Title: QSC definition in N=4

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Collection: Quantum Spectral Curve and Three Point Functions mini-course

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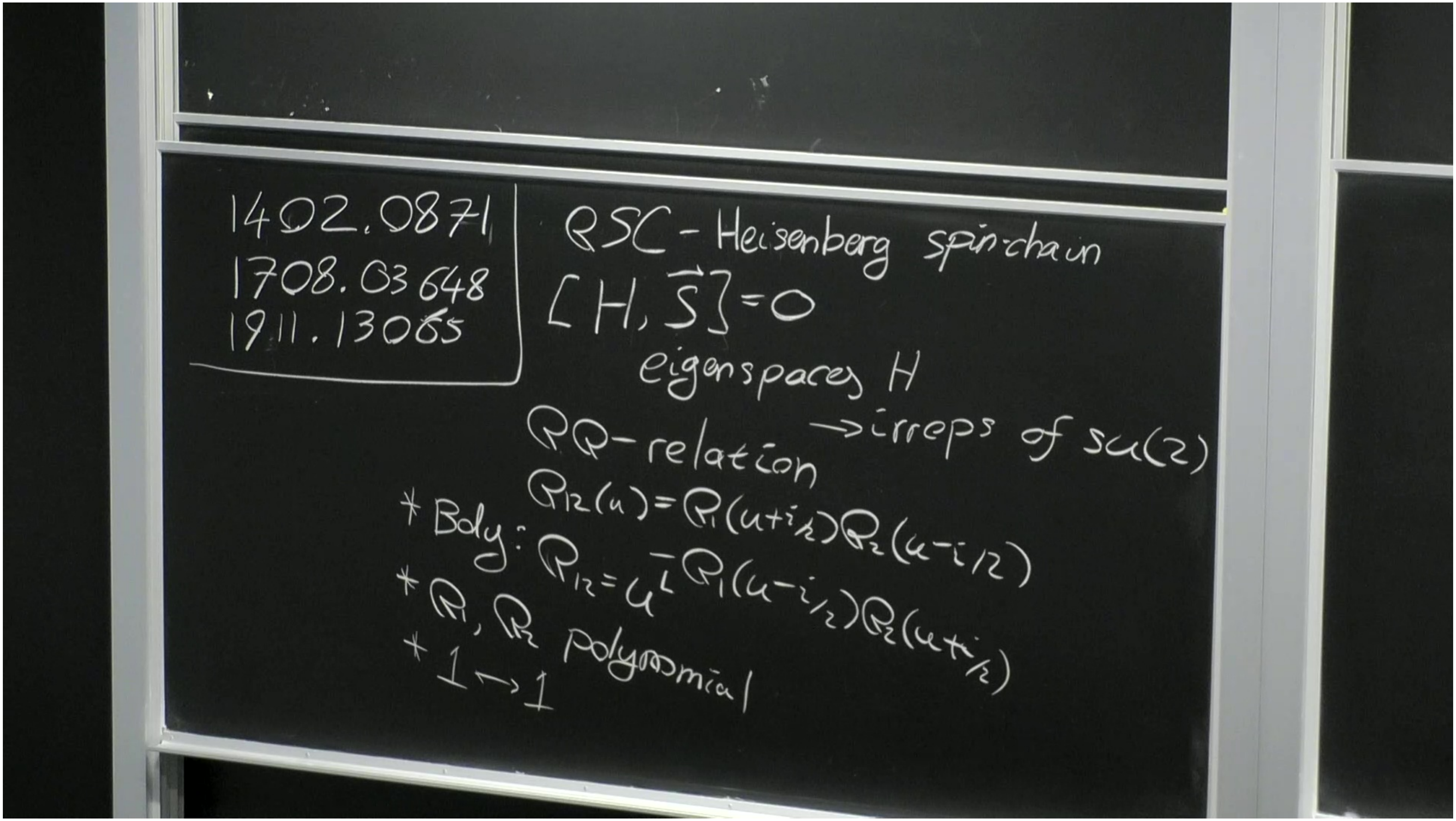
1402.0871
1708.03648
1911.13065
QSC - Heisenberg spin-chain
[H, S⃗] = 0
eigenspaces H
→ irreps of su(2)
QQ-relation
Q12(u) = Q1(u+i/2)Q2(u−i/2) − Q1(u−i/2)Q2(u+i/2)
* Body: Q12 = u^L
* Q1, Q2 polynomial
* 1 → 1

su(2) irrep $S_z \sim L/2 - S$

$$Q_1 \sim A_1 u^{S}, \quad Q_2 \sim A_2 u^{L+1-S}$$

QSC for planar $N=4$ SYM.

- $psu(2,2|4)$

  $\mathbb{R}$ form of $sl(4|4)$

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  $\mathbb{R}$ form of $sl(4|4)$

