Title: Topological modular forms and heretoric string theory

Speakers: Mayuko Yamashita

Series: Mathematical Physics

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Abstract: In this talk I will explain my works with Y. Tachikawa to study anomaly in heterotic string theory via homotopy theory, especially the theory of Topological Modular Forms (TMF). TMF is an E-infinity ring spectrum which is conjectured by Stolz-Teichner to classify two-dimensional supersymmetric quantum field theories in physics. In the previous work (https://arxiv.org/abs/2108.13542), we proved the vanishing of anomalies in heterotic string theory mathematically by using TMF. Furthermore, we have a recent update (https://arxiv.org/abs/2305.06196) on the previous work. Because of the vanishing result, we can consider a secondary transformation of spectra, which is shown to coincide with the Anderson self-duality morphism of TMF. This allows us to detect subtle torsion phenomena in TMF by differential-geometric ways, and leads us to new conjectures on the relation between VOAs and TMF.

Zoom link: https://pitp.zoom.us/j/95550331572?pwd=OW1oYlBvUWVxaGNJRWl5aHVrS0pJZz09

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1
36

Topological Modular Forms and heterotic string theory

Mayuko Yamashita (Kyoto Univ)

j.w/ Yuji Tachikawa (IPMU, Univ. of Tokyo)

Based on

[TY'21] arXiv: 2108.13542

[TY'23] arXiv: 2305.0619

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36

Goal of this talk : Explain our works to connect homotopy theory & string theories.

① The result of [Tachikawa-Y '21] :

vanishing of anomaly in heterotic string theory

$\Leftrightarrow$ $\alpha = 0 : TMF^{d+22} \to I_{\mathbb{Z}} MString^{d+2}$

Topological Modular Forms — Anderson dual

② [TY '23] : $\alpha = 0$ implies we have

$\tilde{\alpha} : TMF^{d+22} \to I_{\mathbb{Z}} MSpin/MString^{d+2}$

"secondary anomaly transformation"

which turns out to be related to

Anderson duality in TMF.

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Plan (phys ↔ math.)

Part I Vanishing of heterotic anomaly

[TY '21] & Topological Modular Forms (TMF)

§1. 2d SUSY QFT & TMF

§2. Anomalies & Anderson duals

§3. Formulation & proof

Part II Secondary anomaly

[TY '23] & Anderson self-duality of TMF

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Part I. Vanishing of heterotic anomaly and Topological Modular Forms [TY '21]

Main Result of [TY '21] (: Physical statement.)

| Heterotic String theories are anomaly-free.

Strategy We use the relations

Physics ⟷ Math (Homotopy theory)

Stolz-Teichner conjecture.
Freed-Hopkins conjecture

to translate Phys. Q to Math. Q.
and solve it mathematically.

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5
36

We use the following diagram:

worldsheet $c = \frac{d+22}{2}$ spacetime d-dim

$$\begin{tikzcd} \text{Phys} & \left\{\begin{matrix}\text{2d SCFTs}\\ \mathcal{N}=(0,1)\end{matrix}\right\} \arrow[r, "\text{heterotic}", "\text{compactification}"'] \arrow[d, "\text{Stolz-Teichner conjecture ['04, '11]}"] & \left\{\begin{matrix}\text{QFTs on}\\ \text{String manifolds}\end{matrix}\right\} \arrow[d, "\text{extract anomalies}", "\text{Freed-Hopkins conjecture ['16]}"'] \\ \text{Math} & TMF^{d+22} \arrow[r, "\alpha"] & I_{\mathbb{Z}} MString^{d+2} \end{tikzcd}$$

Topological Modular Forms — Anderson dual to String bordism

$\rightsquigarrow$ <u>Phys. Problem</u> Show that heterotic anomaly vanishes.

$\Leftrightarrow$ <u>Math Problem</u> (roughly): Show $\alpha = 0$.
under some conditions on α implied by Physics

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§1. 2d SUSY QFT & TMF

TMF (Topological Modular Forms)

… a **Spectrum** (generalized coh) which is a "topological" version of **Modular Forms**.

Stolz-Teichner Conjecture ['04, '11]

$$\left\{ \begin{array}{l} \text{2d } \mathcal{N}=(0,1) \overbrace{\text{SUSY QFTs}}^{\text{unitary, fully extended}} \\ \text{of central charge } c \end{array}\right\} \Big/ \text{deformation} \cong \mathrm{TMF}^{2c}.$$

6
36

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Stolz - Teichner conj : $\{2d\ \mathcal{N}=(0,1)\,\mathrm{SQFT}\} \cong \mathrm{TMF}$

a "topological refinement"

$$\left\{ 2d\ \boxed{\mathcal{N}=(0,1)}\ \mathrm{SQFT} \atop \text{central charge } c \right\} \longrightarrow \mathrm{MF}[\Delta^{-1}]_{-2c}$$

$$\mathcal{T} \longmapsto Z_{\mathcal{T}}(T^2, RR)$$

$\mathcal{N}=(0,1) \rightsquigarrow$ holomorphicity

$\mathrm{MF}[\Delta^{-1}]_k$: weakly $\boxed{\text{holom}}$. $\mathbb{Z}$-modular forms of weight k

$\cap$ $\mathbb{Z}((q))$

$\theta : \mathbb{H} \to \mathbb{C}$ holom, $\theta\left(\frac{a\tau+b}{c\tau+d}\right) = (c\tau+d)^k\, \theta(\tau)$

$\circlearrowleft SL_2(\mathbb{Z})$

$$\theta(\tau) = \sum_n a_n q^n \qquad q = e^{2\pi i \tau}$$

$a_n \in \mathbb{Z}$

$$\mathrm{MF}[\Delta^{-1}]_* = \mathbb{Z}[c_4, c_6, \Delta, \Delta^{-1}] / (c_4^3 - c_6^2 - 1728\Delta)$$

7

36

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Topological refinements.

