Title: An explicit solution

Speakers: Paul Ryan

Collection: Quantum Spectral Curve and Three Point Functions mini-course

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* Symmetries

$$\tilde{P}_a = \mu_{ab} P^b \, , \quad P^b = \chi^{bc} P_c \, , \quad \chi = \begin{pmatrix} & & & -1 \\ & & 1 & \\ & -1 & & \\ 1 & & & \end{pmatrix}$$

$$\mu_{ab}(u+i) - \mu_{ab}(u) = P_a \tilde{P}_b - \tilde{P}_a P_b$$

$$P_a \to H_a{}^b P_b \, , \quad \mu \to H^{-1} \mu \chi H$$

$P_c \,, \quad X = \begin{pmatrix} & & & -1 \\ & & 1 & \\ & -1 & & \\ 1 & & & \end{pmatrix}$

$P_a P_b$

$X H$

$A_1 A_4 \quad (\Delta, L, S)$

$A_2 A_3 = (\quad)$

$L = 2 \,, \quad P_a \sim (u^{-2}, u^{-1}, u^{0}, u)$

$$H = \begin{pmatrix} a & & & \\ & b & & \\ & & b^{-1} & \\ & & & a^{-1} \end{pmatrix}$$

* Symmetries

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$$\mu_{ab}(u+i) - \mu_{ab}(u) = P_a \widetilde{P}_b - \widetilde{P}_a P_b$$

$$P_a \to H_a{}^b P_b \,, \quad \mu \to H^{-1} \mu \chi H$$

$A_1 A_4$ (Δ, L, S)

$A_2 A_3 = (\quad)$

$L = 2$, $P_a \sim (u^{-2}, u^{-1}, u^0, u)$

$$H = \begin{pmatrix} a & & & \\ & b & & \\ & & b^{-1} & \\ & & & a^{-1} \end{pmatrix}$$

* Explicit solution

$\mathcal{O} = \mathrm{Tr}[D^S Z^L]$ + perms   small S

Analytically continue in S

$$\Delta = L + S + \gamma(S,g)$$

$$\gamma(S,g) = S\,\gamma^{(1)}(g) + O(S^2)$$

$$g = \frac{\sqrt{\lambda}}{4\pi} \qquad \gamma^{(1)}(g) = \frac{\sqrt{\lambda}}{L}\,\frac{I_{L+1}(\sqrt{\lambda})}{I_L(\sqrt{\lambda})}$$

$$A_2 A_3 = (\quad)$$

$$A_1 A_4 = \frac{-i}{16L(L+1)} (-\Delta + L - S + 2)(-\Delta + L + S)(\Delta + L - S + 2)(\Delta + L + S)$$

$$\Delta = L + S + \gamma^{(1)} S$$

$$A_1 A_4 \sim (2 - \gamma^{(1)} S)(\gamma^{(1)} S)(2L + 2 + \gamma^{(1)} S)(2L + 2S + \gamma^{(1)} S)$$

