

Title: The Callan Rubakov Effect

Speakers:

Series: Particle Physics

Date: October 03, 2023 - 1:00 PM

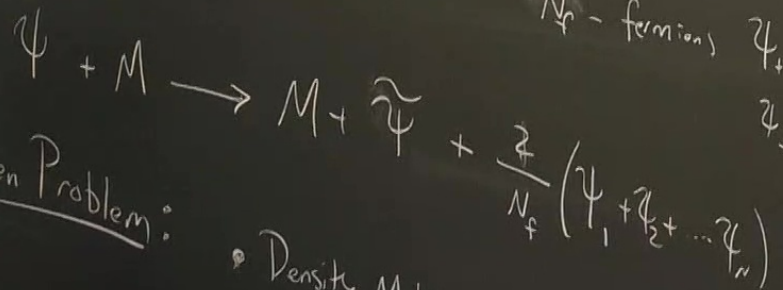
URL: <https://pirsa.org/23100072>

Abstract: The Callan Rubakov Effect describes the interaction between (massless) fermions and a smooth monopole in 4d gauge theory. In this scenario, the fermions can probe the UV physics inside the monopole core which leads to interesting effects such as proton decay in GUT models. However, the monopole-fermion scattering appears to lead to out-states that are not in the perturbative Hilbert space. In this talk, we will review this issue and propose a new physical mechanism that resolves this long-standing confusion.

Zoom link: <https://pitp.zoom.us/j/93551249905?pwd=REJvOUtTMlh6RU0vRjZnRVdZckgyZz09>

The Callan Rubakov Effect

T. Daniel Brennan based on
[hep-th/2309.00680
2109.11207
2109.13820]



N_f - fermions ψ_+
 ψ_-

Open Problem:

- Density Matrix

- \exists n.p. \mathcal{H}

- Extra Interactions - massive fermions

CR Effect

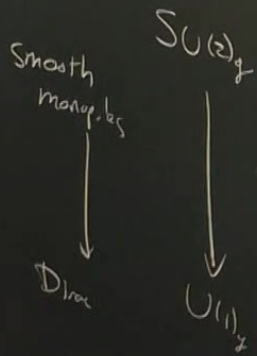
CR Effect (old)

4d $SU(2)_g$ + $2N_f \psi$

+ Φ adj.

No Yukawa

$V(\Phi) = (|\Phi|^2 - v)^2$



$\langle |\Phi|^2 \rangle = v^2$

$m_W = v/g$

Sph Sym (BPS)

$\vec{J} = \vec{L} + \vec{T} \quad \vec{T} - SU(2)_f$

$A_{mono} = T_3 A_{Dirac} \pm \frac{i}{2} T_{\pm} \underbrace{Mer}_\omega \underbrace{(d\theta \mp i \sin\theta d\phi)}_{\omega_{\pm}}$

$\vec{\Phi} = v T_3 \underbrace{hcr}$

$Mer \rightarrow 0$

$hcr \rightarrow 1/r \sim e^{-m_W r}$

$A_{mono} \rightarrow T_3 A_{Dirac}$

IR modes: photon + $\varphi(\pm)$

$$S_H = \int \frac{1}{2m_W} (\dot{\varphi} - 2A_\pm)^2 dt$$

$$\varphi \sim \varphi + 2\pi$$

$$\hat{H} = \frac{m_W}{2} \hat{P}^2 + 2A_\pm \hat{P}$$

$$\rightarrow \mathcal{H} = \left\{ |n\rangle \quad n \in \mathbb{Z} \mid \hat{P}|n\rangle = n|n\rangle \right\}$$

