We proved it under a natural assumption about the R-filtration of composites.

Emerging picture

The VOAs associated to 4D SCFTs can be "built" in terms of "irreducible" lisse blocks $\oplus$ free fields.

This structure should arise in full generality due to the existence of an R-filtration and the constraints of 4D unitarity.

Even the irreducible lisse blocks cannot be arbitrary, because 4D unitarity is very restrictive.

Classification program?

Conclusions

The category of generic VOAs is unwieldy.

VOAs that arise from 4D SCFTs by our map χ are however very special.

For example, they are (conjecturally, but with high degree of confidence) quasi-lisse and (much more tentatively) classically free.

In the fullness of time, axiomatizing all such special properties may lead to a full-fledged classification program.

Hunch: R-filtration and 4D unitarity are extremely constraining.

The rich physics of 4D SCFTs is related to some deep mathematics!