Title: Holography and its implications for quantum gravity - VIRTUAL

Speakers: Johanna Erdmenger

Collection: Puzzles in the Quantum Gravity Landscape: viewpoints from different approaches

Date: October 23, 2023 - 10:20 AM

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Abstract: I will review implications of the AdS/CFT correspondence for quantum gravity, paying particular attention to holographic RG flows and the processing of information. Deformed hyperbolic geometries correspond to boundary field theory RG flows. Conversely, as I will exemplify using the recent approach of discrete holography, boundary RG flows may be used to reconstruct the bulk spacetime, for instance via tensor networks. I will discuss recent approaches towards including gravity dynamics into this reconstruction process.

The presenter will be joining via Zoom for this talk.


Holography and its implications for quantum gravity
Johanna Erdmenger
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ct.qmat
Complexity and Topology
in Quantum Matter
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Overview

I. AdS/CFT correspondence and renormalization group

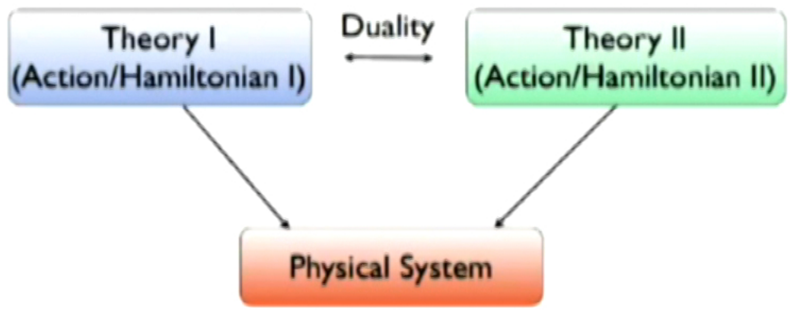
II. Information and gravity

III. Discrete holography

Duality: A physical theory has two equivalent formulations

Same dynamics

One-to-one map between states



Gauge/Gravity Duality:

Gauge Theory Quantum Field Theory	$\Leftrightarrow$	Gravity theory in higher dimensions

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Gauge/gravity duality

- Conjecture which follows from a low-energy limit of string theory

- Duality:

  Quantum field theory at strong coupling

  $\Leftrightarrow$ Theory of gravitation at weak coupling

- Holography:

  Quantum field theory in d dimensions

  $\Leftrightarrow$ Gravitational theory in $d+1$ dimensions

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AdS/CFT correspondence

AdS: Anti-de Sitter space, CFT: Conformal field theory

$\mathcal{N} = 4$ SU(N) Super Yang-Mills theory

Anti-de Sitter space:

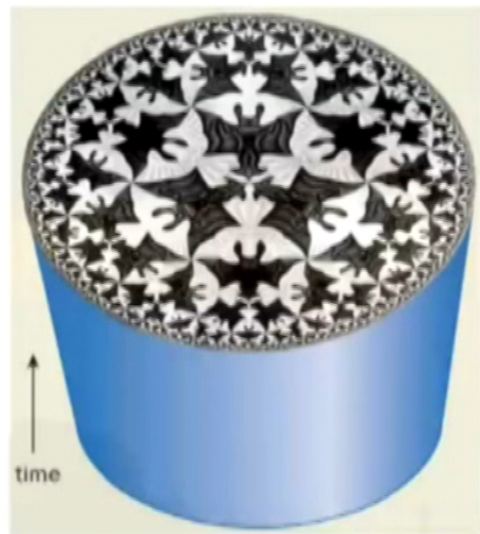
Space of constant negative curvature, has a boundary

In total: $AdS_5 \times S^5$

Isometries of AdS$_{d+1}$ and symmetry of operators in a CFT$_d$:

$$SO(d+1,1)$$

(Euclidean signature)