Recall

$$\mathrm{ABS} : \mathrm{MSpin} \to \mathrm{KO} \quad \overset{\text{refine}}{\rightsquigarrow} \quad \mathrm{Ind} : \Omega^{\mathrm{Spin}}_{*} \to \mathrm{KO}_{*}$$

Witten suggested ('84)

& mathematicians later constructed (~'00s)

(Hopkins, Miller, Goerss, Rezk, ...)

$$\begin{tikzcd} \mathrm{MString} \arrow[r, "\mathrm{Wit}"'] \arrow[d] \arrow[dr, phantom, "\circlearrowleft"] & \mathrm{TMF} \arrow[d] \\ \mathrm{MSpin} \arrow[r, "\mathrm{Wit}"'] & \mathrm{KO}((q)) \end{tikzcd} \quad \overset{\text{refine}}{\rightsquigarrow} \quad \begin{tikzcd} \Omega^{\mathrm{String}}_{*} \arrow[r, "\mathrm{Wit}"'] \arrow[d] \arrow[dr, phantom, "\circlearrowleft"] & \mathrm{MF}[\Delta^{-1}]_{*/2} \arrow[d] \\ \Omega^{\mathrm{Spin}}_{*} \arrow[r, "\mathrm{Wit}"'] & \mathbb{Z}((q)) \end{tikzcd}$$

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Recall how TMF was suggested by Witten('84)

… home for Dirac index on loop spaces.

Dirac index : $\underset{\text{Spin mfd}}{M} \longmapsto \mathrm{Ind}(D_M) \in \mathbb{Z}, \mathbb{Z}/2$ $= KO_*$

"LM version"

- String structure on M $\approx$ Spin str on LM

  = " Spin structure with trivialization of $\frac{p_1}{2}$ "

$$\begin{array}{l} V \quad \text{oriented} \\ \downarrow \quad \mathbb{R}\text{-vec.bdl} \\ X \end{array} \quad \rightsquigarrow \quad \begin{tikzcd} & BString \arrow[d] & \\ & BSpin \arrow[d] \arrow[r, "\frac{p_1}{2}"] & K(\mathbb{Z},4) \\ X \arrow[ruu, dashed, "\text{String str}"] \arrow[ru, dashed, "\text{spin str}"] \arrow[r, "\text{orientation}"'] & BSO \arrow[r, "w_2"] & K(\mathbb{Z}/2,2) \end{tikzcd}$$

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K3

d	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$\Omega_d^{\text{spin}}(\text{pt})$	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	0	$\boxed{\mathbb{Z}}$	0	0	0	$\mathbb{Z}^2$	$(\mathbb{Z}_2)^2$	$(\mathbb{Z}_2)^3$	0	$\mathbb{Z}^3$	0	0	0	$\mathbb{Z}^5$
$\Omega_d^{\text{string}}(\text{pt})$	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\boxed{\mathbb{Z}_{24}}$	0	0	$\mathbb{Z}_2$	0	$\mathbb{Z} \oplus \mathbb{Z}_2$	$(\mathbb{Z}_2)^2$	$\mathbb{Z}_6$	0	$\mathbb{Z}$	$\mathbb{Z}_3$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}^2$
		S^1	$S^1 \times S^1$														

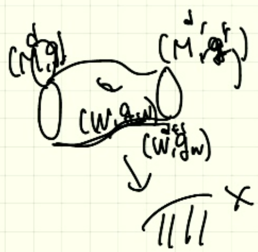
TABLE 1. Table of spin and string bordism groups

$S^3 \cong SU(2)$

Lie grp framing

$\Omega_d^{B}(X)$: B-bordism group

$$\| \qquad \left\{ (M^d, g, f) \;\middle|\; \begin{array}{l} M : d\text{-dim clo. mfd} \\ g : B\text{-str on } M \\ f : M \to X \end{array} \right\} \Big/ \underset{\text{bordism}}{\sim}$$



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Recall how TMF was suggested by Witten ('84)

... home for Dirac index on loop spaces.

