Title: The simplicial Lorentzian path integral and spin foams

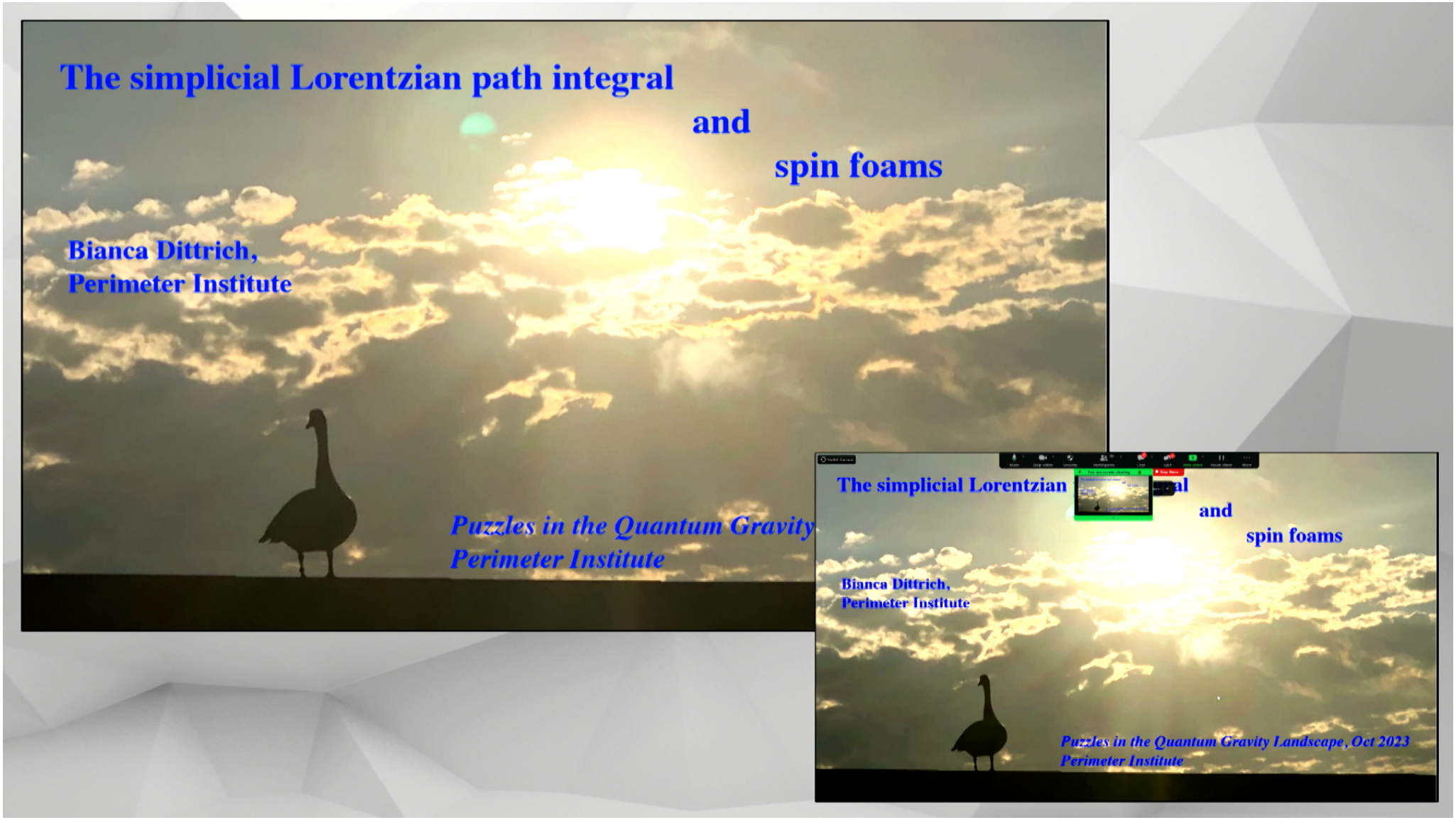
Speakers: Bianca Dittrich

Collection: Puzzles in the Quantum Gravity Landscape: viewpoints from different approaches

Date: October 27, 2023 - 9:35 AM

URL: https://pirsa.org/23100068

Abstract: I will discuss two versions of the simplicial Lorentizian path integral, namely the (Lorentzian) quantum Regge and the spin foam version. I will do so in the simple context of de Sitter cosmology. This simple example will reveal the important role of light cone irregular configurations in the simplicial path integral -- I will show that these can either lead to an exponentially enhanced or an exponentially suppressed amplitude.
I will then highlight an important difference between the spin foams and quantum Regge path integral, which affects the probability for the creation of the (de Sitter) universe.


