Title: Metric signature transitions and the cosmological constant

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Abstract: In classical relativity we usually think of the metric signature as fixed, but quantum cosmology already forces us to consider more general situations such as transitions from Riemannian to Lorentzian signature. I will discuss the less studied phenomenon of an overall "flip" where all metric components change sign simultaneously (e.g., from "East Coast" to "West Coast" signature). Such an overall flip can represent saddle point solutions in quantum cosmology, or appear classically in the Pleba?ski formalism or even a slight extension of Einstein--Hilbert gravity. Interestingly, at such a transition the cosmological constant can change both sign and magnitude, with a pure sign change being the most minimalistic proposal. Cosmological solutions transitioning classically between de Sitter and anti de Sitter can be found immediately, and the quantisation of a minisuperspace model turns out to be simpler than in the fixed signature case: in particular, the gravitational action reduces to a pure boundary term. Various other applications and the relation to unimodular gravity are also discussed.

Zoom link: https://pitp.zoom.us/j/93255772501?pwd=bWhzZUVBOTM2a3QvTGlXVGJ5TlZnUT09

Metric signature transitions and the cosmological constant

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Introduction

In usual classical general relativity, we fix the metric signature as either $(-+++)$ ("East Coast convention") or $(+---)$ ("West Coast convention").

Assuming one is careful in copying over expressions from the opposite convention, any physically relevant statements can be translated from one to the other.

What happens when the signature is allowed to change?

Main motivation comes from quantum treatments: saddle points appearing in a semiclassical expansion are complex. Even if we pretend to start with an East Coast theory, saddle point may be described by a West Coast metric (as one specific case of the more general complex case).

Classical formulations such as Plebański formalism do not work with metric, and do not even allow fixing signature.

An example from quantum cosmology [Hartle & Hertog 2012]

Minisuperspace metric (spherical slices, $k = 1$):

$$\mathrm{d}s^2 = N(\tau)^2 \mathrm{d}\tau^2 + a(\tau)^2 \mathrm{d}\Omega_3^2 \,.$$

Euclidean action for gravity ($\Lambda > 0$)

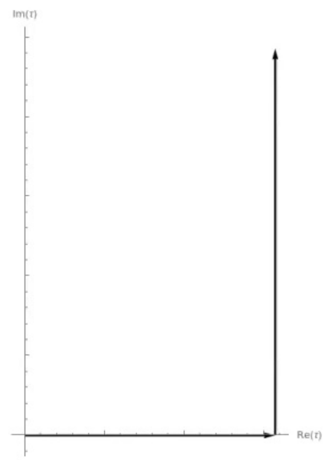
$$I(a) = \frac{3\pi}{4G} \int \mathrm{d}\tau \left(-a\frac{\dot{a}^2}{N} - Na + N\frac{\Lambda}{3}a^3 \right)$$

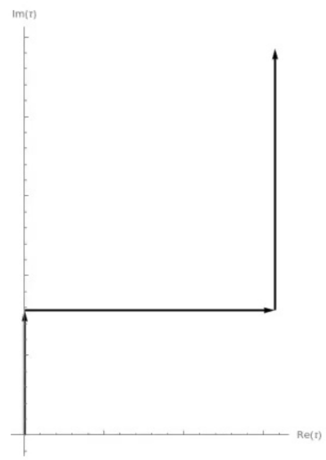
With $N = 1$, unique classical solution is the sphere $a(\tau) = \sqrt{\frac{3}{\Lambda}} \sin\left(\sqrt{\frac{\Lambda}{3}}\tau\right)$.

Classical representation of the no-boundary saddle point solution runs from $\tau = 0$ (south pole) to $\sqrt{\frac{\Lambda}{3}}\tau = \pi/2$ (equator) along the real axis, then follows contour parallel to imaginary τ axis to form a Lorentzian de Sitter solution.

Alternative saddle-point representation

Can deform the contour in the complex τ plane to get an equivalent representation:





Contour deformation leaves total action invariant. However, the representation on the right starts with a portion of negative definite "Euclidean AdS"

$$\mathrm{d}s^2 = -\mathrm{d}\tau^2 - \frac{3}{\Lambda}\sinh^2\left(\sqrt{\frac{\Lambda}{3}}\tau\right)\,\mathrm{d}\Omega_3^2\,.$$

What is going on here?

These saddle point solutions must be solutions to the Einstein equations

$$R_{\mu\nu} = \Lambda g_{\mu\nu}$$

where $g_{\mu\nu}$ should be thought of as complex in general. When changing overall signature (say from $(++++)$ to $(----)$), we have

$$g_{\mu\nu} \to -g_{\mu\nu}\,, \quad R^{\alpha}{}_{\mu\beta\nu} \to R^{\alpha}{}_{\mu\beta\nu}\,,$$

hence changing overall signature is equivalent to sending $\Lambda \to -\Lambda$.
From this point of view, GR with $\Lambda > 0$ contains anti-de Sitter solutions as well.