Dirac index : $M \mapsto \mathrm{Ind}(D_M) \in \mathbb{Z}, \mathbb{Z}/2$
spin mfd $\qquad = KO_*$

"LM version"

- String structure on M $\approx$ Spin str on LM

= " Spin structure with trivialization of $\frac{p_1}{2}$ "

V oriented $\mathbb{R}$-vec.bdl
$\downarrow$
X

$\rightsquigarrow$

$$\begin{tikzcd}
 & & BString \arrow[d] & \\
 & & BSpin \arrow[d] \arrow[r, "\frac{p_1}{2}"] & K(\mathbb{Z},4) \\
X \arrow[rru, dashed, bend left, "String\ str"] \arrow[ru, dashed, "spin\ str"] \arrow[rr, "orientation"'] & & BSO \arrow[r, "w_2"] & K(\mathbb{Z}/2,2)
\end{tikzcd}$$

8
36

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• <u>Witten genus</u> = "S^1-equivariant Dirac index on LM"

$\mathrm{Wit}(M)$ "$:= \mathrm{Ind}_{S^1}(D_{LM})$"

$$:= \underbrace{\mathrm{Ind}_M}_{\substack{\uparrow \\ \text{twisted Dirac index}}}\left(\bigotimes_{m\geq 1}\left(\bigoplus_{k\geq 0} q^{mk}\,\mathrm{Sym}^k(TM \ominus \mathbb{R}^{\dim M})\right)\right) \in \mathbb{Z}[[q]] \text{ if } M:\text{Spin}.$$

q ← formal variable

<u>Fact</u> $\mathrm{Wit}(M) \in MF_{m/2}$ if M : String ($m = \dim M$)

i.e.,

$$\begin{tikzcd} \Omega_*^{\mathrm{String}} \arrow[r, "\mathrm{Wit}"] \arrow[d] \arrow[dr, phantom, "\circlearrowleft"] & MF_{*/2} \arrow[d] \\ \Omega_*^{\mathrm{Spin}} \arrow[r, "\mathrm{Wit}"'] & \mathbb{Z}[[q]] \end{tikzcd}$$

10

36

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Topological refinements.

Recall

$$\text{ABS} : MSpin \to KO \quad \overset{\text{refine}}{\rightsquigarrow} \quad \text{Ind} : \Omega^{Spin}_* \to KO_*$$

Witten suggested ('84)

& mathematicians later constructed (~'00s)

(Hopkins, Miller, Goerss, Rezk, ⋯)

$$\begin{tikzcd} MString \arrow[r, "Wit"] \arrow[d] \arrow[dr, phantom, "\circlearrowleft"] & TMF \arrow[d] \\ MSpin \arrow[r, "Wit"] & KO((q)) \end{tikzcd} \quad \overset{\text{refine}}{\rightsquigarrow} \quad \begin{tikzcd} \Omega^{String}_* \arrow[r, "Wit"] \arrow[d] \arrow[dr, phantom, "\circlearrowleft"] & MF[\Delta^{-1}]_{*/2} \arrow[d] \\ \Omega^{Spin}_* \arrow[r, "Wit"] & \mathbb{Z}((q)) \end{tikzcd}$$

11
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TMF is a spectrum which is a "topological" version of $MF[\Delta^{-1}]$ ...

- Defined as a global section of an E_∞-sheaf on $\mathcal{M}_{ell}/\mathbb{Z}$
- 576-periodic : $TMF_* \cong TMF_{*+576}$
- $TMF_* \xrightarrow{\exists} (MF[\Delta^{-1}])_{*/2} = \mathbb{Z}[c_4, c_6, \Delta, \Delta^{-1}]/(c_4^2 - c_6^2 - 1728\Delta)$

  inducing $TMF_* \otimes \mathbb{Q} \cong MF[\Delta^{-1}]_{*/2} \otimes \mathbb{Q}$.

  but $\exists$ nontrivial cokernels.

  e.g.
  $$TMF_{\pm 24} \to MF[\Delta^{-1}]_{\pm 12}$$
  $$\nexists \dashrightarrow \Delta, \Delta^{-1}$$
  $$\exists \text{ lift} \longmapsto 24\Delta, 24\Delta^{-1}$$

- $\exists$ many 2, 3-power torsions in TMF_*.

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We have $\mathrm{MString} \xrightarrow{\mathrm{Wit}} \mathrm{TMF}$:

d	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$\Omega_d^{\mathrm{spin}}(\mathrm{pt})$	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	0	$\mathbb{Z}$	0	0	0	$\mathbb{Z}^2$	$(\mathbb{Z}_2)^2$	$(\mathbb{Z}_2)^3$	0	$\mathbb{Z}^3$	0	0	0
$\Omega_d^{\mathrm{string}}(\mathrm{pt})$	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_{24}$	0	0	$\mathbb{Z}_2$	0	$\mathbb{Z} \oplus \mathbb{Z}_2$	$(\mathbb{Z}_2)^2$	$\mathbb{Z}_6$	0	$\mathbb{Z}$	$\mathbb{Z}_3$	$\mathbb{Z}_2$	$\mathbb{Z}_2$

TABLE 1. Table of spin and string bordism groups

d	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$\pi_d(\mathrm{tmf})_{(2)}$	[illegible]	[illegible]	[illegible]						[illegible]	[illegible]	[illegible]		[illegible]		