$B = \{1,2,3,4\}$ $A \subset B$, $a, b \in B$

$F = \{1,2,3,4\}$ $I \subset F$, $i, j \in F$

$Q_{A|I}(u)$

$Q_{\phi|\phi}(u) = 1$

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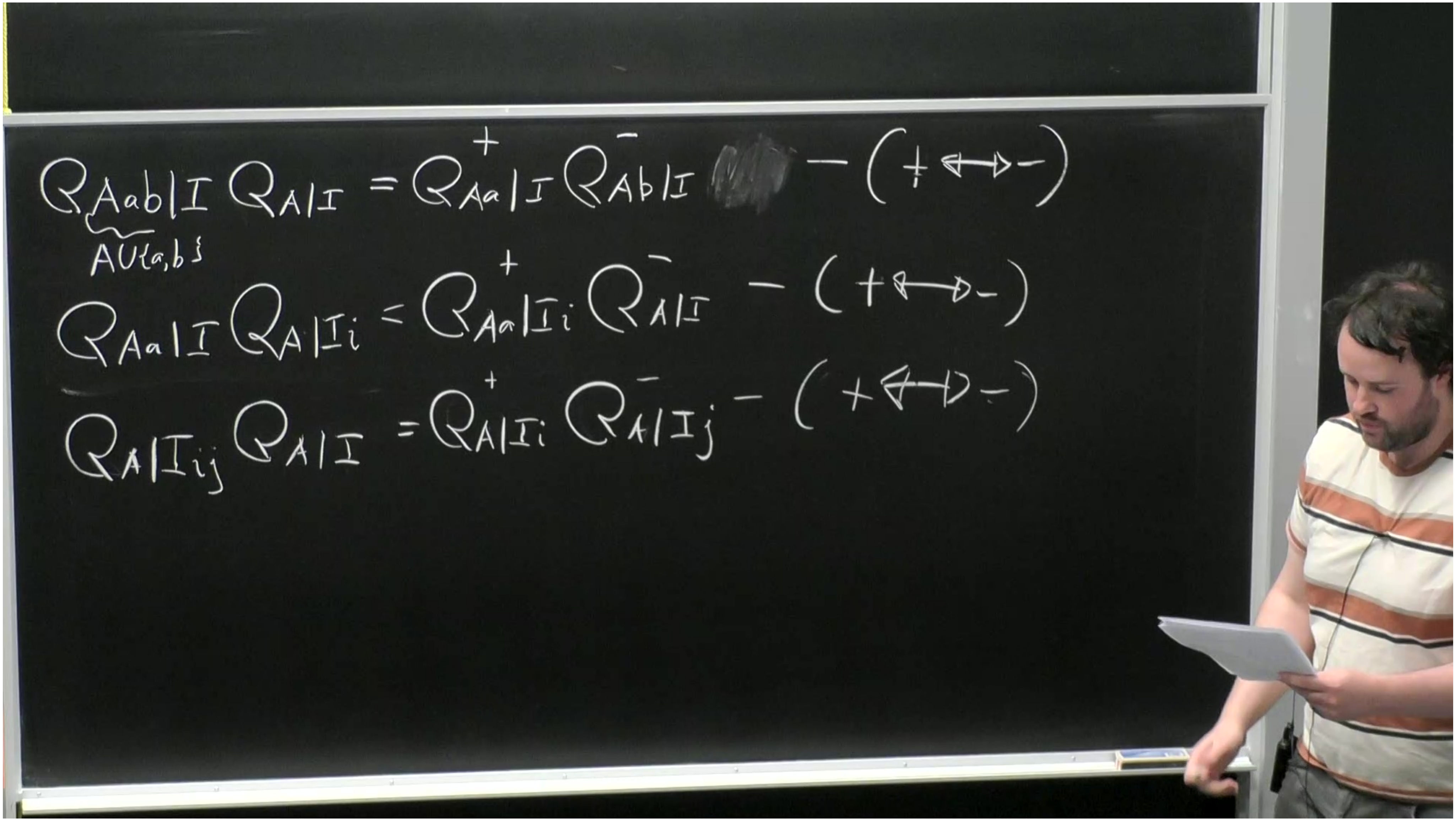
$Q_{A|I}(u)$

