Title: 3pt functions: Yes Q's

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Collection: Quantum Spectral Curve and Three Point Functions mini-course

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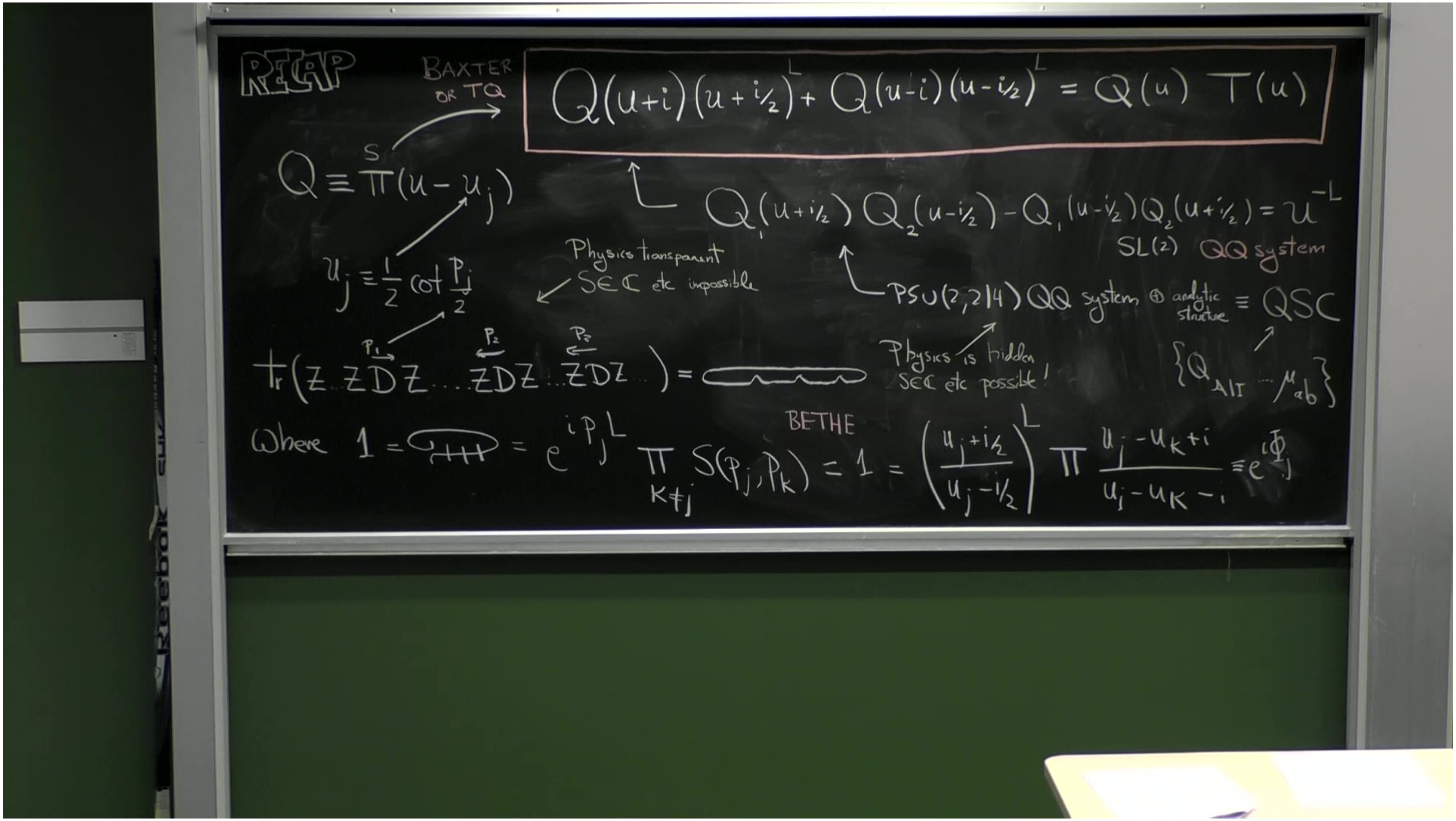
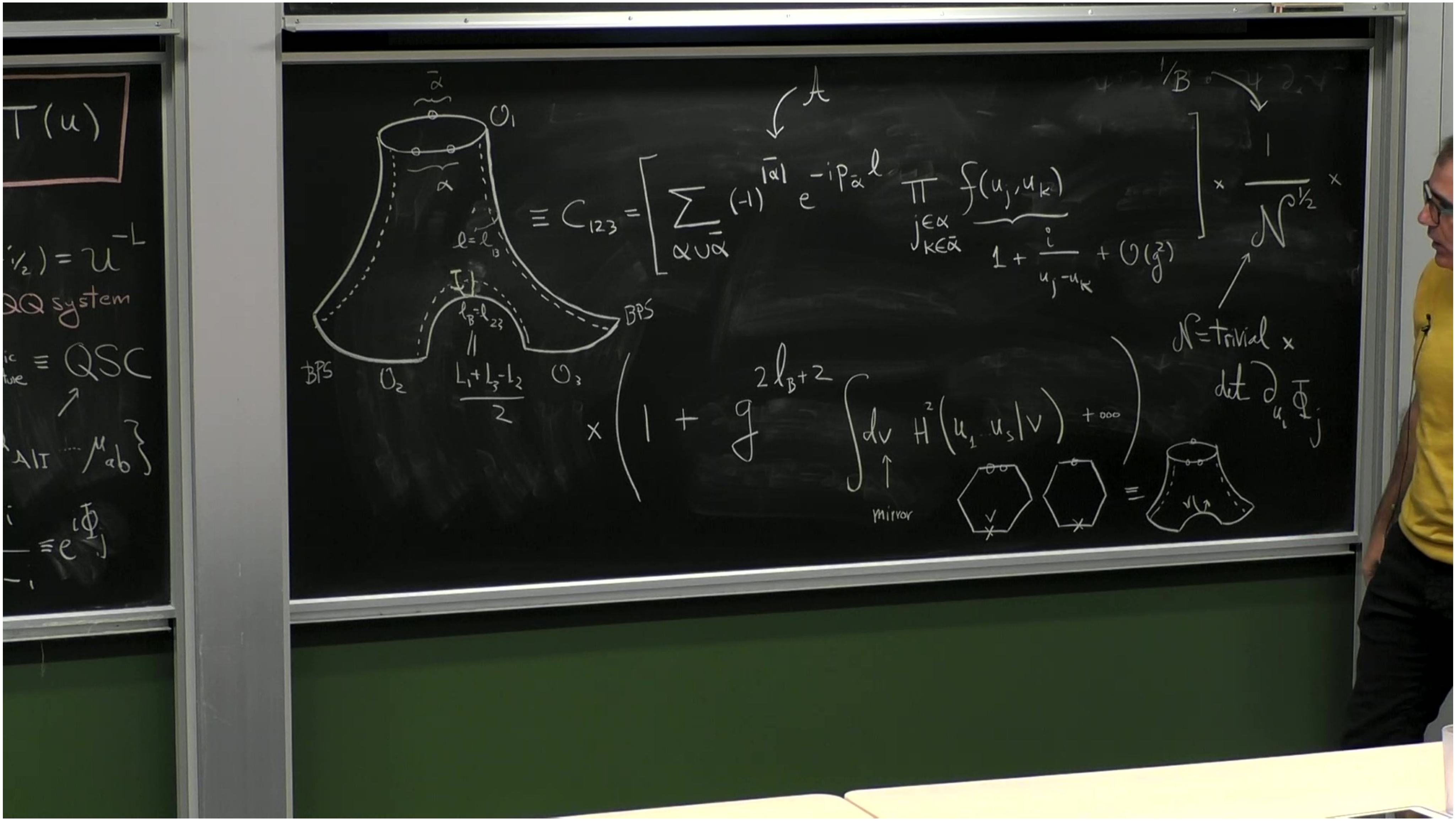