Quelle: Institute of Physics, Copyright: C. Esche

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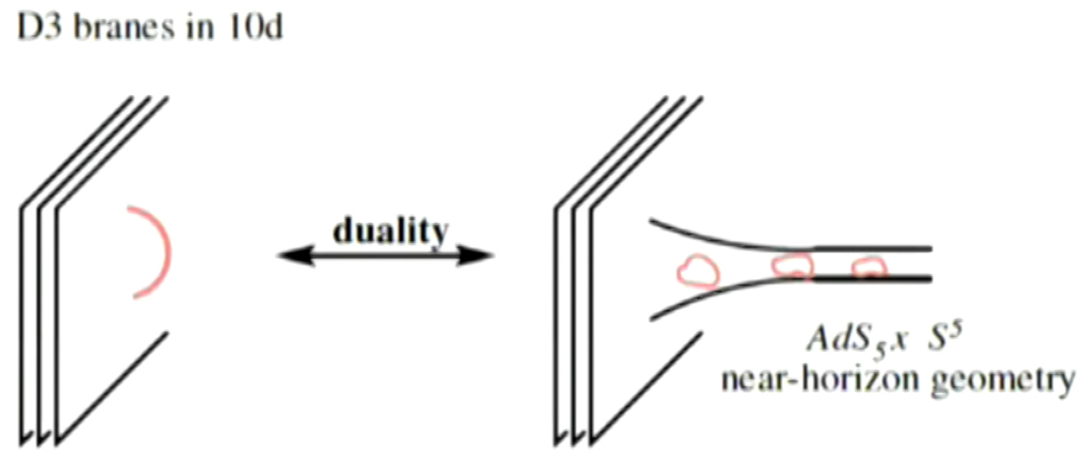
String theory origin of the AdS/CFT correspondence



⇓ Low energy limit

Supersymmetric $SU(N)$ gauge theory in four dimensions $(N \to \infty)$

Supergravity on the space $AdS_5 \times S^5$

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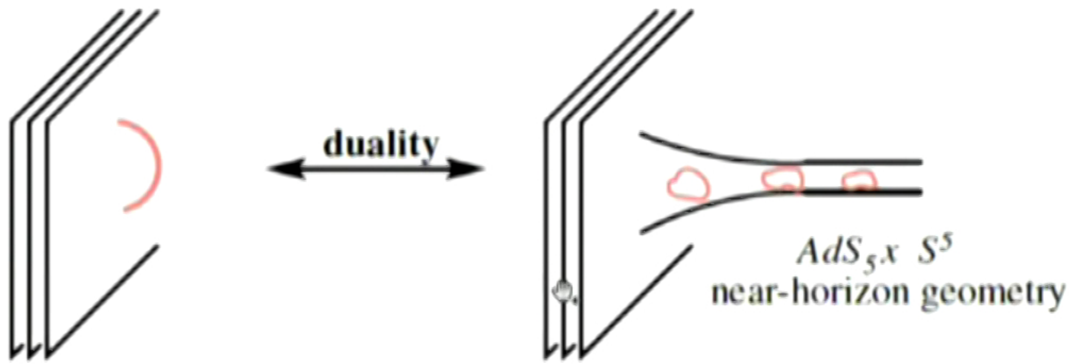
String theory origin of the AdS/CFT correspondence