$Q_{\phi|\phi}(u)=1$

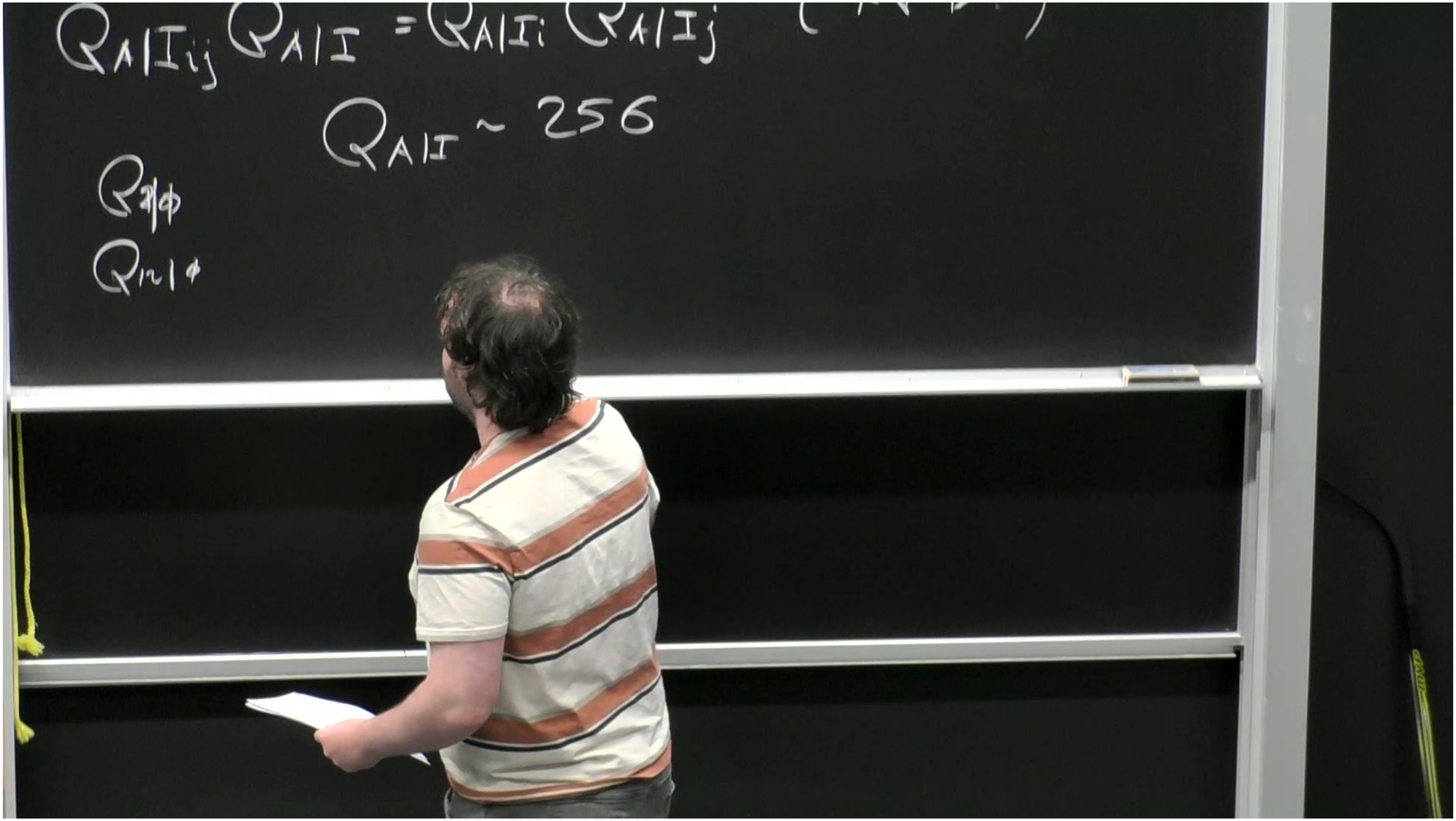
$f^{\pm}(u) := f(u \pm i/2)$



$$Q_{\underbrace{Aab}_{A\cup\{a,b\}}|I}\, Q_{A|I} = Q^{+}_{Aa|I}\, Q^{-}_{Ab|I} - (+ \leftrightarrow -)$$

$$Q_{Aa|I}\, Q_{A|Ii} = Q^{+}_{Aa|Ii}\, Q^{-}_{A|I} - (+ \leftrightarrow -)$$

$$Q_{A|Iij}\, Q_{A|I} = Q^{+}_{A|Ii}\, Q^{-}_{A|Ij} - (+ \leftrightarrow -)$$


QA|Iij QA|I = QA|Ii QA|Ij
QA|I ~ 256

$$P_a = Q_{a|\emptyset}\ , \quad a=1,2,3,4.$$

$$Q_i = Q_{\emptyset|i}\ , \quad i=1,2,3,4.$$