RECAP
BAXTER OR TQ
$Q(u+i)(u+\frac{i}{2})^L + Q(u-i)(u-\frac{i}{2})^L = Q(u)\,T(u)$
$Q \equiv \prod^S (u-u_j)$
$Q_1(u+\frac{i}{2})\,Q_2(u-\frac{i}{2}) - Q_1(u-\frac{i}{2})\,Q_2(u+\frac{i}{2}) = u^{-L}$
SL(2) QQ system
$u_j \equiv \frac{1}{2}\cot\frac{p_j}{2}$
Physics transparent
SEC etc impossible
PSU(2,2|4) QQ system ⊕ analytic structure ≡ QSC
Physics is hidden
SEC etc possible!
$\{Q_{A|I} \dots \mu_{ab}\}$
$\mathrm{tr}(Z..Z\overset{p_1}{D}Z \dots Z\overset{p_2}{D}Z..Z\overset{p_3}{D}Z \dots) =$
BETHE
Where $1 = e^{ip_jL} \prod_{k\neq j} S(p_j,p_k) = 1 = \left(\frac{u_j+\frac{i}{2}}{u_j-\frac{i}{2}}\right)^L \prod \frac{u_j-u_k+i}{u_j-u_k-i} \equiv e^{i\Phi_j}$


T(u)
QQ system
≡ QSC
$\equiv C_{123} = \left[\sum_{\alpha \cup \bar{\alpha}} (-1)^{|\bar{\alpha}|} e^{-i p_{\bar{\alpha}} \ell} \prod_{j\in\alpha, k\in\bar{\alpha}} \frac{f(u_j,u_k)}{1+\frac{i}{u_j-u_k}+O(g^2)} \right] \times \frac{1}{\mathcal{N}^{1/2}} \times$
$\times \left(1 + g^{2\ell_B+2} \int dv\, H^2(u_1 \ldots u_s | v) + \ldots \right)$
mirror
$\mathcal{N}$ = trivial × det $\partial_{u_i} \Phi_j$
$\ell = \ell_{13}$
$\ell_B = \ell_{23} = \frac{L_1+L_3-L_2}{2}$
BPS
O_1 O_2 O_3
A
1/B

$k\neq j \quad (u_j - 1/2) \quad u_j - u_k - i$

* Pairing

$$\langle Q, Q' \rangle$$

goal: $\langle \psi, \psi' \rangle = 0$ if $E \neq E'$

$L = 2$

$Q(u) = Q(-u)$

$$\underbrace{Q(u+i)(u+i/2)^2 + Q(u-i)(u-i/2)^2}_{\psi''}$$

$$= \underbrace{T(u)}_{\text{pol of deg 2}} \; \underbrace{Q(u)}_{\psi}$$

$$T(u) = \underbrace{2u^2}_{V} + \underbrace{f(s)}_{E}$$

$$\mathcal{D} \circ Q = \left(2u^2 + f(s)\right) Q$$

want

$$\langle Q, \mathcal{D} \circ Q' \rangle = \langle \mathcal{D} \circ Q, Q' \rangle$$

