Title: Puzzles, Resolutions and Open Questions in Causal Set Theory - VIRTUAL

Speakers: Sumati Surya

Collection: Puzzles in the Quantum Gravity Landscape: viewpoints from different approaches

Date: October 25, 2023 - 9:35 AM

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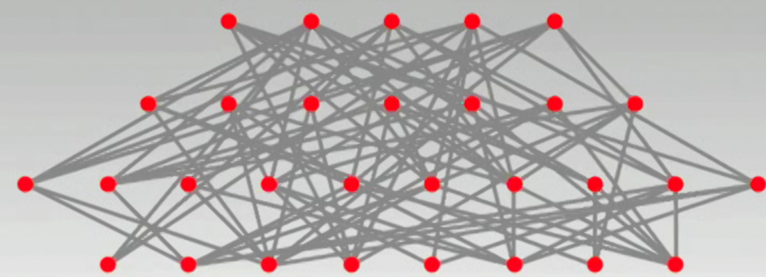
Abstract: I will begin with a broad overview of the main ingredients of causal set theory, and then discuss the partial resolution of a long-standing "puzzle" in the causal set path sum. I will then discuss other causal set curiousities, ending with a brief overview of the challenges in constructing a realistic fully non-perturbative quantum dynamics.

The presenter will be joining via Zoom for this talk.

Puzzles, Resolutions and Open Questions in Causal Set Theory



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Puzzles in the Quantum Gravity Landscape, Oct 23-27, 2023

PERIMETER PI INSTITUTE FOR THEORETICAL PHYSICS

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Outline

- The Causal Set Way : Quantising the causal structure

  -Bombelli, Lee, Meyer and Sorkin, 1987
  -Myrheim, 1978,
  - Kronheimer and Penrose, 1967, Finkelstein, 1969

- The CST path sum
  - The battle between Entropy and Action

    -Loomis and Carlip, 2017,
    -Anand-Singh, Mathur and Surya, 2021,
    -P. Carlip, S. Carlip and S. Surya, 2022
    -P. Carlip, S. Carlip and S. Surya, in preparation

  - The numerical CST lab

    --Surya, 2012,
    --Glaser and Surya, 2016,
    --Glaser 2018,
    --Glaser, O'Connor and Surya, 2018,
    --Cunningham and Surya, 2020

  - *The double path integral and the quantum measure*

    - Rideout and Sorkin, 2001,
    - Dowker, Johnston, Surya, 2010,
    - Surya and Zalel, 2020

- QFT on continuumlike causal sets and EE of horizons

  - Johnston, 2008,
  - Johnston 2009
  - Sorkin, 2011,
  - Dowker, Surya, X, 2017
  - Surya, Yazdi, X, 2019

  - Sorkin and Yazdi, 2018
  - Surya, Yazdi, X, 2021

- New Developments , Open Questions, Work ahead, etc.

  - Rejzner, 2019,
  - Jubb, 2023,
  - Brito, Eichhorn, Pfeiffer, 2023,
  - Brito, Eichhorn, Fausten, 2023,
  - Das, Nasiri, Yazdi, 2023
  - Barton, Counsell, Dowker, Gould and Jubb, 2019

Quantising the Causal Structure..

-Bombelli, Lee, Meyer and Sorkin, 1987

- For every causal space $(M, g) \rightarrow (M, \prec)$, there is a causal structure **poset**

- **Acylic:** $x \preceq y, y \preceq x \Rightarrow x = y$
- **Transitive:** $x \prec y, y \prec z \Rightarrow x \prec z$

- Distinguishes the Lorentzian signature $(-, +, \cdots, +)$

- HKMMKP Theorem: $(M, g) = (M, \prec) + \epsilon$

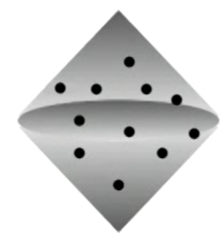
-Hawking - 1975, Hawking-King-MacCarthy-1976, Malament -1977, Kronheimer-Penrose -1967

- Causal Set Proposal: Quantise $(M, \prec)$:

- **Local Finiteness:** $|\mathrm{Fut}(e) \cap \mathrm{Past}(e')| < \infty$

A **causal set** is a locally finite poset

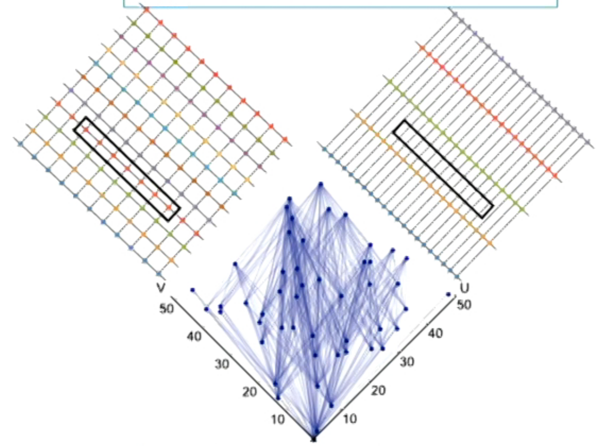
- Order ~ Causal Structure
- Number ~ Volume element



Order + Number ~ Spacetime

The Continuum Approximation

regular lattices don't work!



random lattice does !

