Title: Form Factors - a unifying language for Quantum Gravity

Speakers: Benjamin Knorr

Collection: Puzzles in the Quantum Gravity Landscape: viewpoints from different approaches

Date: October 24, 2023 - 11:50 AM

URL: https://pirsa.org/23100087

Abstract: QFT's remarkable success lies in its ability to yield precise predictions through e.g. scattering amplitudes, capturing the essence of running couplings in a gauge-invariant manner. However, the introduction of gravity disrupts this straightforward notion of momentum dependence, posing a significant challenge to our theoretical framework.

Enter form factors--an ingenious concept that extends the idea of momentum dependence to the curved spacetime of quantum gravity. They form the central ingredient when discussing quantum gravity in a QFT language. Furthermore, within an effective action, form factors offer a unifying language, allowing us to compare and contrast different quantum gravity approaches apples-to-apples.

I will give an overview of the ideas underlying form factors, their application to scattering problems, and their potential to put different approaches on the same footing.

Form Factors - a unifying language for Quantum Gravity

Benjamin Knorr

NORDITA
The Nordic Institute for Theoretical Physics

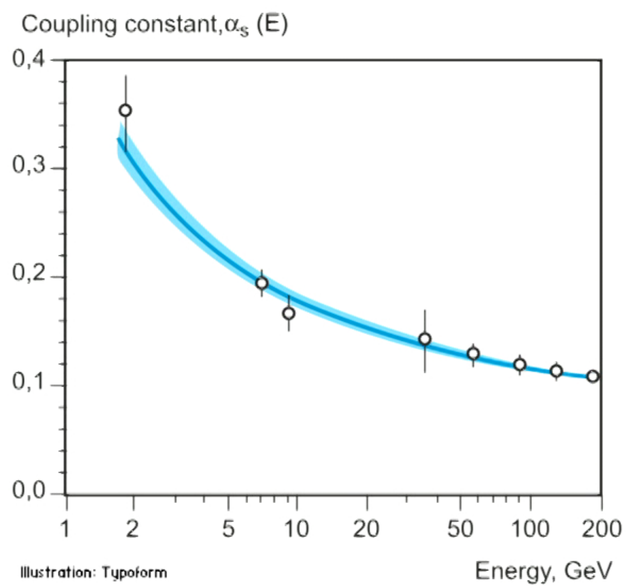
Outline

- Running couplings in a covariant theory - form factors
- Gravity-mediated scattering amplitudes
- Form factors (and the effective action) as a universal language

Running couplings in a covariant theory - form factors

Running coupling constants

- established experimental fact: coupling constants “run with energy”



Nobel prize in Physics 2004
(Gross, Politzer, Wilczek)
“for the discovery of asymptotic freedom in the theory of the strong interaction”

nobelprize.org

Running coupling constants

- established experimental fact: coupling constants “run with energy”
- measure scattering cross sections and compare them to theoretical predictions - coupling “constants” depend on energy scale dictated by their beta functions

$$\beta_{\alpha_s} = -\left(11 - \frac{2}{3}N_f\right)\frac{\alpha_s^2}{2\pi} + \mathcal{O}(\alpha_s^3)$$

Running coupling constants

- What is the fundamental meaning of “running coupling constants”?
  - “fundamental”: discuss in terms of QFT concepts using the language of the effective action Γ
- How do we generalise this notion to a curved spacetime?

Form Factors

- RG running = dependence of a coupling in the effective action on covariant derivatives

$$\text{EM/YM:} \qquad \Gamma = \int \mathrm{d}^4x \sqrt{-g} \left[-\frac{1}{4} \mathcal{F}^{\mu\nu} \frac{1}{\alpha_s(\Box)} \mathcal{F}_{\mu\nu} + \mathcal{O}(\mathcal{F}^3) \right]$$

Form Factors

- RG running = dependence of a coupling in the effective action on covariant derivatives

$$\text{EM/YM:} \qquad \Gamma = \int \mathrm{d}^4x \sqrt{-g} \left[-\frac{1}{4} \mathcal{F}^{\mu\nu} \frac{1}{\alpha_s(\Box)} \mathcal{F}_{\mu\nu} + \mathcal{O}(\mathcal{F}^3) \right]$$

$$\text{gravity:} \qquad \Gamma = \frac{1}{16\pi G_N} \int \mathrm{d}^4x \sqrt{-g} \left[2\Lambda - R + R\, f_R(\Box)\, R + C^{\mu\nu\rho\sigma} f_C(\Box) C_{\mu\nu\rho\sigma} + \mathcal{O}(\mathcal{R}^3) \right]$$