$|n\rangle \rightarrow 2n$ bulk charge

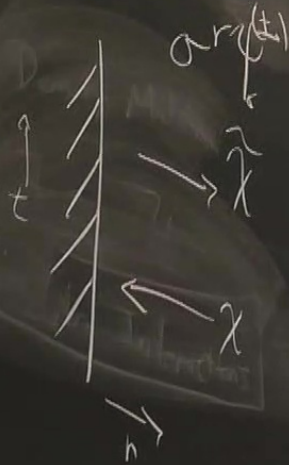
Massless Fermions

- Sph. Sym \rightarrow expand ψ in ang. mom.
 \hookrightarrow restrict to $j=0$

$$\psi^\dagger = \begin{pmatrix} \psi_+^\dagger \\ \psi_-^\dagger \end{pmatrix}$$

$$\psi_+^\dagger = \frac{1}{r} \chi^\dagger(t+r) \psi_0^{(+)}$$

$$\psi_-^\dagger = \frac{1}{r} \tilde{\chi}^\dagger(t-r) \psi_0^{(-)}$$



$$a r(t) = \pm \psi(t)$$

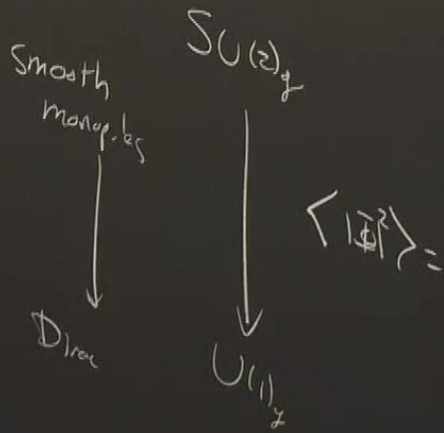
$$\chi = \tilde{\chi} \rightarrow SO(2,1)$$

$$\chi = e^{i\theta} \tilde{\chi} \rightarrow SO(2,1)$$

CR Effect (old)

$$4d \quad SU(2)_g$$

No Yukawa

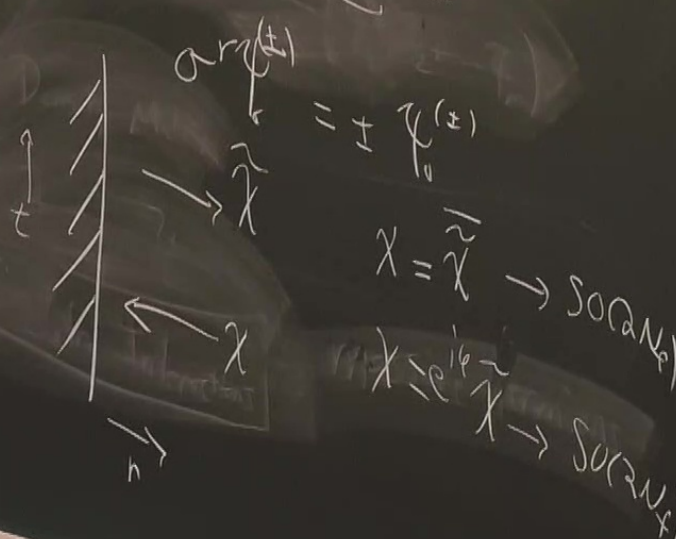


Massless Fermions

- Sph. Sym \rightarrow expand ψ in ang. mom.
 \hookrightarrow restrict to $j=0$

$$\psi^\uparrow = \begin{pmatrix} \psi_+^\uparrow \\ \psi_-^\uparrow \end{pmatrix} \rightarrow \psi_+^\uparrow = \frac{1}{r} \chi^\uparrow(t+r) \psi_0^{(+)}$$

$$\psi^\downarrow = \frac{\tanh(m_{\text{eff}} r)}{r} \begin{pmatrix} \psi_0^{(+)} \\ \psi_0^{(-)} \end{pmatrix} \quad \psi_-^\downarrow = \frac{1}{r} \tilde{\chi}^\uparrow(t-r) \psi_0^{(-)}$$

