Title: QSC definition in N=4

Speakers: Paul Ryan

Collection: Quantum Spectral Curve and Three Point Functions mini-course

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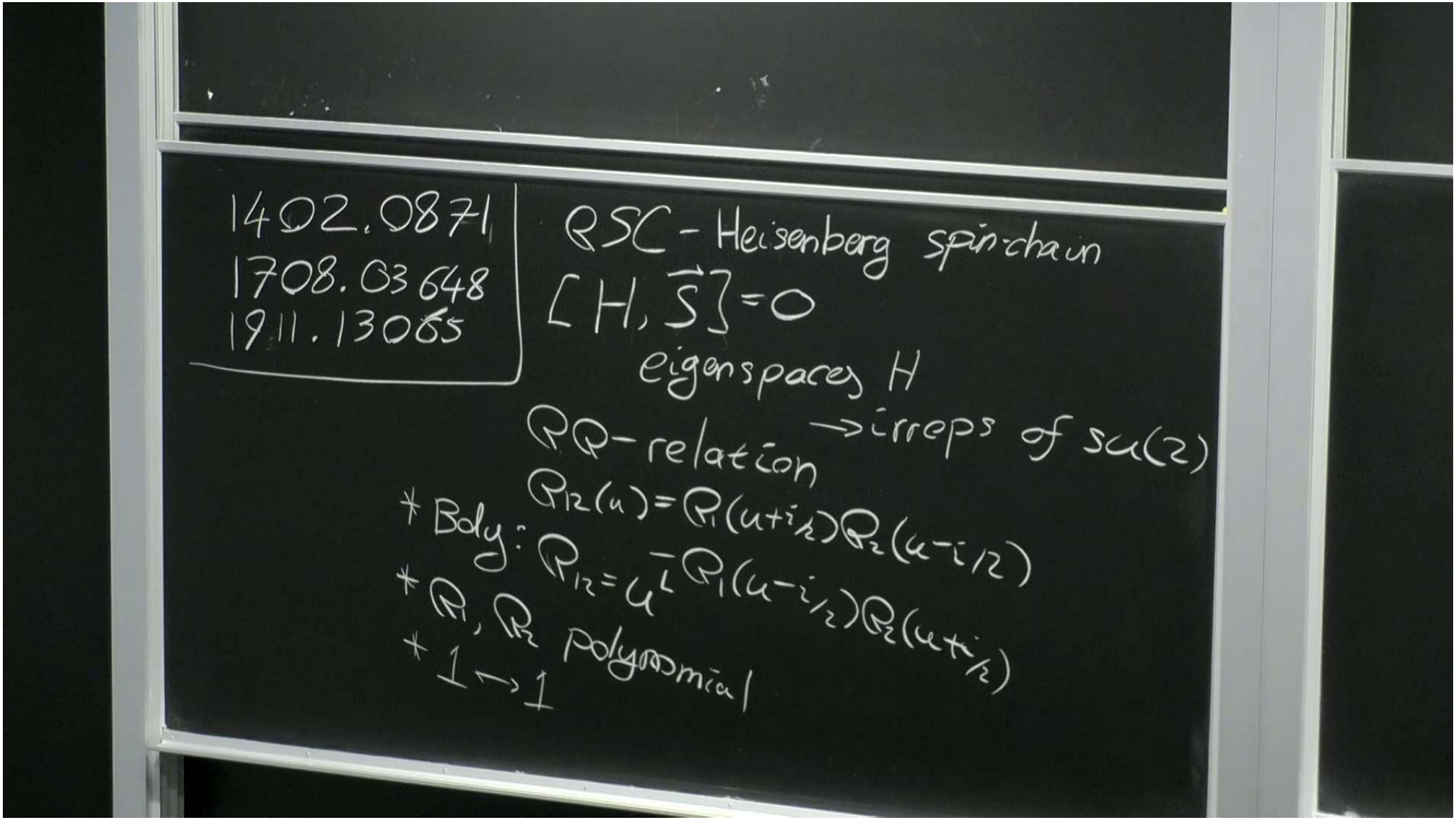
1402.0871
1708.03648
1911.13065
RSC - Heisenberg spin-chain
$[H, \vec{S}] = 0$
eigenspaces H
→ irreps of su(2)
QQ-relation
$R_{12}(u) = R_1(u+i/2) R_2(u-i/2)$
$R_{12} = u^{-1} R_1(u-i/2) R_2(u+i/2)$
* Body:
* R_1, R_2 polynomial
* 1 → 1

$SU(2)$ irrep $S_z \sim L/2 - S$

$Q_1 \sim A_1 u^S$, $Q_2 \sim A_2 u^{L+1-S}$

QSC for planar $N=4$ SYM.

– $PSU(2,2|4)$

$\mathbb{R}$ form of $SL(4|4)$

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QSC for planar $N=4$ SYM.

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$\mathbb{R}$ form of $SL(4|4)$