D3 branes in 10d



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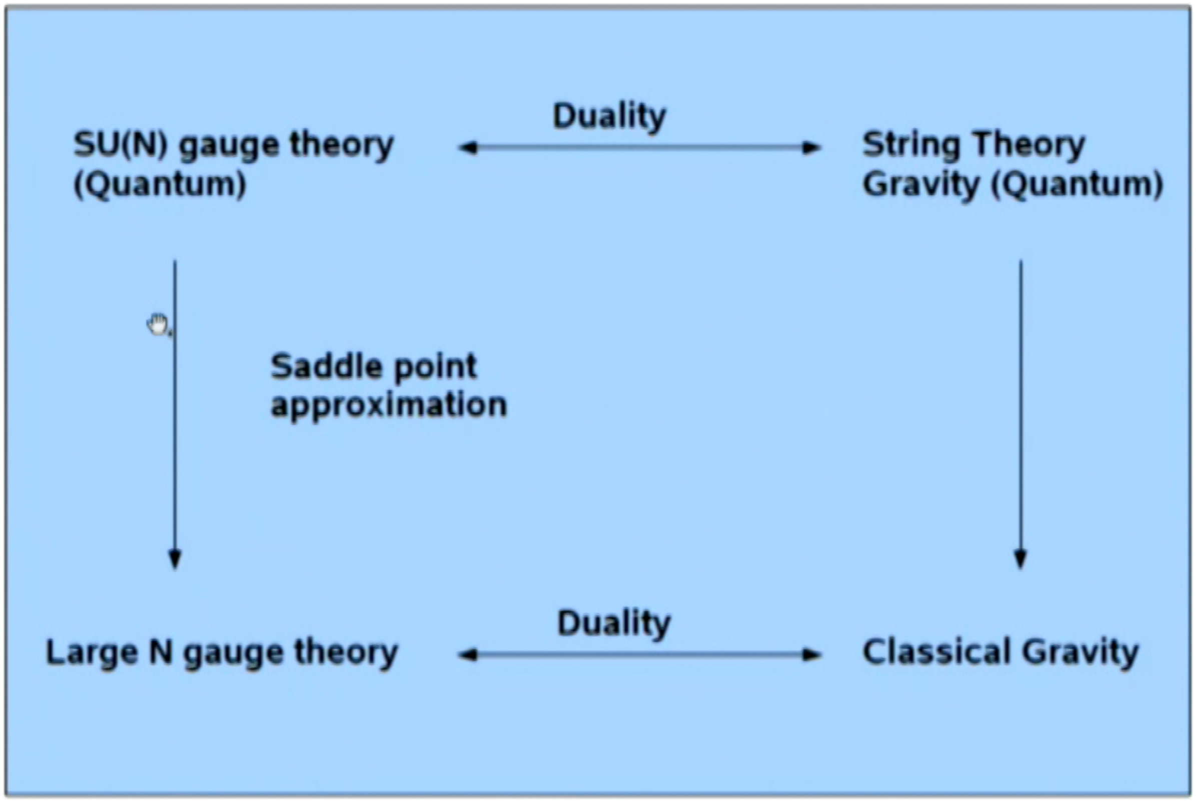
Gauge/Gravity Duality: Low-energy limit



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Different levels of the AdS/CFT correspondence

$$g_{\mathrm{YM}}^2 = 2\pi g_s \qquad \lambda = g_{\mathrm{YM}}^2 N \qquad 2 g_{\mathrm{YM}}^2 N = L^4/\alpha'^2$$

't Hooft coupling

	$\mathcal{N}=4$ SYM theory	IIB theory on $AdS_5 \times S^5$
Strongest form	any N and λ	Quantum string theory, $g_s \neq 0$, $\alpha' \neq 0$
Strong form	$N \to \infty$, λ fixed but arbitrary	Classical string theory, $g_s \to 0$, $\alpha' \neq 0$
Weak form	$N \to \infty$, λ large	Classical supergravity, $g_s \to 0$, $\alpha' \to 0$

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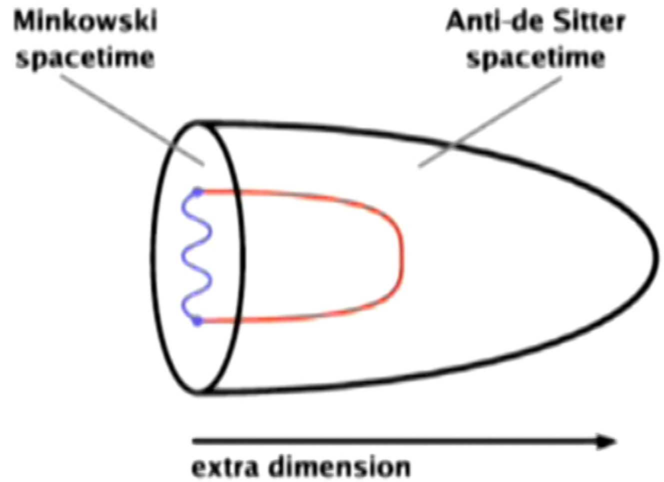

'Dictionary'   Gauge invariant field theory operators
$\Leftrightarrow$ Classical fields in gravity theory

Symmetry properties coincide, generating functionals are identified

Test: (e.g.) Calculation of correlation functions

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Different levels of the AdS/CFT correspondence

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AdS/CFT correspondence

- Field-operator correspondence:

$$\langle e^{\int d^dx\, \phi_0(\vec{x})\mathcal{O}(\vec{x})}\rangle_{CFT} = Z_{sugra}\Big|_{\phi(0,\vec{x})=\phi_0(\vec{x})}$$

Generating functional for correlation functions of particular composite operators in the quantum field theory

coincides with

Classical tree diagram generating functional in supergravity

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Holographic Renormalization Group Flows

Original AdS/CFT correspondence involves $\mathcal{N}=4$ SU(N) Super Yang-Mills theory