then

$$0 = \left(f(s) - f(s')\right) \langle Q, Q' \rangle$$

* Pairing

$$\langle Q , Q' \rangle$$

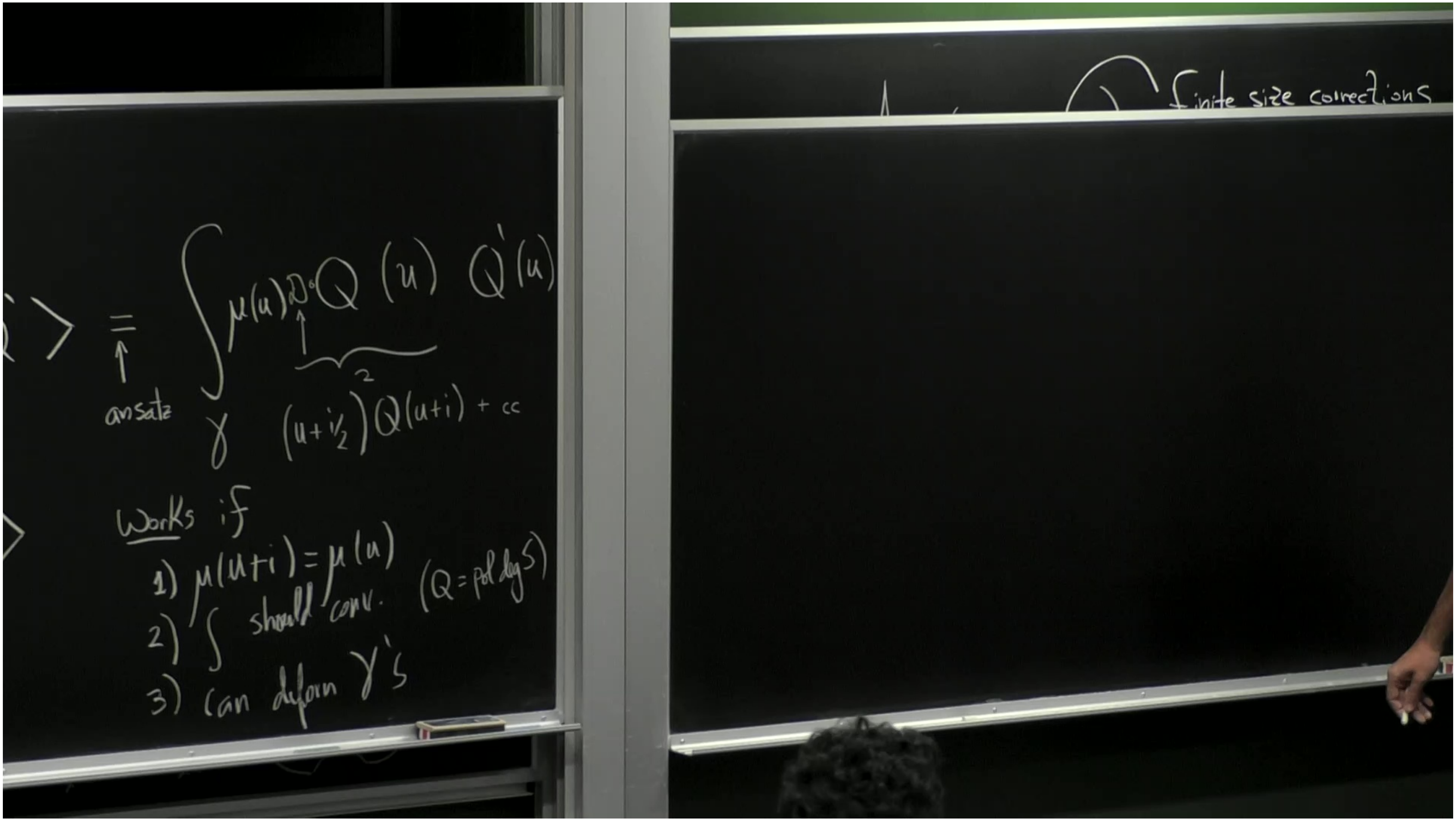
$L = 2$

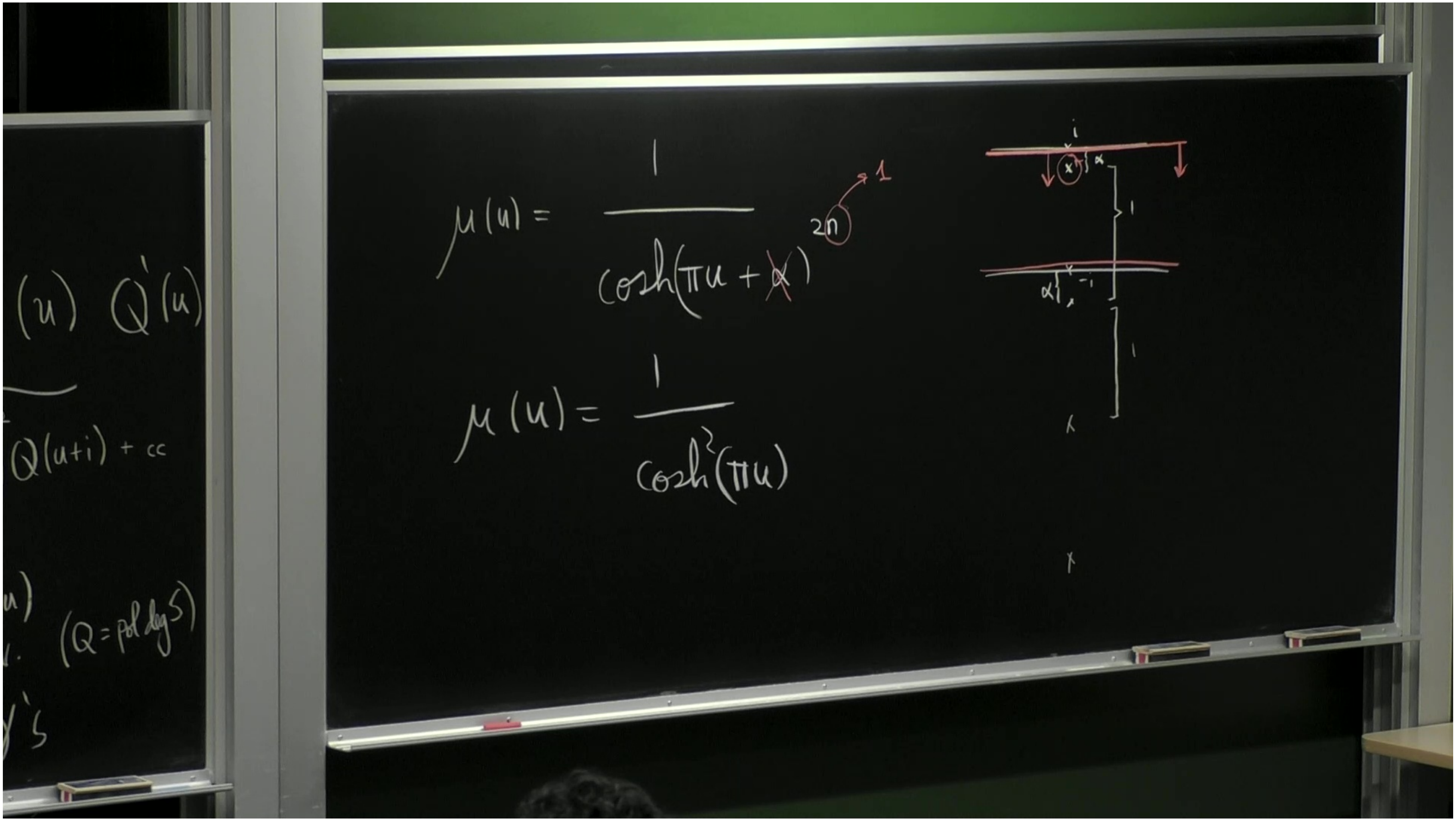
$Q(u) = Q(-u)$

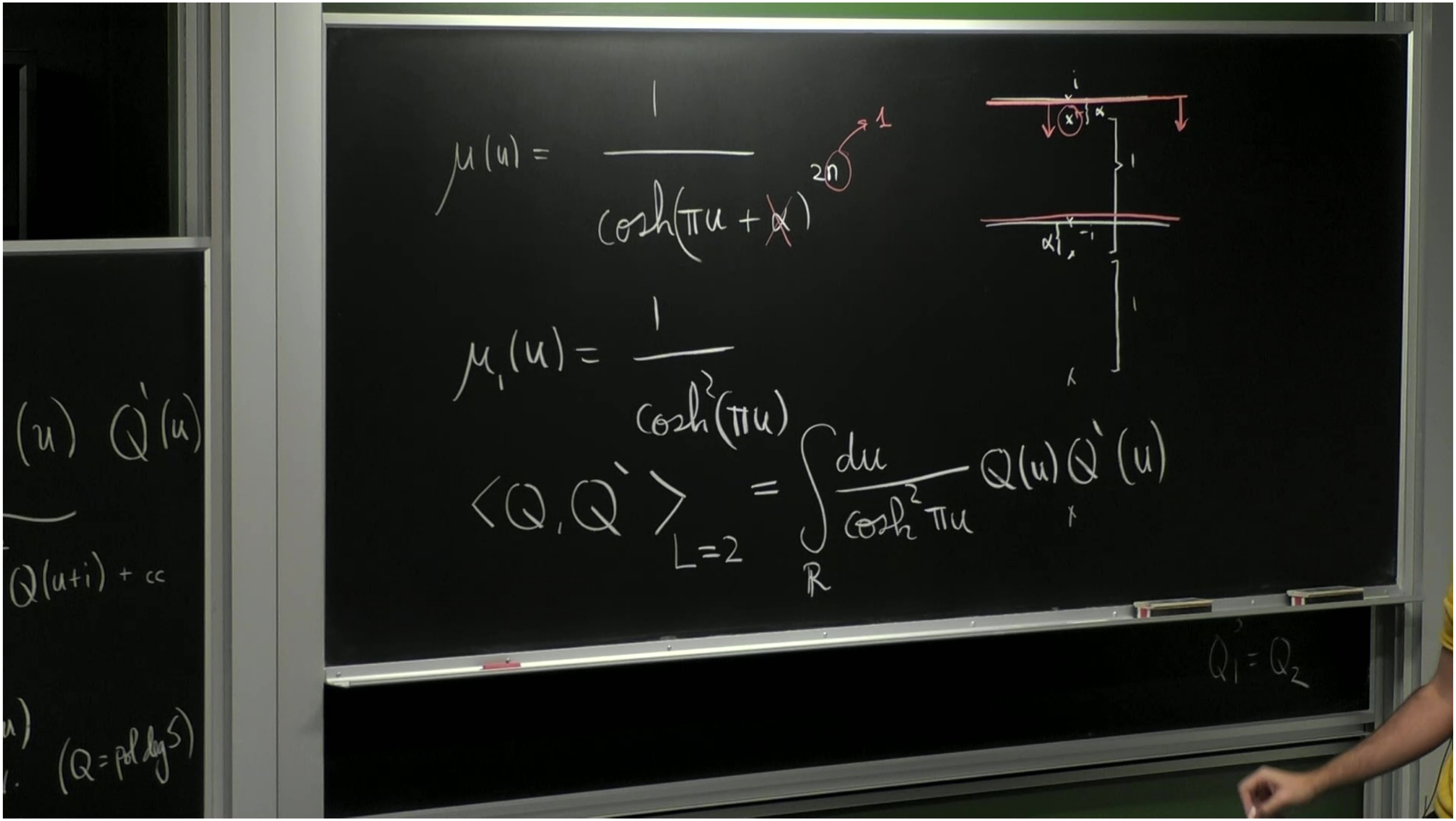
$$\underbrace{Q(u+i)(u+i/2)^2 + Q(u-i)(u-i/2)^2}_{\psi''} = \underbrace{T(u)}_{\text{pol of deg } 2} \; \underbrace{Q(u)}_{\psi}$$

goal: $\langle \psi , \psi' \rangle = 0$ if $E \neq E'$

$$T(u) = \underbrace{2u^2}_{V} + \underbrace{f(s)}_{E}$$


finite size corrections
> = ∫_γ μ(u) D° Q(u) Q'(u)
ansatz
(u + i/2)^2 Q(u+i) + cc
Works if
1) μ(u+i) = μ(u)
2) ∫ should conv. (Q = pol deg S)
3) can deform γ's


$\mu(u) = \frac{1}{\cosh(\pi u + \cancel{\alpha})^{2n}}$
$2n \to 1$
$\mu(u) = \frac{1}{\cosh^2(\pi u)}$
(u) Q'(u)
Q(u+i) + cc
(Q = pol deg S)
i
α
1
-i
1


μ(u) = 1 / cosh(πu + α)^2n
1
μ1(u) = 1 / cosh²(πu)
⟨Q, Q`⟩ L=2 = ∫R du / cosh² πu Q(u) Q`(u)
Q(u+i) + cc
(Q = pol deg 5)
Q1 = Q2