The simplicial Lorentzian path integral
and
spin foams
Bianca Dittrich,
Perimeter Institute
Puzzles in the Quantum Gravity
Perimeter Institute
The simplicial Lorentzian
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Puzzles in the Quantum Gravity Landscape, Oct 2023
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Outlook

- (Effective) Spin foams
- Light cone structure in Lorentzian simplicial path integrals (for cosmology)
- Effective Spin foams for cosmology

Spin foams

Loop quantum gravity:

Rigorous background independent notion of quantum geometry. Closely related to TQFTs with defects.

[Ashtekar, Isham, Lewandowski, Rovelli, Smolin, …, BD, Geiller, …]

Areas have a discrete spectrum.

[Rovelli, Smolin, …]

Quant. extension of length metric space to area metrics. [BD,Ryan; BD Padua-Arguelles]

Forced by discrete area spectrum. [Asante, BD, Haggard]

Characterized by Barbero-Immirzi parameter γ. [BD, Ryan]

γ is an anomaly parameter.

Areas are the (more) fundamental variables.

Common to:

Ryu-Takayanagi, tensor networks, bh entropy, …

Spin foams:

Path integral based on LQG quantum geometry.

Based on Plebanski formulation of gravity.

[Reisenberger, Rovelli, Barret, Crane, … , Engle, Pereira, Rovelli, Livine, Freidel, Kransov, Baratin, Oriti, … Asante, BD, Haggard, Padua-Arguelles…]

Can spin foams lead to a gravitational dynamics?

[Bonzom, Hellmann, Kamiński, Han, …,Engle, Kamiński, Oliveira, Dona, Gozzini, …]

Classical Plebanski formulation requires constraining configuration to length metric space.

"Flatness problem" as a result of γ-anomaly.

Effective Spin foams

Can spin foams lead to a gravitational dynamics?

Transparent encoding of the dynamics. Numerical simulations are faster by several magnitudes: seconds vs months. [Asante, BD, Haggard 2020 PRL]

First explicit computation of expectation values testing discrete EOM: reproduce discrete GR for sufficiently small γ. [Asante, BD, Haggard 2020 CQG]

"Flatness problem" resolved in the discrete.

Perturbative continuum limit on lattice: $\lambda >> a$ [BD 2021, BD, Kogios 2022]

Results:

- Except for effective length metric all dof are Planck massive
- After integrating out all these additional dof:
- Leading order: (Linearized) Einstein-Hilbert action
- Next order: Weyl squared which comes from integrating out effective area metric
- Universality: Does not depend on details of spin foam models or on value of γ.

Very surprising!!!
Resolves "flatness problem"
in the continuum.

Analysis directly in the continuum: Modified Plebanski theory framework. [Krasnov 2008+; Freidel 2008] [BD, Borissova 2022]

Results:

- Derivation of action for area metrics from (modified) Plebanski action.
- Integrating out additional area metric dof's (linearized):

$$L_{eff} = L_{EH} - \frac{1}{4}\frac{1}{\Box - M(\gamma)^2}\text{Weyl}^2$$

Ghost-free!

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3 coupling constants G, Λ, γ: as needed in Asymptotic Safety, CDT, EDT. γ is an anisotropy parameter as in CDT.

Spin foams can lead to a gravitational dynamics in the continuum limit.

Rest of the talk: application to cosmology by symmetry reduction with focus on Lorentzian features.

Our universe is Lorentzian … most of the times.

Spin foams are proper quantum mechanical path integrals.

$$Z \sim \int \mathcal{D}\text{geom}\, \exp(iS(\text{geom}))$$

→ For Euclidean geometries.

→ For Lorentzian geometries.

Lorentzian path integral

→ Light cone structure — Consider Regge calculus.

→ How to compute it?