Similar ideas were used to show that Wheeler–DeWitt wavefunctions for gravity with $\Lambda < 0$ contain de Sitter-like semiclassical, exponentially expanding solutions [Hartle, Hawking & Hertog 2012].

A more general (classical) approach

Einstein–Hilbert action usually defined with certain conventions, e.g.,

$$S[g] = \frac{1}{16\pi G} \int \mathrm{d}^4x \, \sqrt{-g} \, (R - 2\Lambda)$$

for Lorentzian metrics of signature $(-+++)$. Different conventions often made when starting with Euclidean action. In general, need to decide what happens when metric signature is more generally $\sigma(s+++)$.

One proposal [Alexandre, SG & Magueijo 2023]:

$$S[g] = \frac{1}{16\pi G} \int \mathrm{d}^4x \, \sqrt{|g|} \, (\sigma R - 2\Lambda)$$

so Einstein equation is $R_{\mu\nu} = \sigma\Lambda g_{\mu\nu}$. From now on, Λ is the "physical Λ".

Minisuperspace reduction

We could write

$$\begin{aligned} \mathrm{d}s^2 &= \sigma s N^2(t)\,\mathrm{d}t^2 + \sigma\, a^2(t) h^0_{ij}(x)\,\mathrm{d}x^i\,\mathrm{d}x^j \\ &= s\tilde{N}^2(t)\,\mathrm{d}t^2 + \tilde{a}^2(t) h^0_{ij}(x)\,\mathrm{d}x^i\,\mathrm{d}x^j \end{aligned}$$

Can calculate R, add appropriate boundary term to cancel total derivatives.
Choice of signs for lapse N and scale factor a in setting $\sqrt{|g|} = Na^3\sqrt{h^0}$.
Minisuperspace action becomes (convention: $\tilde{N}\tilde{a} = Na$)

$$S = \frac{3V_0}{8\pi G}\int \mathrm{d}t\; Na\left(s\frac{\dot{a}^2}{N^2} + k - \frac{\Lambda}{3}a^2\right) = \frac{3V_0}{8\pi G}\int \mathrm{d}t\; \tilde{N}\tilde{a}\left(s\frac{\dot{\tilde{a}}^2}{\tilde{N}^2} + k - \frac{\Lambda}{3}\mathrm{sgn}(\tilde{a}^2)\tilde{a}^2\right)$$

In terms of the "signed" ~ variables, simpler to keep $\tilde{\Lambda} = \mathrm{sgn}(\tilde{a}^2)\Lambda$ fixed.

Varying Λ?

Einstein's equations are not well-defined when signature changes through either s or σ. Hence, Bianchi identities

$$R_{\mu\nu} = \sigma\Lambda g_{\mu\nu} \quad \Rightarrow \quad \partial_\mu(\sigma\Lambda) = 0$$

still allow for $\Lambda = \Lambda(\sigma, s)$. In minisuperspace, consistency condition becomes

$$a^2 \frac{\mathrm{d}}{\mathrm{d}t}(\sigma\Lambda) = 0\,.$$

Hamiltonian form of minisuperspace action: $(N_0 \equiv \tilde{N}\tilde{a})$

$$S = \frac{3V_0}{8\pi G}\int \mathrm{d}t \left(\dot{\tilde{b}}\tilde{a}^2 - N_0\left[s\tilde{b}^2 - k + \frac{\Lambda}{3}\mathrm{sgn}(\tilde{a}^2)\tilde{a}^2\right]\right)\,.$$

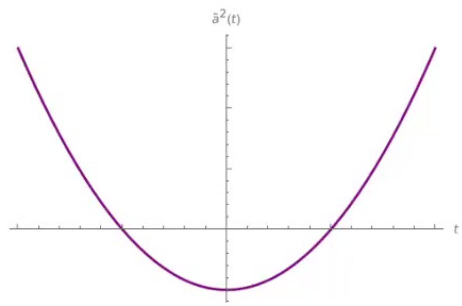
Fundamental variables: connection $\tilde{b}$, and $\tilde{a}^2$ which can be positive or negative.

Simple example: dS–AdS transition

Let us fix $\tilde{\Lambda} = \mathrm{sgn}(\tilde{a}^2)\Lambda > 0$ and $s = -1$ (Lorentzian).
In the gauge $N_0 = 1$, general solution given by

$$\tilde{a}^2(t) = \frac{\tilde{\Lambda}}{3}t^2 + \frac{3}{\tilde{\Lambda}}k\,, \quad \tilde{b}(t) = \frac{\tilde{\Lambda}}{3}t\,.$$

With $k < 0$: transition between de Sitter and anti-de Sitter (with $\tilde{a}^2 < 0$)



Metric smooth even at transition point. No curvature singularities.