* Pairing

$$\langle Q, Q' \rangle$$

goal: $\langle \Psi, \Psi' \rangle = 0$ if $E \neq E'$

$L = 2$ $\;>2$

$Q(u) = Q(-u)$

$$\underbrace{Q(u+i)(u+i/2)^{2 \to L} + Q(u-i)(u-i/2)^{2 \to L}}_{\Psi''}$$

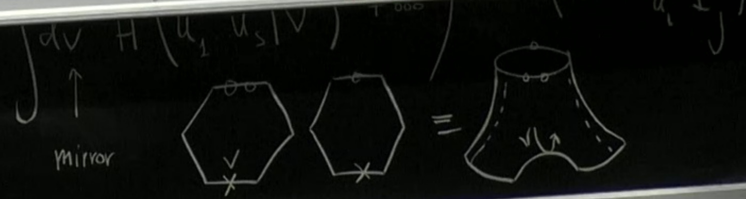
$$= \underbrace{T(u)}_{\text{pol of deg } 2\,L} \; \underbrace{Q(u)}_{\Psi}$$

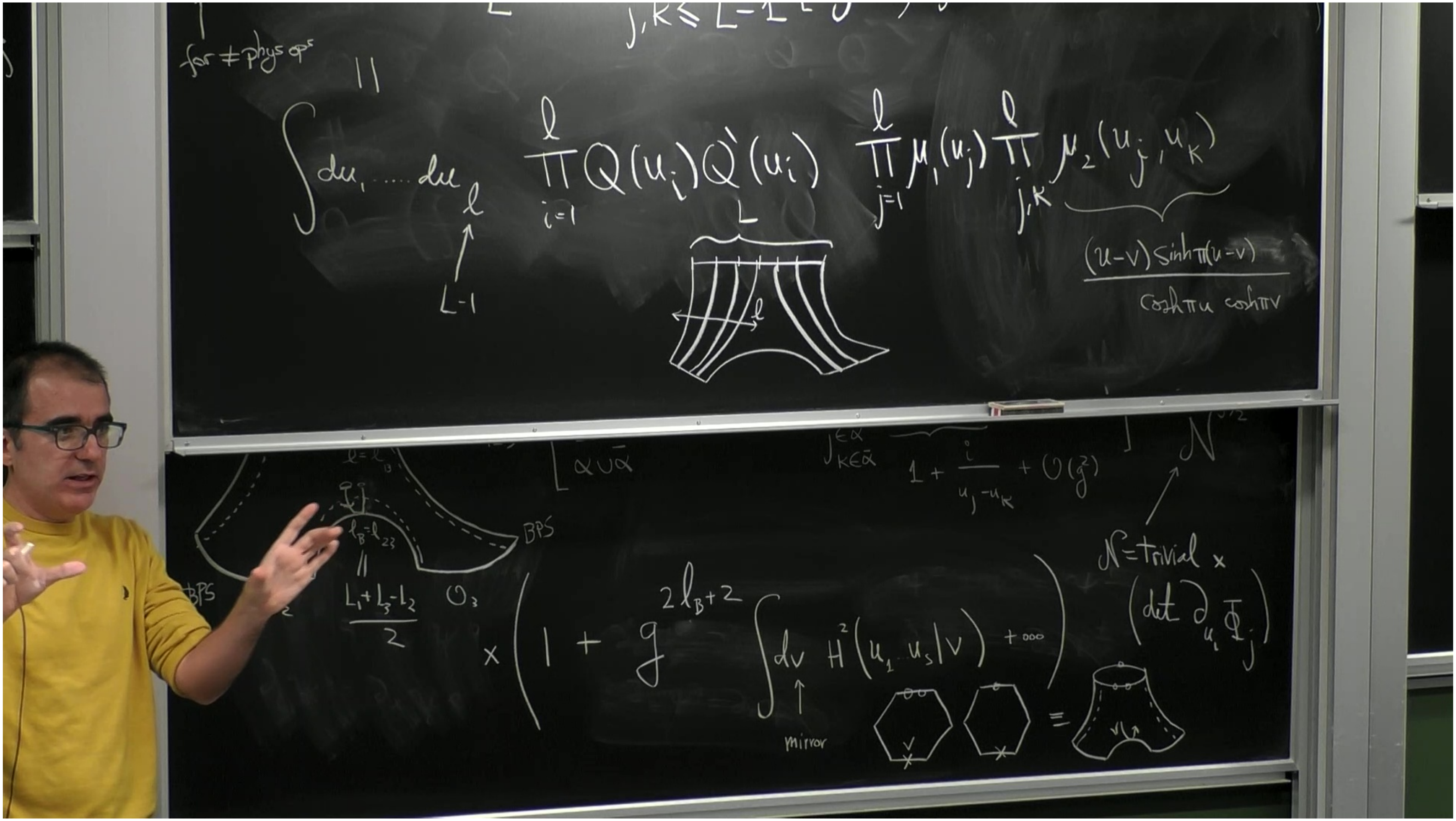
$$T(u) = \underbrace{2u^{2L}}_{V} + \underbrace{f^{(1)}(S)u^{L-2}}_{E} + \dots + u^0 f^{(L-1)}(S)$$

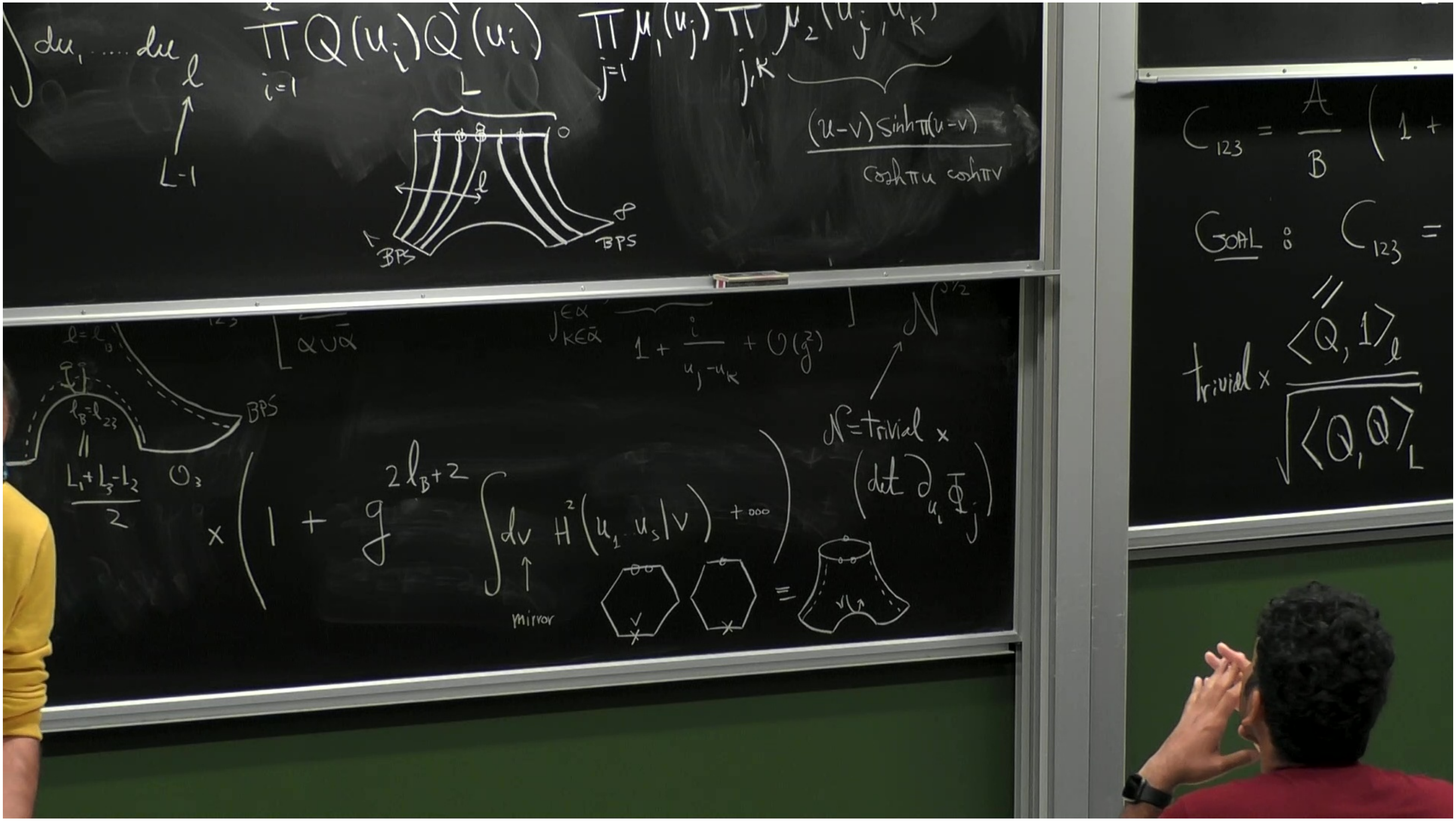
$$\langle Q, \grave{Q} \rangle_L = \det_{j,k \leqslant L-1} \left[\int du\, \mu_j(u)\, u^{k-1}\, Q(u)\, \grave{Q}(u) \right]$$