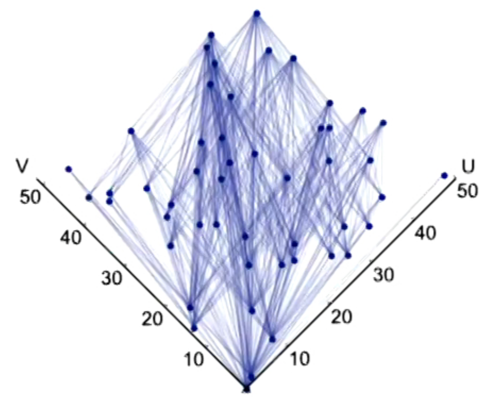
- $n \sim \rho V$ correspondence has to be diffeo invariant

ρ ***sets a scale***

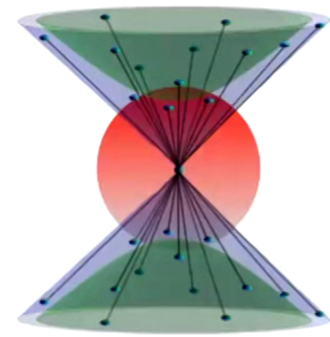
- Random discretisation via a Poisson sprinkling process:
  - $P_V(n) = \frac{(\rho V)^n}{n!} e^{-\rho V}$, $\langle n \rangle = \rho V$, $\Delta n = \sqrt{\rho V}$
  - $C \hookrightarrow_\rho (M, g)$ is order preserving
- For every causal spacetime (M, g) there is a **kinematic ensemble** of causal sets $\mathscr{C}_\rho(M, g) = \{C\}$
- Continuumlike causal sets are those which allow a discrete-continuum correspondence

"First Quantisation"

The Continuum Approximation



- Causal Set Non-locality
  - The nearest neighbours lie all along a collar neighbourhood of $\mathscr{H}^{n-1}$
  - A continuumlike causal set is a graph without a fixed valency



- Lorentz invariance — *Bombelli, Henson and Sorkin, 2006*

Theorem :

There is no measurable map $D : \mathscr{C}_\rho(\mathbb{M}^d) \to \mathscr{H}^{d-1}$ which is equivariant, i.e., $D \circ \Lambda = \Lambda \circ D$.

Proof: If such a map existed, then $\mu_D = \mu \circ D^{-1}$ is a Lorentz invariant probability measure on H which is not possible since is H non-compact.

Causal Set Path Sum

"Second Quantisation"

$$Z_n = \sum_{C \in \Omega_n} \exp\left(\frac{iS(C)}{\hbar}\right)$$

— Benincasa & Dowker 2010,
— Dowker & Glaser 2011
— Glaser 2012

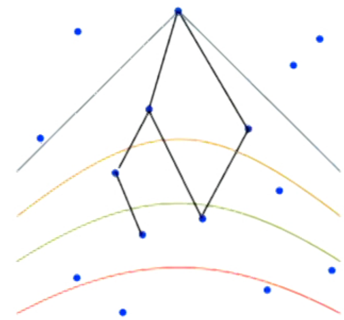
- $S(C)$: Discrete Einstein-Hilbert Action or Benincasa-Dowker-Glaser Action
- Ω_n : Sample space of all n element causal sets (fixing n ~ fixing spacetime volume)
- Example: $n = 3$

- What is the large n behaviour of Z_n ?

The Benincasa-Dowker-Glaser Action

— *Benincasa & Dowker 2010,*
— *Dowker & Glaser 2011*
— *Glaser 2012*

- $S^{(d)}_{BDG}(C) = -\alpha_d \left(\frac{l}{l_p}\right)^{d-2} \left(n + \frac{\beta_d}{\alpha_d} \sum_{j=0}^{j_{max}} C_j^{(d)} N_j\right)$



- $S^{(4)}_{BDG} = \frac{4}{\sqrt{6}} \left(\frac{l}{l_p}\right)^{2} \left(n - N_0 + 9N_1 - 16N_2 + 8N_3\right)$

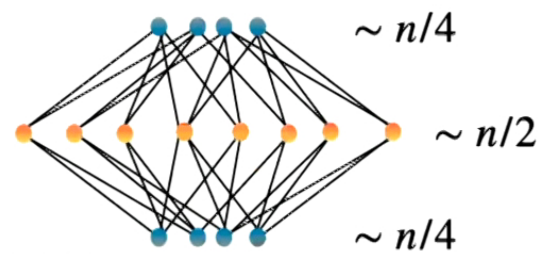
- $\lim_{\rho \to \infty} \langle S^{(d)}_{BDG}(C) \rangle = S_{EH}(g)$ for $C \in \mathscr{C}_\rho(M, g)$

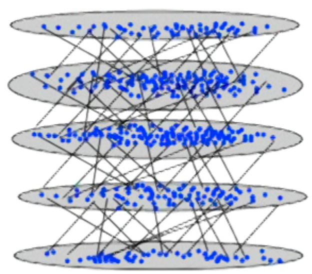
- Example: $n = 3$: $Z_3 = \exp\left(\frac{i}{\hbar}\frac{12}{\sqrt{6}}\right)\left[\exp\left(\frac{i}{\hbar}\frac{28}{\sqrt{6}}\right) + 2\exp\left(-\frac{i}{\hbar}\frac{8}{\sqrt{6}}\right) + \exp\left(-\frac{i}{\hbar}\frac{4}{\sqrt{6}}\right) + 1\right]$

What is a typical causal set in Ω_n at large n?