$B = \{1,2,3,4\}$ $A \subset B$, $a, b \in B$

$F = \{1,2,3,4\}$ $I \subset F$, $i, j \in F$

$Q_{A|I}(u)$

$Q_{\phi|\phi}(u) = 1$

$su(2)$ irrep $S_z \sim L/2 - S$

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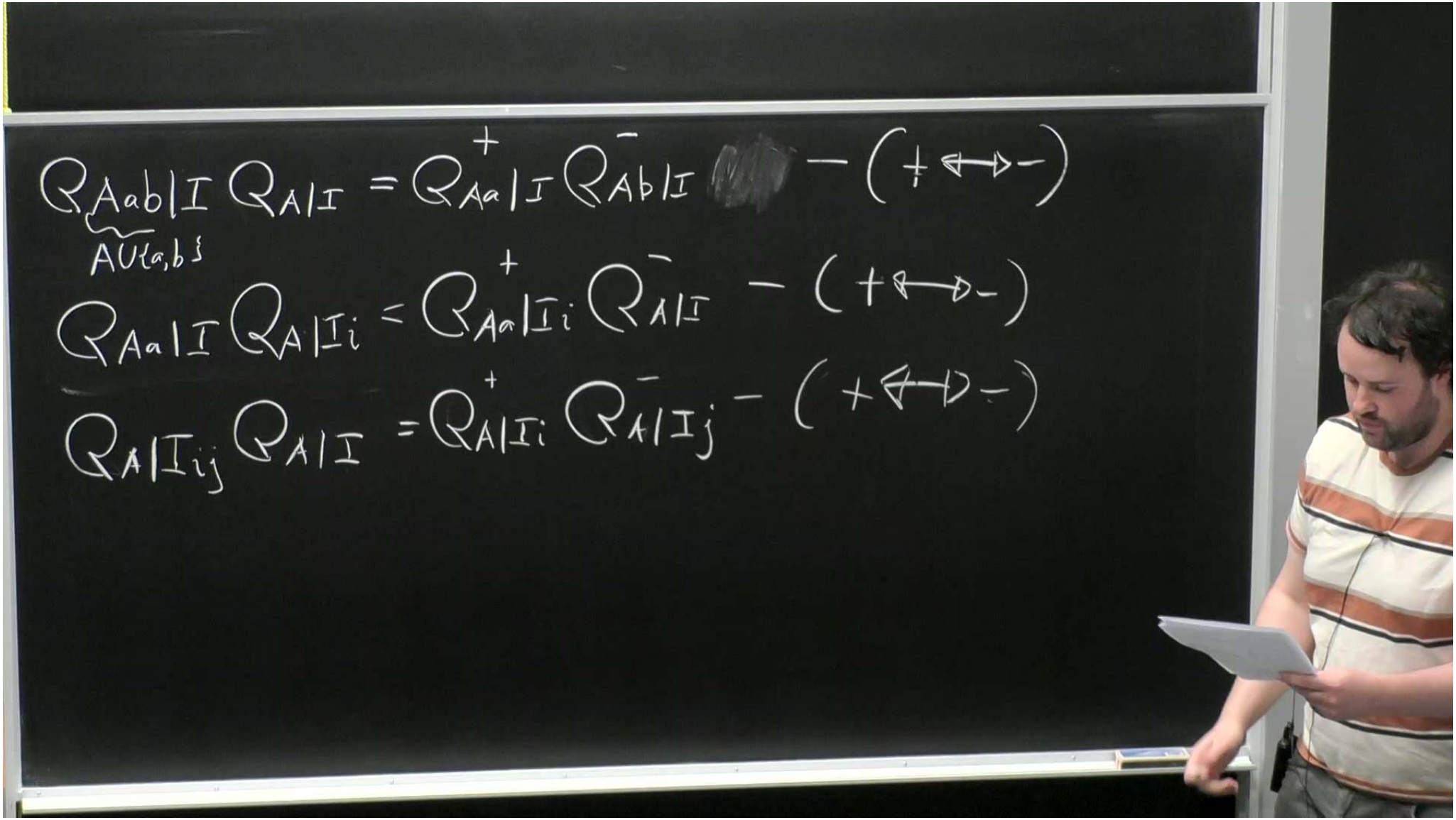
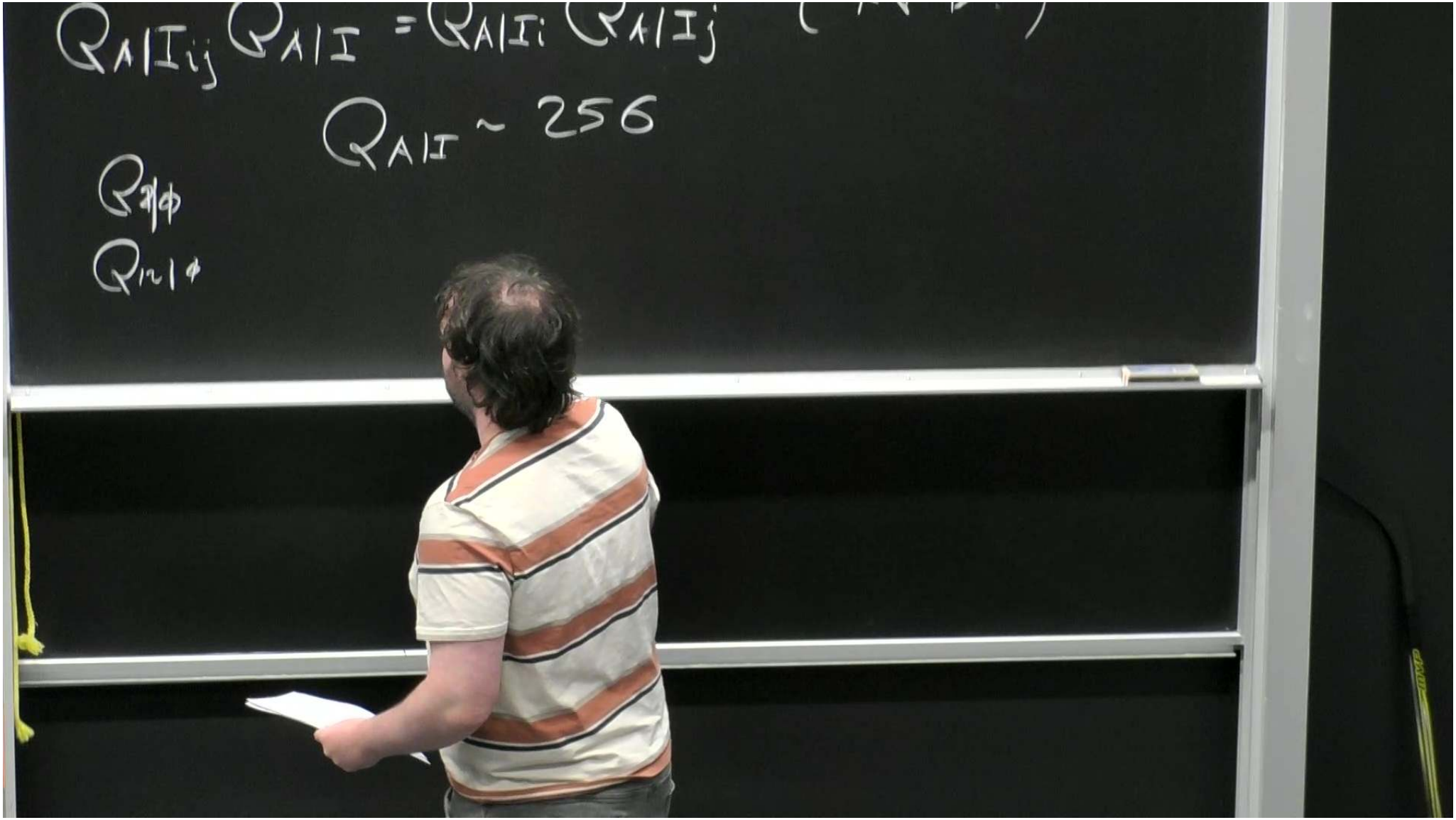
$Q_{A|I}(u)$

$Q_{\phi|\phi}(u) = 1$

$f^{\pm}(u) := f(u \pm i/2)$


$Q_{Aab|I}\, Q_{A|I} = Q^{+}_{Aa|I}\, Q^{-}_{Ab|I} - (+ \leftrightarrow -)$
$A \cup \{a,b\}$
$Q_{Aa|I}\, Q_{A|Ii} = Q^{+}_{Aa|Ii}\, Q^{-}_{A|I} - (+ \leftrightarrow -)$
$Q_{A|Iij}\, Q_{A|I} = Q^{+}_{A|Ii}\, Q^{-}_{A|Ij} - (+ \leftrightarrow -)$


$Q_{A|I_{ij}} Q_{A|I} = Q_{A|I_i} Q_{A|I_j}$
$Q_{A|I} \sim 256$
$Q_{2|\phi}$
$Q_{1\wedge 2|\phi}$

$P_a = Q_{a|4}$, $a = 1,2,3,4$.

$Q_i = Q_{4|i}$, $i = 1,2,3,4$.

$Q^+_{a|i} - Q^-_{a|i} = P_a Q_i$

$PSU(2,2|4)$ → special bdy cond.

$Q_{1234|1234} = 1$

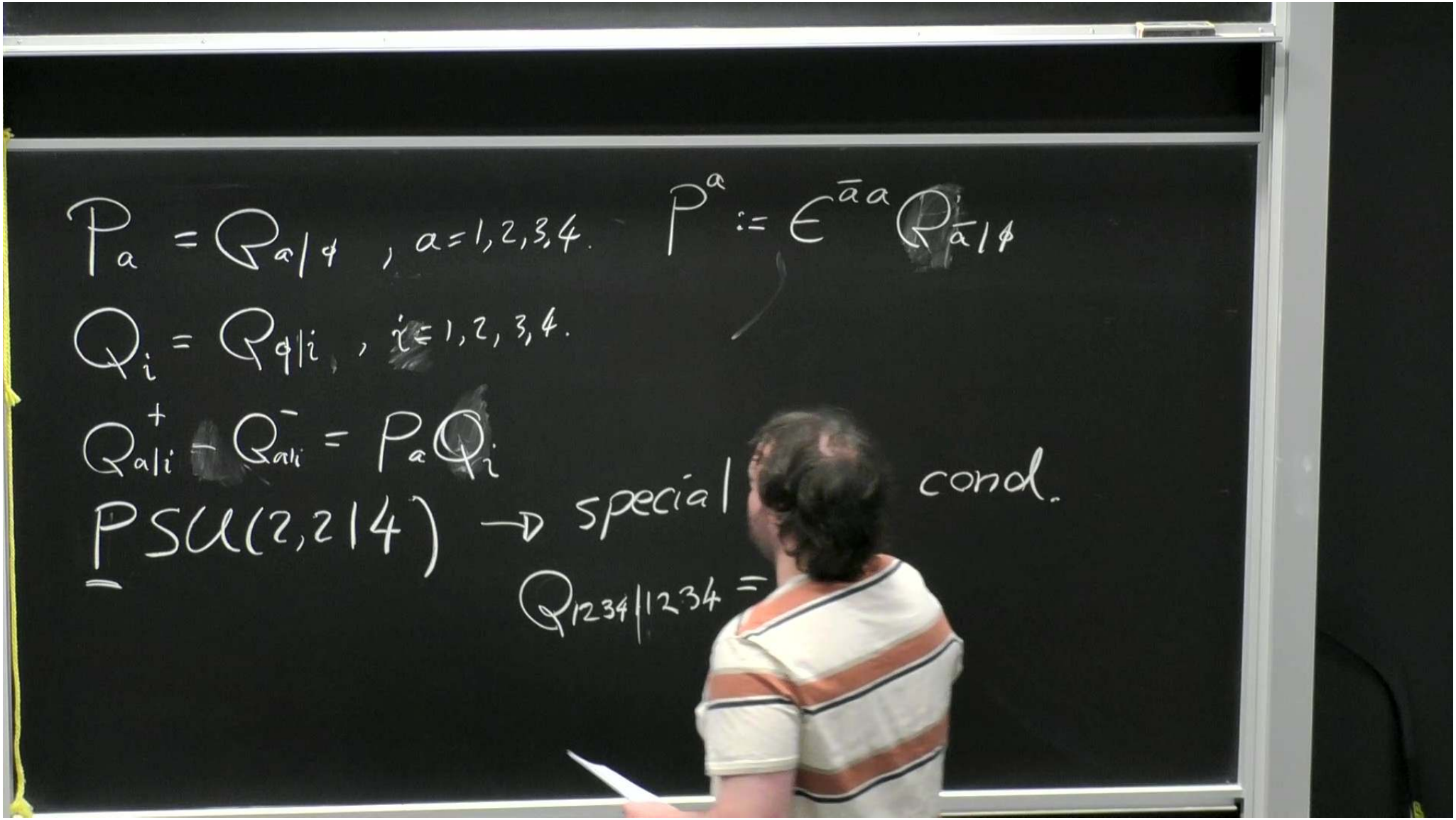
$P_a = Q_{a|4}$, $a = 1,2,3,4$. $P^a := \epsilon^{\bar{a}a} Q^{\dagger}_{\bar{a}|4}$

$Q_i = Q_{4|i}$, $i = 1,2,3,4$.