Form Factors

- interaction terms are more complicated, e.g. three-point function:

$$\Gamma^{(3)} \supset \frac{1}{16\pi G_N} \int \mathrm{d}^4x \sqrt{-g}\, f_{R^3}(\Box_1, \Box_2, \Box_3)\, R\, R\, R$$

- four-point function and higher: operator ordering needs convention (difference is of higher order)

$$\Gamma^{(4)} \supset \frac{1}{16\pi G_N} \int \mathrm{d}^4x \sqrt{-g}\, f_{R^4}\left(\{-D_i \cdot D_j\}\right)\, R\, R\, R\, R$$

Form Factors

- interaction terms are more complicated, e.g. three-point function:

$$\Gamma^{(3)} \supset \frac{1}{16\pi G_N} \int \mathrm{d}^4x \sqrt{-g}\, f_{R^3}(\Box_1, \Box_2, \Box_3)\, R\, R\, R$$

- four-point function and higher: operator ordering needs convention (difference is of higher order)

$$\Gamma^{(4)} \supset \frac{1}{16\pi G_N} \int \mathrm{d}^4x \sqrt{-g}\, f_{R^4}\left(\{-D_i \cdot D_j\}\right)\, R\, R\, R\, R$$

Form Factors

- RG running of couplings generically depends on several momentum scales - there is no unique scale in many processes

see also discussion in 2307.00055 (Buccio, Donoghue, Percacci)

Form Factors

- RG running of couplings generically depends on several momentum scales - there is no unique scale in many processes

see also discussion in 2307.00055 (Buccio, Donoghue, Percacci)

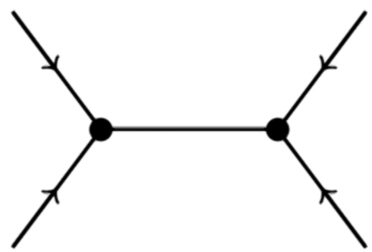
- based on curvature/field strength expansion - can access momentum dependence by considering n-point function around vanishing field configuration

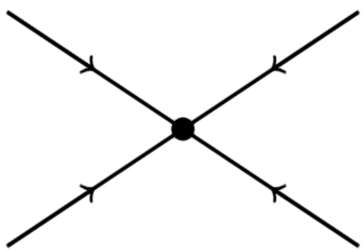
BK-Ripken-Saueressig collaboration: 1907.02903, 2111.12365, 2210.16072

Gravity-mediated scattering amplitudes

Graviton-mediated scattering amplitudes

- first non-trivial example: compute $2 \to 2$ gravitational scattering amplitudes and confront them with theoretical&experimental constraints





Graviton-mediated scattering amplitudes

- first non-trivial example: compute $2 \to 2$ gravitational scattering amplitudes and confront them with theoretical&experimental constraints
- benefits:
  - probe quantum gravity effects
  - direct link to observables
  - independent of arbitrary choices
  - use effective action = tree-level diagrams encode “everything”

Graviton-mediated scattering amplitudes

- first non-trivial example: compute $2 \to 2$ gravitational scattering amplitudes and confront them with theoretical&experimental constraints
- benefits:
  - probe quantum gravity effects
  - direct link to observables
  - independent of arbitrary choices
  - use effective action = tree-level diagrams encode "everything"

Graviton-mediated scattering amplitudes

- strategy for a given scattering amplitude:

  1. parameterise all possible terms in the effective action that contribute to the scattering event