In Regge calculus. (Integral)

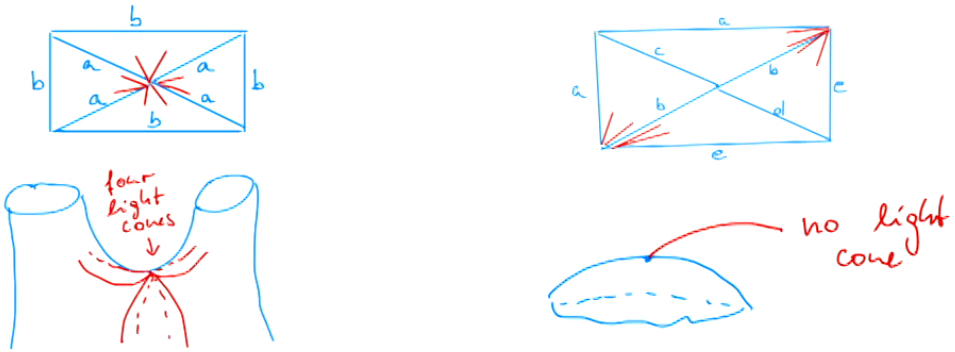
In Spin Foams. (Sum)

Lorentzian simplicial geometries: Regge calculus

[Regge 1961
Sorkin 1975, 2019
Asante, BD, Padua-Arguelles 2021]

- Based on triangulation of space-time. Variables are lengths assigned to edges. Very natural discretization of Einstein-Hilbert action
- Each simplex has a well defined Lorentzian geometry: it is Minkowski-flat.
- Nevertheless it is easy to construct configurations with irregular light cone structure

[Jordan, Loll 2013]



- Such configurations lead to imaginary terms in the Regge action. Sign of this imaginary term seems to depend on choice of convention. [Sorkin 2019]

- Constructing Regge action for complexified length variables reveals: irregular light cone structures lead to branch cuts, explaining these opposite signs

[Asante, BD, Padua-Arguelles 2021]

?? Who ordered these configurations ??

?? What do we do with these configurations ??

Are these just annoying discretization artifacts?

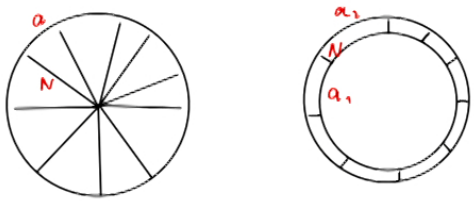
No. They can actually be useful.

- Imaginary metrics / imaginary terms in the action also appear in the continuum, e.g. for topology change. [Louko, Sorkin 1995, ..., Witten 2022, ...]
- Light cone irregularities (co-dimension 2 conical singularities) introduced by hand in Lorentzian continuum path integral [Marolf 2022]

- This talk: Important to get entropy for de Sitter space [BD, Jacobson, Padua-Arguelles TO APPEAR]
- This talk: Branch cut choice makes a more subtle choice in the continuum more obvious

Discretized de Sitter

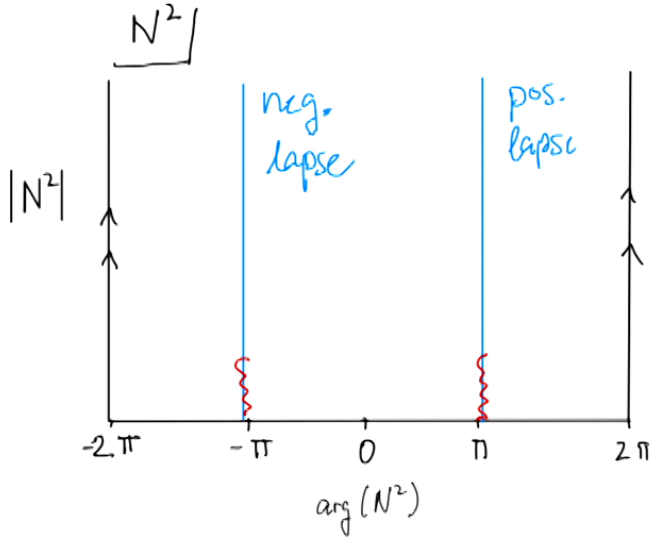
One time evolution step in de Sitter



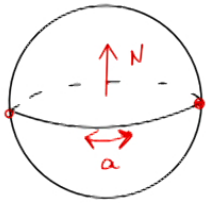
[BD, Gielen, Schander]

With symmetry reduction:

one integration variable ~ lapse



Compactified Lorentzian de Sitter



[Continuum:

Banihashemi, Jacobson]

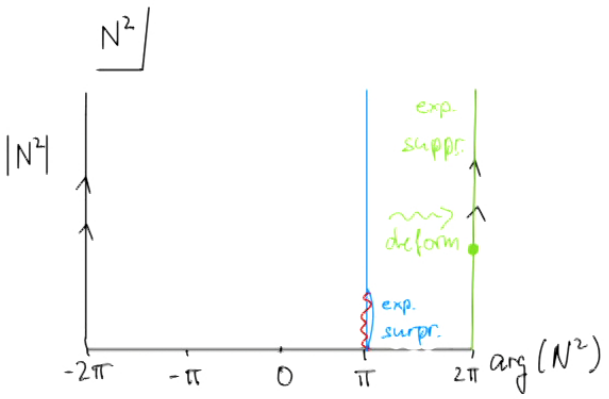
Aim: Entropy from Lorentzian Path integral.