$Q^{+}_{a|i} - Q^{-}_{a|i} = P_a Q_i$

$\underline{P}SU(2,2|4) \rightarrow$ special cond.

$Q_{1234|1234} =$



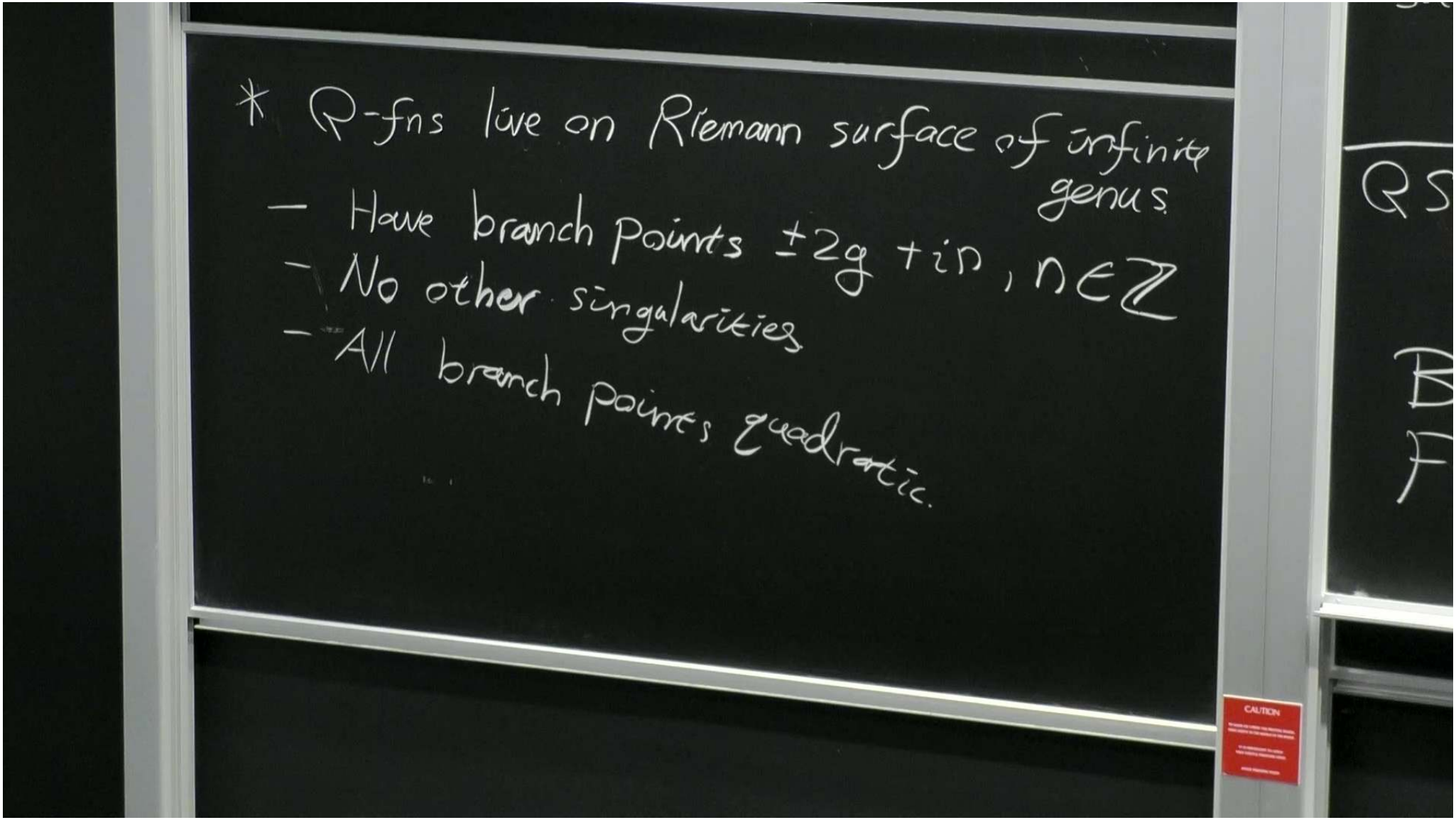
$P_a = Q_{a|4}$, $a = 1,2,3,4$. $\quad P^a := \epsilon^{\bar{a}a} Q^i_{\bar{a}|4}$

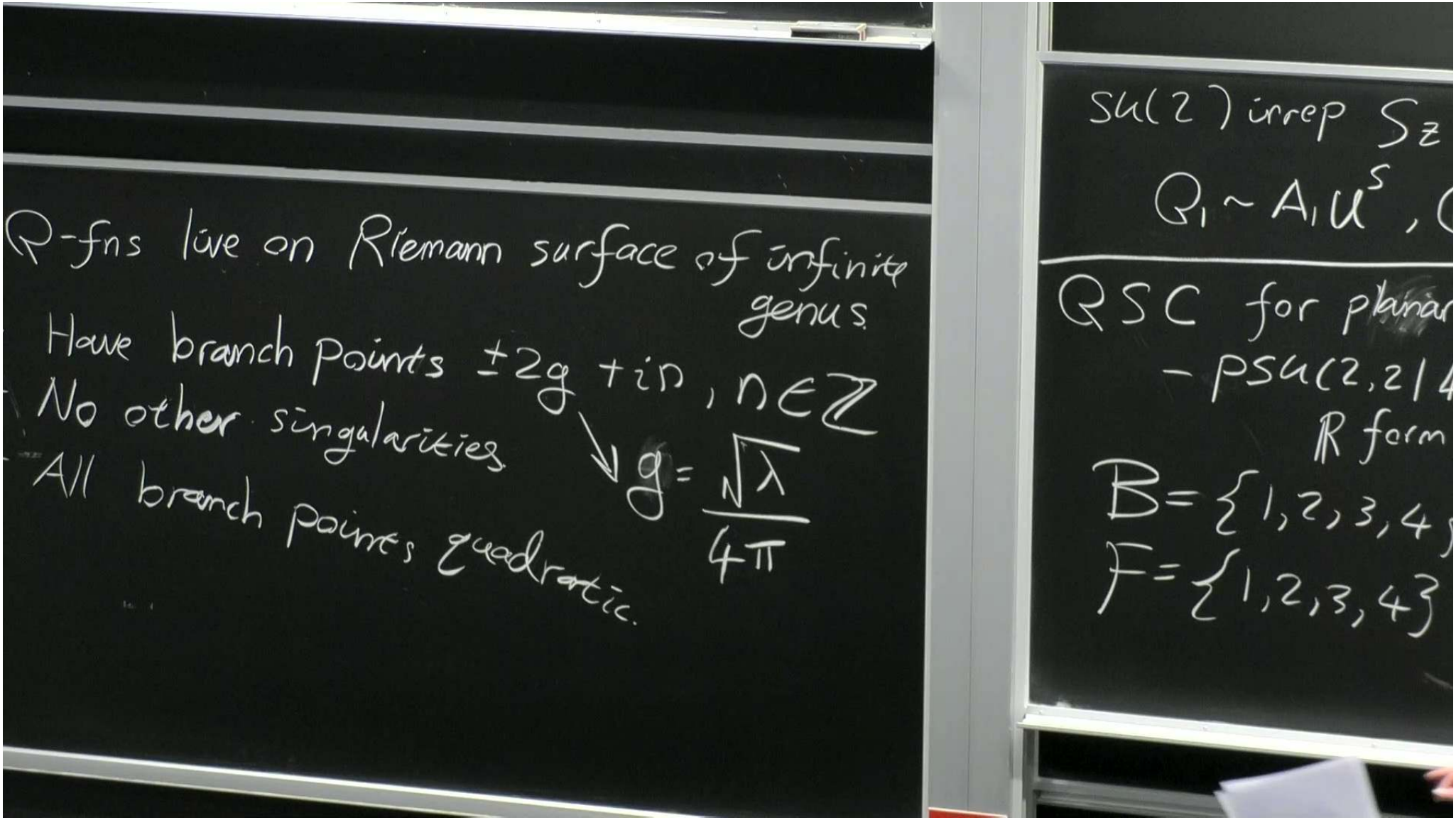
$Q_i = Q_{4|i}$, $i = 1,2,3,4$.

