Title: QFT1 Lecture - 103023

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Collection: Quantum Field Theory 1 2023/24

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Interaction (generic) (LSZ ... Wic

fe

scalar

step 1 $\langle f | S | i \rangle$ $\quad a_1 \equiv a_{k_1}$

$|i\rangle = a_1^\dagger(-\infty)\, a_2^\dagger(-\infty)|\Omega\rangle$

$\langle \Omega | a_4(+\infty)\, a_3(+\infty)\, a_1^\dagger(-\infty) a_2^\dagger(-\infty)|\Omega\rangle$

... Wick)

fermion (2 particle to 2 particle)

$$|i\rangle = b_1^{+}(-\infty)\, b_2^{+}(-\infty)|\Omega\rangle$$

$$b_1 \equiv b_{k_1}^{s_1}$$

$\longrightarrow$ change a to bs

step 2

$$a_1(+\infty) - a_1(-\infty)$$

$$\equiv I_1 = \int_{-\infty}^{\infty} dt\, \partial_0 a_1(t)$$

$$\equiv -i \int d^4x\, e^{-ik_1 x} \underline{(\partial^2 + m^2)\varphi(x)}$$

$$I_1^{+} = i \int d^4x\, e^{+ik_1 x} (\partial^2 + m^2)\, \varphi(x).$$

$$I_{1,f} \equiv b_1(+\infty) - b_1(-\infty)$$

$$\equiv \int_{-\infty}^{\infty} dt\, \partial_0 b_1(t)$$

$$I_{1,f} \equiv b_1(+\infty) - b_1(-\infty)$$

$$\equiv \int_{-\infty}^{\infty} dt\, \partial_0 b_1(t)$$

$$= \# \int d^4x\, e^{-ik_1 x}\, \bar{u}_1 (i\not{\partial} - m)\psi(x)$$

$$= \# \int d^4x\, e^{-ik_1 x}\, u_1^\dagger \gamma^0 (i\gamma^\mu \partial_\mu \psi - m\psi)$$

$$I_{1,f}^{*} = \#^* \int d^4x\, e^{+ik_1 x} (\partial_\mu \psi^\dagger \gamma^0 \gamma^\mu \gamma^0 (-i) \gamma^0 u_1 - m\psi^\dagger \gamma^0 u_1)$$

$$= \# \int d^4x\, e^{+ik_1 x} (\partial_\mu \bar{\psi} \gamma^\mu u_1 (-i) - m\bar{\psi} u_1)$$

$$= \#' \int d^4x\, \bar{\psi}(i\overleftarrow{\not{\partial}} + m) u_1 e^{+ik_1 x}$$

scalar

$$\langle f|S|i\rangle = \prod_{i=1}^{4}\int d^4x_j\, e^{-i\lambda_i p_i x^i}(\partial_j^2+m^2)$$

$$\langle\Omega|T\varphi_1\varphi_2\varphi_3\varphi_4|\Omega\rangle$$

scalar

$$\langle f|S|i\rangle = \prod_{i=1}^{4}\int d^4x_j\, e^{-i\lambda_i p_i x^i}(\partial_j^2+m^2)$$

$$\langle\Omega|T\varphi_1\varphi_2\varphi_3\varphi_4|\Omega\rangle$$

step 3

$$\langle\Omega|T\varphi_1\varphi_2\varphi_3\varphi_4|\Omega\rangle = \frac{\langle 0|T\varphi_1\varphi_2\varphi_3\varphi_4\, e^{i\mathcal{L}}|0\rangle}{\langle 0|e^{i\mathcal{L}}|0\rangle}$$

fermion case

$$= \prod_{j=3}^{4}\int d^4x_j\, e^{-i}$$

$$\langle\Omega|T$$

$$\prod_{j=1}^{2} d^4x$$

$$e^{-ik_i x^i}(\partial_j^2+m^2)$$

$$\varphi_3\varphi_4|\Omega\rangle$$

$$\varphi_3\varphi_4|\Omega\rangle = \frac{\langle 0|T\varphi_1\varphi_2\varphi_3\varphi_4 e^{i\mathcal{L}}|0\rangle}{\langle 0|e^{i\mathcal{L}}|0\rangle}$$

fermion case

$$= \prod_{j=3}^{4}\int d^4x_j\, e^{-ik_j x^j}\,\underline{\bar{u}_j}\,(i\not{\partial}-m)$$

$$\langle\Omega|T\psi\psi\bar{\psi}\bar{\psi}|\Omega\rangle$$

$$\prod_{j=1}^{2}\int d^4x_j\,(i\overleftarrow{\not{\partial}}+m)\,\underline{u_j e^{+ik_i x^i}}$$

$$\langle\Omega|I_4 I_3 I_1^{\dagger} I_2^{\dagger}|\Omega\rangle$$

fermion case

$$= \prod_{j=3}^{4} \int d^4x_j \, e^{-ik_j\cdot x^j} \, \underline{\bar{u}_j} \, (i\not\partial - m)$$

$$\langle \Omega | T \psi \psi \bar{\psi} \bar{\psi} | \Omega \rangle$$

$$\prod_{j=1}^{2} d^4x_j \, (i\overleftarrow{\not\partial} + m) \, \underline{u_j e^{+ik_j\cdot x^j}}$$

$$\langle \Omega | \bar{T} \, I_4 I_3 I_1^{\dagger} I_2^{\dagger} | \Omega \rangle$$

$$\frac{\ldots \varphi_3 \varphi_4 e^{i\alpha} | 0 \rangle}{\ldots | e^{i\alpha} | 0 \rangle} \longrightarrow \text{no change}$$