-Kleitmann and Rothschild, 1975,
-Dhar, 1978
-Promel, Steger and Taraz, 2001

- Ω_n : sample space of all n-element causal sets
- $|\Omega_n| \sim 2^{\frac{n^2}{4}+\frac{3n}{2}+o(n)}$
- Typical causal sets are **Kleitmann-Rothschild**: $|\Omega_{KR}| \sim 2^{\frac{n^2}{4}+\frac{3n}{2}+o(n)}$
- Other K-layered Posets are subdominant for $K \ll n$: $\sim 2^{c(d)n^2+o(n^2)}, \quad c(d) \leq 1/4$





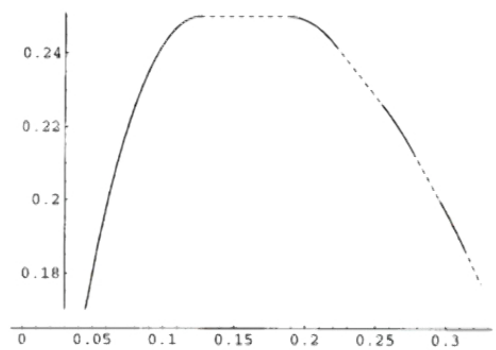
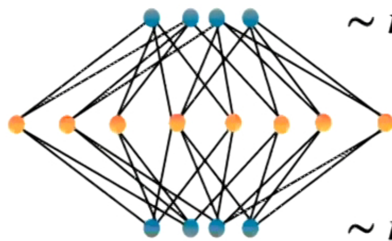
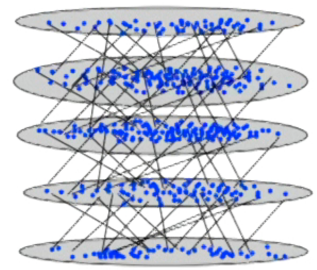
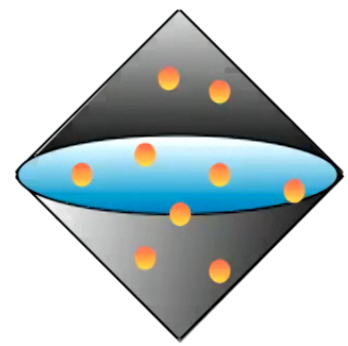


FIG. 5. c(d) in the range [0.05, 0.32].



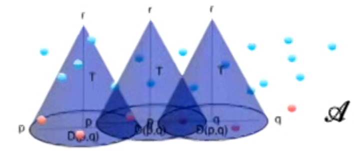


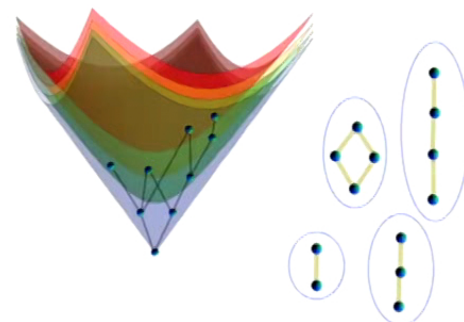
- KR and other $K-$layered Posets are not **continuumlike**
- A maximum of $K = 3$ "moments of time" for KR even for large n. Similarly for general K.
- For a causal diamond $\mathbb{D}^d \subset \mathbb{M}^d$,
  - $\text{Vol} \sim n, \quad \tau \sim h,$
  - $\text{Vol} \propto \tau^d \Rightarrow h \propto n^{\frac{1}{d}}$