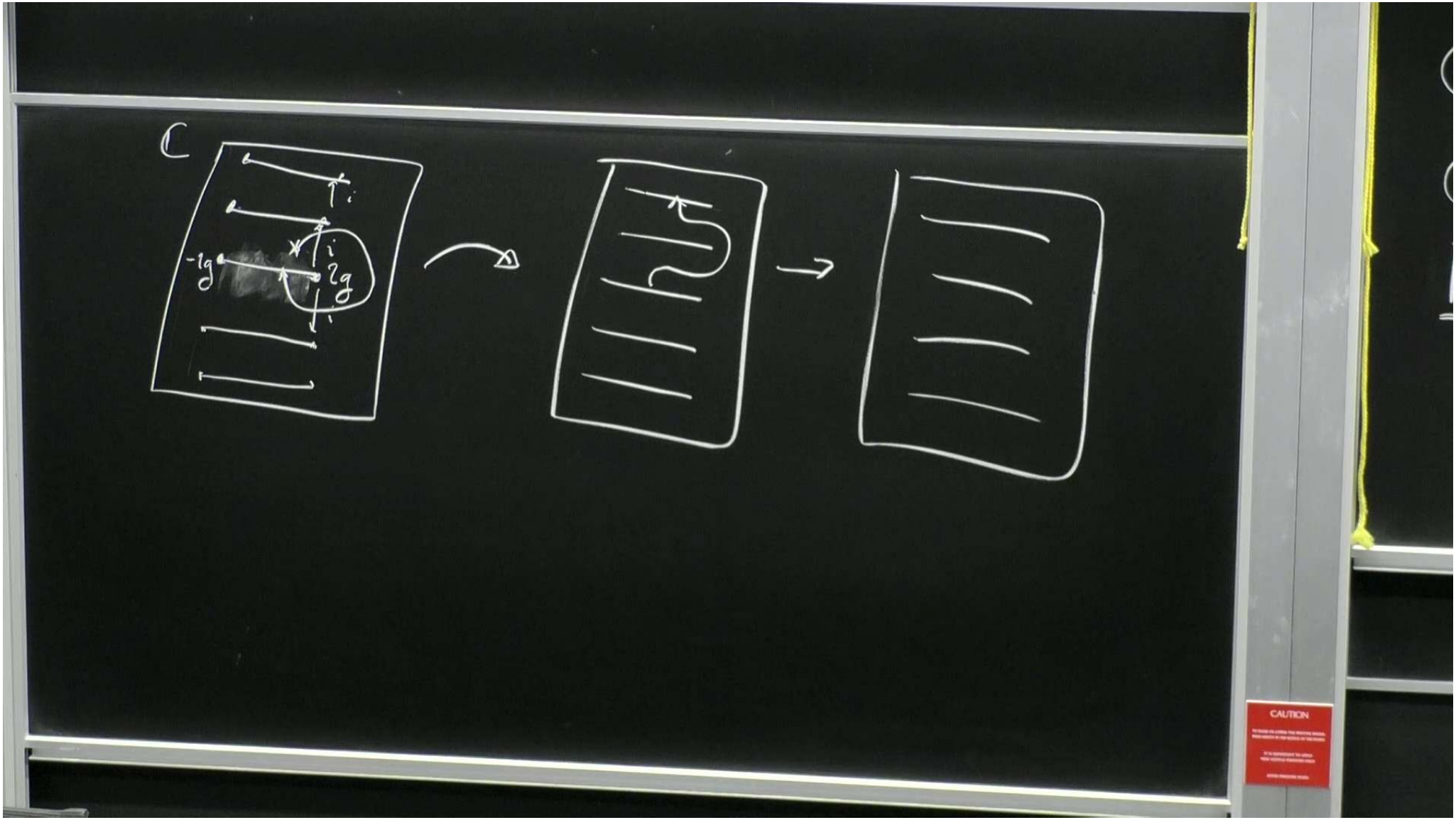
$Q^+_{a|i} - Q^-_{a|i} = P_a Q_i$

$\underline{P}SU(2,2|4) \rightarrow$ special bdy cond.

$Q_{1234|1234} = 1 \Rightarrow Q_i = \sum_a Q^{\pm}_{a|i} P^a$

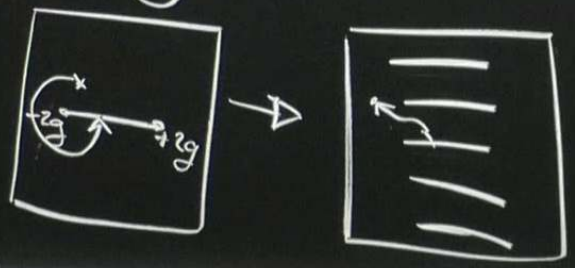

* Q-fns live on Riemann surface of infinite genus
- Have branch points $\pm 2g + in$, $n \in \mathbb{Z}$
- No other singularities
- All branch points quadratic.


Q-fns live on Riemann surface of infinite genus
Have branch points ±2g + in, n∈ℤ
No other singularities
All branch points quadratic.
g = √λ / 4π
su(2) irrep Sz
Q1 ~ A1 u^s
QSC for planar
- PSU(2,2|4
ℝ form
B = {1,2,3,4
F = {1,2,3,4}


C
-1g
2g
CAUTION

Special sheet for P_a

"Defining sheet" – only have single cut.