With symmetry reduction:

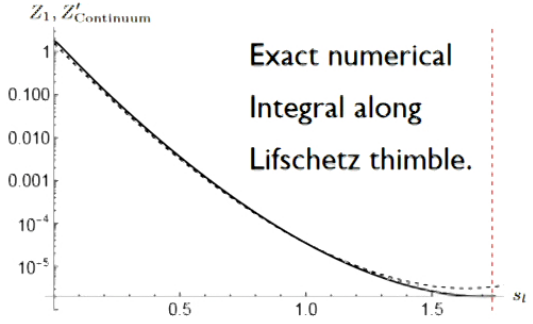
Two integration variables ~ lapse, equator

Discretized de Sitter - two choices

One time evolution step in de Sitter

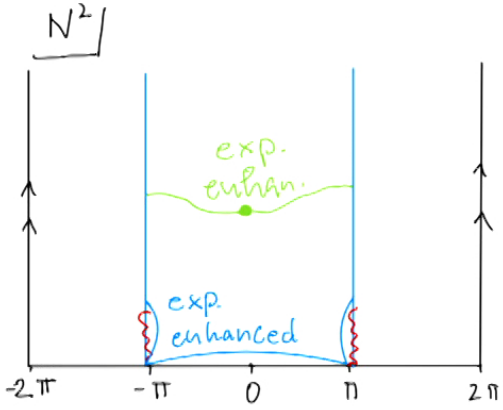


[Continuum-Mini-Super-Space:
Feldbrugge, Lehners, Turok]



Need irregular configurations to agree with continuum result.

Compactified de Sitter



Need irregular configurations to obtain de Sitter entropy.

Lorentzian path integral can give exp. enhancement.

Is their a similar choice in the continuum?

Yes, in choosing how to circumvent the N=0 singularity.

[Continuum-Mini-Super-Space:
Diaz-Dorronsoro, Halliwell, Hartle, Hertog, Janssen]

Irregular light cone structures have an important role.
Surprise: important for entropy calculation.

How to compute the Lorentzian path integral?

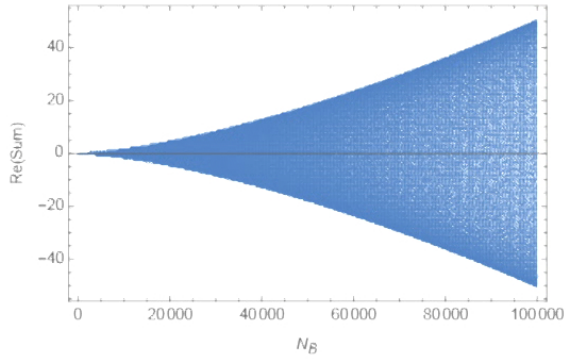
- Monte Carlo: not available.
- For integrals: deformation of contour, for example Lefschetz thimble.
- For sums? E.g. Spin foam sums.

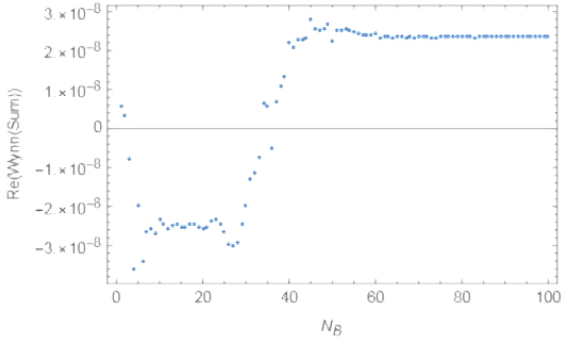
⇒Acceleration techniques for series convergence. For sums and integrals.

In particular: Shanks transform (with Wynn's epsilon algorithm).

[Schmidt 41, Shanks 55, Wynn 56, …] [BD, Padua-Arguelles 23]

Effective spin foam Expectation values:





Rel. Error~10^{-8}

Works very well for sums with actions that are at most linear in the summation variable.
Consistent with quantum mechanics (Bohr quantization) and spin foams.