  2. compute ingredients from first principles

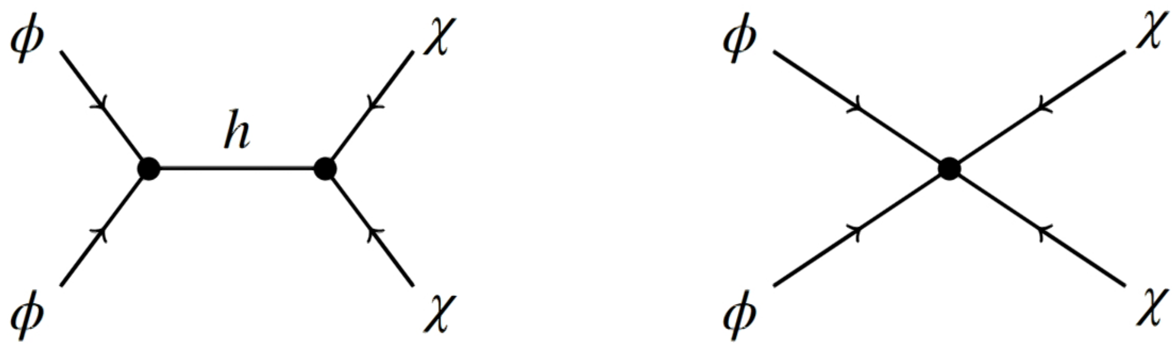
  3. confront with experimental data and theoretical constraints (finiteness, unitarity, causality, …)

recall Andrew Tolley's talk

Parameterisation of amplitudes with form factors

Scalar-scalar scattering

- gravity-mediated scalar scattering:



Scalar-scalar scattering

- necessary ingredients in the effective action:

$$G^{hh}$$

$$\Gamma \simeq \frac{1}{16\pi G_N} \int \mathrm{d}^4x \sqrt{-g} \left[-R + R\, f_R(\Box)\, R + C^{\mu\nu\rho\sigma} f_C(\Box) C_{\mu\nu\rho\sigma} \right]$$

$$+ \int \mathrm{d}^4x \sqrt{-g} \left[\frac{1}{2} \phi\, f_\phi(\Box)\, \phi + f_{R\phi\phi}(\Box_1, \Box_2, \Box_3) R\, \phi\, \phi + f_{Ric\phi\phi}(\Box_1, \Box_2, \Box_3) R^{\mu\nu} \left(D_\mu D_\nu \phi \right) \phi \right] + (\phi \to \chi)$$

$$\Gamma^{\phi\phi h} \qquad \Gamma^{\chi\chi h}$$

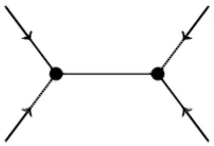
$$+ \frac{1}{(2!)^2} \int \mathrm{d}^4x \sqrt{-g}\, f_{\phi\chi}(\{-D_i \cdot D_j\})\, \phi\, \phi\, \chi\, \chi$$

$$\Gamma^{\phi\phi\chi\chi}$$

full momentum dependence is key

form factor toolbox:
BK, Ripken, Saueressig
1907.02903

Scalar-scalar scattering



$$\mathcal{A}_{\mathfrak{s}}^{\phi\chi} = \frac{4\pi}{3}\Bigg[-\left(1+\mathfrak{s} f_{Ric\phi\phi}(\mathfrak{s}, m_\phi^2, m_\phi^2)\right)\left(1+\mathfrak{s} f_{Ric\chi\chi}(\mathfrak{s}, m_\chi^2, m_\chi^2)\right) G_C(\mathfrak{s}) \left\{\mathfrak{t}^2 - 4\mathfrak{t}\mathfrak{u} + \mathfrak{u}^2 + 2\left(m_\phi^2 - m_\chi^2\right)^2\right\}$$
$$+ \left((\mathfrak{s}+2m_\phi^2)(1+\mathfrak{s} f_{Ric\phi\phi}(\mathfrak{s}, m_\phi^2, m_\phi^2)) - 12\mathfrak{s} f_{R\phi\phi}(\mathfrak{s}, m_\phi^2, m_\phi^2)\right)$$
$$\times \left((\mathfrak{s}+2m_\chi^2)(1+\mathfrak{s} f_{Ric\chi\chi}(\mathfrak{s}, m_\chi^2, m_\chi^2)) - 12\mathfrak{s} f_{R\chi\chi}(\mathfrak{s}, m_\chi^2, m_\chi^2)\right) G_R(\mathfrak{s})\Bigg]$$

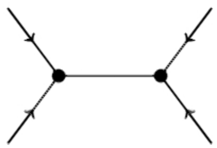
$$G_X(z) = \frac{G_N}{z(1+f_X(z))}$$

$$p_1^2 = p_2^2 = m_\phi^2$$
$$p_3^2 = p_4^2 = m_\chi^2$$

$$\mathfrak{s} = (p_1+p_2)^2$$
$$\mathfrak{t} = (p_1+p_3)^2$$
$$\mathfrak{u} = (p_1+p_4)^2$$