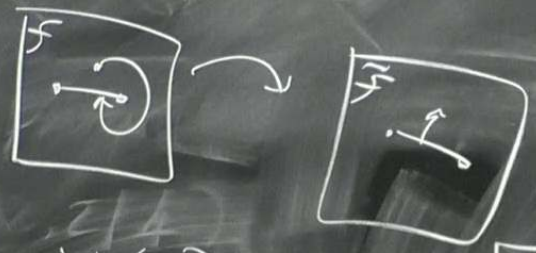


Q_i +

Q_{ali}

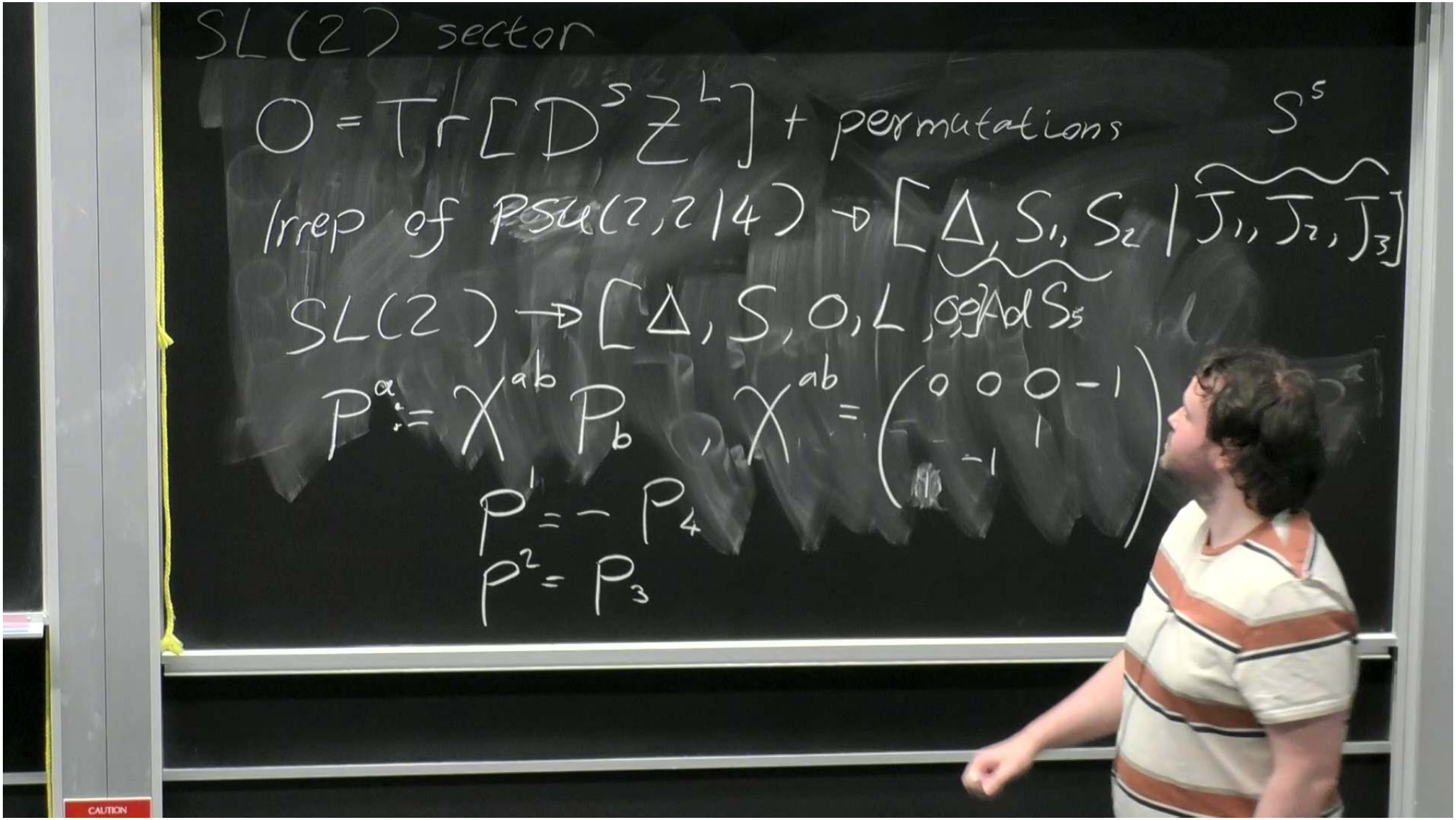
P_S

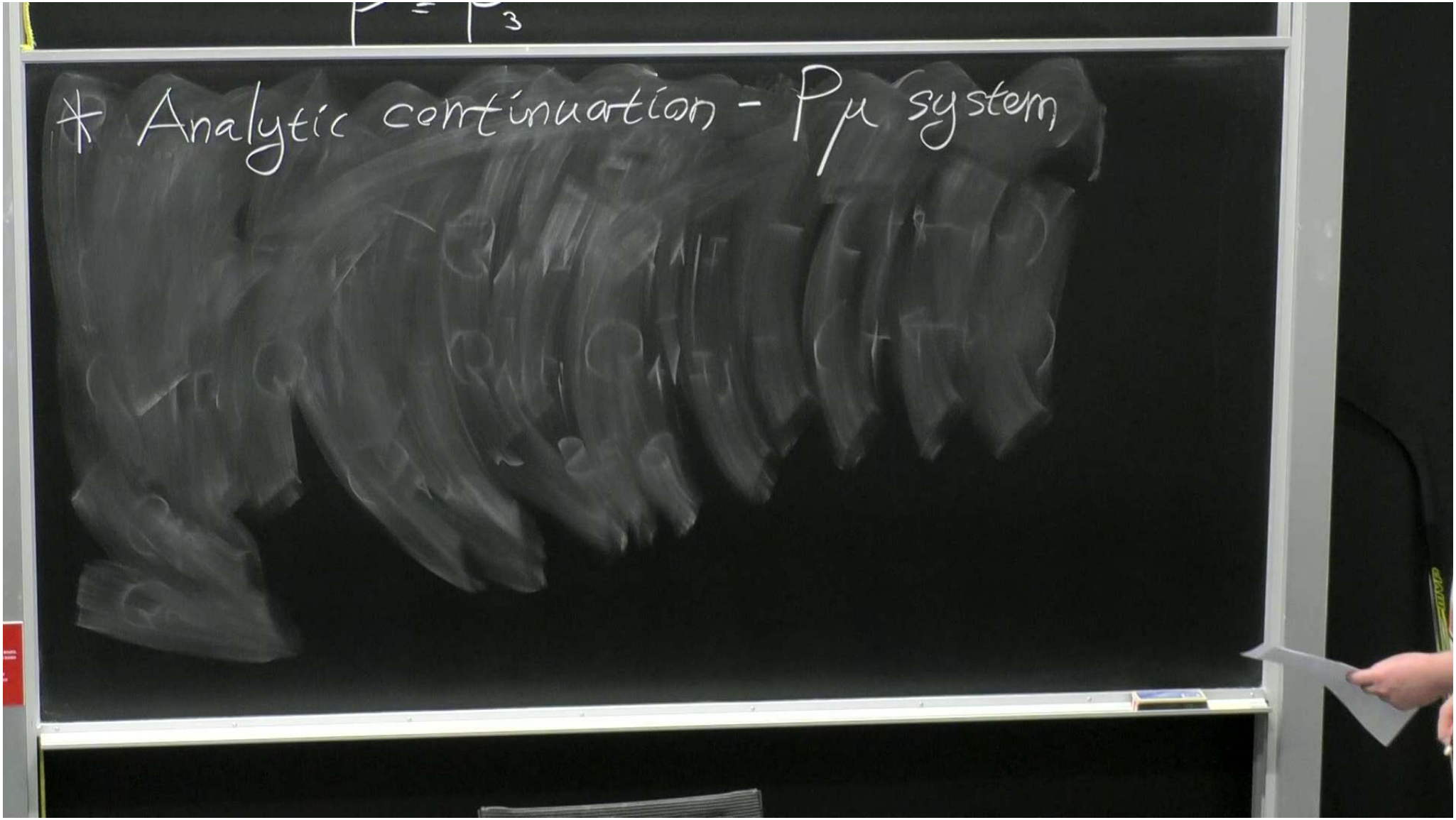
* $f(u)$ with branch points $\pm 2g$

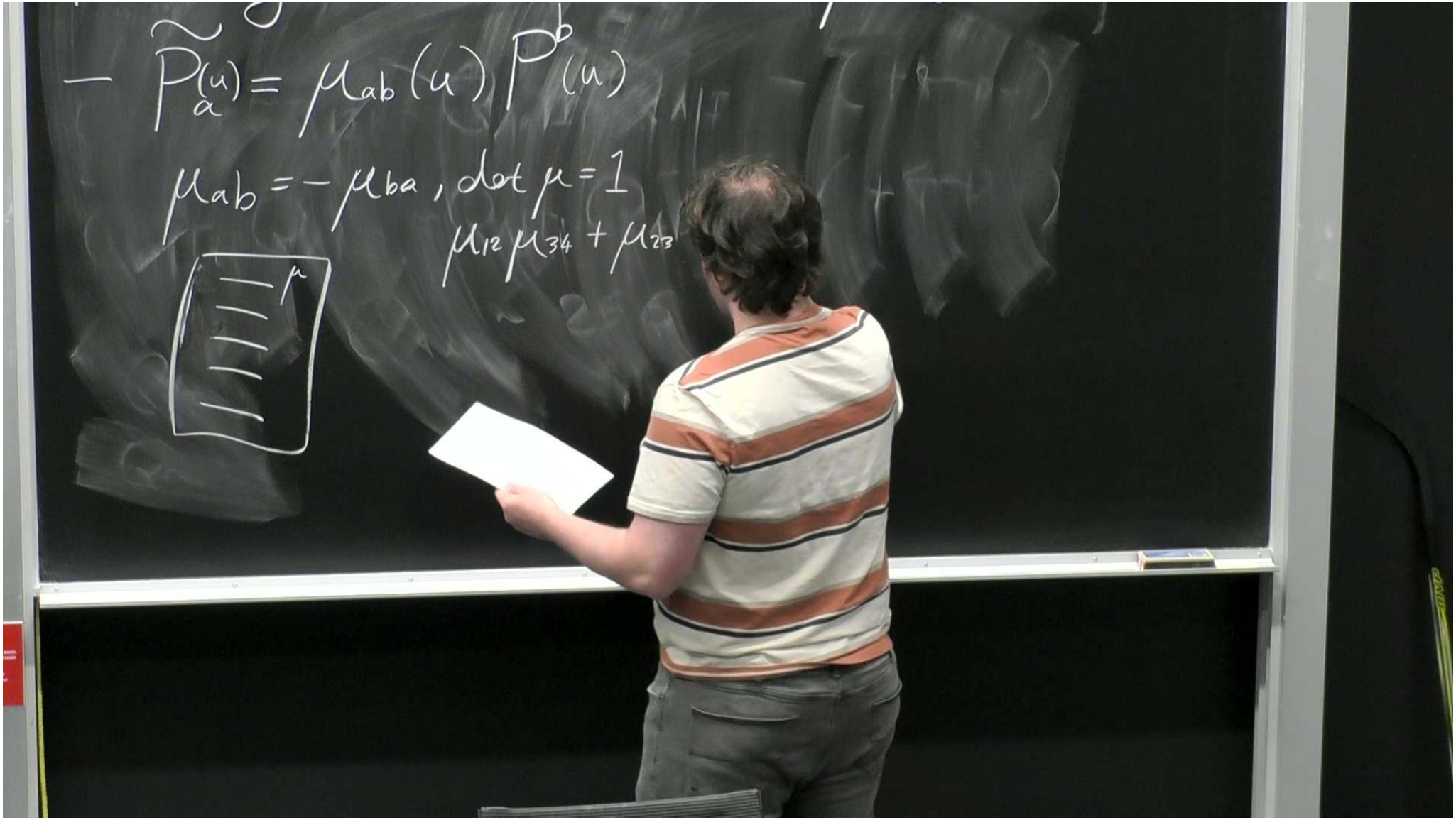
$\widetilde{f(u)}$ 

* Example: $X(u) = \frac{u}{2g} + \sqrt{\frac{u}{2g}+1}\sqrt{\frac{u}{2g}-1}$

$\widetilde{X} = \frac{1}{X}$


SL(2) sector
$O = Tr[D^S Z^L]$ + permutations
S^5
Irrep of $PSU(2,2|4) \to [\Delta, S_1, S_2 | J_1, J_2, J_3]$
$SL(2) \to [\Delta, S, 0, L, \infty]_{AdS_5}$
$P^a_{\cdot} = \chi^{ab} P_b$, $\chi^{ab} = \begin{pmatrix} 0 & 0 & 0 & -1 \\ & & 1 & \\ & -1 & & \\ & & & \end{pmatrix}$
$P^1 = -P_4$
$P^2 = P_3$
CAUTION


* Analytic continuation - Pμ system