$$Q^+_{a|i} - Q^-_{a|i} = P_a Q_i$$

$\underline{PSU(2,2|4)}$ $\rightarrow$ special bdy cond.

$$Q_{1234|1234} = 1$$

$$P_a = Q_{a|\emptyset}, \quad a = 1,2,3,4. \qquad P^a := \epsilon^{\bar{a}a} Q_{\bar{a}|\emptyset}$$

$$Q_i = Q_{\emptyset|i}, \quad i = 1,2,3,4.$$

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$PSU(2,2|4) \to$ special cond.

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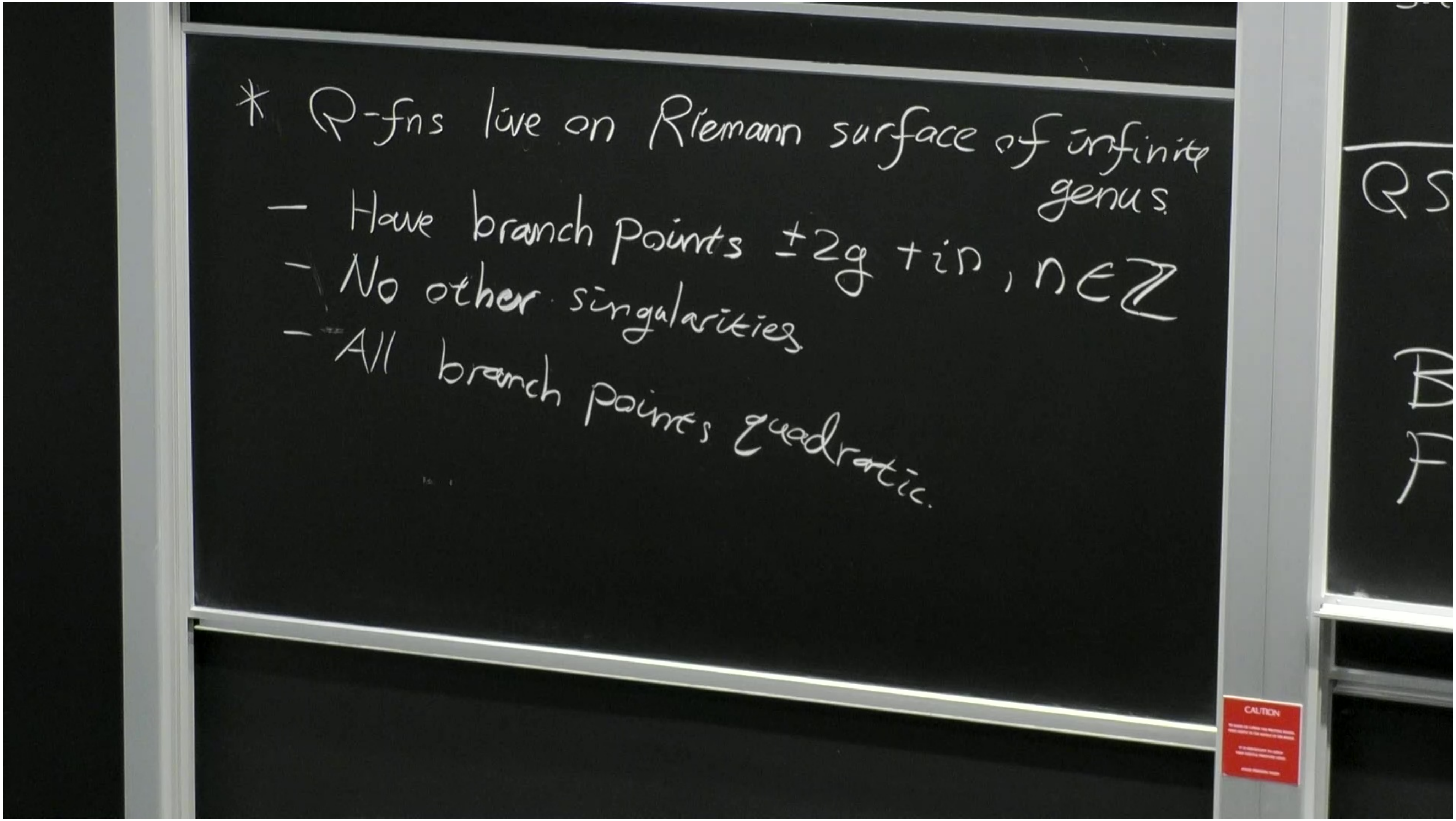
$\underline{P}SU(2,2|4) \rightarrow$ special bdy cond.

