Title: QFT1 Lecture - 103023

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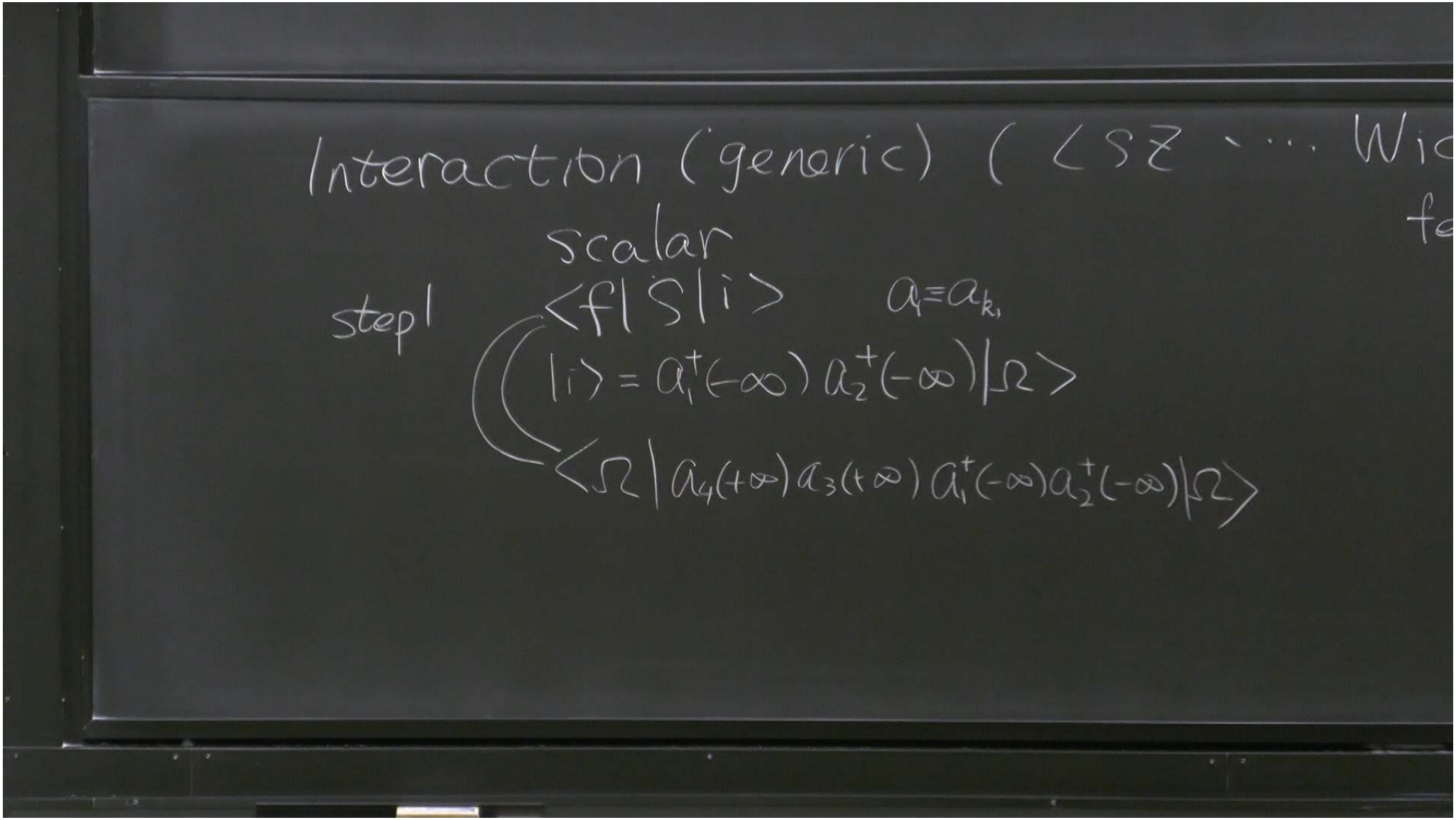
Collection: Quantum Field Theory 1 2023/24