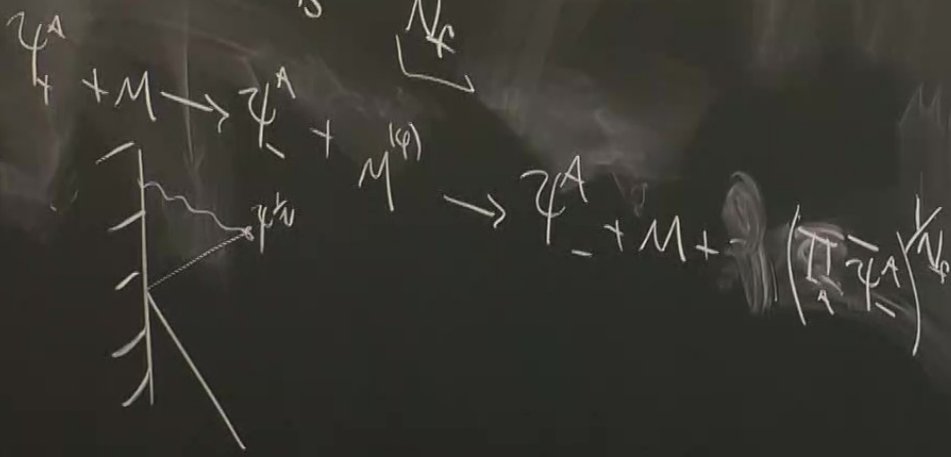


$$\bar{\chi}^A \chi^A = \partial_+ H^A = J^A$$

$$\bar{\chi}^A \tilde{\chi}^A = \partial_- H^A = \tilde{J}^A$$

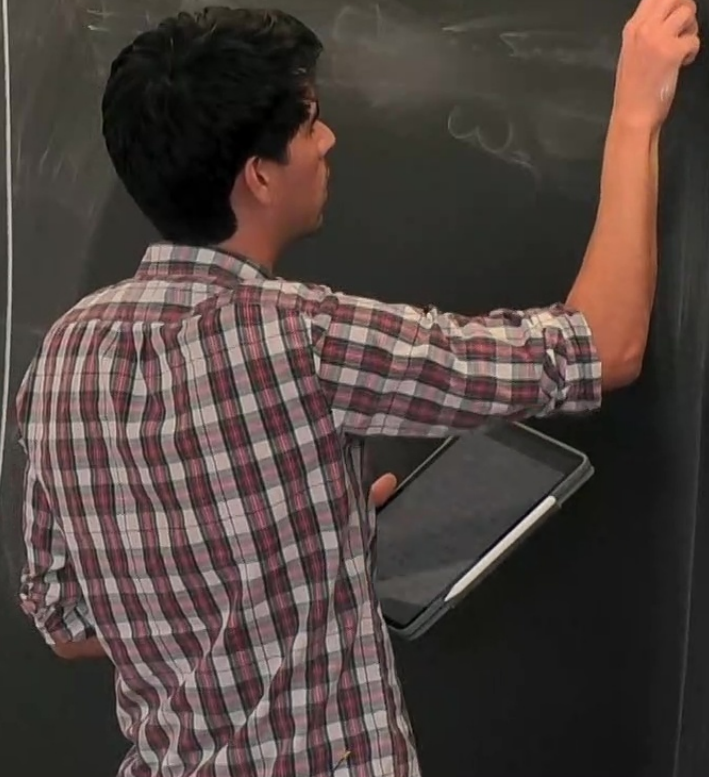
$$(J^A - R_B^A \tilde{J}^B) / \Omega = 0$$

$$R_B^A = J_B^A - \frac{1}{\sqrt{g}} \left[\frac{N_B}{N_A} \right]$$



Callan Rubakov (New)

$$L = \frac{1}{2m_w} (\dot{\varphi} - 2A_0)^2 + \frac{\Theta}{2i} ($$



Callan Rubakov (New)

$$L = \frac{1}{2m_w} (\dot{\varphi} - 2A_0)^2 + \frac{\Theta}{2\pi} (\dot{\varphi} - 2a_0)$$

$$\Theta \sim \Theta + 2\pi$$

$$\hat{p} \rightarrow \hat{p} + \frac{\Theta}{2\pi} \Rightarrow \hat{H} = \frac{m_w}{2} \left(\hat{p} - \frac{\Theta}{2\pi} \right)^2 + 2A_0 \hat{p}$$

$$\Theta \rightarrow \Theta + 2\pi$$

$$|\psi\rangle \rightarrow |\psi\rangle$$

\Rightarrow sources charge

Witten Effect:

