Title: The Case for Renormalizable Quantum Gravity: from local to nonlocal approaches (and back!)

Speakers: Luca Buoninfante

Collection: Puzzles in the Quantum Gravity Landscape: viewpoints from different approaches

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Abstract: In the context of perturbative quantum field theory (QFT), the addition of quadratic-curvature invariants to the Einstein-Hilbert action makes it possible to achieve strict renormalizability in four dimensions. This theory exhibits unusual features due to an additional massive spin-2 ghost which, in general, may cause instabilities. In the first part of this talk, we focus on the possibility of giving up locality as a way to avoid ghost-like degrees of freedom and provide a critical assessment on open questions in nonlocal theories of gravity, such as the uniqueness problem. In the second part of the talk, we take a step back and argue that, despite the presence of the ghost and actually thanks to it, Quadratic Gravity can still provide a consistent local perturbative QFT description of the gravitational interaction and explain new physics beyond Einstein's general relativity, e.g., it offers a natural explanation for the inflationary phase. Finally, we argue that a type of nonlocality in gravity can still occur non-perturbatively and show that a new lower bound on scattering amplitudes indicates that the gravitational interaction is intrinsically nonlocal if black holes form.

The Case for Renormalizable Quantum Gravity:
from local to nonlocal approaches (and back!)

Luca Buoninfante

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Puzzles in the Quantum Gravity Landscape:
viewpoints from different approaches
Perimeter Institute, 24th October 2023

Motivations

Einstein's General Relativity: $S_{EH} = \frac{M_p^2}{2} \int d^4x \sqrt{-g}(R - 2\Lambda)$

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'Unique' (strictly) renormalizable QFT of gravity in $D = 4$:

[Stelle PRD (1977)]

$$S = \frac{1}{2}\int d^4x\,\sqrt{-g}\left(M_p^2(R - 2\Lambda) + \frac{\alpha}{6}R^2 - \frac{\beta}{2}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}\right)$$

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Massive spin-0: $m_0^2 = \frac{M_p^2}{\alpha}$,

$\alpha \sim 10^{10}$: natural explanation for inflation!

[Starobinsky, 1980+]

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[Starobinsky, 1980+]

Massive spin-2 ghost:

$$m_2^2 = \frac{M_p^2}{\beta} + \frac{2}{3}\Lambda\left(2\frac{\alpha}{\beta}+1\right)$$

Ghost instability: quantum level

Scalar toy model:

$$\mathcal{L} = \frac{1}{2}\phi(\Box - m^2)\left(1 - \frac{\Box}{M^2}\right)\phi - V(\phi) \implies \Pi(p) = \frac{1}{p^2 + m^2 - i\epsilon} - \frac{1}{p^2 + M^2 - i\varepsilon}$$

ghost

($\epsilon, \varepsilon > 0$ Feynman prescription)

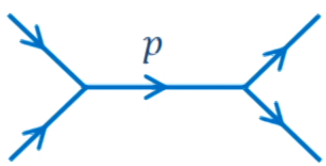
Optical theorem:

$$S^{+}S = 1, \qquad S = 1 + iT,$$

$$1 = \sum_{\{n\}} c_n \, |n\rangle\langle n|, \qquad c_n > 0$$

$$\implies \quad 2Im\{\langle a|T|a\rangle\} = \sum_{\{n\}} c_n |\langle n|T|a\rangle|^2 \geq 0$$

Tree-level example ($V \sim \phi^3$):



$$Im\{\langle a|T|a\rangle\} \sim \theta(p^0)[\delta(p^2 + m^2) - \delta(p^2 + M^2)]$$

It can be negative: violation of unitarity!

Quatum Gravity Puzzle

How to solve the ghost puzzle in Perturbative Quantum Gravity?