$$\sim \# S + O(S^2)$$

$$P_1 \sim \sqrt{S},\ P_4 \sim \sqrt{S} \longrightarrow P_2, P_3 \sim \sqrt{S}$$

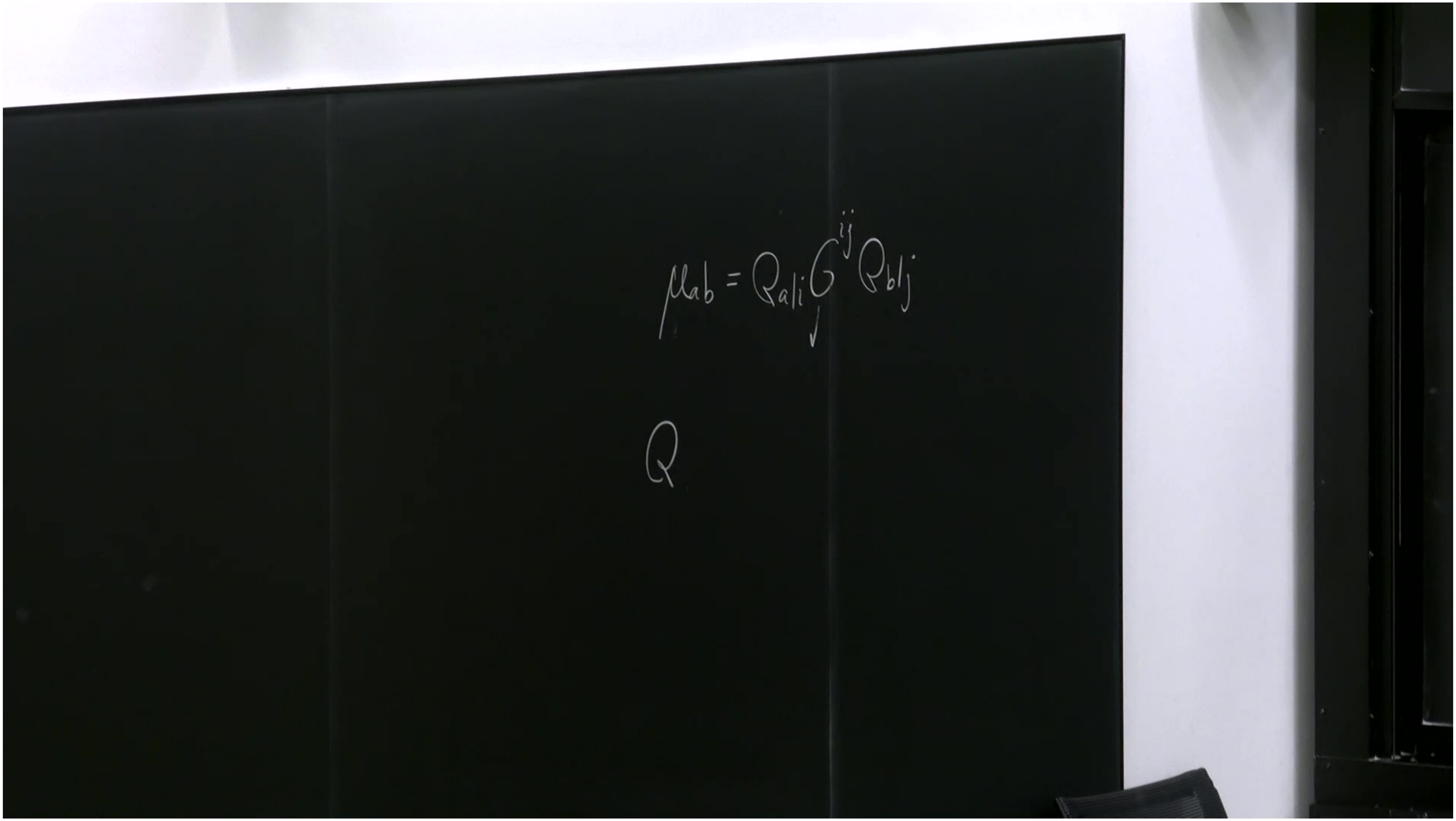
$$\mu_{ab}(u+\epsilon)-\mu_{ab}(u) = P_a \check{P}_b - \tilde{P}_a P_b$$

$$= O(\epsilon) \Rightarrow \mu = \mu^{(0)} + \mu^{(1)}\epsilon + O(\epsilon^2)$$