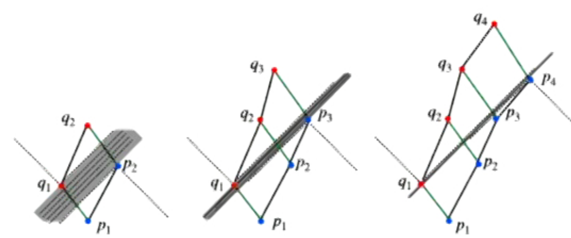


Geometry from counting: Order Invariants as Observables

- ***Dimension Estimators*** — *Myrheim, Myer, Sorkin, Reid, Glaser & Surya, ..*
- ***Timelike Distance*** — *Brightwell & Gregory*
- ***Spatial Homology*** —*Major, Rideout & Surya*
- ***Spatial and Spacelike Distance*** — *Rideout & Wallden, Eichhorn, Mizera & Surya, Eichhorn, Surya & Versteegen*
- ***D'Alembertian*** — *Sorkin, Henson, Benincasa & Dowker, Dowker & Glaser*
- ***Benincasa-Dowker-Glaser Action*** — *Benincasa & Dowker, Dowker & Glaser*
- ***GHY terms in the Action*** — *Buck, Dowker, Jubb & Surya, Machet & Wang, Dowker,*
- ***Locality and Interval Abundance*** — *Glaser & Surya*
- ***Horizon Molecules*** — *Dou & Sorkin,Barton, Counsell, Dowker, Gould & Jubb,*
- ***Scalar Field Greens functions*** —*Sorkin, Johnston, Dowker, Surya & Nomaan X*
- ***Scalar Field Johnston vacuum*** — *Johnston, Sorkin, Yazdi, Nomaan X, Surya*
- ***Entanglement Entropy*** — *Sorkin & Yazdi, Yazdi, Nomaan X, Surya*
- ***Null Geodesics from Ladder molecules*** *--A. Bhattacharya, A. Mathur & Surya*







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Puzzle:

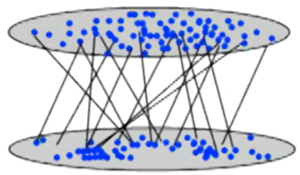
$$Z_n = \sum_{C \in \Omega_n} \exp\left(\frac{i S^{(d)}_{BDG}(C)}{\hbar} \right)$$

- Do layered posets also dominate Z_n?
- Can the BDG action overcome the Entropic dominance in some limit of the theory?
- How can continuumlike causal sets ever emerge from this theory?

—Loomis and Carlip, 2017
— A.Anand Singh, A.Mathur and Surya, 2021
—P. Carlip, S. Carlip and S. Surya, 2022
— P. Carlip, S. Carlip and S. Surya, in preparation

Bilayer Posets

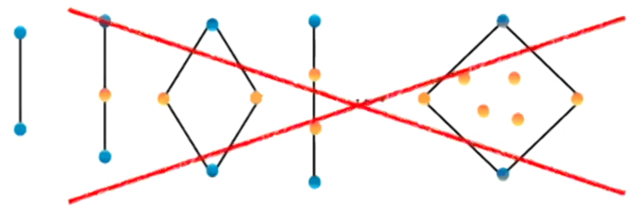
—Loomis and Carlip, 2017



γn

$N_0^{max} = \gamma(1-\gamma)n^2$

$(1-\gamma)n$



$$Z = Z_B + Z_{rest}$$

$$\mathcal{S}(C) = -\alpha_d\left(\frac{l}{l_p}\right)^{d-2}\left(n + \frac{\beta_d}{\alpha_d}\sum_{j=0}^{j_{max}} C_j^{(d)} N_j\right) = \mu_d(n + \lambda_0^{(d)} N_0) = \mathcal{S}_L$$ **Link Action**

$$Z_B[\mu_d, \lambda_0^{(d)}] = \sum_{c \in B} \exp^{i\mu_d(n + \lambda_0^{(d)} N_0)},$$

$$Z_B[\mu_d, \lambda_0^{(d)}] \sim \int_0^{1/2} \mathrm{d}p\, |\mathscr{C}_{p,n}| \exp(iS_L(p)) = e^{i\mu n}\int_0^{1/2} \mathrm{d}p \exp\left[n^2\left(i\mu_d\lambda_0^{(d)} p/2 + h(2p)/4\right) + o(n^2)\right]$$

- $\ln|\mathscr{C}_{p,n}| = \frac{1}{4}h(2p)n^2 + o(n^2),$
- $h(p) = -p\ln p - (1-p)\ln(1-p)$

Dhar's Entropy function.

Suppression for:

$$\tan\left[\frac{1}{2}\beta_d\left(\frac{l}{l_p}\right)^{d-2}\right] > \left(\frac{27}{4}e^{-1/2} - 1\right)^{1/2}$$

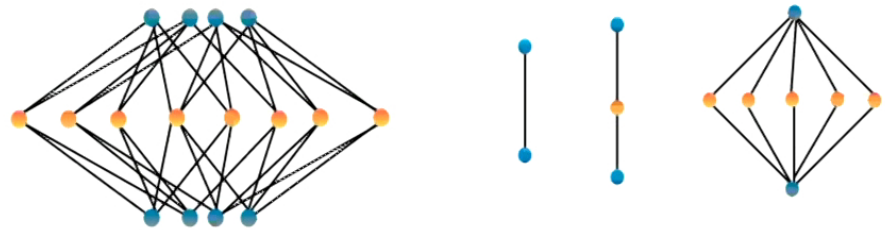
$$d = 4: \mu = \beta_4\left(\frac{l}{l_p}\right)^2 \Rightarrow l \geq 1.86 l_p$$