I consider two types of approaches:

1. Correspondence Principle applies:

2. Correspondence Principle does not apply:

Quatum Gravity Puzzle

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2. Correspondence Principle does not apply:

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1. Correspondence Principle applies: ghost must be classically harmless

(give up locality)

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$$+RF_R(-\Box)R + C_{\mu\nu\rho\sigma}F_C(-\Box)C^{\mu\nu\rho\sigma} + \cdots\Big], \qquad \Box = g^{\mu\nu}\nabla_\mu\nabla_\nu$$

2. Correspondence Principle does not apply:

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2. Correspondence Principle does not apply:

   – Keep locality
   – quantum features needed to make the ghost harmless
   – (consistent) classical limit taken from the quantum theory

Outline

- **Approach 1:** Correspondence Principle applies (give up ‘‘locality’’)
- **Approach 2:** Correspondence Principle does not apply
- Discussion

Approach 1: beyond 4 derivatives

Scalar 4-derivative model:

$$\mathcal{L} = \frac{1}{2}\phi\Box\left(1 - \frac{\Box}{M^2}\right)\phi - V(\phi) \quad \Longrightarrow \quad \Pi(p^2) = \frac{1}{p^2} - \frac{1}{p^2 + M^2}$$

Generalized higher-derivative model:

$$\mathcal{L} = \frac{1}{2}\phi f(-\Box)\Box\phi - V(\phi) \quad \Longrightarrow \quad \Pi(p) = \frac{1}{f(p^2)p^2}$$

Is there any non-trivial $f(-\Box)$ such that the propagator is ghost-free?

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Is there any non-trivial $f(-\Box)$ such that the propagator is ghost-free?

YES! Nonlocality (infinite-order derivatives) can help!

Approach 1: beyond 4 derivatives

Local vs Nonlocal

Local Lagrangian:

$$\mathcal{L}_L \equiv \mathcal{L}_L(\phi, \partial\phi, \partial^2\phi, \dots, \partial^n\phi)$$

Nonlocal Lagrangian:

$$\mathcal{L}_{NL} \equiv \mathcal{L}_{NL}\left(\phi, \partial\phi, \partial^2\phi, \dots, \partial^n\phi, \dots, \log(\Box)\,\phi, e^{\Box}\phi, \frac{1}{\Box}\phi, \dots\right)$$

Approach 1: beyond 4 derivatives

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Typical in standard perturbative QFT

Approach 1: beyond 4 derivatives

Bare scalar field Lagrangian:

$$\mathcal{L} = -\frac{1}{2}\phi F(-\Box)\phi - V(\phi)$$

Entire function
(good IR limit: $F(-\Box) \to -\Box + m^2$)

Weierstrass' theorem:

$$F(-\Box) = e^{-\gamma(-\Box)} \prod_{i=1}^{N} \left(-\Box + m_i^2\right)^{r_i}, \qquad N \leq \infty,$$

$\gamma(-\Box)$ is an entire function

N is the number of zeroes m_i^2; r_i is the multiplicity of each zero

Approach 1: beyond 4 derivatives

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Propagator:

$$\Pi(p^2) = \frac{e^{\gamma(p^2)}}{p^2 + m^2} \prod_{i=2}^{N} \frac{1}{(p^2 + m_i^2)^{r_i}}$$

Approach 1: beyond 4 derivatives

$$F(-\Box) = e^{-\gamma(-\Box)} \prod_{i=1}^{N} (-\Box + m_i^2)^{r_i}, \qquad N \leq \infty,$$

- $N = 1, \ r_i = 1, \ \gamma(-\Box) = 0 \implies$ 2-derivative theory (Klein-Gordon)

$$F(-\Box) = -\Box + m^2$$

- $N = 2, \ r_i = 1, \ \gamma(-\Box) = 0 \implies$ 4-derivative theory with ghost

$$F(-\Box) = (-\Box + m^2)\left(1 - \frac{\Box}{M^2}\right)$$

- $\infty > N \geq 2$ and/or $r_i \geq 2$ (with m_i real) $\implies$ ghosts!