$$\Theta_{4\pi} \int \frac{f_1 f}{8\pi^2} \Rightarrow \text{monopole}$$

$$\Theta_{4\pi} \int \frac{f_1 f}{8\pi^2} = \frac{\Theta}{4\pi} \int F_{\mu\nu} \rightarrow \frac{\Theta}{4\pi} \int (\dot{\varphi} - 2A_0) dt$$

$$q_e = \frac{\Theta}{2\pi} q_m$$

$$m_{\psi} = |m_{\psi}| e^{i\alpha}$$

$$\rightarrow \Theta_{4\pi} = 2$$

\Rightarrow Vacuum charge density
 $\rho(r) = e^{-Im\psi}$

Claim: CR Effect is related!

Massive Fermions

M_ψ $SU(2N_f) \rightarrow SU(N_f)$

$$\Psi^A = \begin{pmatrix} \psi_+^A \\ \psi_-^A \end{pmatrix} = \frac{1}{r} \left(\chi^A(t+r) \Psi_0^{(+)} + \tilde{\chi}^A(t-r) \Psi_0^{(-)} \right)$$

$$S = S_{kin} + \int d^2x \left[\overbrace{\frac{i}{2} \chi^A \not{\partial} \tilde{\chi}^A}^{+ \text{ in-going}} - \overbrace{\frac{i}{2} \tilde{\chi}^A \not{\partial} \chi^A}^{- \text{ out-going}} \right]$$

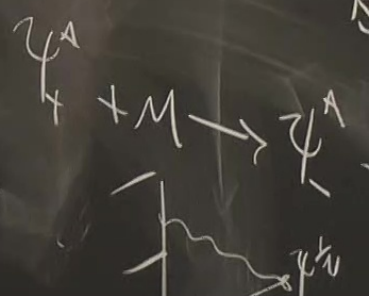
$\chi \rightarrow e^{i\omega t + ikx}, \tilde{\chi} \rightarrow e^{-i\omega t + ikx} \rightarrow \chi^A = e^{i\omega t + ikx} \tilde{\chi}^A$

$$\bar{\chi}^A \chi^A = \partial_+ H$$

$$\bar{\tilde{\chi}}^A \tilde{\chi}^A = \partial_- \tilde{H}$$

$$(J^A - R_B^A \tilde{J}^B)$$

$$R_B^A = \delta_B^A$$



$$\begin{aligned} \vec{\chi}^\dagger \chi^\dagger &= \partial_+ H^\dagger = \vec{J}^\dagger \\ \vec{\chi}^\dagger \vec{\chi}^\dagger &= \partial_- H^\dagger = \vec{J}^\dagger \end{aligned} \left. \begin{aligned} h^\dagger &= H^\dagger + \tilde{H}^\dagger \\ h &= \sum h^A \end{aligned} \right\} \begin{aligned} &E^{M\nu} \partial_\mu h^\dagger_\nu \\ &- h F_{tr} + \frac{r^2}{2g^2} \left(\frac{2}{tr} \right) \end{aligned}$$

$$S_{\text{eff}} = \int d^2x \left(|\partial h|^2 - m_{H^\dagger}^2 \cos(h^\dagger) \right) + \int dt \left(\frac{(\dot{\varphi} - 2A_0)^2}{2m_w^2} + (\dot{\varphi} - 2A_0) \frac{h}{4\pi} \right)$$

$$\vec{\chi}^\dagger \left(\partial_+ - iA_+ - \frac{\varphi - \Theta(r)}{2} \right) \chi$$

Callan Rubakov (New)

$$L = \frac{1}{2m_w} (\dot{\varphi} - 2A_0)^2 + \frac{\Theta}{2\pi} (\dot{\varphi} - 2A_0)$$

$$\Theta \sim \Theta + 2\pi$$

$$\hat{p} \rightarrow \hat{p} + \frac{\Theta}{2\pi} \Rightarrow \hat{H} = \frac{m_w}{2} \left(\hat{p} - \frac{\Theta}{2\pi} \right)^2 + 2A_0 \hat{p}$$