Date: October 30, 2023 - 10:45 AM

URL: https://pirsa.org/23100054



Interaction (generic) (LSZ ... Wic

fe

scalar

step 1 $\langle f | S | i \rangle$ $\quad a_1 \equiv a_{k_1}$

$$|i\rangle = a_1^\dagger(-\infty)\, a_2^\dagger(-\infty)|\Omega\rangle$$

$$\langle \Omega | a_4(+\infty)\, a_3(+\infty)\, a_1^\dagger(-\infty)\, a_2^\dagger(-\infty)|\Omega\rangle$$

... Wick)

fermion (2 particle to 2 particle)

$$|i\rangle = b_1^\dagger(-\infty)\, b_2^\dagger(-\infty)\,|\Omega\rangle$$

$$b_1 \equiv b_{k_1}^{s_1}$$

$\Rightarrow$ ⟶ change a to bs

Step 2

$$a_1(+\infty) - a_1(-\infty)$$

$$\equiv I_1 = \int_{-\infty}^{\infty} dt\, \partial_0 a_1(t)$$

$$\equiv -i \int d^4x\, e^{-ik_1 x} (\partial^2 + m^2) \varphi(x)$$

$$I_1^+ = i \int d^4x\, e^{+ik_1 x} (\partial^2 + m^2) \varphi(x).$$

$$I_{1,f} \equiv b_1(+\infty) - b_1(-\infty)$$

$$\equiv \int_{-\infty}^{\infty} dt\, \partial_0 b_1(t)$$

$$I_{1,f} \equiv b_1(+\infty) - b_1(-\infty)$$

$$\equiv \int_{-\infty}^{\infty} dt\, \partial_0 b_1(t)$$

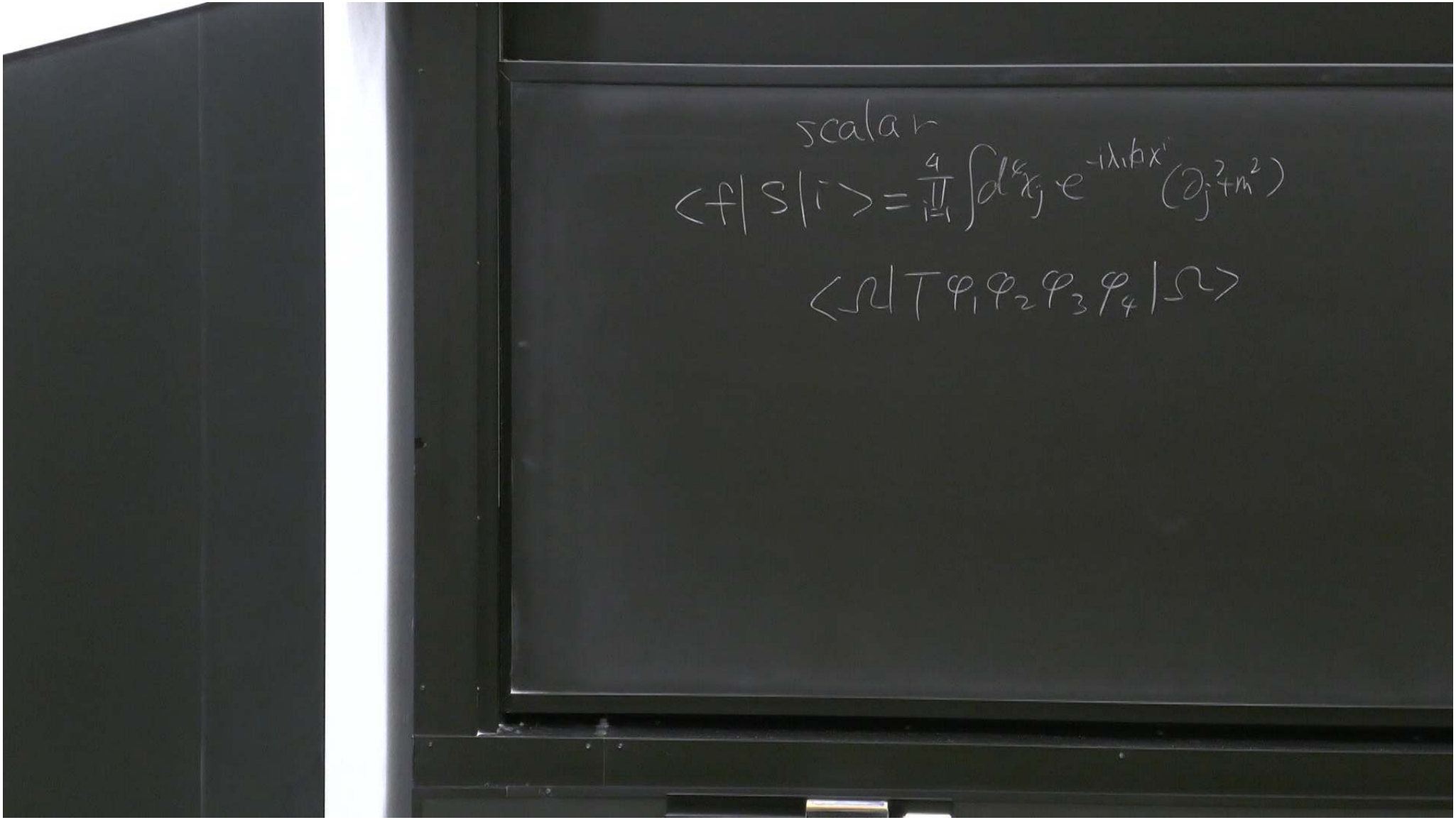
$$= \# \int d^4x\, e^{-ik_1 x}\, \bar{u}_1 (i\not{\partial} - m)\psi(x)$$