$\mu^{(0)}$ periodic $\Rightarrow$ const

Allow μ to have exponential

$$\tilde{P}_a = \underset{\text{const.}}{\mu_{ab}} P^b$$


$\mu_{ab} = Q_{a|i} G^{ij} Q_{b|j}$
Q

$$\mu_{ab} = Q_{a|i}\, G^{ij}\, Q_{b|j}$$

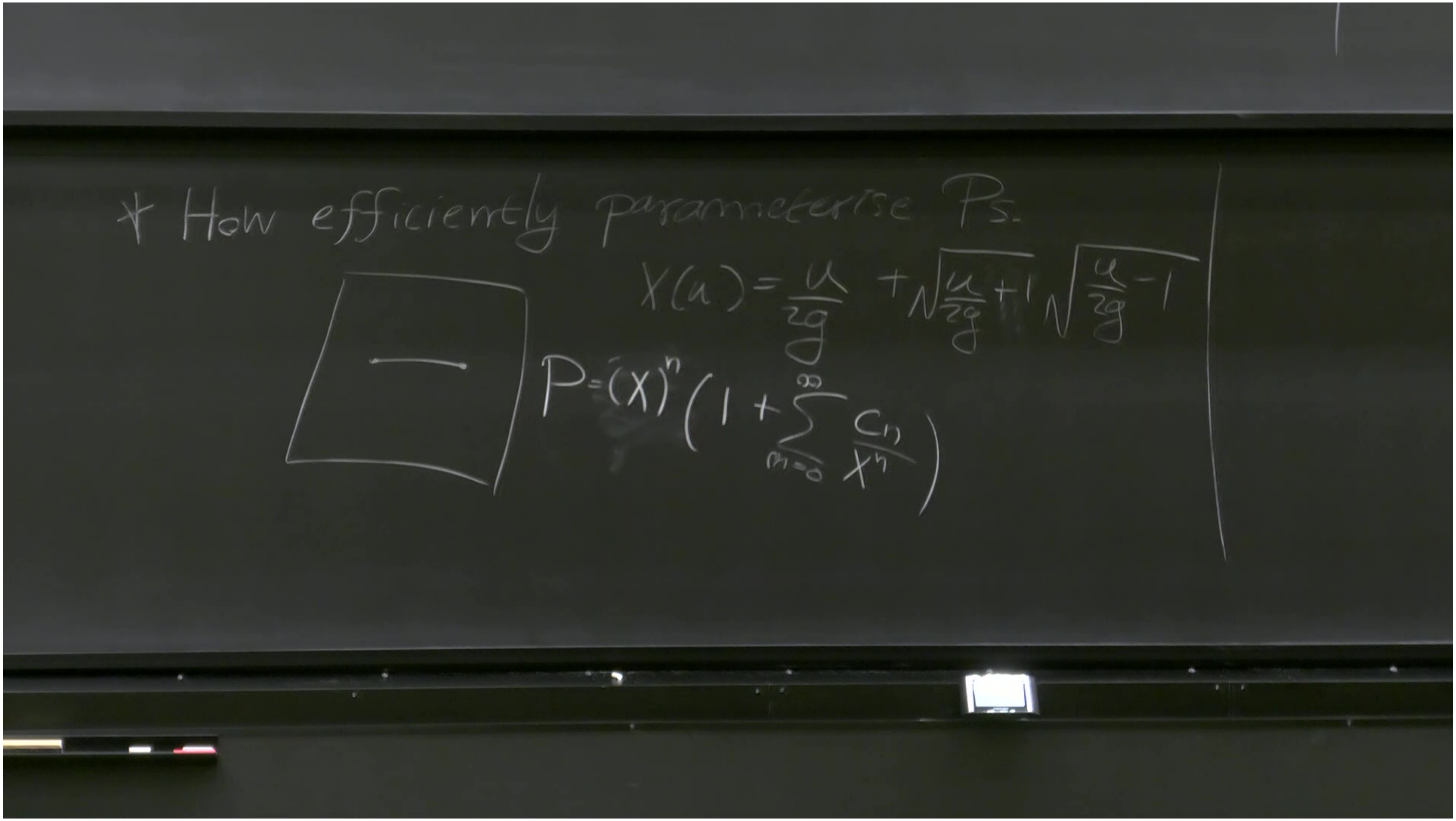
Q

$\sinh(2\pi u)(1-e^{i\pi s})$

Q

$$\tilde{P}_a = \mu_{ab} P^b$$

$$\tilde{P}_4 = P_2, \quad \tilde{P}_1 = P_3 + \sinh(2\pi u) P_2$$


* How efficiently parameterise Ps.
$x(u) = \frac{u}{2g} + \sqrt{\frac{u}{2g}+1}\sqrt{\frac{u}{2g}-1}$
$P = (x)^n \left(1 + \sum_{n=0}^{\infty} \frac{c_n}{x^n}\right)$

...se Ps.

$$+\sqrt{\frac{u}{2g}+1}\sqrt{\frac{u}{2g}-1}$$

$$X \sim \frac{u}{g}$$

$$P_4 \sim A_4 (gX)\left(1+\sum_{n=1}^{\infty}\frac{C_{n,4}}{X^n}\right) \qquad \tilde{X} = \frac{1}{X}$$

$$P_2 \sim A_2 \frac{1}{(gX)}\left(1+\sum_{n=1}^{\infty}\frac{C_{n,2}}{X^n}\right)$$

$$\tilde{P}_4 = A_4 \frac{g}{X}\left(1+\sum_{n=1}^{\infty} C_{n,4} X^n\right) \rightarrow C_{n,2} = C_{n,4} = 0$$

$$\Rightarrow P_4 = A_4 g X,\quad P_2 = \frac{A_2}{gX} \Rightarrow A_4 g^2 = A_2$$