Draper, BK, Ripken, Saueressig
2007.00733, 2007.04396

Scalar-scalar scattering



vertex factors — graviton propagator — contraction factor — **spin 2**

$$\mathcal{A}_{\mathfrak{s}}^{\phi\chi} = \frac{4\pi}{3}\Big[-\left(1+\mathfrak{s}f_{Ric\phi\phi}(\mathfrak{s},m_\phi^2,m_\phi^2)\right)\left(1+\mathfrak{s}f_{Ric\chi\chi}(\mathfrak{s},m_\chi^2,m_\chi^2)\right) G_C(\mathfrak{s}) \left\{\mathfrak{t}^2 - 4\mathfrak{t}\mathfrak{u} + \mathfrak{u}^2 + 2\left(m_\phi^2 - m_\chi^2\right)^2\right\}$$

$$+ \left((\mathfrak{s}+2m_\phi^2)(1+\mathfrak{s}f_{Ric\phi\phi}(\mathfrak{s},m_\phi^2,m_\phi^2)) - 12\mathfrak{s}f_{R\phi\phi}(\mathfrak{s},m_\phi^2,m_\phi^2)\right)$$

$$\times \left((\mathfrak{s}+2m_\chi^2)(1+\mathfrak{s}f_{Ric\chi\chi}(\mathfrak{s},m_\chi^2,m_\chi^2)) - 12\mathfrak{s}f_{R\chi\chi}(\mathfrak{s},m_\chi^2,m_\chi^2)\right) G_R(\mathfrak{s}) \Big]$$

spin 0

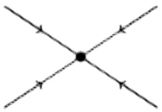
$$G_X(z) = \frac{G_N}{z(1+f_X(z))}$$

$$p_1^2 = p_2^2 = m_\phi^2$$
$$p_3^2 = p_4^2 = m_\chi^2$$

$$\mathfrak{s} = (p_1+p_2)^2$$
$$\mathfrak{t} = (p_1+p_3)^2$$
$$\mathfrak{u} = (p_1+p_4)^2$$

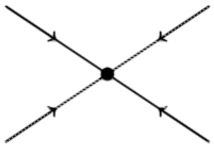
Draper, BK, Ripken, Saueressig
2007.00733, 2007.04396

Scalar-scalar scattering



$$\mathcal{A}_4^{\phi\chi} = f_{\phi\chi}\left(\frac{\mathfrak{s}-2m_\phi^2}{2}, \frac{\mathfrak{t}-m_\phi^2-m_\chi^2}{2}, \frac{\mathfrak{u}-m_\phi^2-m_\chi^2}{2}, \frac{\mathfrak{u}-m_\phi^2-m_\chi^2}{2}, \frac{\mathfrak{t}-m_\phi^2-m_\chi^2}{2}, \frac{\mathfrak{s}-2m_\chi^2}{2}\right)$$

$$p_1^2 = p_2^2 = m_\phi^2$$
$$p_3^2 = p_4^2 = m_\chi^2$$



Scalar-scalar scattering

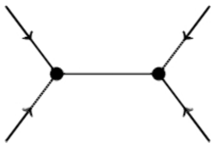
$$\mathcal{A}_4^{\phi\chi} = f_{\phi\chi}\left(\frac{\mathfrak{s}-2m_\phi^2}{2}, \frac{\mathfrak{t}-m_\phi^2-m_\chi^2}{2}, \frac{\mathfrak{u}-m_\phi^2-m_\chi^2}{2}, \frac{\mathfrak{u}-m_\phi^2-m_\chi^2}{2}, \frac{\mathfrak{t}-m_\phi^2-m_\chi^2}{2}, \frac{\mathfrak{s}-2m_\chi^2}{2}\right)$$

$$p_1^2 = p_2^2 = m_\phi^2$$
$$p_3^2 = p_4^2 = m_\chi^2$$

$$\mathfrak{s} = (p_1 + p_2)^2$$
$$\mathfrak{t} = (p_1 + p_3)^2$$
$$\mathfrak{u} = (p_1 + p_4)^2$$