$\mu_{ab} = -\mu_{ba}$, $\det \mu = 1$
$\mu_{12}\mu_{34} + \mu_{23}$
μ

$-\mu_{ab} = -\mu_{ba}$, set $\mu = 1$

$$-\mu_{12}\mu_{34} + \mu_{23}\mu_{14} - \mu_{13}\mu_{24} = 1$$

$$-\widetilde{\mu_{ab}(u)} = \mu_{ab}(u+i)$$

$$\Rightarrow \mu_{ab}(u+i) - \mu_{ab}(u) = P_a \widetilde{P}_b - P_b \widetilde{P}_a$$

* Analytic continuation - Pμ system

- $\widetilde{P}_a^{(u)} = \mu_{ab}(u) P^b(u)$

- $\mu_{ab} = -\mu_{ba}$, $\det \mu = 1$

$$-\mu_{12}\mu_{34} + \mu_{23}\mu_{14} - \mu_{13}\mu_{24} = 1$$

- $\widetilde{\mu_{ab}(u)} = \mu_{ab}(u+i)$

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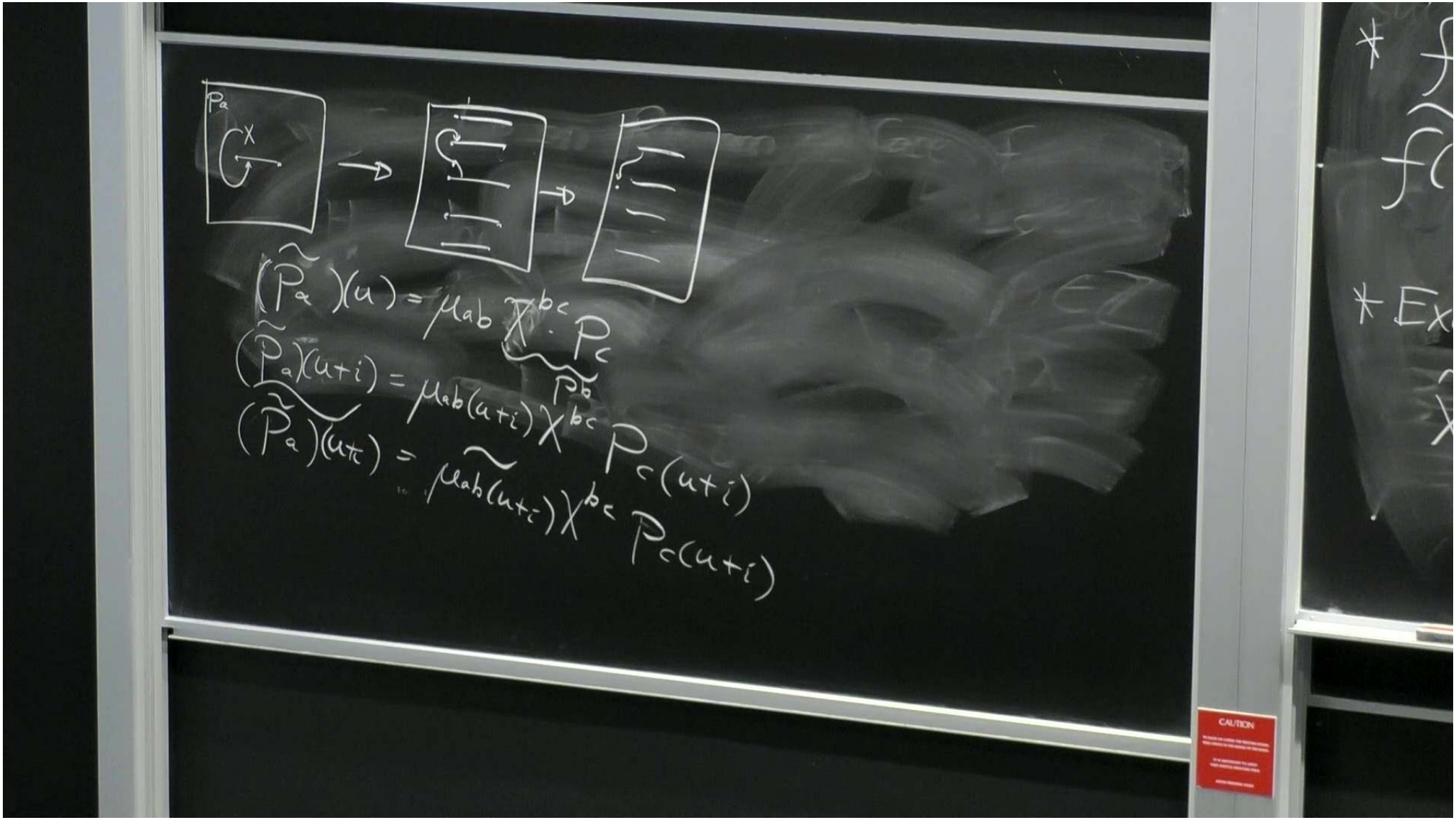
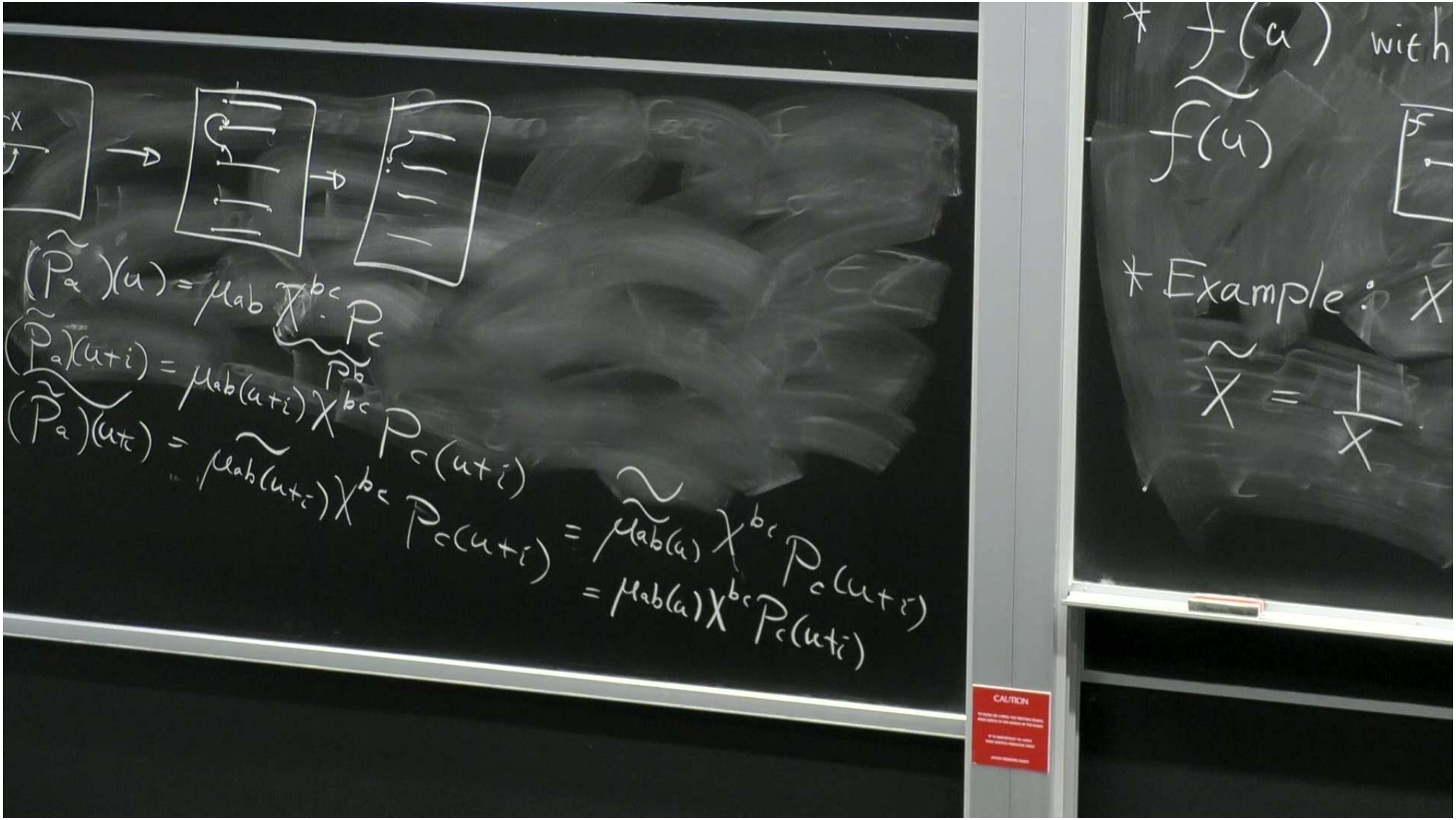