Approach 1: beyond 4 derivatives

$$F(-\Box) = e^{-\gamma(-\Box)} \prod_{i=1}^{N} (-\Box + m_i^2)^{r_i}, \qquad N \leq \infty,$$

- $N = 1,\ r_i = 1,\ \gamma(-\Box) \neq 0$

$\Longrightarrow$ infinite-order derivatives and one real zero:

$$F(-\Box) = e^{-\gamma(-\Box)}(-\Box + m^2)$$

Propagator:

$$\Pi(p^2) = \frac{e^{\gamma(-p^2)}}{p^2 + m^2 - i\epsilon}$$

Approach 1: nonlocal field theories

Nonlocal scalar field models:

$$\mathcal{L} = \frac{1}{2}\phi e^{-\gamma(-\Box/M_s^2)}(\Box - m^2)\phi - V(\phi),$$

Ghost-free propagator:

$$\Pi(p^2) = \frac{e^{\gamma(p^2/M_s^2)}}{p^2 + m^2 - i\epsilon}$$

Perturbative unitarity (optical theorem and Cutkosky rules) holds

[Pius & Sen 2015; Briscese & Modesto 2018; Chin & Tomboulis 2018; Koshelev & Tokareva 2021; Buoninfante 2022]

Approach 1: nonlocal QFTs of gravity

Generalized quadratic gravity action:

$$S = S_{EH} + \int d^4x \sqrt{-g}\left(RF_1(-\Box)R + R_{\mu\nu}F_2(-\Box)R^{\mu\nu} + R_{\mu\nu\rho\sigma}F_3(-\Box)R^{\mu\nu\rho\sigma} + \cdots\right)$$

Analytic form factors:

$$F_i(-\Box/M_s^2) = \sum_{n=0}^{N\leq\infty} f_{i,n}\left(\frac{-\Box}{M_s^2}\right)^n,$$

$N = \infty \iff$ nonlocal

M_s: energy scale

[Krasnikov, Kuz'min, Tomboulis, Koshelev, Siegel, Biswas, Mazumdar, Modesto, Calcagni, Briscese, Rachwal, Frolov, Zelnikov, Starobinsky, Kumar, Tokareva, Boos, Kolar, Lambiase, Buoninfante,......]

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[Asymptotic Safety Community: Saueressig, Knorr, Ripken, Platania, Schiffer, Reichert, Pawlowski, Litim, Bonanno,...]

Approach 1: nonlocal QFTs of gravity

Consider

$$S = S_{EH} + \frac{1}{2}\int d^4x\,\sqrt{-g}\big(RF_0(-\Box)R + C_{\mu\nu\rho\sigma}F_2(-\Box)C^{\mu\nu\rho\sigma}\big)$$

Propagator:

$$\Pi_{\mu\nu\rho\sigma}(k^2) = \frac{\mathcal{P}^{(2)}_{\mu\nu\rho\sigma}}{f_2(k^2)k^2} - \frac{1}{2}\frac{\mathcal{P}^{(0)}_{\mu\nu\rho\sigma}}{f_0(k^2)k^2},$$

$$f_0(k^2) = 1 + 6F_0(k^2)k^2/M_p^2$$

$$f_2(k^2) = 1 - 2F_2(k^2)k^2/M_p^2$$

Approach 1: nonlocal QFTs of gravity

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$$f_2(k^2) = 1 - 2F_2(k^2)k^2/M_p^2$$

No-ghost condition:

$$f_2(k^2) = e^{-\gamma_2(k^2)}, \qquad f_0(k^2) = e^{-\gamma_0(k^2)}(1 + k^2/m_0^2)$$

Entire functions

Approach 1: nonlocal QFTs of gravity

Ghost-free nonlocal gravity:

$$S = S_{EH} + \frac{1}{2}\int d^4x\,\sqrt{-g}\big(RF_0(-\Box)R + C_{\mu\nu\rho\sigma}F_2(-\Box)C^{\mu\nu\rho\sigma}\big)$$

$$F_0(-\Box) = M_p^2\,\frac{e^{-\gamma_0(-\Box)}(1-\Box/m_0^2)-1}{6\Box}, \qquad F_2(-\Box) = M_p^2\,\frac{1-e^{-\gamma_2(-\Box)}}{2\Box}$$

Approach 1: nonlocal QFTs of gravity

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Simplest case: no spin-0 dof

$$F_2(-\Box) = -3F_0(-\Box) = M_p^2\,\frac{1-e^{-\gamma_2(-\Box)}}{2\Box}$$

$$\Longrightarrow \quad \Pi_{\mu\nu\rho\sigma}(k^2) = e^{\gamma_2(k^2)}\,\Pi^{EH}_{\mu\nu\rho\sigma}(k^2)$$