Draper, BK, Ripken, Saueressig
2007.00733, 2007.04396

Scalar-scalar scattering



vertex factors — graviton propagator — contraction factor — **spin 2** / **spin 0**

$$\mathcal{A}_{\mathfrak{s}}^{\phi\chi} = \frac{4\pi}{3}\Big[-\left(1+\mathfrak{s} f_{Ric\phi\phi}(\mathfrak{s}, m_\phi^2, m_\phi^2)\right)\left(1+\mathfrak{s} f_{Ric\chi\chi}(\mathfrak{s}, m_\chi^2, m_\chi^2)\right) G_C(\mathfrak{s}) \left\{\mathfrak{t}^2 - 4\mathfrak{t}\mathfrak{u} + \mathfrak{u}^2 + 2\left(m_\phi^2 - m_\chi^2\right)^2\right\}$$
$$+\left((\mathfrak{s}+2m_\phi^2)(1+\mathfrak{s} f_{Ric\phi\phi}(\mathfrak{s}, m_\phi^2, m_\phi^2)) - 12\mathfrak{s} f_{R\phi\phi}(\mathfrak{s}, m_\phi^2, m_\phi^2)\right)$$
$$\times\left((\mathfrak{s}+2m_\chi^2)(1+\mathfrak{s} f_{Ric\chi\chi}(\mathfrak{s}, m_\chi^2, m_\chi^2)) - 12\mathfrak{s} f_{R\chi\chi}(\mathfrak{s}, m_\chi^2, m_\chi^2)\right) G_R(\mathfrak{s})\Big]$$

$$G_X(z) = \frac{G_N}{z(1+f_X(z))}$$

$$p_1^2 = p_2^2 = m_\phi^2$$
$$p_3^2 = p_4^2 = m_\chi^2$$

$$\mathfrak{s} = (p_1+p_2)^2$$
$$\mathfrak{t} = (p_1+p_3)^2$$
$$\mathfrak{u} = (p_1+p_4)^2$$

Draper, BK, Ripken, Saueressig
2007.00733, 2007.04396

Beyond scalar-scalar scattering

- similar computations can be done for any scattering event
  - scalar-photon, photon-photon
  - to do: fermions

BK, Pirlo, Ripken, Saueressig, 2205.01738
book chapter: BK, Ripken, Saueressig, 2210.16072

- using field redefinitions and focussing on **essential** couplings/form factors will be helpful, reduces complexity severely

Form factors (and the effective action) as a universal language

Form factors in different approaches

Stelle gravity

$$f_{R,C}(\Box) = c_0 + c_1 \ln \Box$$

Non-local gravity

recall Luca Buoninfante's talk

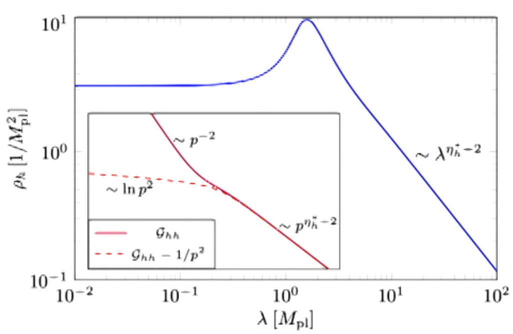
$$f_{R,C}(\Box) \simeq \frac{e^{\Box/M^2} - 1}{\Box}$$

LQG

$$f_C(\Box) \simeq -\frac{1}{8}\left[\frac{\gamma_+}{m_+^2 + \Box} + \frac{\gamma_-}{m_-^2 + \Box}\right]$$

Borissova, Dittrich
2207.03307

Asymptotic Safety



Fehre, Litim, Pawlowski, Reichert
2111.13232

CDT

$$f_R(\Box) \simeq \frac{b}{\Box^2}$$

BK, Saueressig
1804.03846

Form Factors in CDT

- use correlation function for which CDT data is available, e.g.