$$Q_{1234|1234} = 1 \Rightarrow Q_i = -\sum_a Q^{\pm}_{a|i} P^a$$


* Q-fns live on Riemann surface of infinite genus
— Have branch points ±2g + in, n ∈ ℤ
— No other singularities
— All branch points quadratic.
CAUTION

Q-fns live on Riemann surface of infinite genus

- Have branch points $\pm 2g + in$, $n \in \mathbb{Z}$ $\qquad g = \frac{\sqrt{\lambda}}{4\pi}$
- No other singularities
- All branch points quadratic.

SU(2) irrep S_z

$Q_1 \sim A_1 u^S$, ...

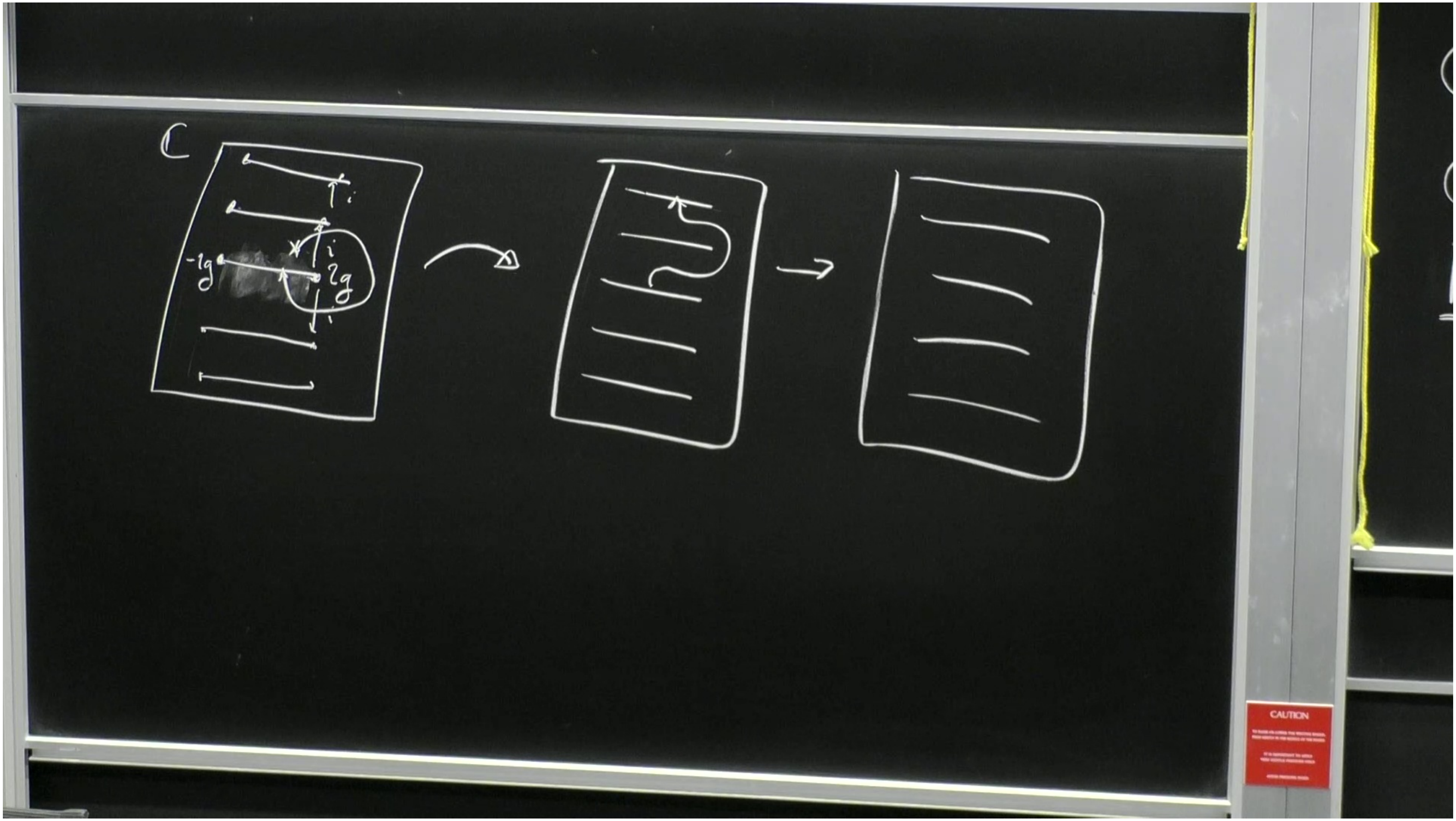
QSC for planar

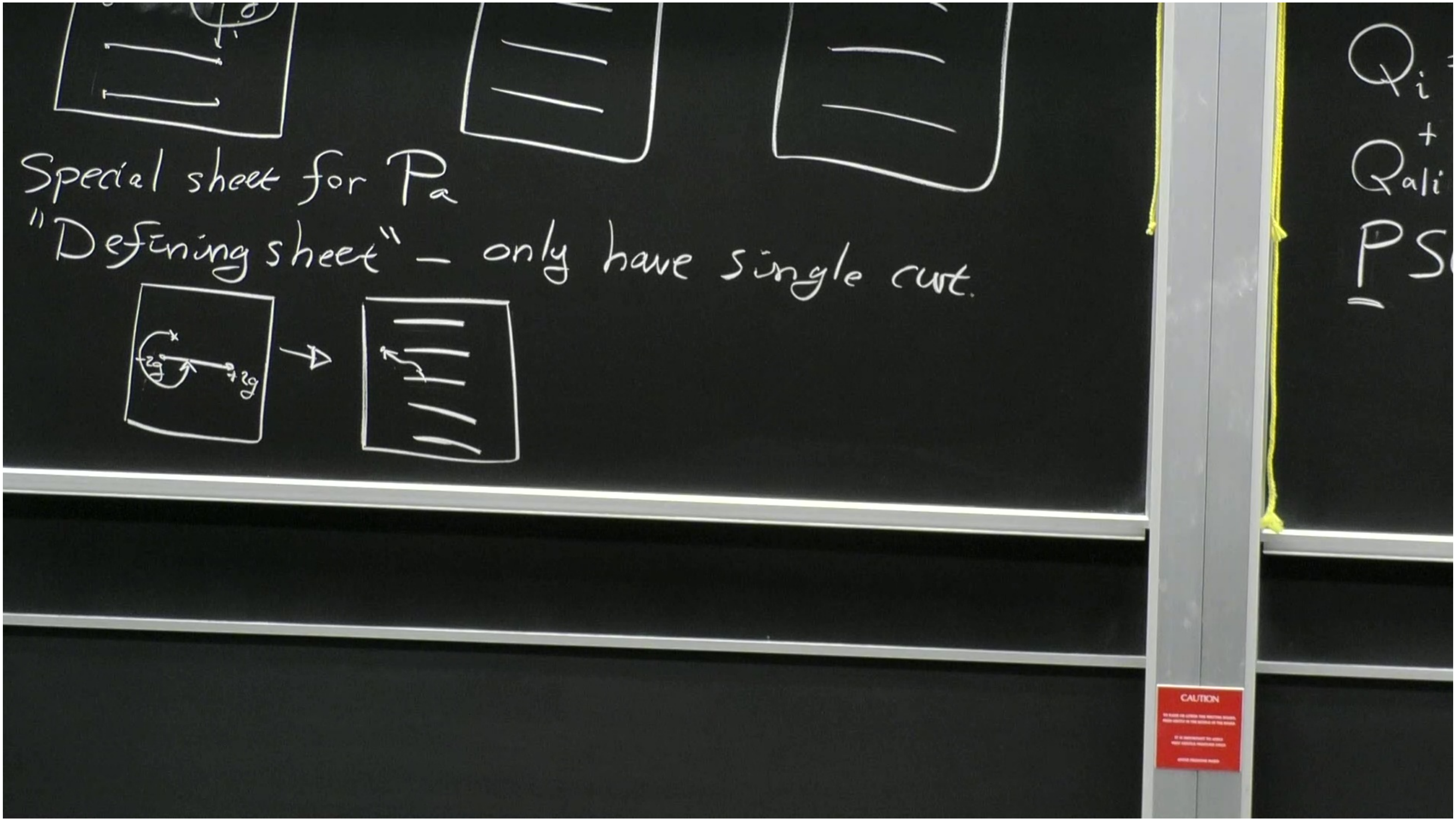
- PSU(2,2|4

$\mathbb{R}$ form

$B = \{1,2,3,4\}$

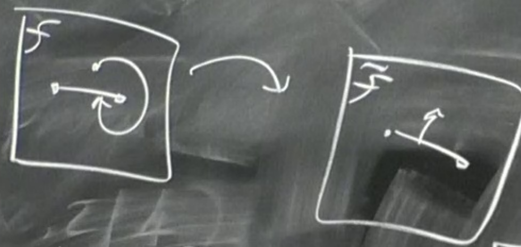
$F = \{1,2,3,4\}$


C
-1g
?g


Special sheet for Pa
"Defining sheet" – only have single cut.