Wick theorem

$$T = :\;: + \sqcap$$

$$T(\varphi(x)\varphi(y)) = :\varphi(x)\varphi(y): + \overbracket{\varphi(x)\varphi(y)}$$

$$\langle 0| T(\varphi_1(x)\varphi_2(y)|0\rangle = \langle 0| \overbracket{\varphi(x)\varphi(y)}|0\rangle$$

$$\Delta_F(x-y) = \int d^4p \frac{i}{p^2-m^2+i\epsilon} e^{-ip(x-y)}$$

① define time ordering.

② split $\varphi \to \varphi_+$ and φ_-

③ compute $[\varphi_-, \varphi_+]$

④ assemble with step function

starting point

$$T(\psi_a(x)\bar{\psi}_b(y)) = :\psi_a(x)\bar{\psi}_b(y): + \psi_a(x)\bar{\psi}_b(y) \leftarrow \text{goal.}$$

step 1 time ordering $$T(\psi(x)\bar{\psi}(y)) = \begin{cases} \psi(x)\bar{\psi}(y) & x^0 > y^0 \\ \bar{\psi}(y)\psi(x) & y_0 > x_0 \end{cases}$$

suppose $x_0 > y_0$

$$T\psi_a(x)\psi_b(x) = -T\psi_b(y)\psi_a(x) \qquad \psi = \int u b e^{-ikx} + v c^{\dagger} e^{+ikx}$$

$$\psi_a(x)\psi_b(y) = -\psi_a(x)\psi_b(y) = 0 \,?!!$$

first $x_0 > y_0$

choose $T(\psi(x)\bar{\psi}(y)) = \psi(x)\bar{\psi}(y)$

$$= \psi_+(x)\bar{\psi}_+(y) + \psi_+(x)\bar{\psi}_-(y)$$

problem $\rightarrow$ $+\psi_-(x)\bar{\psi}_+(y) + \psi_-(x)\bar{\psi}_-(y)$

$:cc^\dagger: = -c^\dagger c$ $\quad -\bar{\psi}_+(y)\psi_-(x) + \bar{\psi}_+(y)\psi_-(x)$

$$= :\psi(x)\bar{\psi}(y): + \{\psi_-(x), \bar{\psi}_+(y)\}$$

$$\chi = \int dV_k\, u\, b_k e^{-ikx} + v\, c_k^+ e^{+ikx}$$

$$\chi^+(y) = \int dV_p\, u^+ b_p^+ e^{+ipx} + v^+ c_p e^{-ipx}$$

$$\{\psi_-(x), \bar{\psi}_+(y)\} = \{\int dV_k\, u\, b_k e^{-ikx}, \int dV_q\, \bar{u}\, b_q^+ e^{+iq\cdot y}\}$$

$$= \int dV_k\, u\, e^{-ikx} \int dV_q\, \bar{u}\, e^{+iqy} \underbrace{\{b_k, b_q^+\}}_{N_0 \delta(k-q)}$$

$$= \int dV_k\, u\bar{u}\, e^{-ik(x-y)}$$

$$= \int dV_k\, (\cancel{k}+m)\, e^{-ik(x-y)}$$

$$= (i\cancel{\partial}_x + m) \int dV_k\, e^{-ik(x-y)}$$

$e^{+iq\cdot y}\}$

$$\underbrace{\{b_k, b_q^\dagger\}}_{N_D\,\delta(k-q)}$$

$$S_F(x-y) = (i\partial\!\!\!/_x + m)\Delta_F(x-y)$$

$$= (i\partial\!\!\!/_x + m)\int d^4p \, \frac{i}{p^2 - m^2 + i\epsilon} e^{-ip(x-y)}$$

$$= \int d^4p \, \boxed{\frac{i(p\!\!\!/ + m)}{p^2 - m^2 + i\epsilon}} \, e^{-ip(x-y)}$$

$$(\partial^2+m^2)\Delta_F(x-y)=i\delta(x-y)$$

$$(i\not{\partial}+m)(i\not{\partial}-m)$$

$$(i\not{\partial}-m)(i\not{\partial}+m)\Delta_F(x-y)=i\delta(x-y)$$

$$(i\not{\partial}-m)S_F(x-y)=i\delta(x-y)$$

Wick contraction for many fermion field

all contractions

$$\cdots\ \psi_1\psi_2\psi_3 \cdots$$

$$\cdots\ \psi_1\psi_2\psi_3 \cdots = -\psi_1\psi_3(\cdots\psi_2\cdots)$$

① bring together $(-)^s$

③ $\psi\psi$