$$\Theta \rightarrow \Theta + 2\pi$$

$$|\psi\rangle \rightarrow |\psi\rangle$$

\Rightarrow sources charge

Witten Effect:

$$\Theta_{4d} \int \frac{f_1 f}{8\pi^2} \Rightarrow \text{monopole}$$

$$\Theta_{4d} \int \frac{f_1 f}{8\pi^2} = \frac{\Theta}{4\pi} \int F_{tr} \rightarrow \frac{\Theta}{4\pi} \int (\dot{\varphi} - 2A_0)$$

$$q_e = \frac{\Theta}{2\pi} q_m$$

$$\bar{\chi}^A \chi^A = 2H^A = J^A$$

$$\bar{\chi}^A \tilde{\chi}^A = 2\tilde{H}^A = \tilde{J}^A$$

$$h^A = H^A + \tilde{H}^A$$

$$h = \sum h^A$$

$$S_{\text{eff}} = \int d^3x \left(|Dh|^2 - m_W^2 \cos(h^A) - \frac{1}{4} F_{\mu\nu}^2 + \frac{r^2}{2g^2} \left(\frac{2}{5} \right) \right)$$

$$+ \int dt \left(\frac{(\dot{\varphi} - 2A_0)^2}{2m_W} + (\dot{\varphi} - 2A_0) \left(\frac{h}{4\pi} \right) \right)$$

$$\bar{\chi} (D_+ - iA_+ - \frac{\varphi - \Theta(h)}{2}) \chi$$

Fermion Scattering $\rightarrow H, \tilde{H}$ (kink) solitons

h -kink $\rightarrow 4\pi$

$h|_{b.r.} = 0$

$\Delta\Theta_{\text{eff}} = 4\pi$

Callan Rubakov (New)

$$L = \frac{1}{2m_W} (\dot{\varphi} - 2A_0)^2 + \frac{\Theta}{2\pi} (\dot{\varphi} - 2A_0)$$

$$\Theta \sim \Theta + 2\pi$$

$$\hat{p} \rightarrow \hat{p} + \frac{\Theta}{2\pi} \Rightarrow \hat{H} = \frac{m_W}{2} \left(\hat{p} - \frac{\Theta}{2\pi} \right)^2$$

Witten Effect:

$$\Theta \rightarrow \Theta + 2\pi$$

$$|q\rangle \rightarrow |q\rangle \Rightarrow \text{sources charge}$$

$$\int_{\mathbb{R}^3} \frac{f_{\mu\nu}}{8\pi} = \frac{\Theta}{4\pi} \int F_{\mu\nu} \rightarrow \frac{\Theta}{4\pi} \int (\dot{\varphi} - 2A_0)$$

$$\int_{\mathbb{R}^3} \frac{f_{\mu\nu}}{8\pi} \Rightarrow \text{monopole}$$

$$\bar{\chi}^A \chi^A = \partial_+ H^A = J^A$$

$$\bar{\tilde{\chi}}^A \tilde{\chi}^A = \partial_- \tilde{H}^A = \tilde{J}^A$$

$$h^A = H^A + \tilde{H}^A$$

$$h = \sum h^A$$

Inflaton

$$S_{\text{eff}} = \int d^4x \left(|Dh|^2 - m_h^2 \cos(h/f) - \frac{1}{4} F_{\mu\nu}^2 + \frac{r^2}{2g^2} \text{tr} \left(\frac{F}{f} \right)^2 \right)$$

$$+ \int dt \left(\frac{(\dot{\varphi} - 2A_0)^2}{2m_w} + (\dot{\varphi} - 2A_0) \frac{h}{4\pi} \right)$$

$$\bar{\chi} \left(D_+ - iA_+ + \frac{g}{2} \Theta(n) \right) \chi$$

$$H^A = \epsilon_{AB} \tilde{H}^B$$

Fermion Scattering $\rightarrow H, \tilde{H}$ (kink) solitons

h -kink $\rightarrow 4\pi$

$|0\rangle \rightarrow |1\rangle$

$h|0\rangle = \Theta_{\text{eff}}$

$\Delta \Theta_{\text{eff}} = 4\pi$

In Unit 4 ↓

$$\psi_+ \rightarrow e^{i\alpha} \psi_+ \quad \alpha \rightarrow 0 \rightarrow 2\pi$$