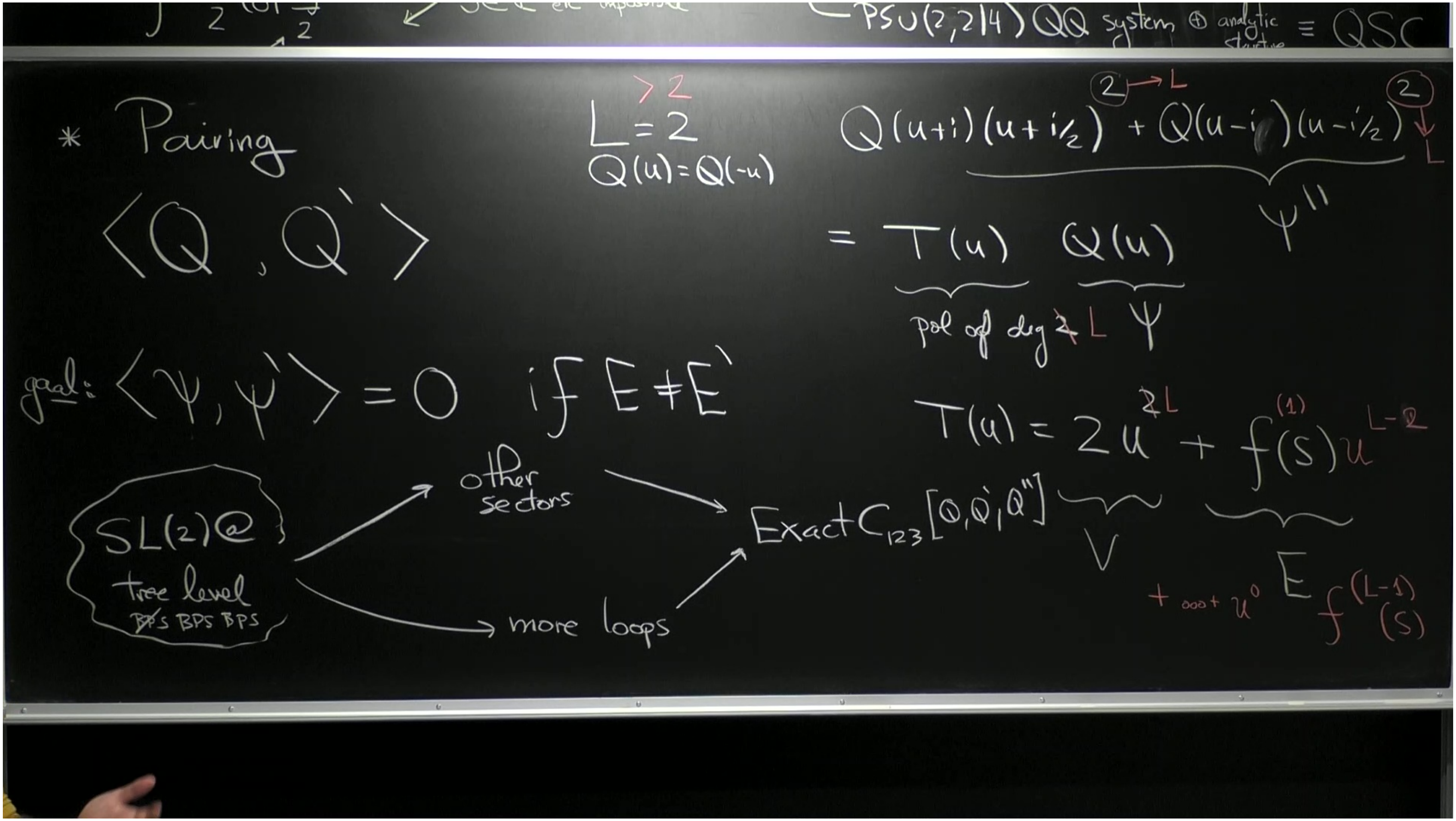
$$\parallel$$

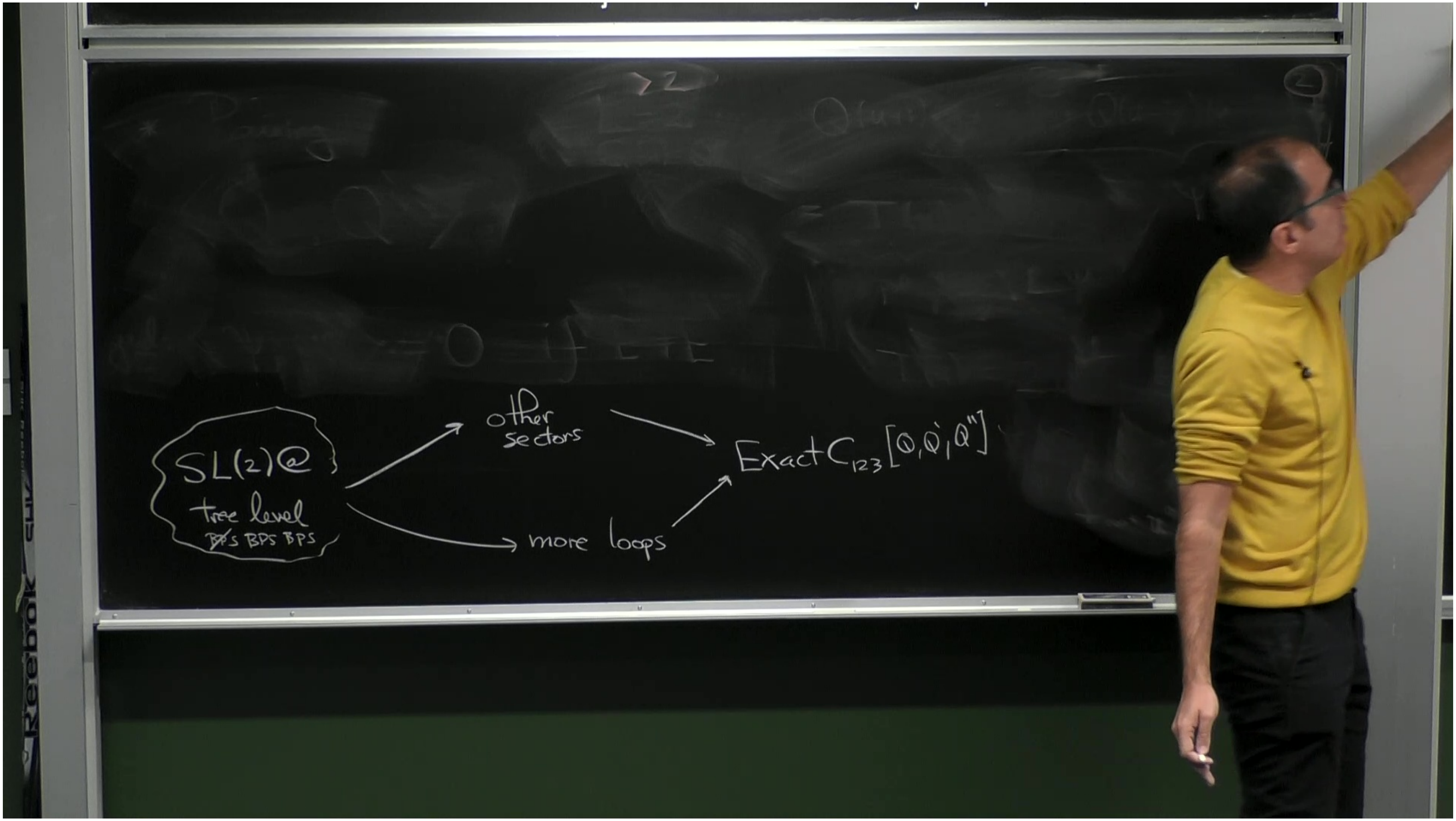
$$\int du_1 \ldots\ldots du_{\underset{\uparrow\, L-1}{\ell}} \;\prod_{i=1}^{\ell} Q(u_i)\grave{Q}(u_i)\; \prod_{j=1}^{\ell} \mu_1(u_j) \prod_{j,k}^{\ell} \underbrace{\mu_2(u_j, u_k)}_{\frac{(u-v)\sinh\pi(u-v)}{\cosh\pi u\, \cosh\pi v}}$$