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d	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	“chromatic height”
$\pi_d(\mathrm{tmf})_{(2)}$	$\mathbb{Z}_{(2)}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$						$\mathbb{Z}_{(2)}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$		$\mathbb{Z}_{(2)}$				← 1
				$\mathbb{Z}_8$			$\mathbb{Z}_2$		$\mathbb{Z}_2$	$\mathbb{Z}_2$					$\mathbb{Z}_2$	$\mathbb{Z}_2$	← 2
d	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	
$\pi_d(\mathrm{tmf})_{(2)}$	$\mathbb{Z}_{(2)}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$		$\mathbb{Z}_{(2)}$				$\mathbb{Z}_{(2)}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$		$\mathbb{Z}_{(2)}$				
		$\mathbb{Z}_2$			$\mathbb{Z}_8$	$\mathbb{Z}_2$	$\mathbb{Z}_2$		$\mathbb{Z}_{(2)}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_4$	$\mathbb{Z}_2$				
d	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	
$\pi_d(\mathrm{tmf})_{(2)}$	$\mathbb{Z}_{(2)}^2$	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^2$		$\mathbb{Z}_{(2)}^2$				$\mathbb{Z}_{(2)}^2$	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^2$		$\mathbb{Z}_{(2)}^2$				
	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$				$\mathbb{Z}_2$	$\mathbb{Z}_4$	$\mathbb{Z}_2$	$\mathbb{Z}_2$			$\mathbb{Z}_2$	$\mathbb{Z}_2$		
d	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	
$\pi_d(\mathrm{tmf})_{(2)}$	$\mathbb{Z}_{(2)}^2$	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^2$		$\mathbb{Z}_{(2)}^2$				$\mathbb{Z}_{(2)}^3$	$\mathbb{Z}_2^3$	$\mathbb{Z}_2^3$		$\mathbb{Z}_{(2)}^3$				
	$\mathbb{Z}_{(2)}$		$\mathbb{Z}_2$	$\mathbb{Z}_8$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_4$			$\mathbb{Z}_2$		$\mathbb{Z}_2$	$\mathbb{Z}_4$				
d	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	
$\pi_d(\mathrm{tmf})_{(2)}^3$	$\mathbb{Z}_{(2)}^3$	$\mathbb{Z}_2^3$	$\mathbb{Z}_2^3$		$\mathbb{Z}_{(2)}^3$				$\mathbb{Z}_{(2)}^3$	$\mathbb{Z}_2^3$	$\mathbb{Z}_2^3$		$\mathbb{Z}_{(2)}^3$				
		$\mathbb{Z}_2^2$	$\mathbb{Z}_2$		$\mathbb{Z}_2$		$\mathbb{Z}_2$		$\mathbb{Z}_{(2)}$			$\mathbb{Z}_2$					
d	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	
$\pi_d(\mathrm{tmf})_{(2)}$	$\mathbb{Z}_{(2)}^4$	$\mathbb{Z}_2^4$	$\mathbb{Z}_2^4$		$\mathbb{Z}_{(2)}^4$				$\mathbb{Z}_{(2)}^4$	$\mathbb{Z}_2^4$	$\mathbb{Z}_2^4$		$\mathbb{Z}_{(2)}^4$				
	$\mathbb{Z}_2$					$\mathbb{Z}_2$					$\mathbb{Z}_2$						
d	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	
$\pi_d(\mathrm{tmf})_{(2)}$	$\mathbb{Z}_{(2)}^4$	$\mathbb{Z}_2^4$	$\mathbb{Z}_2^4$		$\mathbb{Z}_{(2)}^5$				$\mathbb{Z}_{(2)}^5$	$\mathbb{Z}_2^5$	$\mathbb{Z}_2^5$		$\mathbb{Z}_{(2)}^5$				
	$\mathbb{Z}_{(2)}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_8$	$\mathbb{Z}_2$		$\mathbb{Z}_2$		$\mathbb{Z}_2$	$\mathbb{Z}_2^2$					$\mathbb{Z}_4$	$\mathbb{Z}_2$	

Stolz-Teichner Conjecture ['04, '11]

$$\left\{ \begin{matrix} \overbrace{\text{2d } \mathcal{N}=(0,1) \text{ SQFT}}^{\text{unitary, fully-extended}} \\ \text{of central charge } c \end{matrix} \right\} \Big/ \text{deformation} \cong TMF^{2c}$$

$\boxed{\text{Phys}}$ $\boxed{\text{Math}}$

$$\{\text{String mfd}\} \xrightarrow{\sigma\text{-model}} \{\text{2d } \mathcal{N}=(0,1) \text{ SQFT}\} \longleftrightarrow TMF \xleftarrow{\text{Wit}} MString$$

$$\{S^1\text{-Hilb.sp}\} \xleftarrow{(S^1,R)} \{\text{2d } \mathcal{N}=(0,1) \text{ SQFT}\} \xrightarrow{(T^2,RR)} MF[\Delta^{-1}]$$

$$KO((q)) \longleftarrow TMF \longrightarrow MF[\Delta^{-1}]$$

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How to construct the map

$$\left\{\begin{array}{c} \text{2d } \mathcal{N}=(0,1) \text{ SQFT} \\ \text{central charge } n/2 \end{array}\right\} \longrightarrow \mathrm{TMF}_{-n} \;?$$

Two Easy cases :

① If $n \equiv 0 \mod 4$ & $\mathrm{TMF}_{-n} \overset{\text{inj}}{\rightarrowtail} \mathrm{MF}[\Delta^{-1}]_{-n/2}$,

the map is just $\mathcal{T} \mapsto Z_{\mathcal{T}}(T^2; R\dot{R})$

(easy!)

Witten genus.