Signature change in Plebański gravity

Another motivation for signature change: Chiral Plebański gravity. Gravity encoded in self-dual 2-forms Σ^i and complex $SU(2)$ connection.

$$S = \frac{1}{\sqrt{s}\,8\pi G}\int \Sigma^i \wedge F_i - \frac{1}{2}M_{ij}\Sigma^i \wedge \Sigma^j + \frac{1}{2}\nu\left(\mathrm{tr}\,M - \tilde{\Lambda}\right)$$

encompasses both Euclidean and Lorentzian cases. Usually $\tilde{\Lambda}$ seen as fixed.

For Lorentzian solutions, impose reality conditions; Euclidean theory formulated in terms of real $SO(3)$ variables. Simplicity constraint

$$\Sigma^i \wedge \Sigma^j = \delta^{ij}\nu$$

has multiple solution sectors, which result in a tetrad that is real up to a global $\{0, e^{\mathrm{i}\pi/4}, \mathrm{i}, e^{3\mathrm{i}\pi/4}\}$ (Lorentzian), or real up to a possible i (Euclidean theory). Reconstructed (Urbantke) metric can have either sign: $(-+++)$ or $(+---)$ in Lorentzian case, $(++++)$ or $(----)$ in Euclidean case.

Signature change in Plebański gravity

The trace constraint

$$\mathrm{tr}\, M = \tilde{\Lambda}$$

encodes the trace part of $R_{\mu\nu} = \tilde{\Lambda} g_{\mu\nu}$ where $g_{\mu\nu}$ is Urbantke metric.

Integrate out the self-dual 2-forms:

$$S = \frac{1}{\sqrt{s}\, 16\pi G} \int (M^{-1})_{ij} F^i \wedge F^j + \nu \left(\mathrm{tr}\, M - \tilde{\Lambda} \right) .$$

Minisuperspace reduction leads to [SG & Nash 2022]

$$S = \int \mathrm{d}t \left(\dot{c}p - \bar{\lambda} \left[p - \frac{k - sc^2}{3\tilde{\Lambda}_P} \right] \right) , \quad \Lambda_P = 8\pi G \tilde{\Lambda}$$

equivalent to the previous model, with p corresponding to the variable $\tilde{a}^2$. p is defined from M_{ij} and has no reason to be positive.

Signature change in Plebański gravity

Urbantke metric in minisuperspace:

$$(8\pi G)^{-1}\mathrm{d}s^2 = -\frac{\bar{\lambda}(t)^2}{\Lambda_P^2\,p(t)}\mathrm{d}t^2 + p(t)\,\mathrm{d}\Omega_3^2\,.$$

Popular parametrisation of minisuperspace cosmology [Halliwell 1988].

The variable p can be "integrated out":

$$S = \int \mathrm{d}t\;\dot{c}\,\frac{k - sc^2}{3\tilde{\Lambda}_P} = \left[c\frac{3k - sc^2}{9\tilde{\Lambda}_P}\right]_{t=t_i}^{t=t_f}\,.$$

"Pure connection" Lagrangian is a pure boundary term! This result can also be obtained by reducing Krasnov's pure connection theory [Krasnov 2011] to minisuperspace. This process requires allowing for $p < 0$.

Path integral evaluation

Path integral with fixed connection boundary data (gauge $\bar{\lambda} = \text{const}; \; s = -1$):

$$G(c_f|c_i) = \int \mathrm{d}\bar{\lambda} \int \mathcal{D}c\, \mathcal{D}p\, \exp\left(\mathrm{i}\int_0^1 \mathrm{d}t \left[\dot{c}p - \bar{\lambda}\left\{p - \frac{k+c^2}{3\tilde{\Lambda}_P}\right\}\right]\right)$$
$$= \int \mathrm{d}\bar{\lambda}\, \delta(c_f - c_i - \bar{\lambda}) \exp\left(\mathrm{i}\bar{\lambda}\int_0^1 \mathrm{d}t\, \frac{k + (c_i + \bar{\lambda}t)^2}{3\tilde{\Lambda}_P}\right)$$
$$= \exp\left(\mathrm{i}\frac{(c_f^3 - c_i^3) + 3k(c_f - c_i)}{9\tilde{\Lambda}_P}\right)$$

Reduces to the Chern–Simons/Kodama $e^{\mathrm{i}S}$ form whose Fourier transform(s) give the Hartle–Hawking/Vilenkin wavefunctions (e.g., [Magueijo 2020]).

Again notice that this simple exact result requires a p that can go negative.