* $f(u)$ with branch points $\pm 2g$

$\tilde{f}(u)$



* Example: $x(u) = \frac{u}{2g} + \sqrt{\frac{u}{2g}+1}\,\sqrt{\frac{u}{2g}-1}$

$$\tilde{x} = \frac{1}{x}$$

SL(2) sector

$$\mathcal{O} = \mathrm{Tr}[D^S Z^L] + \text{permutations}$$

$$\text{Irrep of } PSU(2,2|4) \to [\Delta, S_1, S_2 \,|\, \overbrace{J_1, J_2, J_3}^{S^5}]$$

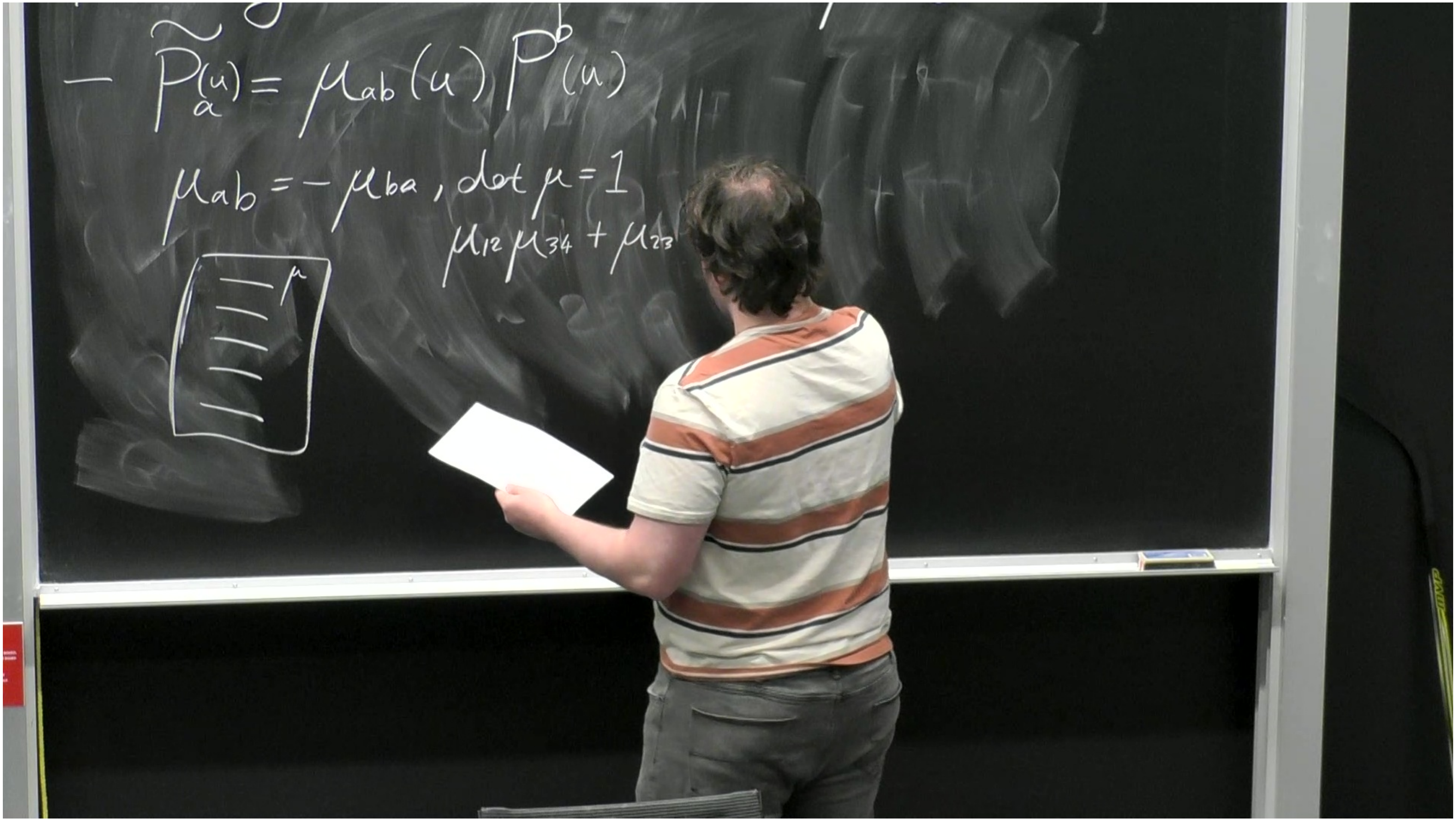
$$SL(2) \to [\Delta, S, 0, L, 0, 0] \quad AdS_5$$

$$P^a = \chi^{ab} P_b \,, \quad \chi^{ab} = \begin{pmatrix} 0 & 0 & 0 & -1 \\ & & 1 & \\ & -1 & & \\ 1 & & & \end{pmatrix}$$

$$P^1 = -P_4$$

$$P^2 = P_3$$

* Analytic continuation - $P\mu$ system


$-\ \tilde{P}_a(u) = \mu_{ab}(u)\, P^b(u)$
$\mu_{ab} = -\mu_{ba}$, $\det \mu = 1$
$\mu_{12}\mu_{34} + \mu_{23}$
μ