② If $n \equiv -1$ or $-2 \mod 8$ & $\mathrm{TMF}_{-n} \overset{\text{inj}}{\rightarrowtail} KO_{-n}((q))$

$\simeq \mathbb{Z}/2((q))$,

the map is just $\mathcal{T} \mapsto Z_{\mathcal{T}}^{\text{mod } 2}(T^2; RR)$

mod 2 - Witten genus.

(easily computed !)

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How to construct the map

$$\left\{\begin{matrix} \text{2d } \mathcal{N}=(0,1) \text{ SQFT} \\ \text{central charge } n/2 \end{matrix}\right\} \longrightarrow \mathrm{TMF}_{-n} \ ?$$

Two Easy cases :

① If $n \equiv 0 \mod 4$ & $\mathrm{TMF}_{-n} \xrightarrow{\text{inj}} \mathrm{MF}[\Delta^{-1}]_{-n/2}$,

the map is just $\mathcal{T} \mapsto Z_{\mathcal{T}}(T^2, R\dot{R})$

Witten genus. (easy !)

② If $n \equiv -1$ or $-2 \mod 8$ & $\mathrm{TMF}_{-n} \xrightarrow{\text{inj}} KO_{-n}((q)) \simeq \mathbb{Z}/2((q))$,

the map is just $\mathcal{T} \mapsto Z_{\mathcal{T}}^{\text{mod } 2}(T^2, RR)$

mod 2 - Witten genus.
(easily computed !)

16
36

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Beyond Easy Cases :

Examples of torsions in TMF_* with corresponding lattice SVOAs we expect

... results from [TY '23], as we explain later.

Lattice	$n = 2c$	in $\mathrm{KO}((q))^n(\mathrm{pt})$	we conjecture...
E_8	$n = 16$	c_4/Δ	
$\widetilde{D_{12}}$	$n = 24$	$24/\Delta$	
$\widetilde{E_7 \times E_7}$	$n = 28$	0	nontrivial in $A^{28} \simeq \mathbb{Z}/2$
$\widetilde{A_{15}}$	$n = 30$	0	nontrivial in $A^{30} \simeq \mathbb{Z}/2$
$\widetilde{D_8 \times D_8}$	$n = 32$	0	nontrivial in $A^{32} \simeq \mathbb{Z}/3$
$\widetilde{D_{16}}$	$n = 32$	$(c_4/\Delta)^2$	

TABLE 1. Examples of lattice SVOAs and conjectured image in TMF.

chromatic height 2

$$A^n := \mathrm{Ker}(\mathrm{TMF}^n \to \mathrm{KO}^n((q)))$$

17
36
Go to next page (→)

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Recall : [TY 21] is about

$$
\begin{tikzcd}[column sep=huge, row sep=huge]
\text{Phys} & \left\{ \begin{matrix} \text{2d SCFTs} \\ \mathcal{N}=(0,1) \end{matrix} \right\} \arrow[r, "\text{heterotic}"', "\text{compactification}"] \arrow[d, "\begin{matrix}\text{Stolz-Teichner}\\ \text{conjecture}\end{matrix}"] & \left\{ \begin{matrix} \text{QFTs on} \\ \text{String manifolds} \end{matrix} \right\} \arrow[d, "\begin{matrix}\text{extract}\\ \text{anomalies}\end{matrix}", "\begin{matrix}\text{Freed-Hopkins}\\ \text{conjecture}\end{matrix}"'] \\
\text{Math} & TMF^{d+22} \arrow[r, "\alpha"] & I_{\mathbb{Z}} MString^{d+2}
\end{tikzcd}
$$

§2. Anomalies and Anderson duals.

$I_{\mathbb{Z}} MTB$: Anderson dual to B-bordism theory

— B : tangential structure (SO, Spin, …)

… a generalized cohomology theory

important in classification of $\left\{ \begin{array}{l} \text{SPT phases} \\ \text{Anomalies} \end{array} \right.$ by :

Conjecture [Freed-Hopkins '16] (roughly)

$$\left\{ \begin{array}{l} \text{invertible } (d+1)\text{-dim QFT} \\ \text{on } B\text{-manifolds} \end{array} \right\} \Big/ \sim_{\text{deformation}} \;\cong\; I_{\mathbb{Z}} MTB^{d+2}$$

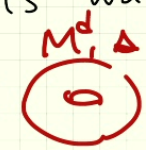
$$\cong$$

$$\left\{ \begin{array}{l} \text{anomalies in } d\text{-dim QFT} \\ \text{on } B\text{-manifolds} \end{array} \right\} \Big/ \sim_{\text{def}}$$

Example of anomaly : massless Fermions

$d = 2k$, $\mathcal{S} = \mathrm{Spin}^c_\nabla$

physicists want to define a QFT α_d with

$\alpha_d\Big($  $\Big) =$ "$\mathrm{Det}(D_M)$"

Actually, we can formulate

$$\mathrm{Det}(D_M) \in \mathfrak{D}\mathrm{et}(D_M)$$: Quillen's determinant line

Fact (Dai - Freed)

$\exists$ invertible $(2k+1)$-dim QFT $T_{d+1} : \mathrm{Bord}^{\mathrm{Spin}^c_\nabla}_{d+1} \to \mathrm{sLine}_{\mathbb{C}}$

with $T_{d+1}(M^d, A) = \mathfrak{D}\mathrm{et}(D_M)$.

$$T_{d+1}(W^{d+1}, A) = \exp\left(2\pi i\, \bar{\eta}(D_M)\right)$$

← reduced eta inv

Anderson duals.