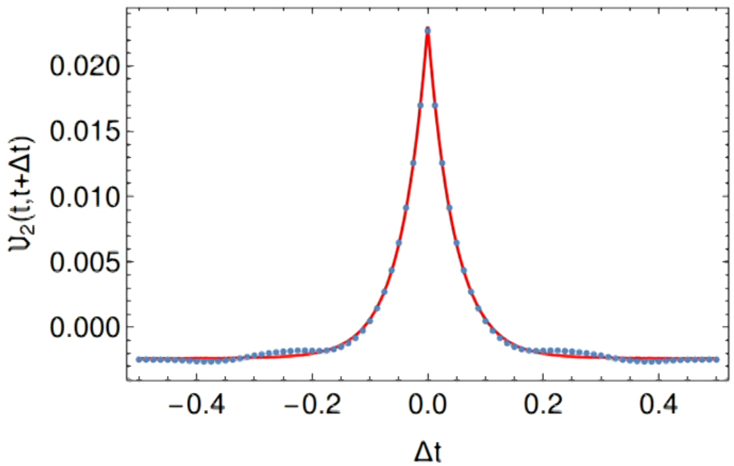
$$\mathfrak{V}_2 = \langle \delta V_3(t) \delta V_3(t + \Delta t) \rangle$$

BK, Saueressig
1804.03846

Form Factors in CDT

- use correlation function for which CDT data is available, e.g.

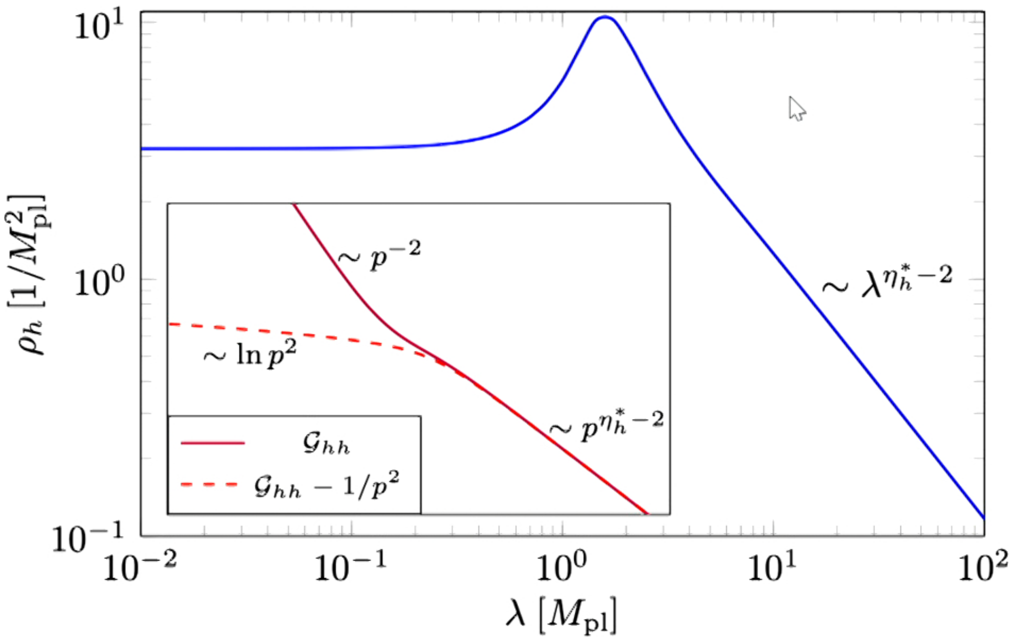
$$\mathfrak{V}_2 = \langle \delta V_3(t) \delta V_3(t+\Delta t) \rangle$$