Field content: One gauge field, four fermions, six real scalars

Beta function vanishes identically to all orders in perturbation theory

Theory at UV fixed point

Perturb by relevant operator preserving $\mathcal{N}=1$ supersymmetry

$$W_{\rm LS} \equiv h\,{\rm Tr}\,(\Phi_3\,[\Phi_1\,,\,\Phi_2]) + \frac{m}{2}\,{\rm Tr}\,\Phi_3^2$$ Leigh-Strassler flow

IR fixed point: $\beta(g) \sim 2N\,(\gamma_1+\gamma_2+\gamma_3)$ $\beta_h = \gamma_1+\gamma_2+\gamma_3,$ $\beta_m = 1 - 2\,\gamma_3$

$$\gamma_1 = \gamma_2 = -\frac{\gamma_3}{2} = -\frac{1}{4}$$

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Holographic renormalization group flows: field-theory side

Leigh-Strassler flow
$$W_{LS} \equiv h \,\mathrm{Tr}\,(\Phi_3 [\Phi_1 , \Phi_2]) + \frac{m}{2} \mathrm{Tr}\, \Phi_3^2$$

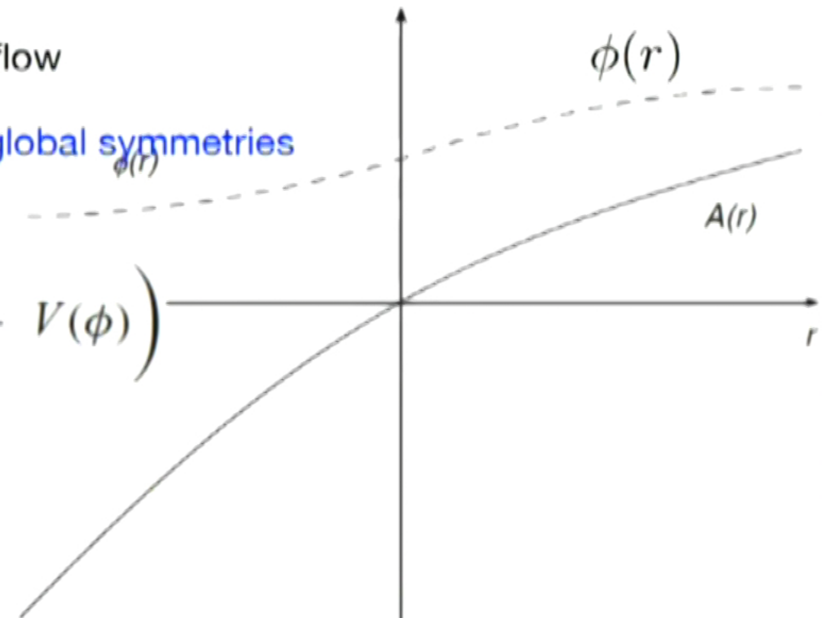
C theorem:
$$\frac{a_{IR}}{a_{UV}} = \frac{c_{IR}}{c_{UV}} = \frac{27}{32}$$

For coefficients of conformal anomaly in conformal Ward identity

$$\langle T_\mu{}^\mu \rangle = \frac{c}{8\pi^2} \left(R^{\mu\nu} R_{\mu\nu} - \frac{1}{3} R^2 \right), \qquad c = \frac{N^2}{4}$$

Holographic Renormalization Group Flows

Gravity side of correspondence: Dual of Leigh-Strassler flow



Look for a solution of supergravity preserving the same global symmetries

Domain wall flow:

$$S = \int \mathrm{d}^{d+1}x \sqrt{-g} \left(\frac{R}{16\pi G} - \frac{1}{2} \partial_m\phi \, \partial^m\phi - V(\phi) \right)$$

$$\mathrm{d}s^2 = e^{2A(r)} \eta_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu + dr^2$$

Conformal anomaly at IR fixed point exactly reproduced

$$\langle T_\mu{}^\mu \rangle = \frac{L^3}{64\pi G_5} \left(R^{\mu\nu} R_{\mu\nu} - \frac{1}{3} R^2 \right)$$

First order RG equation from gradient flow

$$V(\phi) = \frac{1}{2} \left(\frac{\mathrm{d}W}{\mathrm{d}r} \right)^2 - \frac{d}{d-1} W^2 \qquad \sqrt{8\pi G} \frac{\mathrm{d}\phi}{\mathrm{d}r} = \frac{\mathrm{d}W}{\mathrm{d}\phi}, \qquad A' = -\frac{\sqrt{8\pi G}}{(d-1)} W$$