$\boxed{\text{Conjecture}}$ [Freed - Hopkins '16] + "Anomaly inflow"

$$\left\{ \begin{array}{l} \text{anomalies in } d\text{-dim QFT} \\ \text{on } B\text{-mfds w/ target } X \end{array} \right\} \Big/ \underset{\text{deformation}}{\sim} \quad \xleftrightarrow{\text{bij}} \quad I_{\mathbb{Z}} MTB^{d+2}(X)$$

$I_{\mathbb{Z}} MTB$ is a generalized cohomology theory which fits into

$$0 \to \underset{\underset{\mathrm{Hom}(\Omega^B_{d+1}(X), \mathbb{R}/\mathbb{Z}) / \mathrm{Hom}(\Omega^B_{d+1}(X), \mathbb{R})}{\cong}}{\mathrm{Ext}(\Omega^B_{d+1}(X), \mathbb{Z})} \to I_{\mathbb{Z}} MTB^{d+2}(X) \to \mathrm{Hom}(\Omega^B_{d+2}(X), \mathbb{Z}) \to 0 \quad \textbf{(exact)}$$

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Exact sequence of $I_{\mathbb{Z}}MT\mathcal{B}$: Physical interpretation

$$0 \to \underbrace{\mathrm{Ext}(\Omega^{\mathcal{B}}_{d+1}(X), \mathbb{Z})} \to I_{\mathbb{Z}}MT\mathcal{B}^{d+2}(X) \to \mathrm{Hom}(\Omega^{\mathcal{B}}_{d+2}(X), \mathbb{Z}) \to 0$$

$$\mathrm{Ext}(\Omega^{\mathcal{B}}_{d+1}(X), \mathbb{Z}) = \mathrm{Hom}(\Omega^{\mathcal{B}}_{d+1}(X), \mathbb{R}/\mathbb{Z}) / \mathrm{Hom}(\Omega^{\mathcal{B}}_{d+1}(X), \mathbb{R})$$

Topological d-dim anomalies

$$\mathrm{Hom}(\Omega^{\mathcal{B}}_{d+2}(X), \mathbb{Z}) \subset H^{d+1}(X \wedge MT\mathcal{B}; \mathbb{R}) \ni [\omega_\alpha]$$

"Anomaly polynomial" of d-dim anomalous QFT

Cf. [Y-Yonekura '21] for more explanation.

Example $d = 2$ massless fermion

$= $ a generator of $(I_{\mathbb{Z}}M\mathrm{Spin}^c)^4 \simeq \mathbb{Z}$.

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25
36

§ 3. Formulation & proof

Recall the diagram:

$$
\begin{tikzcd}[column sep=huge, row sep=huge]
\text{Phys} & \left\{\begin{matrix}\text{2d SCFTs} \\ \mathcal{N}=(0,1)\end{matrix}\right\} \arrow[r, "\text{heterotic}", "\text{compactification}"'] \arrow[d, "\substack{\text{Stolz-Teichner} \\ \text{conjecture} \\ ['04,\ '11]}"] & \left\{\begin{matrix}\text{QFTs on} \\ \text{String manifolds}\end{matrix}\right\} \arrow[d, "\substack{\text{extract} \\ \text{anomalies}}", "\substack{\text{Freed-Hopkins} \\ \text{conjecture} \\ ['16]}"'] \\
\text{Math} & TMF^{d+22} \arrow[r, "\alpha"] & I_{\mathbb{Z}}MString^{d+2}
\end{tikzcd}
$$

We want to show vanishing of heterotic anomalies

$\Leftrightarrow$ $\alpha = 0$., under assumptions coming from physics.

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Assumptions for α :

①
$$\begin{tikzcd} TMF \arrow[r, "\alpha"] \arrow[d] \arrow[dr, phantom, "\circlearrowright"] & \Sigma^{-20} I_{\mathbb{Z}} MString \\ KO((q)) \arrow[r, "\exists \alpha_{spin}"'] & \Sigma^{-20} I_{\mathbb{Z}} MSpin \arrow[u] \end{tikzcd}$$

... fermions detect anomalies

② α_{spin} is a MSpin-module map ... compatibility with compactification

③ $\alpha_{spin}(pt) : KO((q))^{20}(pt) \to (I_{\mathbb{Z}} MSpin)^{0}(pt)$

$$KO((q))^{20}(pt) \cong \mathbb{Z}((q)), \qquad (I_{\mathbb{Z}} MSpin)^{0}(pt) \cong \mathbb{Z}.$$

is given by $\Phi(q) \longmapsto \Delta(q)\Phi(q)|_{q\text{-coeff}}$.

... formula for anomaly poly (80's)

Thm [TY21]

Under assumptions ①②③, $\alpha = 0$.