$$f_R(\Box) \simeq \frac{b}{\Box^2}$$

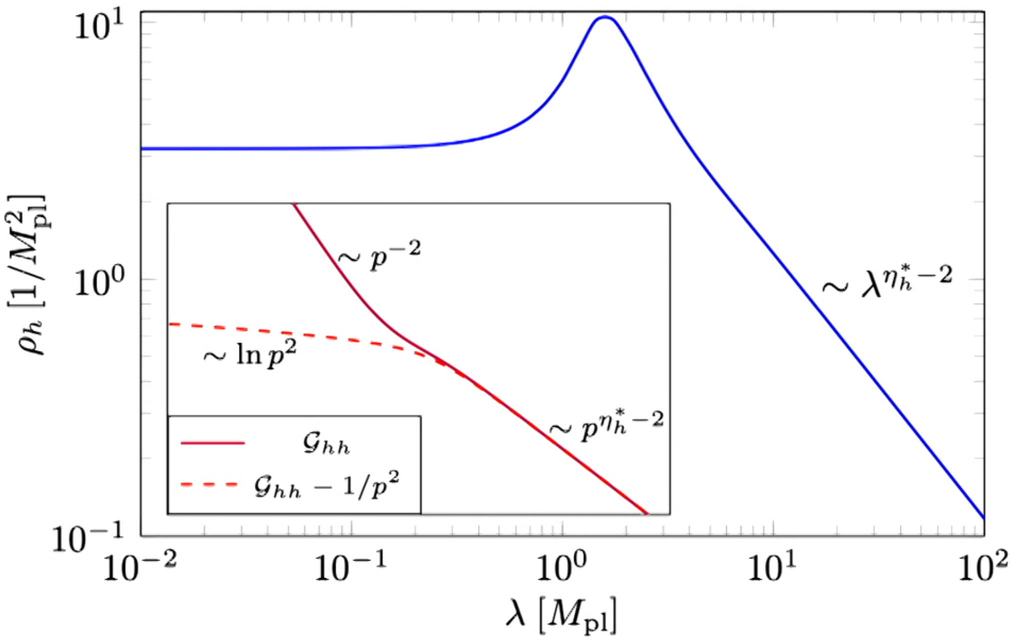
BK, Saueressig
1804.03846

Graviton spectral function



Fehre, Litim, Pawlowski, Reichert
2111.13232

Graviton spectral function



Lorentzian computation!
Matches EFT in IR!

Fehre, Litim, Pawlowski, Reichert
2111.13232

Comparison of amplitudes

scalar toy model

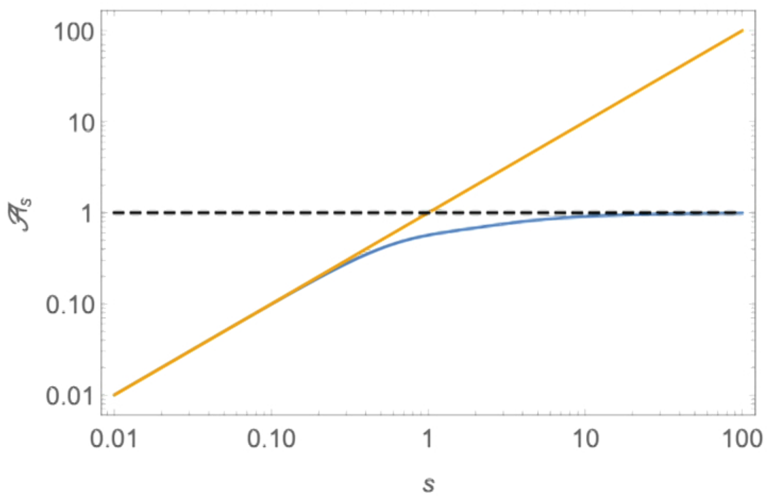
Asymptotic Safety

Non-local gravity

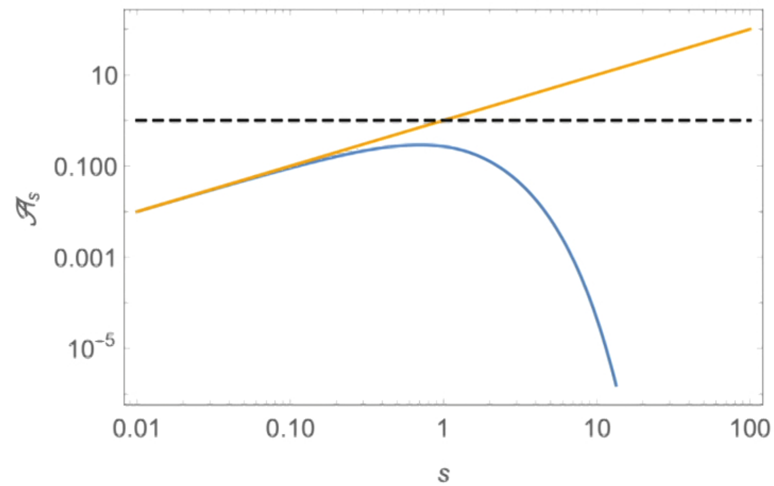
BK, Ripken, Saueressig
2111.12365

Comparison of amplitudes

scalar toy model



Asymptotic Safety



Non-local gravity

BK, Ripken, Saueressig
2111.12365

Summary

Summary

- falsifiability is at the heart of science, and it should also be at the heart of quantum gravity research
- scattering amplitudes are a useful way to probe quantum gravity
- ingredients can be computed ab initio, no need to guess
- form factors and the effective action are promising tools to **compute and compare QG predictions**