- $\mu_{ab} = -\mu_{ba}$, $\det \mu = 1$

- $\mu_{12}\mu_{34} + \mu_{23}\mu_{14} - \mu_{13}\mu_{24} = 1$

- $\tilde{\mu}_{ab}(u) = \mu_{ab}(u+i)$

$\to \mu_{ab}(u+i) - \mu_{ab}(u) = P_a \tilde{P}_b - P_b \tilde{P}_a$

* Analytic continuation - $P\mu$ system

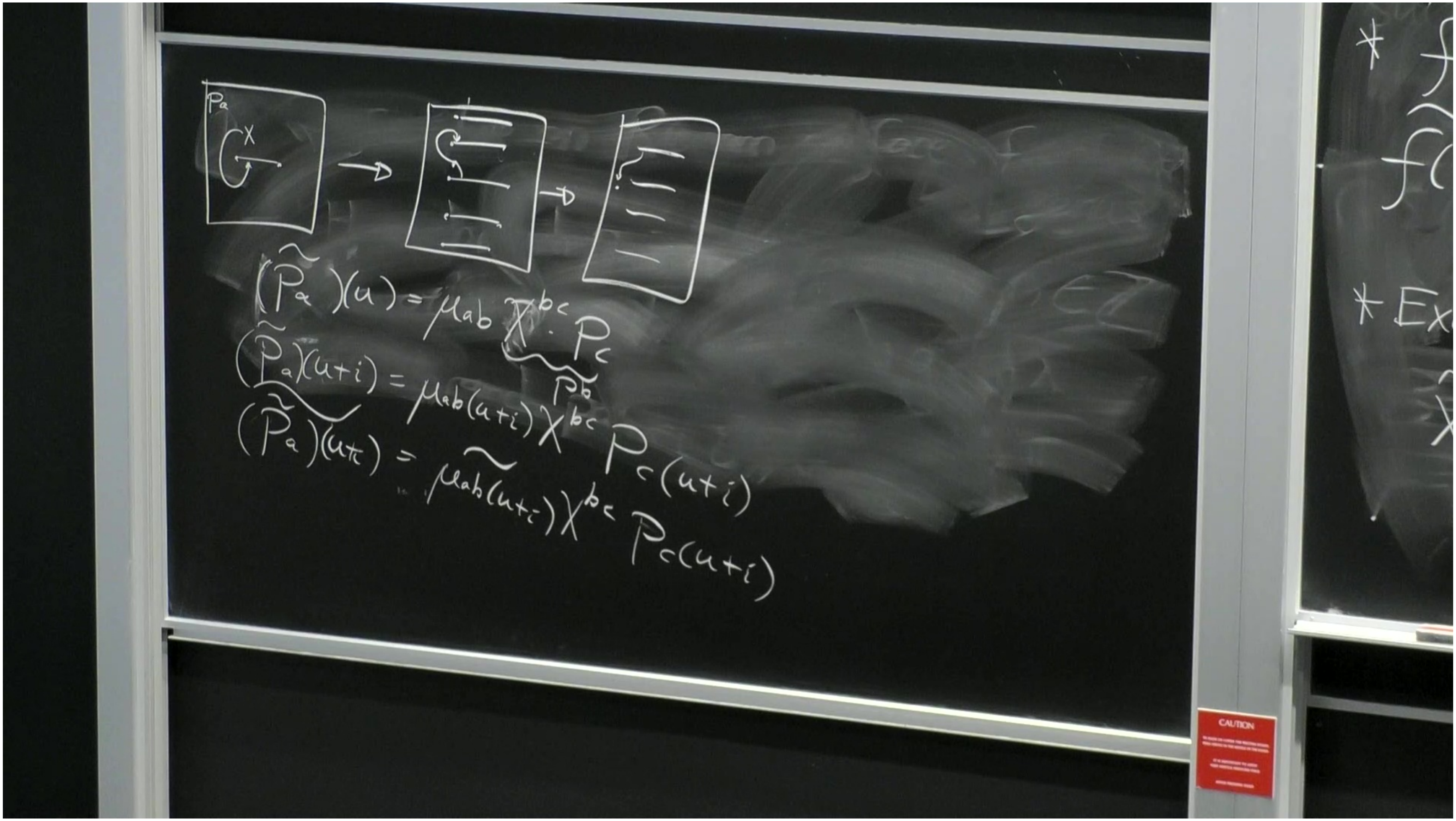
- $\tilde{P}_a(u) = \mu_{ab}(u)\, P^b(u)$

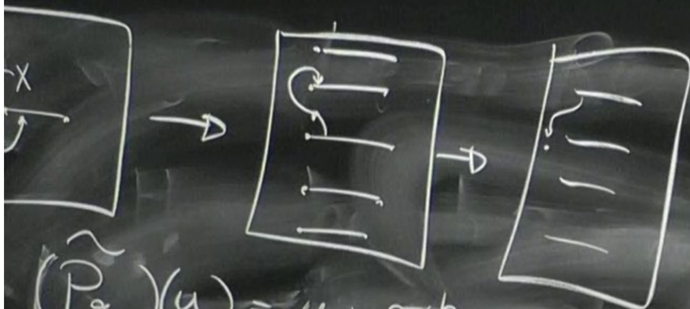
- $\mu_{ab} = -\mu_{ba}$, $\det \mu = 1$

  — $\mu_{12}\mu_{34} + \mu_{23}\mu_{14} - \mu_{13}\mu_{24} = 1$

  — $\tilde{\mu}_{ab}(u) = \mu_{ab}(u+i)$

  $\rightarrow \mu_{ab}(u+i) - \mu_{ab}(u) = P_a \tilde{P}_b - P_b \tilde{P}_a$


P_a
X
$(\widetilde{P_a})(u) = \mu_{ab} \chi^{bc} P_c$
$(\widetilde{P}_a)(u+i) = \mu_{ab}(u+i)\chi^{bc} P_c(u+i)$
$(\widetilde{P}_a)(u_{\tilde{\iota}}) = \widetilde{\mu_{ab}(u+i)}\chi^{bc} P_c(u+i)$



$$(\tilde{P}_a)(u) = \mu_{ab}\,\underbrace{\chi^{bc} P_c}_{P^b}$$

$$(\tilde{P}_a)(u+i) = \mu_{ab}(u+i)\,\chi^{bc} P_c(u+i)$$

$$(\tilde{P}_a)(u+i) = \widetilde{\mu_{ab}(u+i)}\,\chi^{bc} P_c(u+i) = \tilde{\mu}_{ab}(u)\,\chi^{bc} P_c(u+i)$$