K-layer orders

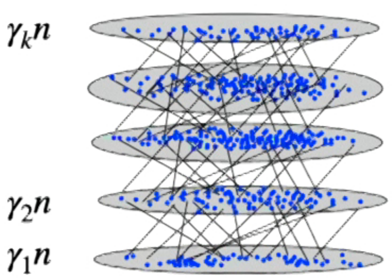
$$Z = Z_K + Z_{rest}\,,\; K \ll n$$

— A.Anand Singh, A.Mathur and Surya, 2021
—P. Carlip, S. Carlip and S. Surya, 2022
— P. Carlip, S. Carlip and S. Surya, in preparation

Intervals in the KR poset:



$$S^{(4)}_{BDG} = \frac{4}{\sqrt{6}}\left(\frac{l}{l_p}\right)^2\left(n - N_0 + 9N_1 - 16N_2 + 8N_3\right)$$



The discrete Einstein Hilbert action in any dimension suppresses all K-layer orders for $K \ll n$:

Action wins over Entropy

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Continuumlike contributions

-- *S. Carlip and S. Surya, work in progress*

Kinematic Ensemble $\mathscr{C}_\rho(M,g)$, $\quad N_n(M) \equiv |\mathscr{C}_\rho(M,g)|$

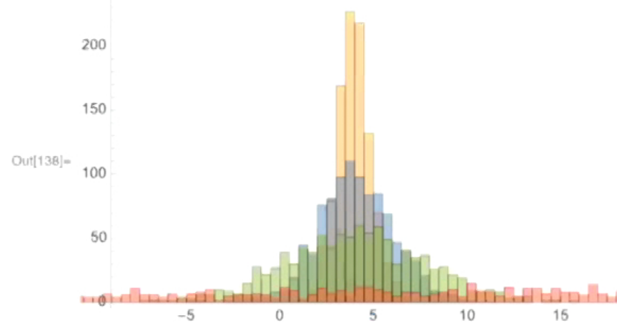
$$S(c) = \langle S(M)\rangle + \Delta S,$$

$$Z_M = \sum_{c\in\Omega_n(M)} e^{\frac{i}{\hbar}(\langle S(M)\rangle + \Delta S)} \sim e^{\frac{i}{\hbar}\langle S(M)\rangle} \int_{-\infty}^{\infty} d(\Delta S)\, e^{\frac{i}{\hbar}\Delta S} F(\Delta S)$$

$$F(\Delta S) = N_n(M)\sqrt{\frac{\alpha}{\pi}}\, e^{-\alpha(\Delta S)^2}$$

$$|Z_M| \sim N_n(M)\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{4\alpha\hbar^2}}$$

$$\lim_{n\to\infty}\frac{1}{\alpha} \to 0 \Rightarrow \lim_{n\to\infty}\frac{1}{|\Omega_n(M)|}|Z_M| = \frac{1}{\sqrt{2\pi}}$$



$\langle N_J\rangle \simeq f(J)\, n^{2-\frac{2}{d}} + o(n^{2-\frac{2}{d}})$ in $\mathbb{M}^d$: $d \neq d_0$ is purely oscillatory

Spacetime-like causal sets may just have a fighting chance..

The CST Lab: Lorentzian Statistical Geometry

--*Surya, 2012,*
--*Glaser and Surya, 2016,*
--*Glaser 2018,*
--*Glaser, O'Connor and Surya, 2018,*
--*Cunningham and Surya, 2020*