$$\tilde{P}_4 = A_4 g / X =$$

$$u = g\left(x + \frac{1}{x}\right)$$

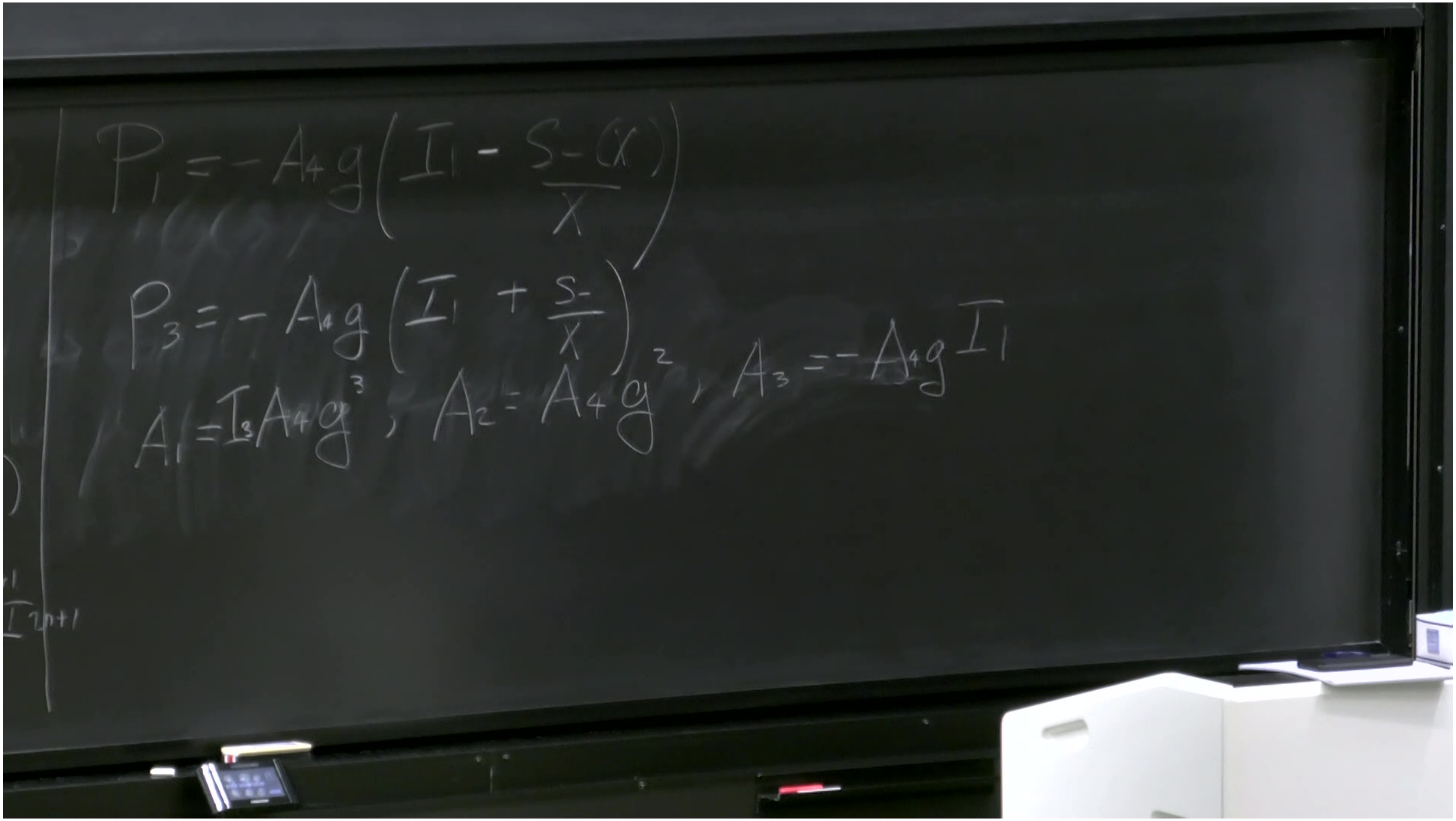
$$\exp(2\pi u) = \exp\left(2\pi g\left(x + \frac{1}{x}\right)\right)$$