for ≠ phys ops
$\int du_1 \ldots du_l \prod_{i=1}^{l} Q(u_i)\dot{Q}(u_i) \prod_{j=1}^{l} \mu_1(u_j) \prod_{j,k}^{l} \mu_2(u_j,u_k)$
L-1
$\frac{(u-v)\sinh\pi(u-v)}{\cosh\pi u \cosh\pi v}$
BPS
$\frac{L_1+L_3-L_2}{2}$
$\times\left(1+g^{2l_B+2}\int dv\, H^2(u_1..u_5|v) + \ldots\right)$
mirror
$\mathcal{N}$ = trivial ×
$(\det \partial_{u_i} \bar{\Phi}_j)$


∫ du_1 ..... du_l ∏ Q(u_i) Q(u_i) ∏ μ_1(u_j) ∏ μ_2(u_j, u_k)
i=1
j=1
j,k
L-1
L
(u-v) sinh π(u-v)
cosh πu cosh πv
BPS
BPS
C_123 = A / B (1 +
GOAL : C_123 =
trivial × ⟨Q, 1⟩_l / √⟨Q,Q⟩_L
l = l_B = l_23
L_1+L_3-L_2 / 2
O_3
BPS
j∈α
k∈ᾱ
α∪ᾱ
1 + i/(u_j-u_k) + O(g^2)
N^{-1/2}
N = trivial ×
(det ∂_{u_i} Φ_j)
× (1 + g^{2 l_B + 2} ∫ dv H^2(u_1 .. u_3 | v) + ...)
mirror
=


PSU(2,2|4) QQ system ⊕ analytic structure ≡ QSC
* Pairing
⟨Q , Q'⟩
L = 2 > 2
Q(u) = Q(-u)
Q(u+i)(u+i/2)^2 + Q(u-i)(u-i/2)^2 → L
ψ''
= T(u) Q(u)
pol of deg 2 L ψ
goal: ⟨ψ, ψ'⟩ = 0 if E ≠ E'
T(u) = 2u^2 L + f^(1)(S) u^(L-2)
+ … + u^0
V
E
f^(L-1)(S)
SL(2)@ tree level BPS BPS BPS
other sectors
more loops
Exact C_123 [Q, Q', Q'']


SL(2)@
tree level
BPS BPS BPS
other sectors
more loops
Exact C123 [Q,Q',Q'']

LO, NLO

$$x(v) + \frac{1}{x(v)} = \frac{v}{g}$$

$$\frac{1}{\cosh^2(\pi u)} = \mu_1$$

$$= \int \frac{N}{x(v)} \frac{dv}{\cosh^2 \pi(u-v)} = \mu_1\left(1 + 3g^2\, \mathrm{th}^2 \pi u + \dots\right)$$

$$\hat{\mu}_2 = \mu_2\left(1 + g^2\, \#\, \mathrm{th}^2 \pi u + \dots\right)$$

4g

SL(2)@
tree level
~~BPS~~ BPS BPS

other sectors

more loops

Exact $C_{123}[Q, Q', Q'']$

$$C_{123} = \frac{A}{B}\left(1 + \dots\right)$$

finite size corrections

Goal : $C_{123} = \langle Q_1, Q_2, Q_3 \rangle \Big/ \sqrt{\prod_{j=1}^{3} \langle Q_j, Q_j \rangle}$

✓ @ LO, NLO, NNLO

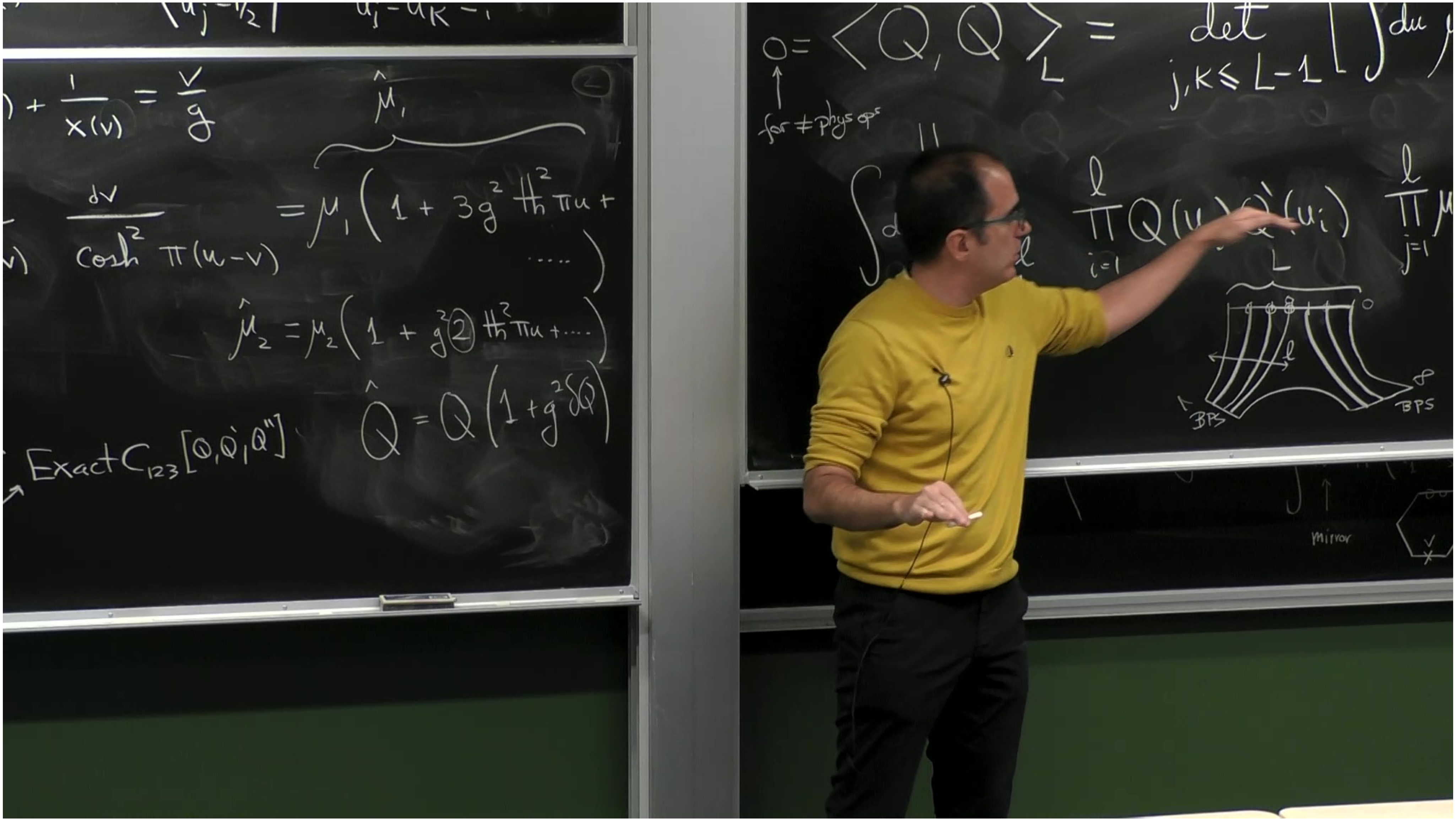
fails for $L>2$, nice $L=2$

$$\text{trivial} \times \boxed{\frac{\langle Q, 1 \rangle_{\ell}}{\sqrt{\langle Q, Q \rangle_{L}}}}$$