$$Z_\beta = \sum_{c \in \Omega} \exp(i\beta S_{BDG}(c)),$$

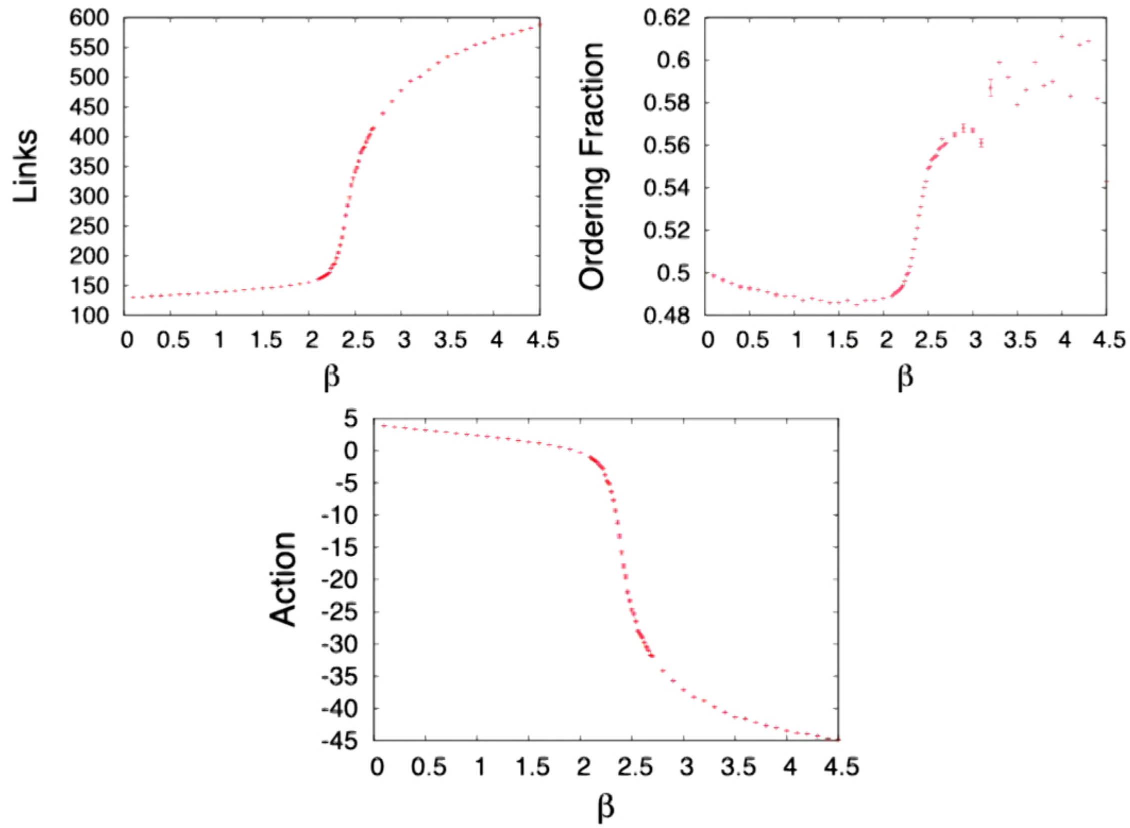
Inverse "temp": $\beta \to i\beta$

$$Z_\beta \to \tilde{Z}_\beta = \sum_{c \in \Omega} \exp(-\beta S_{BDG}(c))$$

- Use MCMC techiniques to "walk through" Ω
- Track observables $\mathcal{O}$ and calculate $\langle \mathcal{O} \rangle$
- Computationally simpler if Ω is dimensionally and topologically restricted

--*Henson, Rideout, Sorkin & Surya, 2017*

Track Observables $\mathcal{O}$



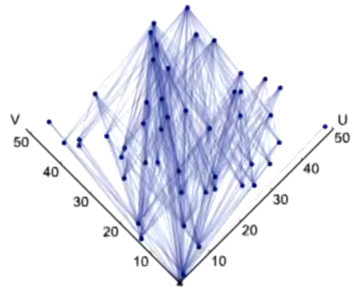
- Myrheim-Myer dimension estimator
- Interval Abundance
- Action
- Height

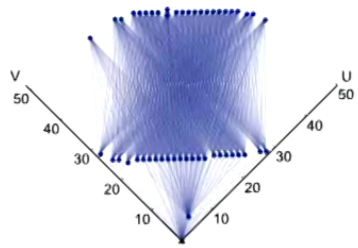
A first order phase transition

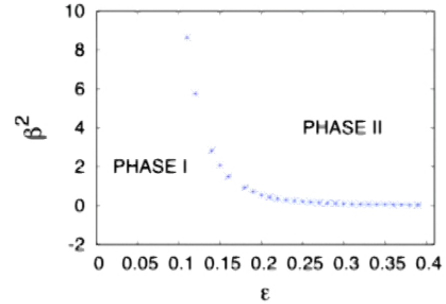
$\beta < \beta_c$: *Continuum phase*

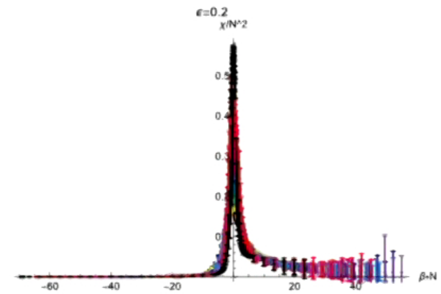
$\beta > \beta_c$: *"Connected/Quantum" Phase*

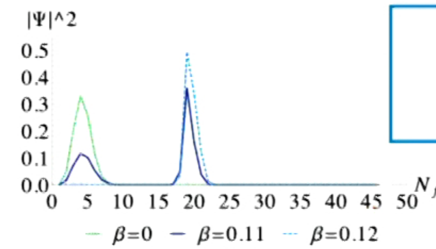
$\mathbb{D}^2 \subset \mathbb{M}^2$





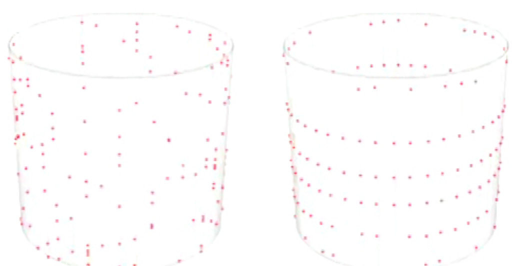




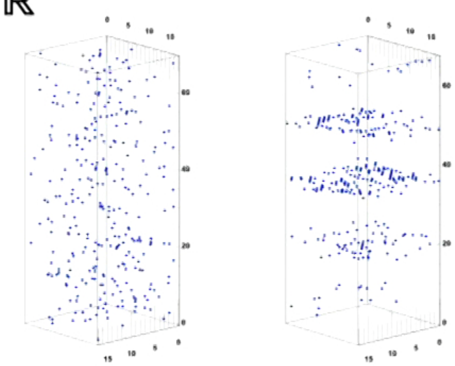


Hartle-Hawking Wave Function

$S^1 \times \mathbb{R}$



$T^2 \times \mathbb{R}$



Extension to d=4 ?!