Freedman, Gubser, Pilch, Warner 1999

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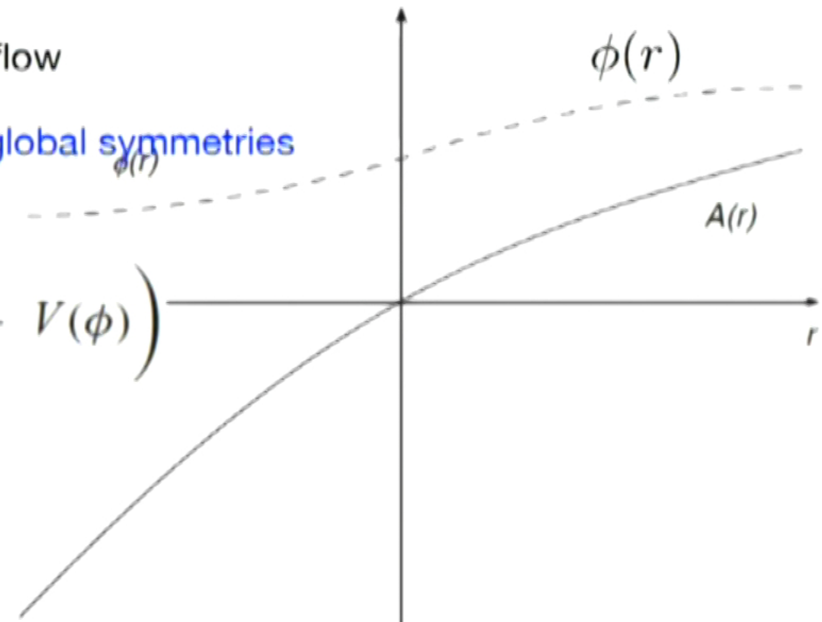

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Freedman, Gubser, Pilch, Warner 1999

Relation to quantum information

Entanglement entropy

Consider product Hilbert space $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$

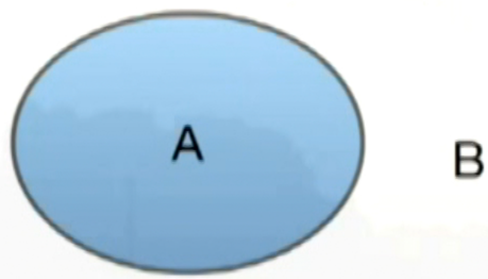
Reduced density matrix

$$\rho_A = \mathrm{Tr}_B \rho_{\mathrm{tot}}$$

Entanglement entropy

$$S_A = -\mathrm{Tr}_A \rho_A \ln \rho_A$$

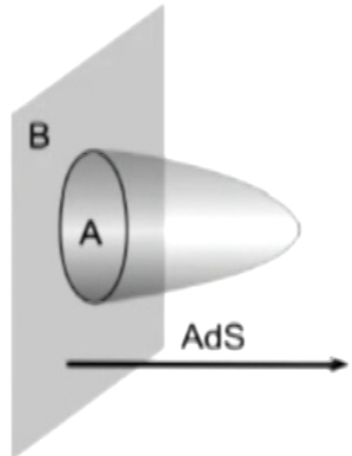
Entanglement for space regions



Entanglement entropy in Gauge/Gravity Duality

(Ryu, Takayanagi Phys.Rev.Lett. 96 (2006) 181602)

Leading term in entanglement entropy given by
area of minimal surface in holographic dimension



Entanglement entropy: Quantum field theory

Conformal field theory in 1+1 dimensions (Cardy, Calabrese, J.Stat.Mech. 0406 (2004) P06002):