P_a
X
$(\widetilde{P_a})(u) = \mu_{ab} X^{bc} \cdot P_c$
$\widetilde{P_b}$
$(\widetilde{P_a})(u+i) = \mu_{ab(u+i)} X^{bc} P_c(u+i)$
$(\widetilde{P_a})(u+\epsilon) = \widetilde{\mu_{ab(u+\epsilon)}} X^{bc} P_c(u+i)$
CAUTION


$(\widetilde{P}_a)(u) = \mu_{ab} X^{bc} \cdot P_c$
$(\widetilde{P}_a)(u+i) = \mu_{ab(u+i)} X^{bc} P_c(u+i)$
$(\widetilde{P}_a)(u+\epsilon) = \widetilde{\mu_{ab(u+\epsilon)}} X^{bc} P_c(u+i) = \widetilde{\mu_{ab(u)}} X^{bc} P_c(u+i)$
$= \mu_{ab(u)} X^{bc} P_c(u+i)$
$\widetilde{f(u)}$
* Example:
$\widetilde{X} = \frac{1}{X}$
CAUTION

* Analytic continuation - Pμ system

$$Q^{+}_{ali} - Q^{-}_{ali} = P_a Q_i$$

$$\mu_{ab} = Q_{ali}\, G^{ik}\, Q_{blj}$$

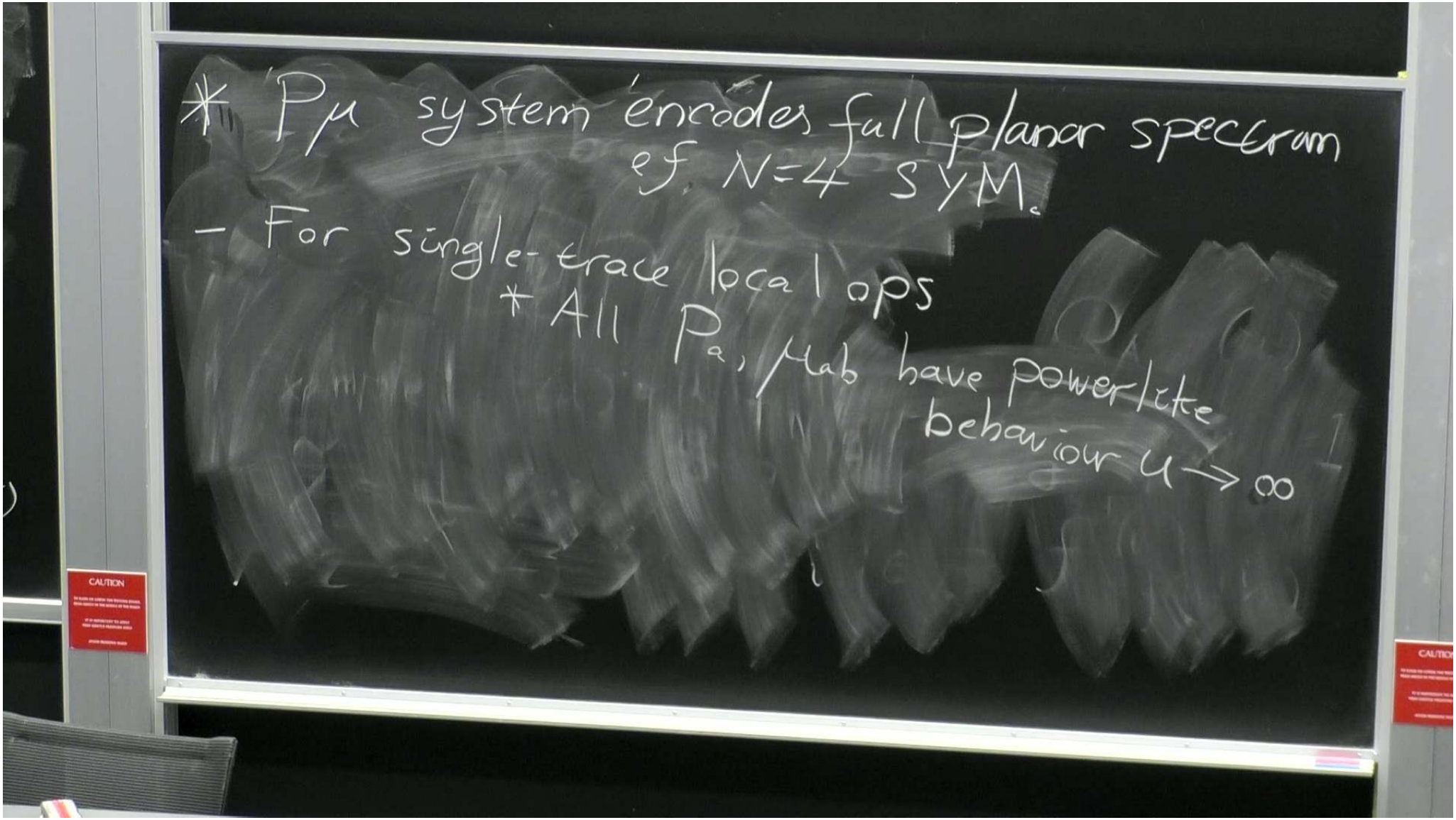
const.