26
36

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Part II. Secondary anomaly & Anderson self-duality of TMF

[TY'23]

by the result of [TY21] + ε, ∃! lift $\tilde{\alpha}$ of α_{spin} :

$$
\begin{tikzcd}
 & TMF \arrow[dl, dashed, "\exists!\,\tilde{\alpha}"'] \arrow[d, "\alpha_{spin}"] \arrow[dr, "0 \text{ by [TY21]}"] & \\
\Sigma^{-20} I_{\mathbb{Z}} MSpin/MString \arrow[r] & \Sigma^{-20} I_{\mathbb{Z}} MSpin \arrow[r] & \Sigma^{-20} I_{\mathbb{Z}} MString
\end{tikzcd}
$$

(: diag of MString - modules.)

which we call the **secondary anomaly transformation.**

$\tilde{\alpha}$ turns out to be directly related to the **Anderson duality in TMF.**

27
36

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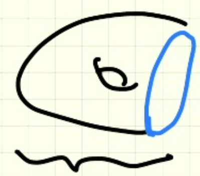
- MSpin/MString : relative Spin / String bordism.

$$\Omega_d^{\mathrm{Spin/String}} = \left\{ \quad \right\} \Big/ \sim \ \text{bordism}$$



$$\Omega_d^{\mathrm{Spin/String}} \cong \pi_d(M\mathrm{Spin}/M\mathrm{String})$$

homotopy fiber exact sequence :

$$\cdots \to \Omega_d^{\mathrm{String}} \to \Omega_d^{\mathrm{Spin}} \to \Omega_d^{\mathrm{Spin/String}} \to \Omega_{d-1}^{\mathrm{String}} \to \cdots$$

(exact)

$\therefore$ $\Omega_*^{\mathrm{Spin/String}}$ = difference between Ω_*^{Spin} & $\Omega_*^{\mathrm{String}}$.

28
36
Go to next page (→)

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$\Omega_*^{\mathrm{Spin/String}}$ = difference between Ω_*^{Spin} & $\Omega_*^{\mathrm{String}}$:

d		$\Omega_d^{\mathrm{String}}$		Ω_d^{Spin}		$\Omega_d^{\mathrm{Spin/String}}$
8	$\longrightarrow$	$\mathbb{Z}/2 \oplus \mathbb{Z}$	$\longrightarrow$	$\mathbb{Z}^2$	$\longrightarrow$	$\mathbb{Z}$
7	$\longrightarrow$	0	$\longrightarrow$	0	$\longrightarrow$	$\mathbb{Z}/2$
6	$\longrightarrow$	$\mathbb{Z}/2$	$\longrightarrow$	0	$\longrightarrow$	0
5	$\longrightarrow$	0	$\longrightarrow$	0	$\longrightarrow$	0
4	$\longrightarrow$	0	$\longrightarrow$	$\mathbb{Z}$ [K3]	$\xrightarrow{24\times}$	$\mathbb{Z}$ $[D^4, S^3 \cong SU(2)]$
3	$\longrightarrow$	$\mathbb{Z}/24$ $S^3 \cong SU(2)$	$\longrightarrow$	0	$\longrightarrow$	0
2	$\longrightarrow$	$\mathbb{Z}/2$	$\longrightarrow$	$\mathbb{Z}/2$	$\longrightarrow$	0
1	$\longrightarrow$	$\mathbb{Z}/2$	$\longrightarrow$	$\mathbb{Z}/2$	$\longrightarrow$	0
0	$\longrightarrow$	$\mathbb{Z}$	$\longrightarrow$	$\mathbb{Z}$	$\longrightarrow$	0

Relation with the Anderson duality of TMF

We have

$$\begin{tikzcd} MString \arrow[r] \arrow[d, "Wit"'] \arrow[dr, phantom, "\circlearrowleft"] & MSpin \arrow[r] \arrow[d, "Wit"] \arrow[dr, phantom, "\circlearrowleft"] & MSpin/MString \arrow[d, "Wit"] \\ TMF \arrow[r] & KO((q)) \arrow[r] & KO((q))/TMF \end{tikzcd}$$

Thm (TY '23)

Anderson duality in TMF !

$\exists$ isom $\quad KO((q))/TMF \simeq \Sigma^{-20} I_{\mathbb{Z}} TMF$

and the dual of $\tilde{\alpha} : TMF \to \Sigma^{-20} I_{\mathbb{Z}} MSpin/MString$ factors as

$$\tilde{\alpha}^{\vee} : MSpin/MString \xrightarrow[Wit]{} KO((q))/TMF \simeq \Sigma^{-20} I_{\mathbb{Z}} TMF .$$

$\cdot\dot{x}\cdot$ the isom is essentially another version of

Fact (Stojanoska '13) $\quad Tmf \simeq \Sigma^{-21} I_{\mathbb{Z}} Tmf .$

☺ $\tilde{\alpha}$ is very nontrivial !!

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How to "see" $\tilde{\alpha} : TMF \to \Sigma^{-20} I_{\mathbb{Z}} MSpin/MString$?