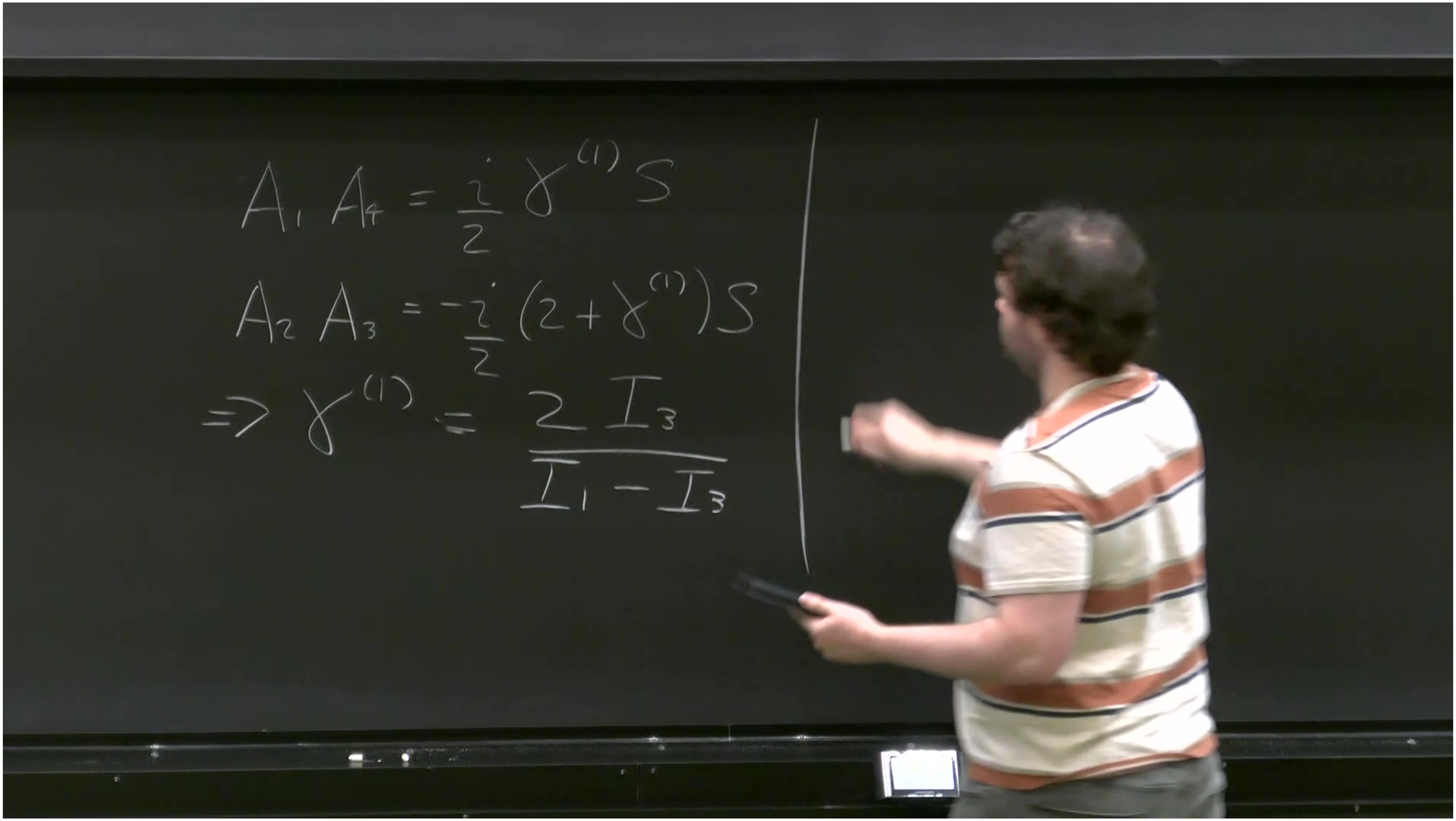
$$= \sum_{n=-\infty}^{\infty} I_n(4\pi g)\, x^n$$

$$\Rightarrow \sinh(2\pi u) = \sum_{n=-\infty}^{\infty} I_{2n+1}\, x^{2n+1} = S_-(x) + S_+(x)$$