Approach 1: nonlocal QFTs of gravity

Some remarks

- Infinite class of viable entire functions $\gamma_1(-\Box)$ and $\gamma_2(-\Box)$
- Smaller (but still infinite) class of super-renormalizable models
  [Kuz'min, Tomboulis, Modesto, Rachwal, Calcagni, Briscese, Giacchini, de Paula Netto,...]
- Applications to black holes and compact objects
  [Biswas, Mazumdar, Siegel, Moffat, Modesto, Frolov, Zelnikov, Boos, Giacchini, de Paula Netto, Kolar, Koshelev, Lambiase, Buoninfante,....]
- Applications to the early universe cosmology (constraint $M_s \gtrsim 10^{14}$GeV)
  [Koshelev, Starobinsky, Kumar, Calcagni, Modesto, Rachwal, Tokareva,....]

Approach 1: nonlocal QFTs of gravity

Open issues

- Hamiltonian for nonlocal theories? (non-perturbative classical stability?)

- Dressed propagator? (non-perturbative quantum stability?)
  [Shapiro (2015)]

- Quantification of causality violation?
  [Some attempts: Tomboulis (2015); Carone (2016); Modesto (2018);
  Buoninfante, Lambiase, Mazumdar (2018)]

- Huge freedom in the choice of the entire functions !?!
  (predictivity?)

- Nonlocal Lagrangians from first principles…?

Approach 1: nonlocal QFTs of gravity

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- Hamiltonian for nonlocal theories? (non-perturbative classical stability?)
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- Huge freedom in the choice of the entire functions !?! (predictivity?)
- Nonlocal Lagrangians from first principles...?

} Most important!

Outline

- ~~**Approach 1:** correspondence principle applies (give up "locality")~~

- **Approach 2:** correspondence principle does not apply

- Discussion

Approach 2: keep locality

Locality + (strict) renormalizability are very restrictive:

$$S = \frac{1}{2}\int d^4x\,\sqrt{-g}\left(M_p^2(R-2\Lambda)+\frac{\alpha}{6}R^2-\frac{\beta}{2}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}\right)$$

Approach 2: possible solutions

$$\Pi(p^2) = \frac{1}{p^2 - i\epsilon} - \frac{1}{p^2 + m_2^2 - i\varepsilon}$$

$$S^{+}S = 1, \qquad S = 1 + iT,$$

$$1 = \sum_{\{n\}} c_n \, |n\rangle\langle n|,$$

Optical theorem:

$$2Im\{\langle a|T|a\rangle\} = \sum_{\{n\}} c_n |\langle n|T|a\rangle|^2$$

Tree level example:

$$Im\{\langle a|T|a\rangle\} \sim \theta(p^0)[\delta(p^2) - sign(\varepsilon)\delta(p^2 + M^2)]$$

Approach 2: possible solutions

Causal propagation & negative norms [Holdom, Salvio, Strumia,...]

$$\Pi(p^2) = \frac{1}{p^2 - i\epsilon} - \frac{1}{p^2 + m_2^2 - i\varepsilon}$$

$$(\epsilon > 0, \ \varepsilon > 0)$$

$$S^{+}S = 1, \qquad S = 1 + iT,$$

$$1 = \sum_{\{n\}} c_n \, |n\rangle\langle n|, \qquad c_n^{normal} > 0 \qquad c_n^{ghost} < 0$$

Optical theorem:

$$2Im\{\langle a|T|a\rangle\} = \sum_{\{n\}} c_n |\langle n|T|a\rangle|^2$$

Tree level example:

$$Im\{\langle a|T|a\rangle\} \sim \theta(p^0)[\delta(p^2) - sign(\varepsilon)\delta(p^2 + M^2)]$$

Unitarity is preserved!