... via Pairings induced by $\tilde{\alpha}$:

① Non-torsion pairings

$$\langle \, , \, \rangle_{\tilde{\alpha}} : TMF^d \otimes \Omega^{Spin/String}_{d-20} \to \mathbb{Z}.$$

② Torsion pairings

$$\langle \, , \, \rangle_{\tilde{\alpha}} : TMF^d_{tor} \otimes (\Omega^{Spin/String}_{d-21})_{tor} \to \mathbb{Q}/\mathbb{Z}.$$

They are

- very nontrivial, and
- computable by differential-geometric ways!
  (e.g. characteristic forms / eta invariants)

$\tilde{\alpha} : TMF^{d} \to I_{\mathbb{Z}} MSpin/MString^{d-20}$ induces :

① Non-torsion pairings :

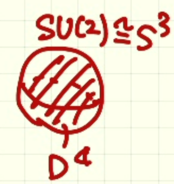
by $I_{\mathbb{Z}} MSpin/MString^{d-20} \to \mathrm{Hom}(\Omega^{Spin/String}_{d-20}, \mathbb{Z})$,

$$\langle\, , \rangle_{\tilde{\alpha}} : TMF^{d} \otimes \Omega^{Spin/String}_{d-20} \to \mathbb{Z}.$$

__Example__ $d = 24$

$$TMF^{24} \ni 24/\Delta \qquad (\Delta \in MF_{12} \text{ modular discriminant})$$

$$\Omega^{Spin/String}_{4} \simeq \mathbb{Z} \ni [D^4, S^3 \simeq SU(2)]$$



__Prop__

$$\langle 24/\Delta, [D^4, S^3 \simeq SU(2)] \rangle_{\tilde{\alpha}} = 1.$$

__proof__ : computation of relative Witten genus.

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② Torsion pairings :

by $\left(\left(I\Omega^{Spin/String}\right)^{d-20}\right)_{torsion} \simeq \mathrm{Ext}\left(\Omega^{Spin/String}_{d-21}, \mathbb{Z}\right)$,

$$\langle\ ,\ \rangle_{\alpha} : \mathrm{TMF}^{d}_{tor} \otimes \left(\Omega^{Spin/String}_{d-21}\right)_{tor} \to \mathbb{Q}/\mathbb{Z}.$$

Example $d = 28$ chromatic height $= 2$!

$\mathrm{TMF}^{28}_{tor} \simeq \mathbb{Z}/2.$ $\qquad \Omega^{Spin/String}_{7} \simeq \mathbb{Z}/2.$

$\in$ $\qquad\qquad \in$

$[E_7 \times E_7]$ $\qquad [D^4 \times S^3,\ S^3 \times S^3]$

$\in$ VOA $\qquad S^3 \simeq SU(2)$, $S^3 \simeq SU(2)$

Prop [TY23]

$$\langle [E_7 \times E_7],\ [D^4 \times S^3,\ S^3 \times S^3] \rangle_{\alpha} = \frac{1}{2}.$$

Proof : Compute eta invariants. We use $\mathrm{TMF}^{\tau+\tau'}_{E_7 \times E_7}$

33
36

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Applications

Math: detect power operations in TMF via differential-geometric methods.

Phys: extract chromatic height-2 information from 2d SQFTs.

e.g.

e.g. $(d=31)$

$$\mathrm{TMF}^{16} \xrightarrow{\cup} \mathrm{TMF}^{32}(B\Sigma_2) \xrightarrow{\text{Möbius}} \mathrm{TMF}^{32}(S^1) \xrightarrow{\int_{S^1}} \mathrm{TMF}^{31}$$

$$\mathrm{TMF}^{16} \ni [E_8 \text{ level } 1] \longmapsto [E_8 \text{ level } 2] \in \mathrm{TMF}^{31}$$

$$[E_8 \text{ level } 1] \in \mathrm{TMF}^{16} \xrightarrow{\cong} \mathrm{MF}[\Delta^{-1}]_{-8} \ni c_4/\Delta$$

$$X \in [D^4 \times S^3 \times S^3,\ S^3 \times S^3 \times S^3] \in \Omega_{10}^{\mathrm{Spin}/\mathrm{String}}$$

$$\langle\, ,\, \rangle_{\tilde{\alpha}} \downarrow$$

nontrivial!

⚠ Actually we need TMF_G^{τ} : equivariant twisted TMF.

34
36

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This leads us to the explicit relation between VOAs and TMF_* :

Lattice	$n = 2c$	in $\mathrm{KO}((q))^n(\mathrm{pt})$	we conjecture...
E_8	$n = 16$	c_4/Δ	
$\widetilde{D_{12}}$	$n = 24$	$24/\Delta$	
$\widetilde{E_7 \times E_7}$	$n = 28$	0	nontrivial in $A^{28} \simeq \mathbb{Z}/2$
$\widetilde{A_{15}}$	$n = 30$	0	nontrivial in $A^{30} \simeq \mathbb{Z}/2$
$\widetilde{D_8 \times D_8}$	$n = 32$	0	nontrivial in $A^{32} \simeq \mathbb{Z}/3$
$\widetilde{D_{16}}$	$n = 32$	$(c_4/\Delta)^2$	

height 2

TABLE 1. Examples of lattice SVOAs and conjectured image in TMF.

$$A^n := \mathrm{Ker}(\mathrm{TMF}^n \to \mathrm{KO}^n((q)))$$

35
36

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Open questions

- Construct mathematicaly,

$$\{\mathcal{N}=(0,1)\ \text{SCFT}\} \longrightarrow \text{TMF}$$

$\cap$ can be formulated via SVOAs

$$\{\mathcal{N}=(0,1)\ \text{SQFT}\}$$ Stolz-Teichner

- Develop & compute TMF^{τ}_{G}.

  For example $G=E_8$ is relevant to computation of torsion pairing.

- Find more physical applications!

36
36