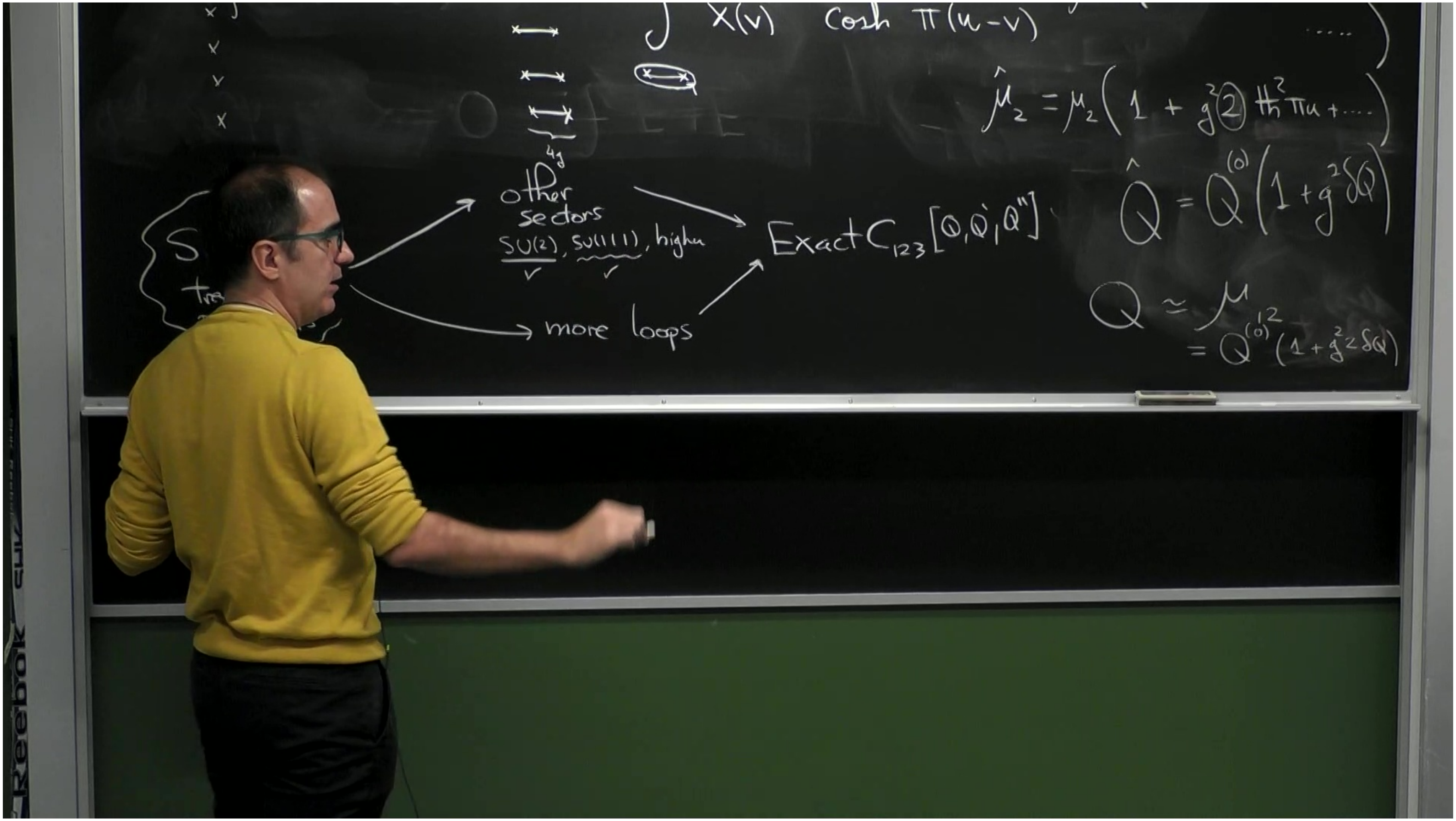
$$\langle Q_j, Q_j \rangle \sim \langle Q_1, Q_2, Q_3 \rangle$$

$Q_3 = 1$

$Q_1 = Q_2$



$\frac{1}{x(v)} = \frac{v}{g}$
$\hat{\mu}_1$
$\frac{dv}{\cosh^2 \pi(u-v)}$
$= \mu_1 \left(1 + 3g^2 \operatorname{th}^2 \pi u + \ldots\right)$
$\hat{\mu}_2 = \mu_2 \left(1 + g^2 2 \operatorname{th}^2 \pi u + \ldots\right)$
$\hat{Q} = Q\left(1 + g^2 \delta Q\right)$
Exact $C_{123}[Q, Q', Q'']$
$0 = \langle Q, Q \rangle_L = \det_{j,k \leq L-1} \left[\int du \right.$
for ≠ phys op's
$\prod_{i=1}^{\ell} Q(u_i) Q'(u_i) \quad \prod_{j=1}^{\ell}$
L
BPS
BPS
mirror


X(v) Cosh π(u-v)
other sectors
SU(2), SU(1|1), higher
Exact C123 [Q, Q', Q'']
more loops
Q ≃ μ

where $I = \ldots = e^{i\phi_j}$ $\quad \prod_{k\neq j} S(p_j,p_k) = 1 = \left(\frac{u_j+1/2}{u_j-1/2}\right) \prod \frac{u_j-u_k+i}{u_j-u_k-i} \equiv e^{i\phi_j}$

$L=2$,

$$= C_{123} = \frac{\overbrace{\langle Q, 1\rangle_{\ell=1}}^{o\ \text{int.}} = 1}{\sqrt{\underbrace{\langle Q, Q\rangle_{L=2}}_{\int du\ \hat{\mu}(u) Q(u) Q(u)}}}$$

- Found it
- State dep

$\langle Q_1, Q_2\rangle = \delta_{12}$

$\mu = \ldots \quad g^4 \frac{1}{\cosh^2\pi u} \overset{n=1,+}{Q_1} \overset{n=2,-}{Q_2} + \ldots$