→ axionic mode

$$\psi_+^A + M_{10}^{\theta_0} \rightarrow \psi_-^A + M_{11}^{\theta_{2\pi}} \approx \psi_-^A + D_e$$

$$R_{charge} \sim 1/m_{\gamma} \rightarrow E \sim m_{\gamma}$$

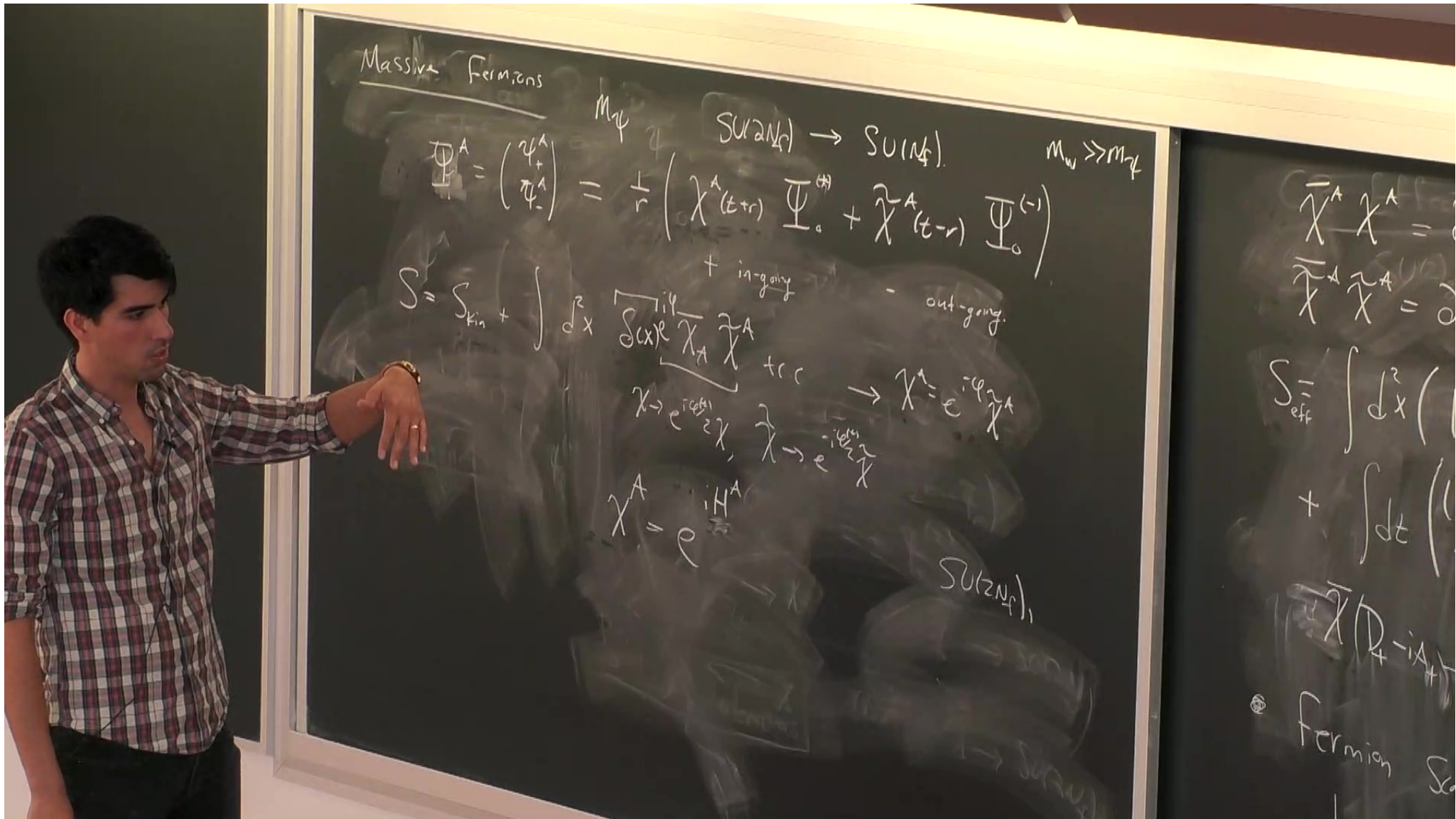
Photon

$$m_{\gamma} = |m_{\gamma}| e^{i\alpha}$$

$$\rightarrow \theta_{ind} = \alpha$$

⇒ Vacuum charge density,
 $\rho(r) \sim e^{-|m_{\gamma}|r}$

Claim: CR Effect is related!