Approach 2: Some Remarks

Beyond tree-level

The massive spin-2 ghost gets a width: [Donoghue & Menezes 2018+]

$$\frac{-1}{p^2 + m_2^2 + i\varepsilon} \quad \longrightarrow \quad \frac{-1}{p^2 + m_{2,ph}^2 + i(\varepsilon + m_{2,ph}\Gamma)}$$

($\varepsilon \geq 0$ anti-Feynman) $\qquad$ ($\Gamma \sim m_2^3/M_p^2 \geq 0$)

Ghost life-time: $\tau_{decay} \sim 1/\Gamma \sim M_p^2/m_2^3$

If $m_2 > 2m_0$ & $m_0 \sim 10^{13} GeV \quad \Longrightarrow \quad \tau_{decay} \lesssim 10^{-3} GeV^{-1} \sim 10^{-28}\text{sec}$

Approach 2: Some Remarks

Classical limit of Quadratic Gravity ?

Approach 2: Some Remarks

Classical limit of Quadratic Gravity ?

- Is $\hbar \longrightarrow 0 \implies \Gamma \sim O(\hbar) \longrightarrow 0 \implies \tau_{decay} \longrightarrow \infty$? NO, too naive!

- Is just a low-energy limit $(E \ll m_2)$?

Approach 2: Some Remarks

Classical limit of Quadratic Gravity ?

- Is $\hbar \longrightarrow 0 \implies \Gamma \sim O(\hbar) \longrightarrow 0 \implies \tau_{decay} \longrightarrow \infty$? NO, too naive!

- Is just a low-energy limit $(E \ll m_2)$? I don't think so.

Approach 2: Some Remarks

Classical limit of Quadratic Gravity ?

Two time-scale regimes to consider (in my opinion):

1. $\Delta t \geq \tau_{decay}$:

Classical limit must be taken consistently with the "quantum projection"

2. $\Delta t < \tau_{decay}$:

Ghost is still alive and can propagate, no projection

[may it be related to the high-energy limit ($E \gg m_2$)?]

Approach 2: Some Remarks

Tree-level 2-2 graviton scattering amplitude

Despite renormalizability, it's the same as Einstein's general relativity:

[Modesto et al. (2015); Holdom (2021)]

$$\mathcal{M}_{2-2}(s) \sim G\, s \sim E^2/M_p^2$$

Perturbativity (not unitarity) breaks at $E \sim M_p$

This may indicate that non-perturbative effects must be taken into account (e.g. black-hole formation !)

Approach 2: Some Remarks

Implications of a non-zero (positive) cosmological constant?

$$S = \frac{1}{2}\int d^4x\,\sqrt{-g}\left(M_p^2(R-2\Lambda) + \frac{\alpha}{6}R^2 - \frac{\beta}{2}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}\right)$$

Approach 2: Some Remarks

$$\Lambda > 0: \qquad \lim_{\beta \to \infty} S = \ ?$$

- Massless spin-2 & $\pm 2, \pm 1$ helicities of massive spin-2 ghost decouple
- Massive spin-0 (ϕ) & helicity-0 (χ) of spin-2 ghost survive

[Buoninfante 2308.11324]

$$S_{\phi\chi}[g,\phi,\chi] = \int d^4x \sqrt{-g}\left[\frac{1}{2}\left(\partial_\mu\chi\partial^\mu\chi - \partial_\mu\phi\partial^\mu\phi\right) - V(\phi,\chi)\right]$$

$$V(\phi,\chi) = \frac{\Lambda}{36\bar{M}_p^2}\left(\chi^2 - \phi^2 - 6\bar{M}_p^2\right)^2 + \frac{m_0^2}{12\bar{M}_p^2}\phi^2(\chi+\phi)^2$$

$$\bar{M}_p^2 = M_p^2 + \frac{4}{3}\alpha\Lambda$$

Constraint:

$$T^{(\phi\chi)} = 0, \qquad T_{\mu\nu}^{(\phi\chi)} = \frac{-2}{\sqrt{-g}}\frac{\delta S_{\phi\chi}}{\delta g^{\mu\nu}}$$

Summary

Ghost puzzle in Perturbative Quantum gravity

Approach 1: give up locality and kill the ghost at the classical level

- Correspondence principle still holds
- Bigger price to pay: uniqueness is lost (predictivity?)

Approach 2: keep locality + strict renormalizability

- Unique Lagrangian
- Correspondence principle does not hold
- Some open questions: classical limit; non-perturbative stability; more investigations in curved spacetimes