$$S_+(x) = \sum_{n=0}^{\infty} x^{2n+1} I_{2n+1}$$

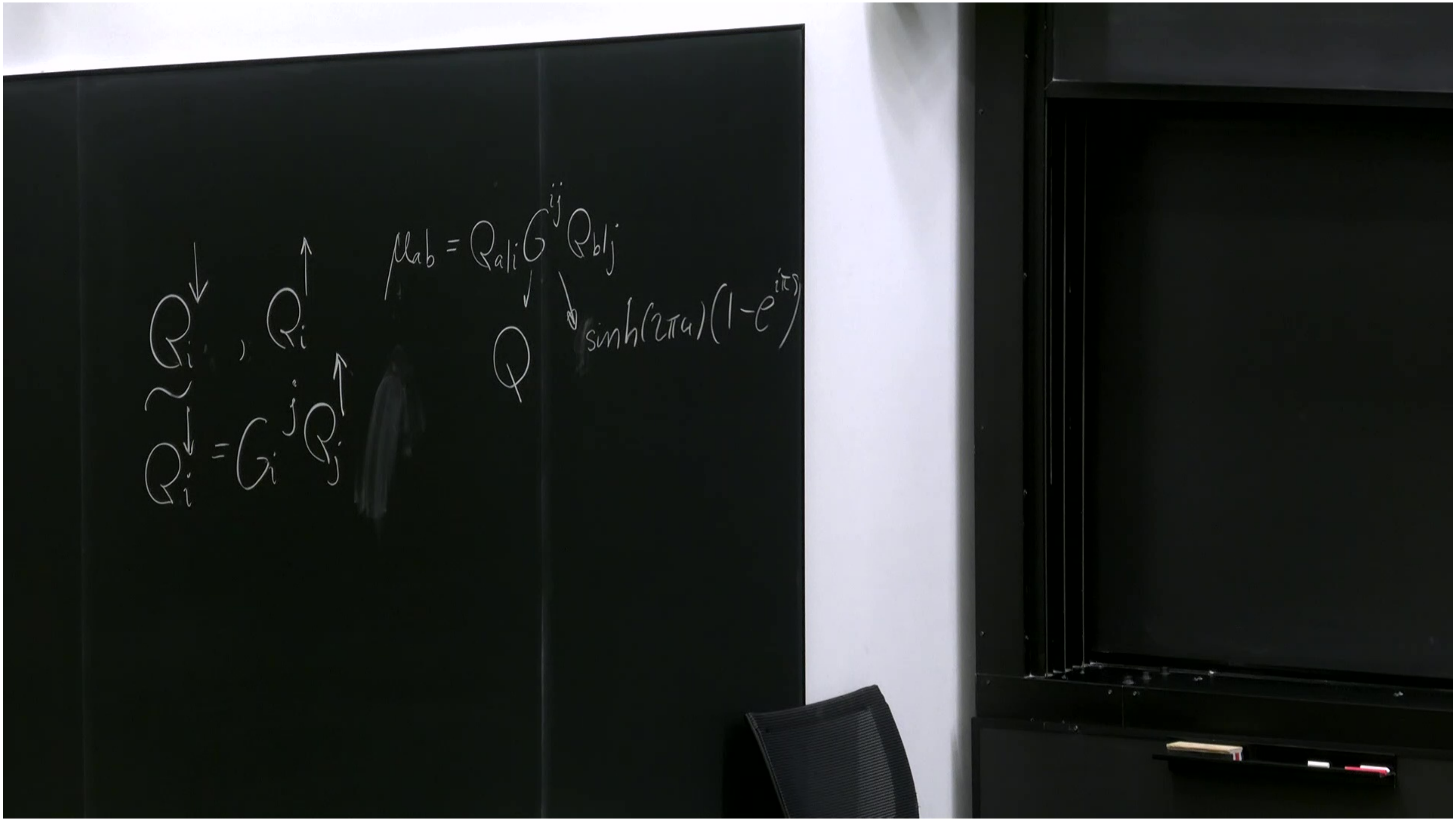
$$S_-(x) = \sum \frac{1}{x^{2n+1}} I_{2n+1}$$


$P_1 = -A_4 g \left(I_1 - \frac{S_-(X)}{X} \right)$
$P_3 = -A_4 g \left(I_1 + \frac{S_-}{X} \right)$
$A_1 = I_3 A_4 g^3 , \quad A_2 = A_4 g^2 , \quad A_3 = -A_4 g I_1$


$A_1 A_4 = \frac{i}{2} \gamma^{(1)} S$
$A_2 A_3 = -\frac{i}{2} (2 + \gamma^{(1)}) S$
$\Rightarrow \gamma^{(1)} = \frac{2 I_3}{I_1 - I_3}$

$$z\left(I_1(z) - I_3(z)\right) = 4\, I_2(z)$$

$$\rightarrow \gamma^{(1)} = 2g\pi \frac{I_3(4g\pi)}{I_1(4g\pi)}$$


$\mu_{ab} = Q_{a|i} G^{ij} Q_{b|j}$
Q
$\sinh(2\pi u)(1-e^{i\pi})$
$Q_i^{\downarrow}$, $Q_i^{\uparrow}$
$\tilde{Q}_i^{\downarrow} = G_i^{\ j} Q_j^{\uparrow}$