$\frac{1}{\cosh^2\pi u} + g^2 \checkmark + g^4 (?)$

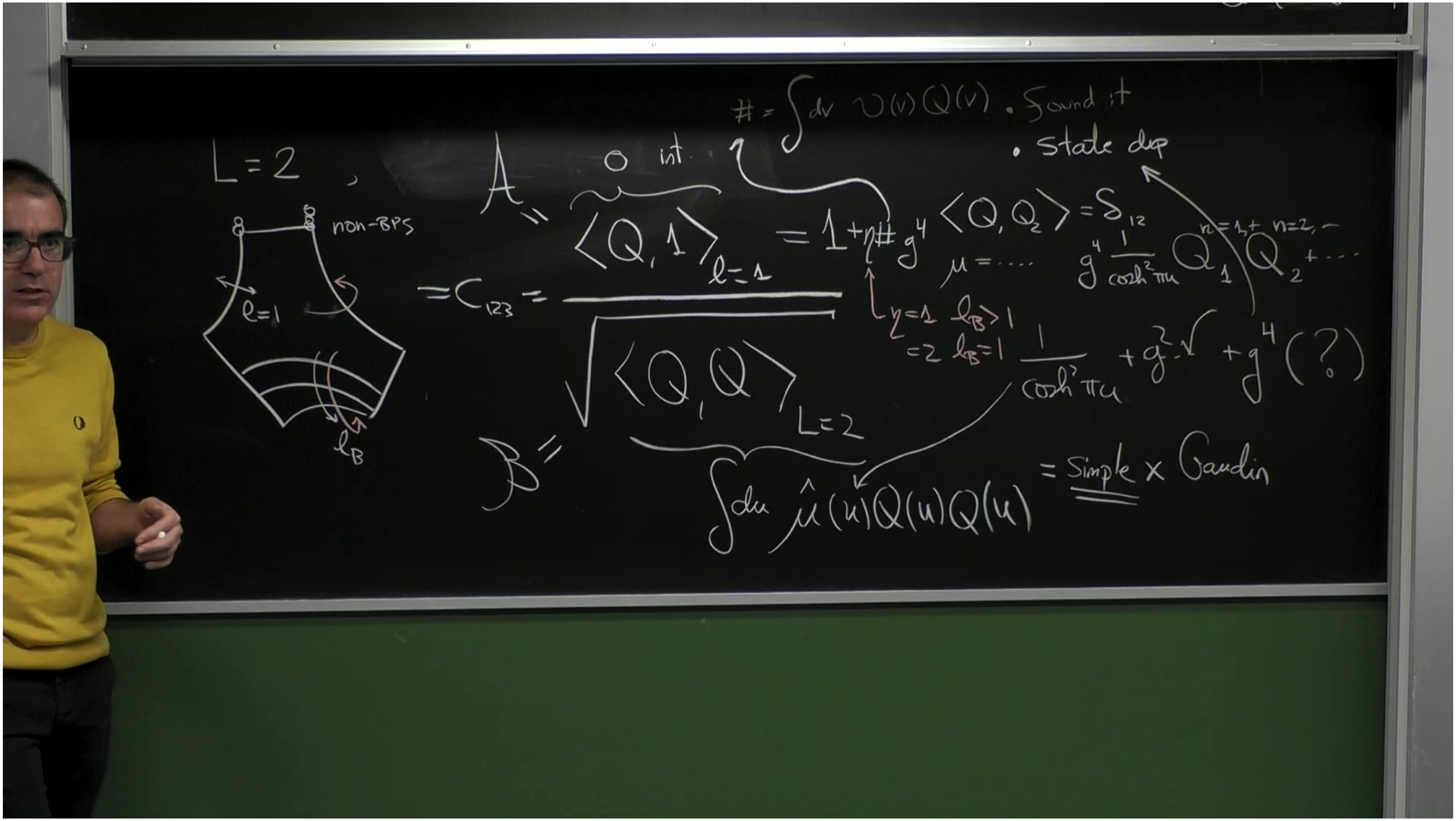
LO ✓, NLO

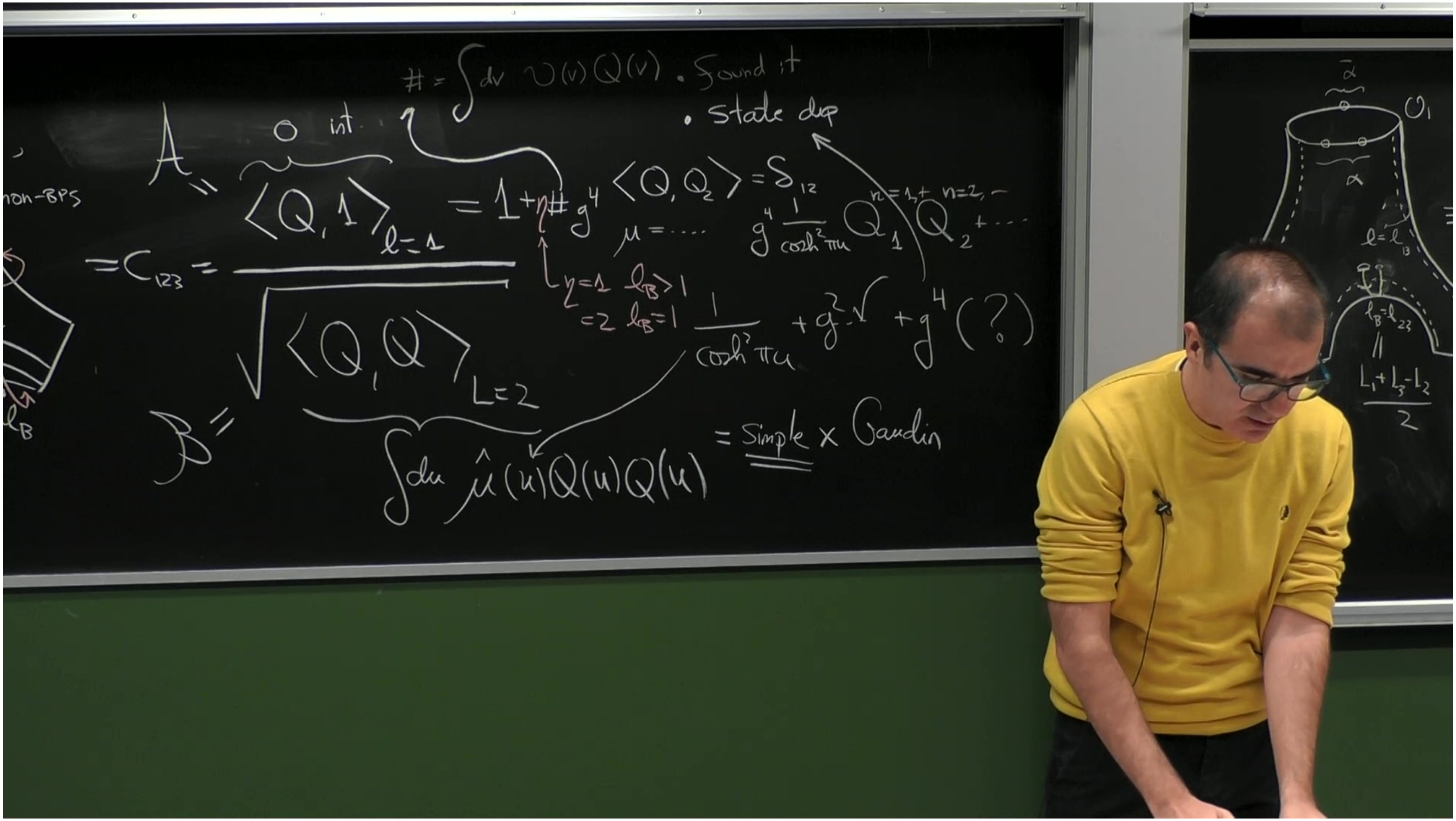
$x(v) + \frac{1}{x(v)} = \frac{v}{g}$

× $\frac{1}{\cosh^2(\pi u)} \equiv \mu_1$

× ξ_i $\quad = \int \frac{N}{x(\ldots)} \frac{dv}{\cosh^2 \pi(u-v)}$

$= \overbrace{\mu_1 \left(1 + 3g^2 \tanh^2 \pi u + \ldots\right)}^{\hat{\mu}_1}$


L=2
non-BPS
ℓ=1
ℓB
A = ⟨Q,1⟩ ℓ=1 / √⟨Q,Q⟩ L=2
= C123 =
O int
= ∫dv ν(v)Q(v)
• found it
• State dep
= 1 + η # g4
⟨Q1 Q2⟩ = δ12
μ = ...
g4 1/cosh2 πu Q1 Q2 + ...
n=1, + n=2, −
η=1 ℓB > 1
=2 ℓB = 1
1/cosh2 πu + g2 √ + g4 (?)
B =
∫du μ̂(u)Q(u)Q(u)
= Simple x Gaudin



$\# = \int dv\, \nu(v) Q(v)$
• found it
• State dep
$A =$
O int.
$\langle Q, 1 \rangle_{\ell=1}$
$= 1 + \eta \# g^4$
$\langle Q_1 Q_2 \rangle = \delta_{12}$
$n=1,\ n=2, \ldots$
$\mu = \ldots$
$g^4 \frac{1}{\cosh^2 \pi u} Q_1 Q_2 + \ldots$
non-BPS
$= C_{123} = \frac{\langle Q, 1 \rangle_{\ell=1}}{\sqrt{\langle Q, Q \rangle_{L=2}}}$
$\eta = 1 \quad \ell_B > 1$
$= 2 \quad \ell_B = 1$
$\frac{1}{\cosh^2 \pi u} + g^2 \cdot \nu + g^4 (?)$
$B =$
$\int du\, \hat{\mu}(u) Q(u) Q(u)$
= Simple × Gaudin
ℓ_B
O_1
$\ell = \ell_{13}$
$\ell_B = \ell_{23}$
$\frac{L_1 + L_3 - L_2}{2}$