$$= \# \int d^4x\, e^{-ik_1 x}\, u_1^\dagger \gamma^0 (i\gamma^\mu \partial_\mu \psi - m\psi)$$

$$I_{1,f} = \# \int d^4x\, e^{+ik_1 x} (\partial_\mu \psi^\dagger \gamma^0 \gamma^\mu \gamma^0 (-i) \gamma^0 u_1 - m\psi^\dagger \gamma^0 u_1)$$

$$= \pm \int d^4x\, e^{+ik_1 x} (\partial_\mu \bar{\psi} \gamma^\mu u_1 (-i) - m\bar{\psi} u_1)$$

$$= \# \int d^4x \; \bar{\psi} (i\overleftarrow{\not{\partial}} + m) u_1\, e^{+ik_1 x}$$


scalar
$\langle f|S|i\rangle = \prod_{j=1}^{4} \int d^4x_j \, e^{-ip_j x^j} (\partial_j^2 + m^2)$
$\langle \Omega | T \varphi_1 \varphi_2 \varphi_3 \varphi_4 | \Omega \rangle$

scalar

$$\langle f|S|i\rangle = \prod_{j=1}^{4} \int d^4x_j \, e^{-ip_j x^j} (\partial_j^2 + m^2)$$

$$\langle \Omega | T\varphi_1\varphi_2\varphi_3\varphi_4 | \Omega \rangle$$

fermion case

$$= \prod_{j=3}^{4} \int d^4x_j \, e^{-i}$$

$$\langle \Omega | T$$

$$\prod_{j=1}^{2} d^4x$$

step 3

$$\langle \Omega | T\varphi_1\varphi_2\varphi_3\varphi_4 | \Omega \rangle = \frac{\langle 0 | T\varphi_1\varphi_2\varphi_3\varphi_4 e^{i\mathcal{L}} | 0 \rangle}{\langle 0 | e^{i\mathcal{L}} | 0 \rangle}$$