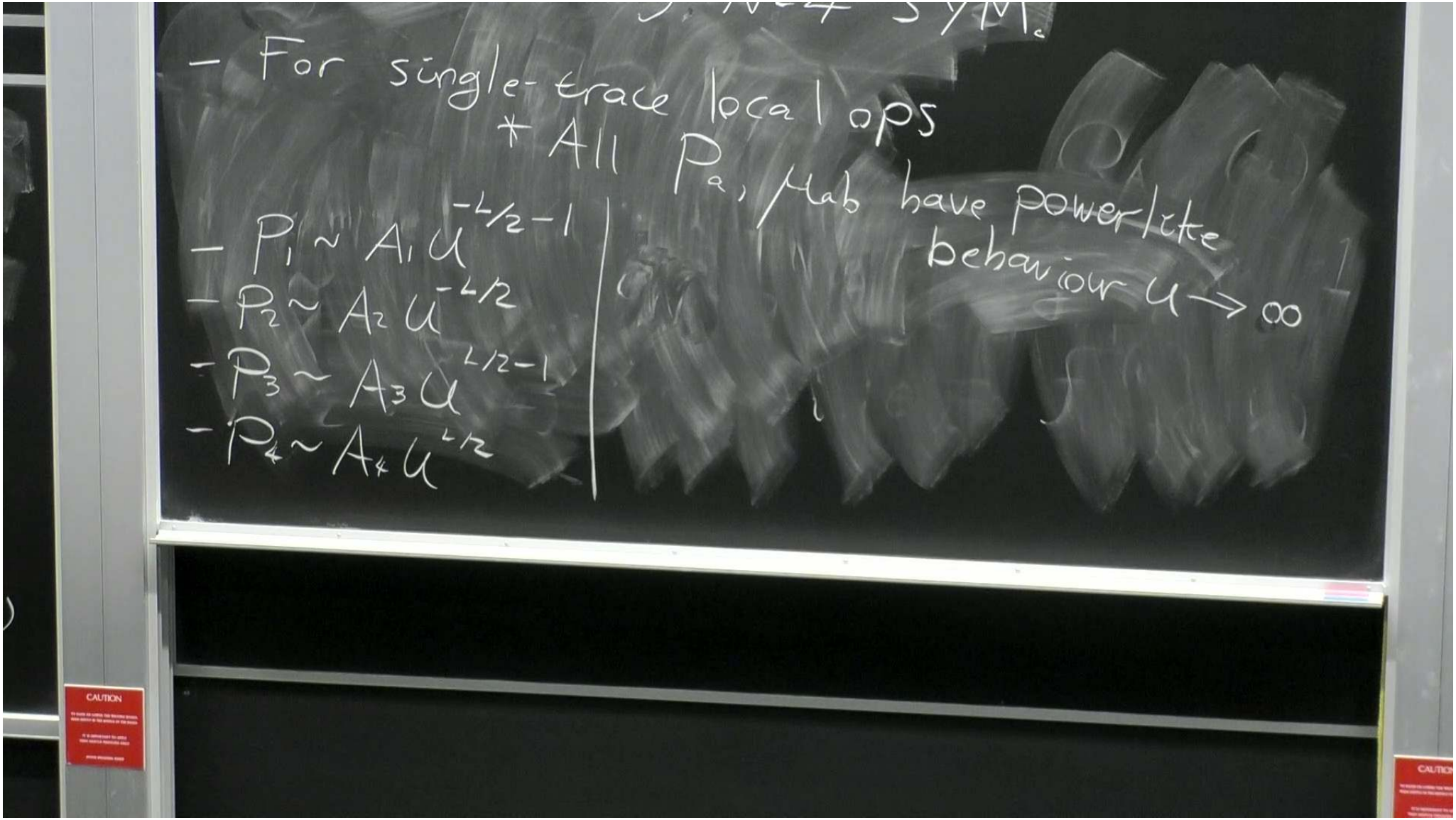
- $\widetilde{P}^{(u)}_a = \mu_{ab}(u)\, P^b(u)$
- $\mu_{ab} = -\mu_{ba}$, $\det \mu = 1$

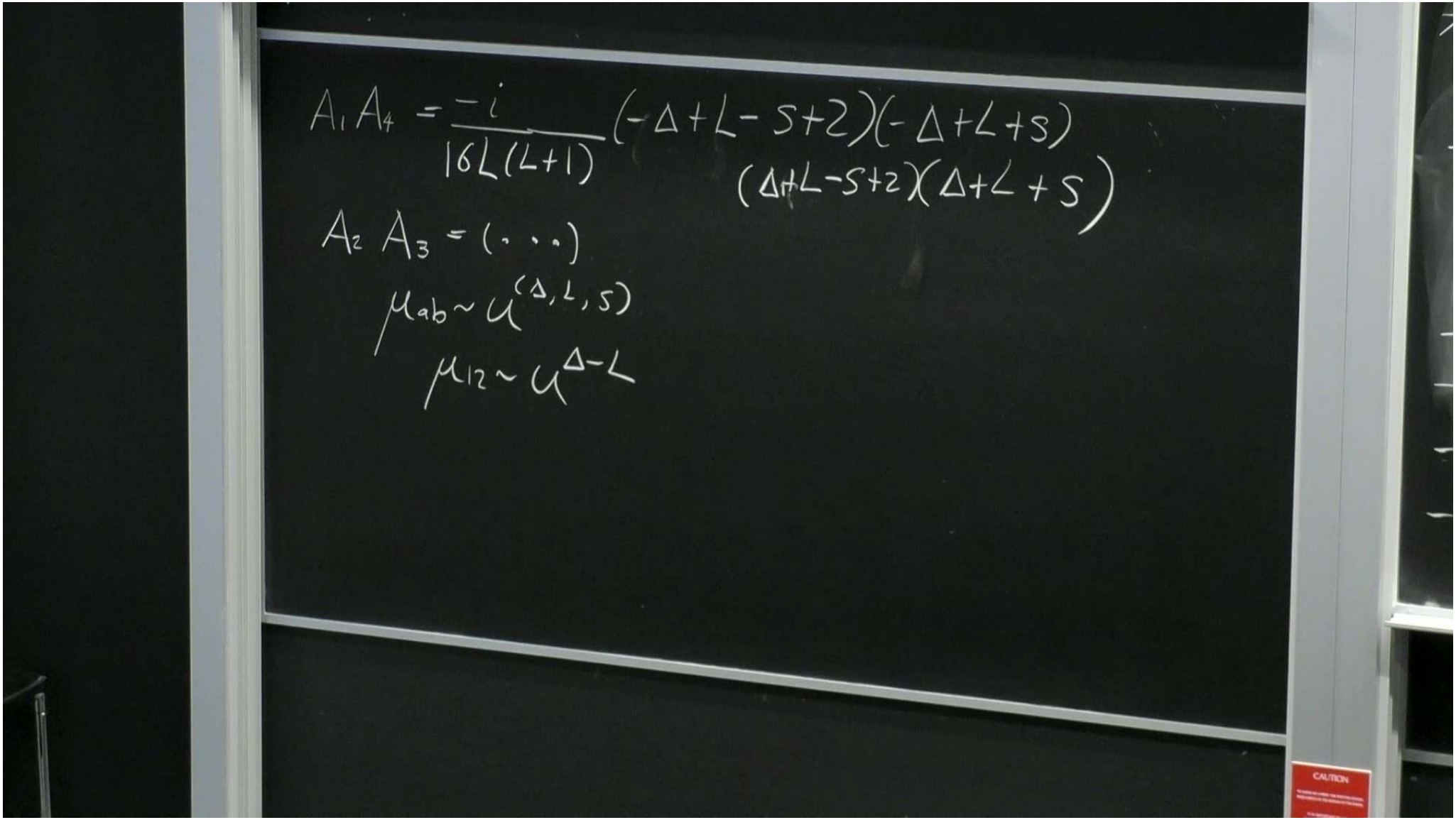
$$\mu_{12}\mu_{34} + \mu_{23}\mu_{14} - \mu_{13}\mu_{24} = 1$$

- $\widetilde{\mu_{ab}(u)} = \mu_{ab}(u+i)$

$$\Rightarrow \mu_{ab}(u+i) - \mu_{ab}(u) = P_a \widetilde{P}_b - P_b \widetilde{P}_a$$


* P_μ system encodes full planar spectrum of $N=4$ SYM.
- For single-trace local ops
* All P_a, M_{ab} have power like behaviour $\mu \to \infty$


- For single-trace local ops
* All $P_{a_1 \ldots a_\ell}$ have power-like behaviour $u \to \infty$
- $P_1 \sim A_1 u^{-\ell/2-1}$
- $P_2 \sim A_2 u^{-\ell/2}$
- $P_3 \sim A_3 u^{\ell/2-1}$
- $P_4 \sim A_4 u^{\ell/2}$


$A_1 A_4 = \frac{-i}{16L(L+1)}(-\Delta+L-s+2)(-\Delta+L+s)$
$(\Delta+L-s+2)(\Delta+L+s)$
$A_2 A_3 = (\cdots)$
$\mu_{ab} \sim u^{(\Delta, L, s)}$
$\mu_{12} \sim u^{\Delta-L}$
CAUTION