$$S = \frac{c}{3} \ln(\ell \Lambda)$$

Reproduced by Ryu-Takayanagi result

$\Lambda \propto 1/\epsilon$, ϵ boundary cut-off in radial direction

$c = 3L/(2G_3)$ central charge of CFT

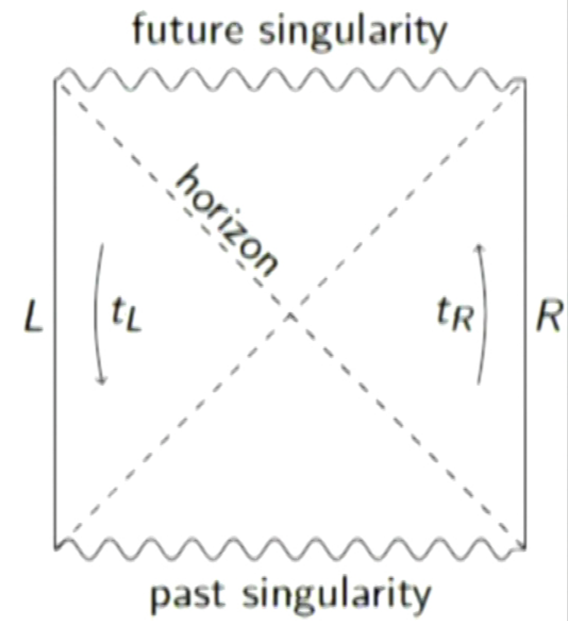
Finite temperature (at small ℓ):

$$S(\ell) = \frac{c}{3} \ln \left(\frac{1}{\pi \epsilon T} \sinh(2\pi \ell T) \right)$$

Von Neumann algebras, quantum gravity and AdS/CFT

Factorization puzzle Maldacena+Maoz '13; Harlow '16

- The two CFTs have disjoint Hilbert spaces since there is no interaction between them $\mathcal{H}_L \otimes \mathcal{H}_R$
- The wormhole Hilbert space does not factorize
- Apparent contradiction?



Eternal AdS black hole

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von Neumann algebras

von Neumann 1930

Jefferson; Liu, Leutheusser; Witten; Chandrasekaran, Penington, Witten

Concept of algebraic QFT
for classifying operator algebras w. r. t. entanglement properties

Type I - density matrix and trace (as in quantum mechanics),

admits irreducible representations

Type II - trace prescription, but does not act irreducibly

Type III - no trace prescription (eg. free QFTs)

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Type II vs. type III von Neumann algebra for eternal black hole

Including 1/N corrections gives rise to type II algebra with vanishing center

Allows for well-defined trace functional for a particular state Witten '21

Geometric phase approach: Non-exact symplectic form leads to Berry phase

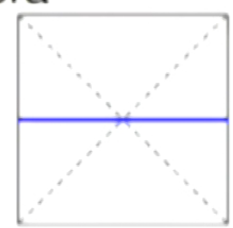
Trace functional proportional to Berry phase

Banerjee, Dorband, J.E., Weigel arXiv:2306.00055

Non-factorization

-> non-zero geometric phase -> no trace definition -> type III vN algebra

Maximally entangled state -> geometric phase vanishes -> type II vN algebra



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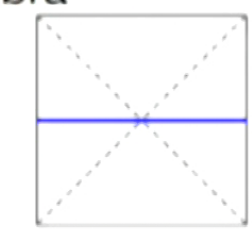

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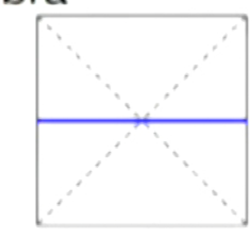

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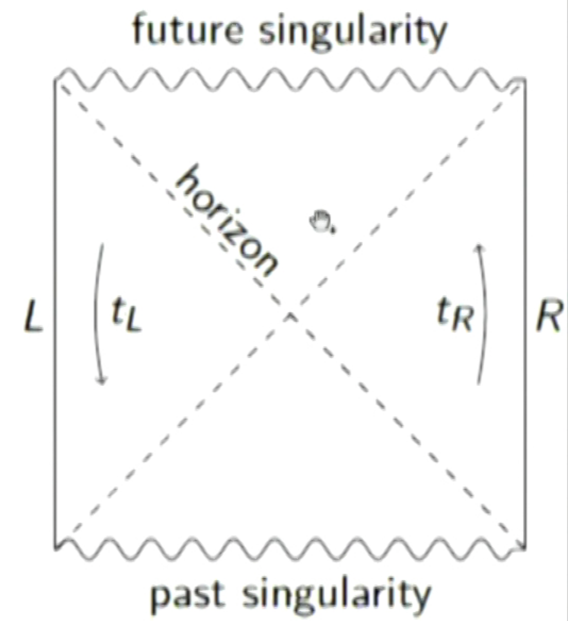