$$= \mu_{ab}(u)\,\chi^{bc} P_c(u+i)$$

* $f(u)$ with

$\tilde{f}(u)$

* Example: χ

$$\tilde{\chi} = \frac{1}{\chi}$$

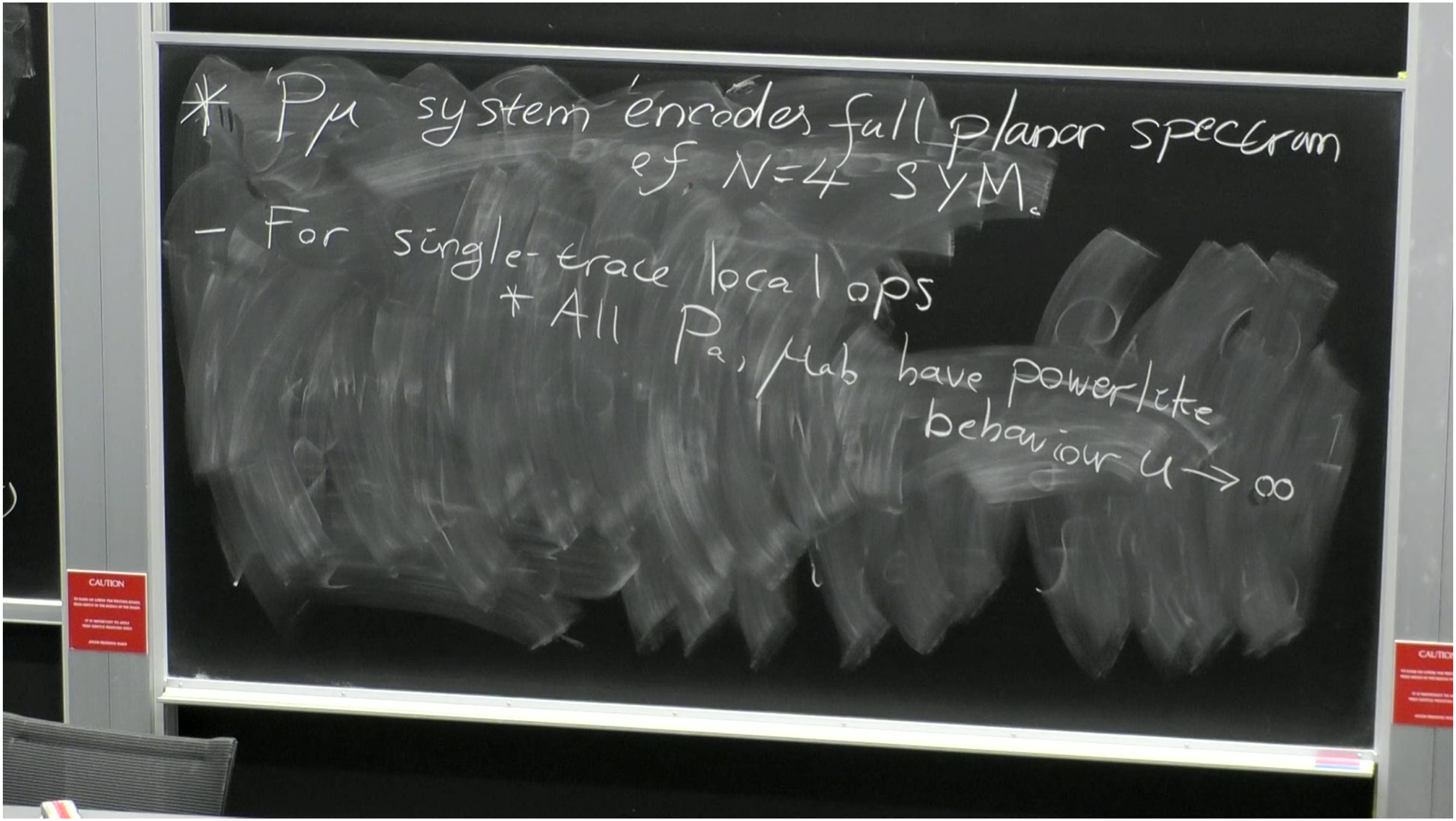
* Analytic continuation - $P\mu$ system

$$\left[\begin{array}{l} Q^{+}_{a|i} - Q^{-}_{a|i} = P_a Q_i \\ \mu_{ab} = Q_{a|i}\, \underset{\substack{\uparrow \\ \text{const.}}}{G^{ik}}\, Q_{b|j} \end{array}\right]$$

- $\tilde{P}_a(u) = \mu_{ab}(u)\, P^b(u)$
- $\mu_{ab} = -\mu_{ba}$, $\det \mu = 1$
  - $\mu_{12}\mu_{34} + \mu_{23}\mu_{14} - \mu_{13}\mu_{24} = 1$

- $\tilde{\mu}_{ab}(u) = \mu_{ab}(u+i)$

$\rightarrow \mu_{ab}(u+i) - \mu_{ab}(u) = P_a \tilde{P}_b - P_b \tilde{P}_a$


* Pμ system encodes full planar spectrum of N=4 SYM.
- For single-trace local ops
* All P_a, μ_{ab} have powerlike behaviour $u \to \infty$
CAUTION

$\mathcal{N}=4$ SYM.

- For single-trace local ops
  - All P_a, μ_{ab} have powerlike behaviour $u \to \infty$

- $P_1 \sim A_1 u^{-L/2-1}$
- $P_2 \sim A_2 u^{-L/2}$
- $P_3 \sim A_3 u^{L/2-1}$
- $P_4 \sim A_4 u^{L/2}$

$$A_1 A_4 = \frac{-i}{16L(L+1)} (-\Delta+L-S+2)(-\Delta+L+S)$$
$$(\Delta+L-S+2)(\Delta+L+S)$$

$$A_2 A_3 = (\cdots)$$

$$\mu_{ab} \sim u^{(\Delta, L, S)}$$

$$\mu_{12} \sim u^{\Delta-L}$$