$e^{-ik_1 \cdot x}(\partial_j^2 + m^2)$

$\varphi_3 \varphi_4 |\Omega\rangle$

$$\varphi_3 \varphi_4 |\Omega\rangle = \frac{\langle 0 | T \varphi_1 \varphi_2 \varphi_3 \varphi_4 e^{ikx} | 0 \rangle}{\langle 0 | e^{ikx} | 0 \rangle}$$

fermion case

$$= \prod_{j=3}^{4} \int d^4x_j \, e^{-ik_j x_j} \bar{u}_j (i\not{\partial} - m)$$

$$\langle \Omega | T \psi \psi \bar{\psi} \bar{\psi} | \Omega \rangle$$

$$\prod_{j=1}^{2} d^4x_j \, (i\not{\partial} + m) u_j e^{+i k_i x_i}$$

$$\langle \Omega | I_4 I_3 I_1^+ I_2^+ | \Omega \rangle$$

fermion case

$$= \prod_{j=3}^{4} \int d^4x_j \, e^{-ik_j x_j} \, \bar{u}_j \, (i\not{\partial} - m)$$

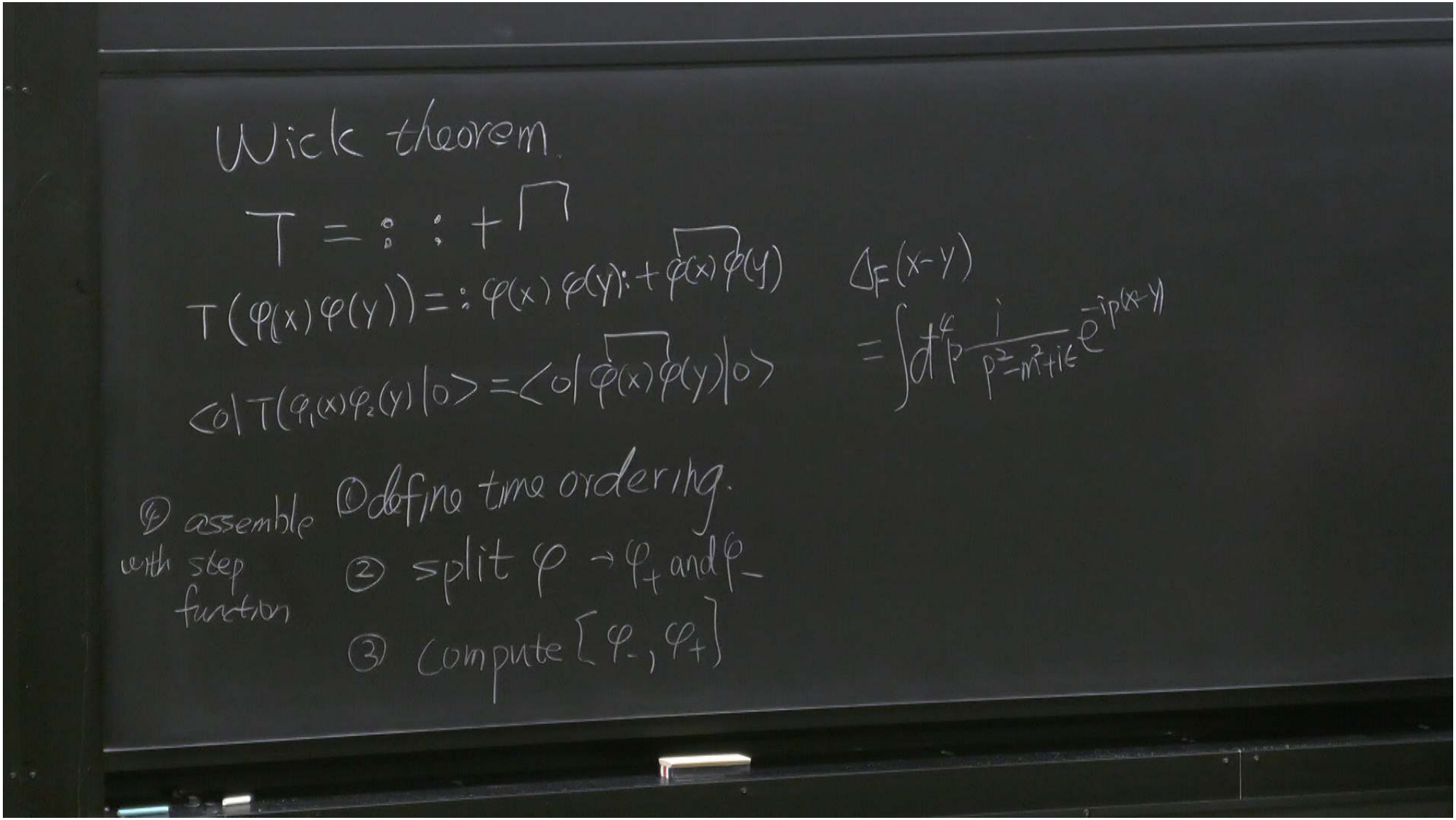
$$\langle \Omega | T \, \psi_4 \psi_3 \bar{\psi} \bar{\psi} | \Omega \rangle$$