* QSC – $\mathcal{N}=4$ SYM

– ABJM

– $AdS_3 \times S^3 \times T^4$ *

$P_1 =$

$P_3 =$

$A_1 =$

A_4

* QSC - $\mathcal{N}=4$ SYM

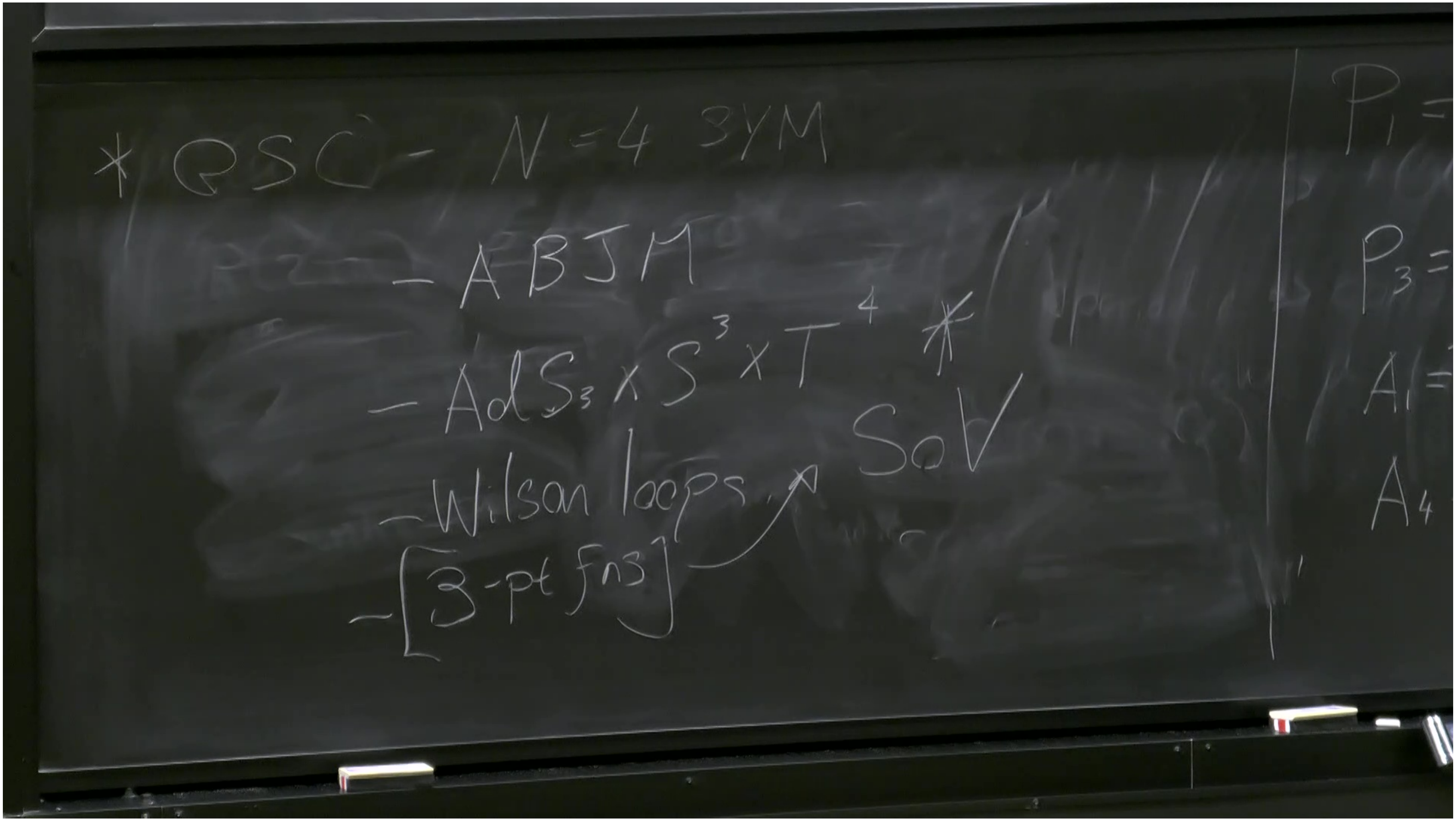
- ABJM
- $AdS_3 \times S^3 \times T^4$ *
- Wilson loops