Massive Fermions

m_ψ $SU(2N_f) \rightarrow SU(N_f)$ $m_w \gg m_\psi$

$$\Psi^A = \begin{pmatrix} \psi_+^A \\ \psi_-^A \end{pmatrix} = \frac{1}{r} \left(\chi^A(t+r) \Psi_0^{(+)} + \tilde{\chi}^A(t-r) \Psi_0^{(-)} \right)$$

$$S = S_{kin} + \int d^2x \left[\overbrace{S_{Dirac} \chi_A \tilde{\chi}^A}^{+ \text{ in-going}} - \overbrace{S_{Dirac} \tilde{\chi}^A \chi_A}^{- \text{ out-going}} \right] + \text{tr } c$$

$$\chi \rightarrow e^{i\frac{m_\psi}{2} \gamma_0} \chi, \quad \tilde{\chi} \rightarrow e^{-i\frac{m_\psi}{2} \gamma_0} \tilde{\chi}$$

$$\chi^A = e^{iH^A} \chi^A$$

$SU(2N_f)$

$$\bar{\chi}^A \chi^A = \dots$$

$$\bar{\tilde{\chi}}^A \tilde{\chi}^A = \dots$$

$$S_{eff} = \int d^2x \left(\dots \right) + \int dt \left(\dots \right)$$

$\bar{\chi} (\not{D} - iA_+)$
Fermion S

In 4D

$$\psi_+ \rightarrow e^{i\alpha} \psi_+ \quad \alpha \rightarrow 0 \rightarrow 2\pi$$

→ axion mode

$$\psi_+^A \xrightarrow{+M_{10}^{\theta_2}} \psi_+^A + M_{10}^{\theta_{2en}} \approx \psi_+^A + D_{10}$$

$$R_{charge} \sim \frac{1}{m_{\chi}} \rightarrow E \sim m_{\chi}$$

$$\frac{U(1)_g \times SU(N_f)}{Z_{N_f}} \rightarrow \text{protects dyon from decay mod } N_f$$

$$P_{\psi_+} \rightarrow M + \sum_A \bar{\psi}_+^A$$

$$D_2 \rightarrow M + \sum_A \psi_+^A$$

$$m_{\psi_+} = |m_{\psi_+}| e^{i\alpha}$$

$$\rightarrow \theta_{ind} = \alpha$$

⇒ Vacuum charge density,
 $\rho(r) \sim -\text{Im} \psi_+$

Claim: CR Effect is related!

In 4d

$$\psi_+ \rightarrow e^{ikx} \psi_+ \quad \theta \rightarrow 0 \rightarrow 2\pi$$

→ axion mode

$$\psi_+^A \xrightarrow{+M_{10}} \psi_-^A + M_{11}^{\text{open}} \cong \psi_-^A + D_2$$

$$R_{\text{charge}} \sim \frac{1}{m_{\text{pl}}} \rightarrow E \sim m_{\text{pl}}$$

$$\frac{U(1)_g \times SU(N_f)}{\mathbb{Z}_{N_f}}$$

protects dyon from decay mod \mathbb{Z}_N

$$D_{2N_f} \rightarrow M + \sum_A \bar{\psi}_-^A$$

$$D_2 \rightarrow M + \sum_A \bar{\psi}_-^A$$

• Matches CR exactly

• $j > 0$

• $m_{\text{pl}} \rightarrow 0 \rightarrow R_c \rightarrow \infty$

$$j=0, k=0 \quad \psi_{(4)} = \frac{1}{r} \psi_0^{(4)}$$