$$\prod_{j=1}^{2} d^4x_j \, (i\overleftarrow{\not{\partial}} + m) u_j e^{+i k_j x_j}$$

$$\langle \Omega | T \, I_4 I_3 I_1^\dagger I_2^\dagger | \Omega \rangle$$

$\psi_3 \psi_4 e^{ikx} |0\rangle$

$e^{ikx} |0\rangle$ → no change


Wick theorem
T = : : + ⌐¬
T(φ(x)φ(y)) = : φ(x)φ(y) : + φ(x)φ(y)
⟨0|T(φ1(x)φ2(y)|0⟩ = ⟨0|φ(x)φ(y)|0⟩
Δ_F(x-y)
= ∫ d^4p i/(p^2-m^2+iε) e^{-ip(x-y)}
① define time ordering.
② split φ → φ+ and φ-
③ compute [φ-, φ+]
④ assemble with step function

starting point

$$T(\psi_a(x)\bar{\psi}_b(y)) = :\psi_a(x)\bar{\psi}_b(y): + \psi_a(x)\bar{\psi}_b(y) \leftarrow \text{goal.}$$

step 1 time ordering $T(\psi(x)\bar{\psi}(y)) = \begin{cases} \psi(x)\bar{\psi}(y) & x^0 > y^0 \\ \bar{\psi}(y)\psi(x) & y_0 > x_0 \end{cases}$

suppose $x_0 > y_0$

$$T\psi_a(x)\psi_b(x) = -T\psi_b(y)\psi_a(x) \qquad \psi = \int dp\, u b e^{-ikx} + v c^\dagger e^{+ikx}$$

$$\psi_a(x)\psi_b(y) = -\psi_a(x)\psi_b(y) = 0 \text{ ????}$$

first $x_0 > y_0$.

choose $T(\psi(x)\bar{\psi}(y)) = \psi(x)\bar{\psi}(y)$

$= \psi_+(x)\bar{\psi}_+(y) + \psi_+(x)\bar{\psi}_-(y)$

problem $\longrightarrow + \psi_-(x)\bar{\psi}_+(y) + \psi_-(x)\bar{\psi}_-(y)$

$:cc^\dagger: = -c^\dagger c \quad -\bar{\psi}_+(y)\psi_-(x) + \bar{\psi}_+(y)\psi_-(x)$

$= :\psi(x)\bar{\psi}(y): + \{\psi_-(x), \bar{\psi}_+(y)\}$

$$\psi = \int dV_k \, u \, b_k e^{-ikx} + v \, c_k^\dagger e^{+ikx}$$

$$\psi^\dagger(y) = \int dV_p \, u^\dagger b_p^\dagger e^{+ipx} + v^\dagger c_p e^{-ipx}$$