Expensive.. :)

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QFT on causal sets and EE of horizons

- Johnston, 2008,
- Johnston 2009
- Sorkin, 2011,
- Dowker, Surya, X, 2017
- Surya, Yazdi, X, 2019

- Sorkin and Yazdi, 2018
- Surya, Yazdi, X, 2021

Johnston's prescription for free scalar field theory:

- Peierls bracket: $[\Phi(x), \Phi(x')] = i\Delta(x, x') = i(G_R(x, x') - G_A(x, x'))$
- Eigenvalues: $i\Delta \circ u_k(x) = \lambda_k u_k(x)$, Pairs: $(\lambda_k, -\lambda_k)$
- $\Phi(x) = \sum_k \sqrt{\lambda_k}\left(\mathbf{a}_k u_k(x) + \mathbf{a}_k^\dagger u_k^*(x)\right)$
- Wightmann Function $W(x, x') = \mathrm{Pos}(i\Delta)$
- Examples in $\mathbb{M}^d, \mathrm{dS}_d$, $d = 2{,}4$:
  - $K_0^{(2)}(e, e') = \frac{1}{2}C_0(e, e')$, $K_0^{(4)}(x, x') = \frac{1}{2\pi}\sqrt{\frac{\rho}{6}}L_0(x, x')$

$$C_0(e, e') \begin{cases} = 1, & e' \prec e \\ = 0, & \text{otherwise} \end{cases}$$

$$L_0(e, e') \begin{cases} = 1, & e' \prec_L e \\ = 0, & \text{otherwise} \end{cases}$$

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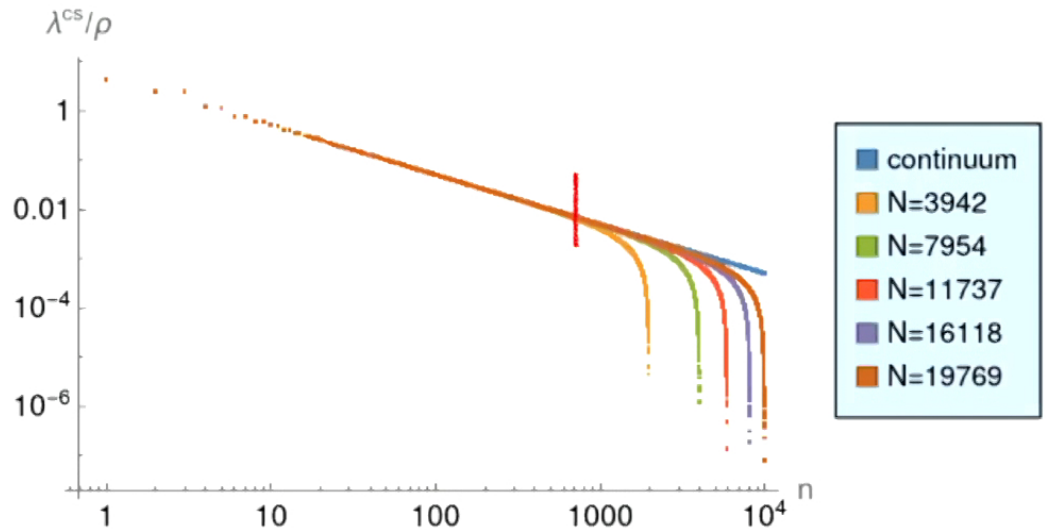
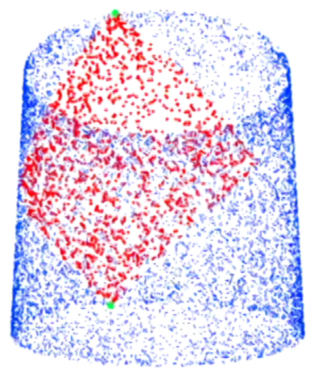
The deep UV regime does not scale



$$\lambda^{cs} = \begin{cases} \dfrac{\beta_1}{n^{\alpha_1}}, & n < n_0 \\ -\alpha_2 n + \beta_2, & n > n_0\,. \end{cases}$$

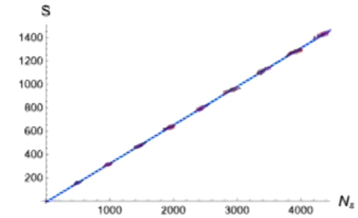
Puzzle/Question: How does this modify RG flows, or Asymptotic Safety?