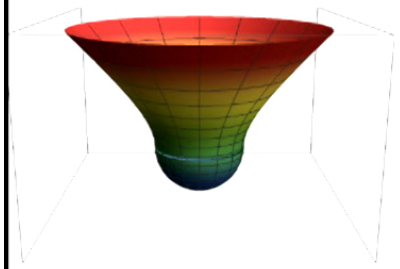
Effective spin foams for de Sitter

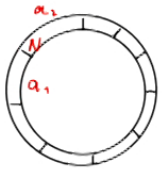
[BD, Padua-Arguelles 2023]

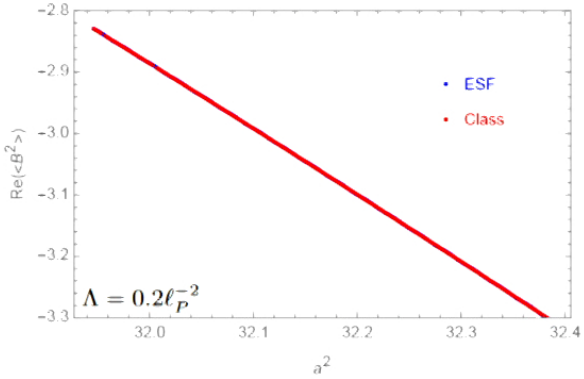
Due to symmetry reduction only difference between Regge path integral and effective spin foams: integral vs sum.

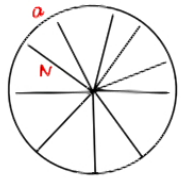
What is the effect of the discrete area spectrum?

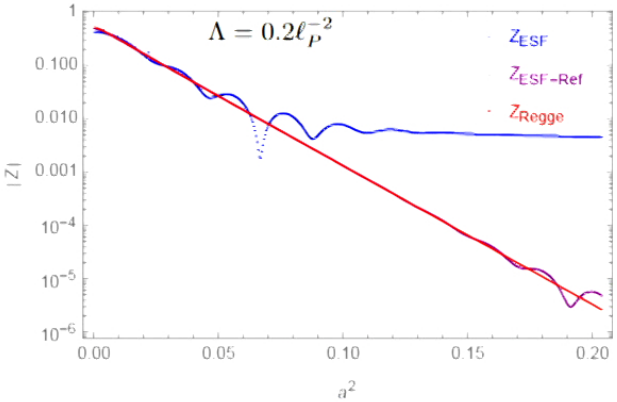
Quantum deSitter











Lorentzian regime transition: very small quantum effects.
Almost no difference between Regge and spin foam.

Euclidean regime transition:
Significant differences between Regge and spin foams.

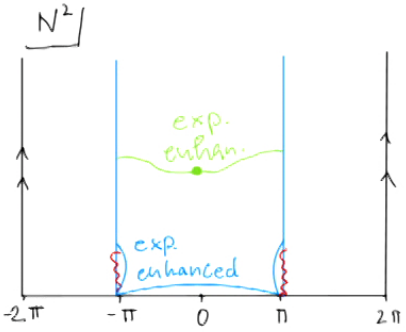
Discrete spectra: Make tunnelling amplitudes less suppressed.

Should be confirmed by using more time steps.

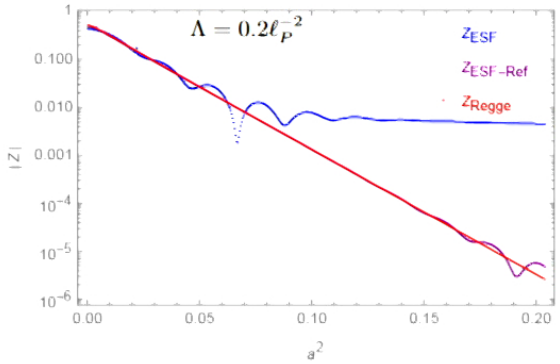
Summary

- Continuum limit of spin foams can lead to general relativity.
- Effective action from spin foams: Weyl curvature squared term from quantum extension of configuration space to area metrics
- Ghost free

$$L_{eff} = L_{EH} - \frac{1}{4}\frac{1}{\Box - M(\gamma)^2}\text{Weyl}^2$$



- Lorentzian simplicial path integral: configurations with light cone irregularities
- Lead to branch cuts and imaginary terms in the action: suppressed or enhanced quantum amplitudes
- Important role for thermodynamic interpretation
- Appears in the continuum in a much more subtle way



- Effective spin foam path integral for de Sitter
- Shank transform to deal with sums (and integrals)
- Weakening of decay of no-boundary probability amplitude

Lorentzian path integral

Spin foams, simplicial Regge, continuum, CDT, causal sets, …

Lots of things to understand!

Computational challenges …

Intriguing conceptual questions.