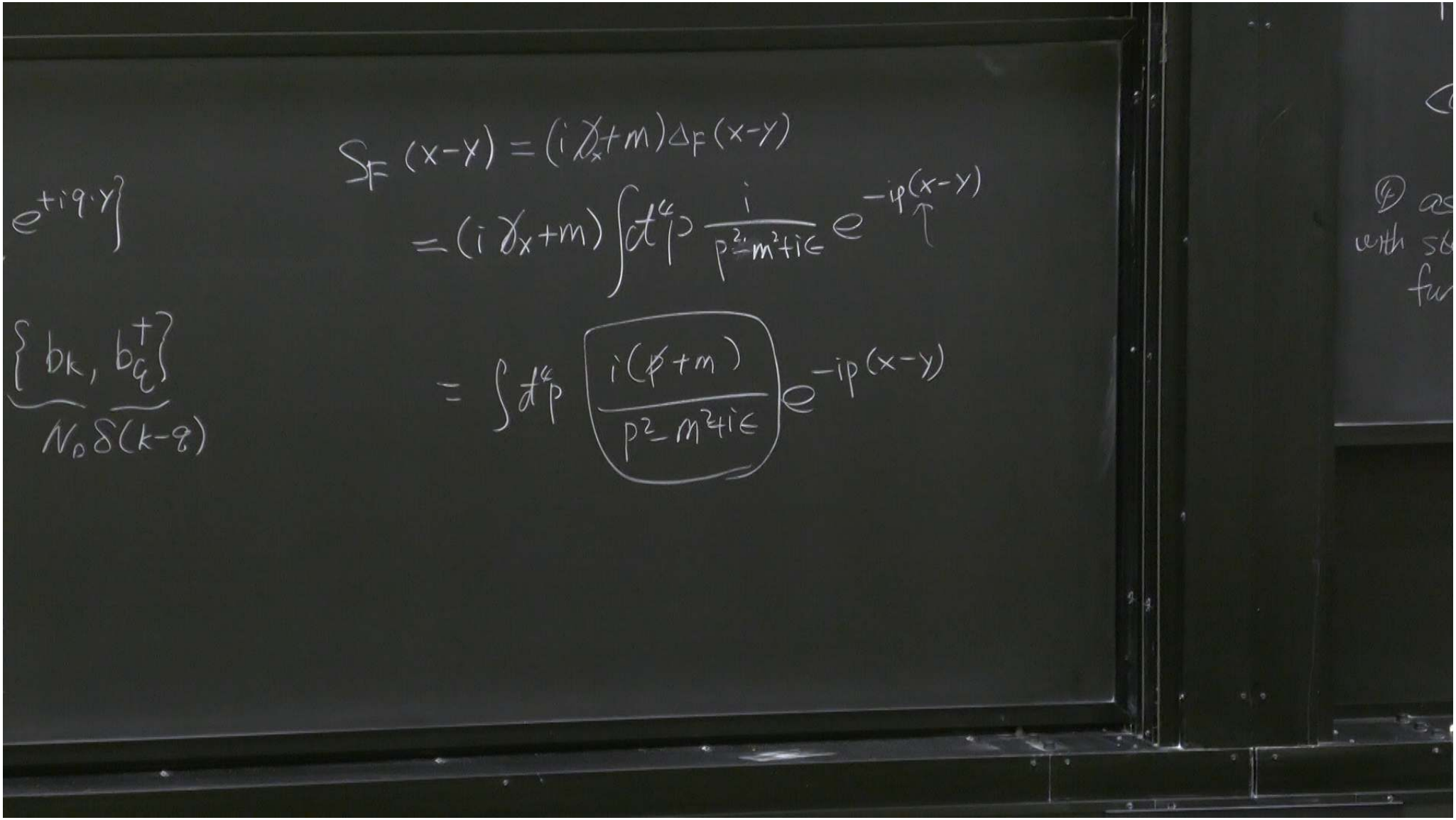
$$\{\psi_-(x), \bar{\psi}_+(y)\} = \Big\{ \int dV_k \, u \, b_k e^{-ikx}, \int dV_q \, \bar{u} \, b_q^\dagger e^{+iq\cdot y} \Big\}$$