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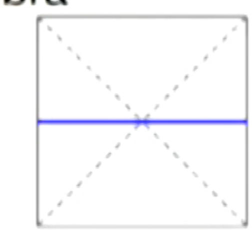

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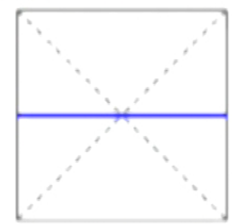

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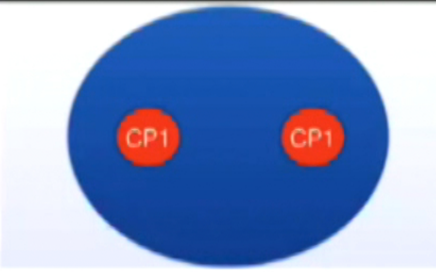
Analogy: Interacting two-spin system:
Berry phase and type I Von Neumann algebra



$$H = J\boldsymbol{S}_1 \cdot \boldsymbol{S}_2 - 2\mu_B B S_{1z} \qquad \tan\alpha = 2\mu_B \frac{B}{J}$$

Entanglement entropy $\quad S_{\mathrm{EE}} = -\sum_{i=\uparrow,\downarrow} \kappa_i^2 \ln \kappa_i^2 = \sin\alpha \ln \frac{1-\sin\alpha}{\cos\alpha} - \ln\frac{\cos\alpha}{2}$

No entanglement (J=0) $\quad S_{EE} = 0 \quad$ Entanglement orbit $\quad \mathbb{C}\mathrm{P}^1 \times \mathbb{C}\mathrm{P}^1$

Berry phase maximal

Maximal entanglement (J very large) $\quad S_{EE} = \ln 2 \quad \frac{SU(2)}{\mathbb{Z}_2} = \mathbb{R}\mathrm{P}^3$

Berry phase vanishes

Intermediate entanglement $\quad \mathbb{C}\mathrm{P}^1 \times \mathbb{R}\mathrm{P}^3$

$$\int \Omega = \frac{\sin\alpha}{2} V(S^2) = 2\pi \sin\alpha$$

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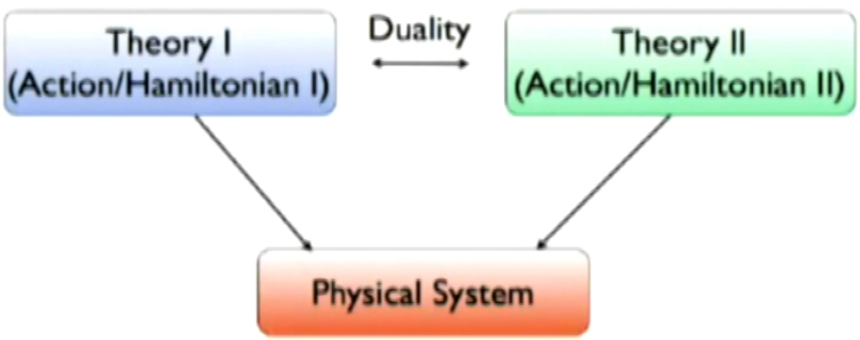
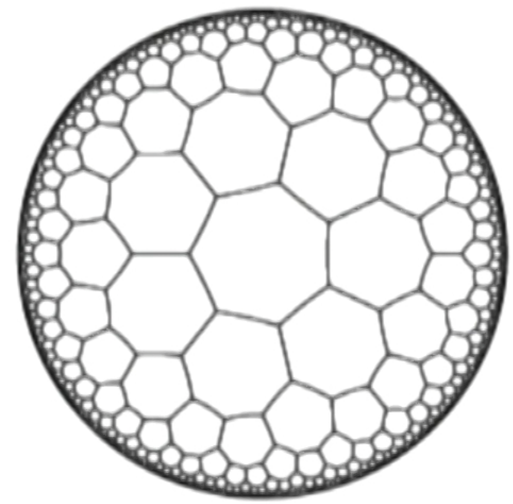
Discrete Holography

Goal: Establish holographic duality on hyperbolic tiling





arXiv:2205.05081, Phys. Rev. Lett. '23 (Basteiro, Dusel, J.E., Hinrichsen, Meyer, Herdt, Schrauth)

arXiv:2205.05693, Sci Post Physics (Basteiro, Di Giulio, J.E., Karl, Meyer, Xian)

arXiv: 2212.11292 (Basteiro, Das, Di Giulio, J.E.)