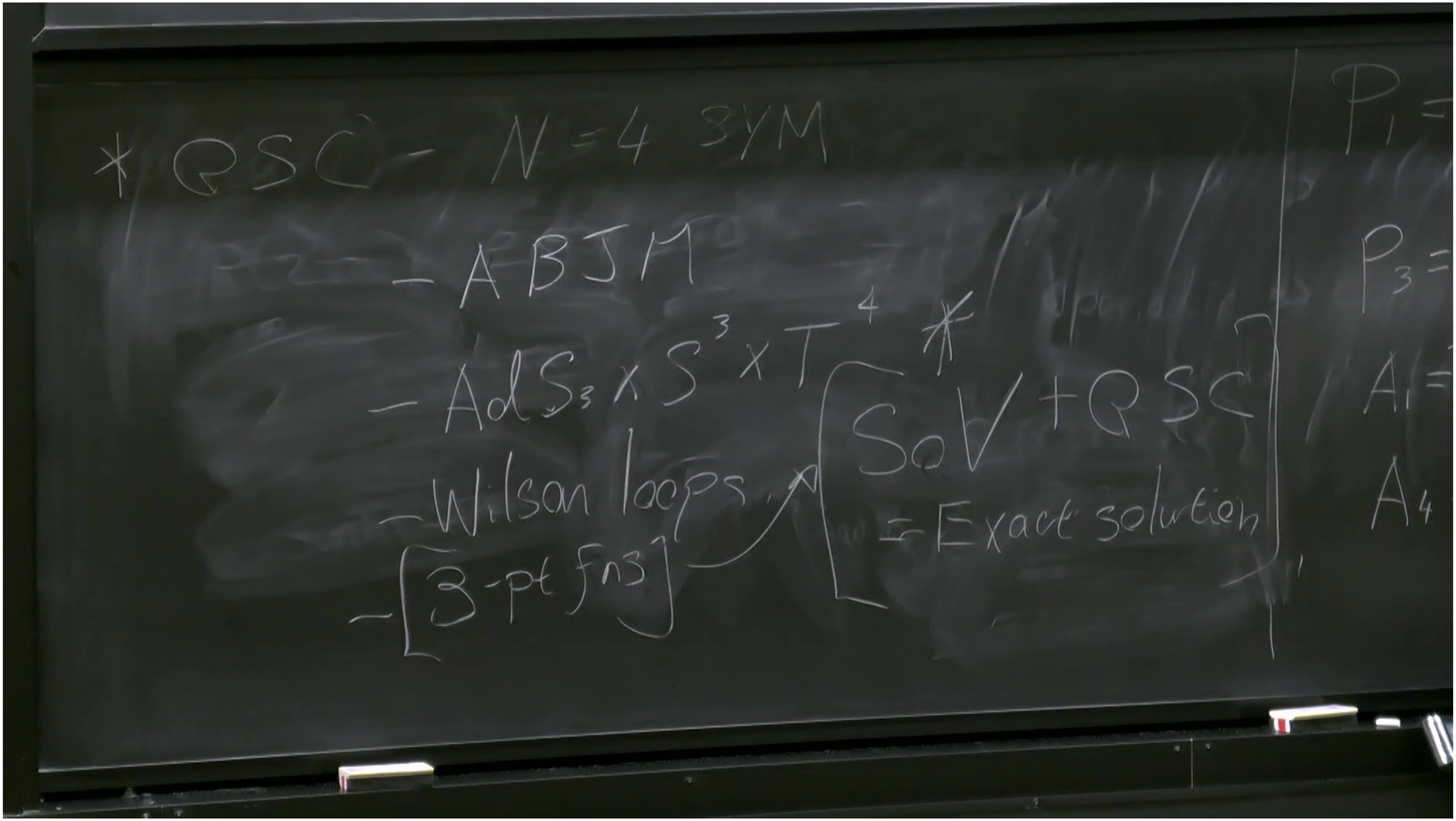
$P_1 =$

$P_3 =$

$A_1 =$

A_4


* QSC - $\mathcal{N}=4$ SYM
- ABJM
- $AdS_3 \times S^3 \times T^4$ *
- Wilson loops
- [3-pt fns]
SoV
$P_1 =$
$P_3 =$
$A_1 =$
A_4


* QSC - N=4 SYM
- ABJM
- AdS3 x S3 x T4
- Wilson loops
- [3-pt fns]
SoV + QSC
= Exact solution
P1 =
P3 =
A1 =
A4