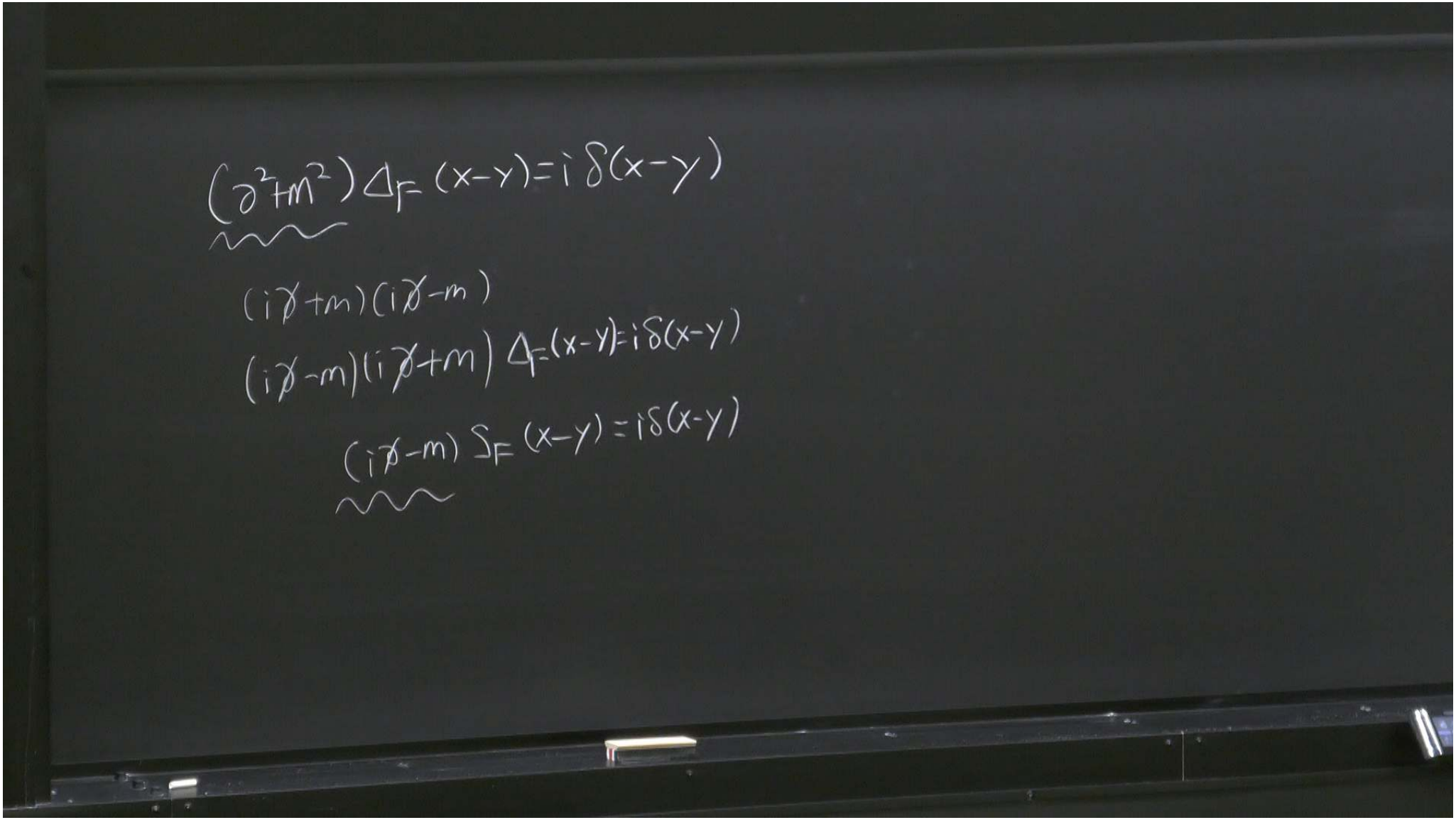
$$= \int dV_k \, u \, e^{-ikx} \int dV_q \, \bar{u} \, e^{+iqy} \underbrace{\{b_k, b_q^\dagger\}}_{N_0 \delta(k-q)}$$