Situation in unimodular gravity

In the standard unimodular formulation of general relativity, we fix the volume form $\sqrt{|g|} = \omega$. We only have the trace-free equations

$$R_{\mu\nu} - \frac{1}{4} R g_{\mu\nu} = 0 \,.$$

The Ricci scalar R is undetermined, but Bianchi identities force $R = \mathrm{const}$ where the Einstein equations are well-defined. If ω is degenerate somewhere, R and hence the effective $\tilde{\Lambda}$ can change (in an unspecified way).

In the alternative "parametrised" formulation due to Henneaux–Teitelboim,

$$S[g, \tilde{\Lambda}, T] = \frac{1}{8\pi G} \int \mathrm{d}^4 x \left[\sqrt{|g|} \left(\frac{R}{2} - \tilde{\Lambda} \right) + \tilde{\Lambda}\, \partial_\mu T^\mu \right] \,,$$

we have an explicit constraint $\tilde{\Lambda} = \mathrm{const}$ coming from variation with respect to T^μ. One can change this to $\Lambda = \mathrm{const}$, or keep any function of Λ, σ, s fixed.

Unimodular Plebański gravity [SG & Nash 2023]

The same unimodular extensions exist for Plebański gravity:

$$S[\Sigma, A, M; \omega_0] = \frac{1}{\sqrt{s}\, 8\pi G} \int \Sigma^i \wedge F_i - \frac{1}{2} M_{ij} \Sigma^i \wedge \Sigma^j + \frac{1}{2} \omega_0 \, \mathrm{tr}\, M$$

where ω_0 is now a fixed volume form which will (through the simplicity constraints) be proportional to the volume element of the Urbantke metric. The trace constraint $\mathrm{tr}\, M = \mathrm{const}$ comes from Bianchi identities if ω_0 is non-degenerate.

There exists a parametrised Henneaux–Teitelboim-like version,

$$S[\Sigma, A, M, T] = \frac{1}{\sqrt{s}\, 8\pi G} \int \Sigma^i \wedge F_i - \frac{1}{2} M_{ij} \Sigma^i \wedge \Sigma^j + \frac{1}{2} \mathrm{d}T \, \mathrm{tr}\, M$$

Here $\mathrm{d}\, \mathrm{tr}\, M = 0$ arises directly as a field equation forcing the effective $\tilde{\Lambda}$ to be constant everywhere.

Observational relevance?

Relaxing cosmological tensions with a sign switching cosmological constant: Improved results with Planck, BAO, and Pantheon data

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We present a further observational analysis of the Λ_sCDM model proposed in Akarsu *et al.* [Phys. Rev. D 104, 123512 (2021)]. This model is based on the recent conjecture suggesting the Universe has transitioned from anti-de Sitter vacua to de Sitter vacua (viz., the cosmological constant switches sign from negative to positive), at redshift $z_\dagger \sim 2$, inspired by the graduated dark energy model proposed in Akarsu *et al.* [Phys. Rev. D 101, 063528 (2020)]. Λ_sCDM was previously claimed to simultaneously relax five cosmological discrepancies, namely, the H_0, S_8, and M_B tensions along with the Ly-α and ω_b anomalies, which prevail within the standard ΛCDM model as well as its canonical/simple extensions. In the present work, we extend the previous analysis by constraining the model using the Pantheon data (with and without the SH0ES M_B prior) and/or the *completed* BAO data along with the full *Planck* CMB data. We find that Λ_sCDM exhibits a better fit to the data compared to ΛCDM, and simultaneously relaxes the six discrepancies of ΛCDM, viz., the H_0, M_B, S_8, Ly-α, t_0, and ω_b discrepancies, all of which are discussed in detail. When the M_B prior is included in the analyses, Λ_sCDM performs significantly better in relaxing the H_0, M_B, and S_8 tensions with the constraint $z_\dagger \sim 1.8$ even when the Ly-α data (which imposed the $z_\dagger \sim 2$ constraint in the previous studies) are excluded. In contrast, the presence of the M_B prior causes only negligible improvements for ΛCDM. Thus, the Λ_sCDM model provides remedy to various cosmological tensions simultaneously, only that the galaxy BAO data hinder its success to some extent.

A cosmological model in which Λ switches sign (but not magnitude) at redshift $z \sim 2$ could explain cosmological observations better than constant Λ!

Conclusions

- Allowing the overall metric signature to flip opens up new possibilities for changes in the cosmological constant Λ. A scenario in which Λ switches sign seems simplest and coming directly from $R_{\mu\nu} = \Lambda g_{\mu\nu}$.
- Such solutions arise as saddle points in quantum cosmology but can be incorporated into classical Lorentzian formulations.
- In chiral Plebański gravity there is no way of fixing metric signature, so such solutions would always appear.
- In minisuperspace, leads to a a^2 variable of indefinite sign, conjugate to the connection/extrinsic curvature. Greatly simplifies the quantum theory.
- Tentative connection to cosmological phenomenology: how could the Universe have flipped signature?