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Boyle, Dickens, Flicker 1805.02665

Hyperbolic tilings through inflation rules

Starting point: central tile = 0-th layer

↓

we construct concentric layers of tiles

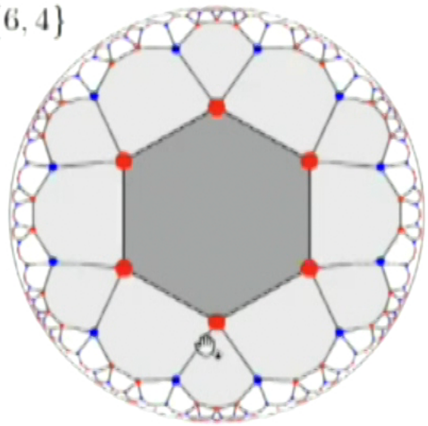
↓

After n layers: outermost set of edges and vertices = **boundary of the tiling**

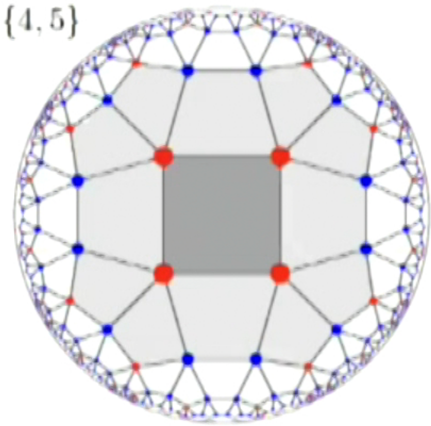
2 internal neighbours = ● ⟷ a

3 internal neighbours = ● ⟷ b

$\{6,4\}$



$\{4,5\}$



Inflation rule

$$\sigma_{\{p,q\}} : \begin{cases} a \to w_a(a,b) \\ b \to w_b(a,b) \end{cases}$$

$$\sigma_{\{6,4\}} : \begin{cases} a \to aabaaab \\ b \to aab \end{cases}$$

$$\sigma_{\{4,5\}} : \begin{cases} a \to babab \\ b \to bab \end{cases}$$

The boundary of the tiling after a large number of inflation steps is characterised by a long **aperiodic sequence**, whose size grows by a factor $\lambda_+(p,q)$ at each step

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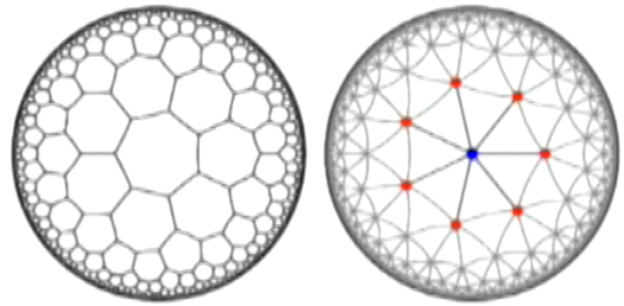
Discrete holography



Spin chain at boundary realises bulk tiling construction principle at boundary

Aperiodic spin chain

Ground state of spin chain Hamiltonian determined using RG approach

Associated tensor network representing this ground state ➡ bulk reconstruction

Gravity dynamics on bulk side — dynamical triangulations in hyperbolic space?

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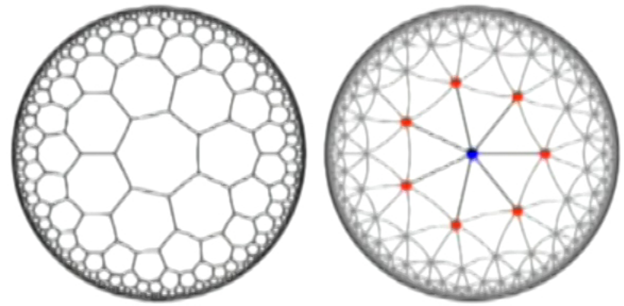
Discrete holography



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Conclusion and Outlook

Gauge/Gravity Duality

Holographic principle

Holographic RG flows implemented from supergravity equations of motion

(Gradient flow) - Constructed to match global symmetries

New relations between quantum information and gravity

Entanglement entropy, mutual information, computational complexity

von Neumann algebras, Berry phases

Reconstrucion of spacetime, discrete holography