$$= \int dV_k \, u\bar{u} \, e^{-ik(x-y)}$$

$$= \int dV_k \, (\not{k} + m) \, e^{-ik(x-y)}$$

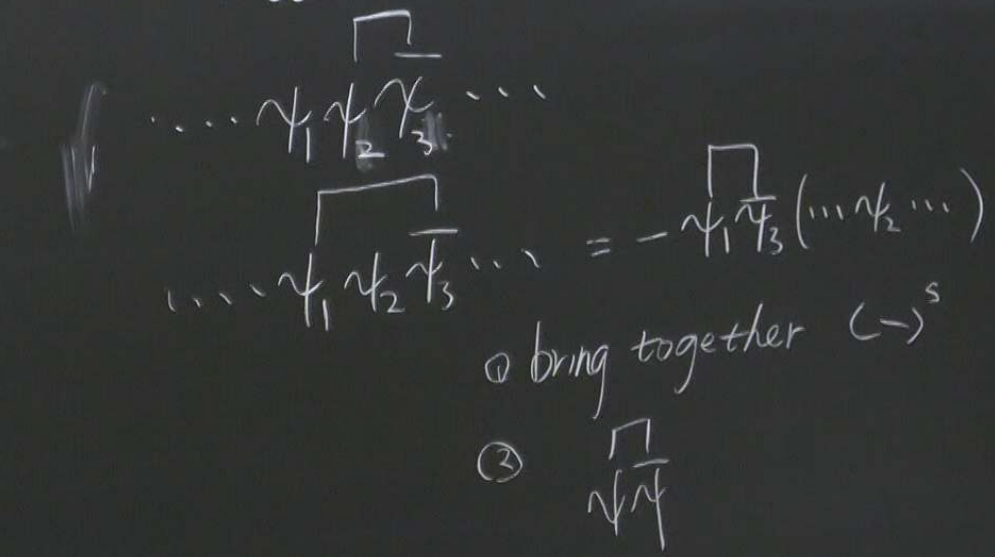
$$= (i\partial_x + m) \int dV_k \, e^{-ik(x-y)}$$


$S_F(x-y) = (i\not{\partial}_x + m)\Delta_F(x-y)$
$= (i\not{\partial}_x + m)\int d^4p \frac{i}{p^2 - m^2 + i\epsilon} e^{-ip(x-y)}$
$e^{+iq\cdot y}$
$\{b_k, b_q^\dagger\}$
$N_0\delta(k-q)$
$= \int d^4p \frac{i(\not{p}+m)}{p^2 - m^2 + i\epsilon} e^{-ip(x-y)}$


$(\partial^2+m^2)\Delta_F(x-y)=i\delta(x-y)$
$(i\not\partial+m)(i\not\partial-m)$
$(i\not\partial-m)(i\not\partial+m)\Delta_F(x-y)=i\delta(x-y)$
$(i\not\partial-m)S_F(x-y)=i\delta(x-y)$

Wick contraction for many fermion field

all contractions



$$\cdots \psi_1 \psi_2 \bar{\psi}_3 \cdots$$

$$\cdots \psi_1 \psi_2 \bar{\psi}_3 \cdots = -\psi_1 \bar{\psi}_3 (\cdots \psi_2 \cdots)$$

① bring together $(-)^s$

② $\psi \bar{\psi}$