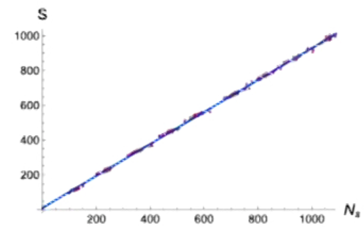
Sorkin's Spacetime Entanglement Entropy

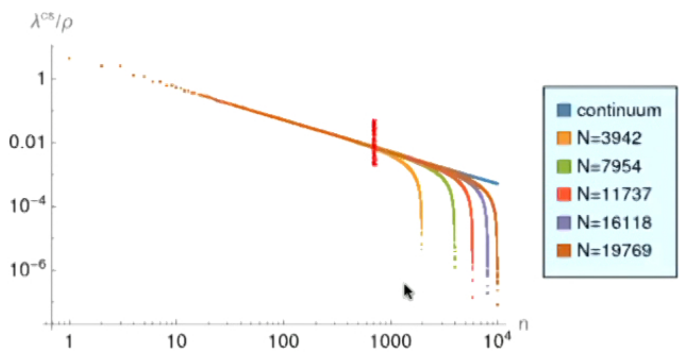


No Truncation

$\mathcal{S} \propto N$

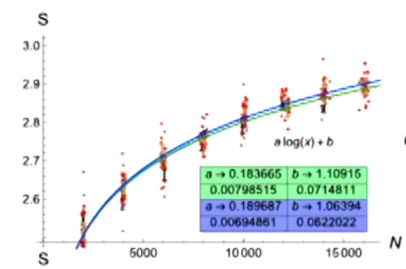




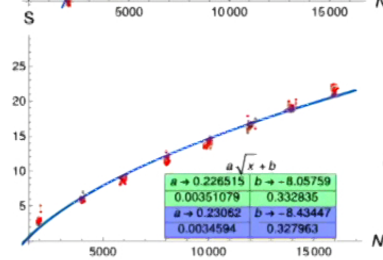


With Truncation

$\mathcal{S} \propto A$



$$dS_2 : \quad \mathcal{S} = \frac{1}{6} \ln N + b\,.$$



$$dS_4 : \quad \mathcal{S} = a\sqrt{N} + b\,.$$

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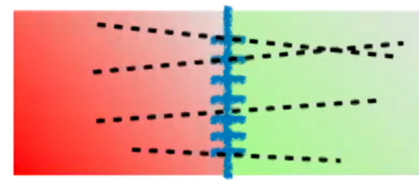
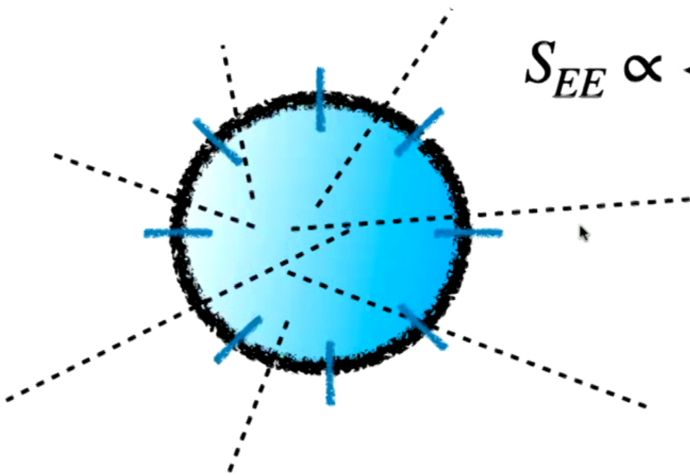
Is there a Fundamental Volume Law for EE?



- Causal Sets are non-local in the continuum approximation.
- Locality is emergent at some meso-scale $\ell_m \gg \ell_p$
- Volume Laws in Non-local QFTs, Long Range Ising and Kitaev Models

-Shiba and Takayanagyi —2014,
-B. Basa, G. La Nave, and P. W. Phillips —2020,
-D. Vodola, L. Lepori, E. Ercolessi, and G. Pupillo -2015



$$S_{EE} \propto \begin{cases} A, & n_{cut-off} \sim n_{cont} \\ V, & n_{cut-off} = n_{max}. \end{cases}$$

- Energy dependent "Entanglement Probes"

-Abu Ashik Md Irfan, Patrick Blackstone, Roger Pynn, Gerardo Ortiz — 2021
-S.J. Kuhn, S. McKay, J. Shen, N. Geerits, R.M. Dalgliesh et al.-2021

- Probe Dependent EE ?

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New Developments in 2023..

- Perturbative QFT: pAQFT techniques for causal sets
  - Rejzner, 2019,
  - Jubb, 2023,
- Generating Higher derivative terms in the Causal Set Action
  - Brito, Eichhorn, Pfeiffer, 2023,
- QFT: Bounding the Higgs Mass in Causal Set Quantum Gravity -- new physics at the nonlocality scale?
  - Brito, Eichhorn, Fausten, 2023,
- Ever-Present Lambda Models : recent developments and comparisons with SN Ia and CMB data
  - Das, Nasiri, Yazdi, April 2023, July 2023
- Using Horizon Molecules for Entropy -- not just measuring geometry, but measuring number of "states"
  - Barton, Counsell, Dowker, Gould and Jubb, 2019
  - Dowker et al -- see QG@RRI day 1

Open Questions, Work Ahead..