

Title: QFT1 Lecture - 102523

Speakers: Gang Xu

Collection: Quantum Field Theory 1 2023/24

Date: October 25, 2023 - 10:45 AM

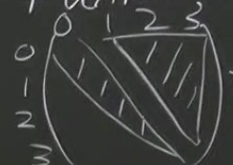
URL: <https://pirsa.org/23100052>

Tutorial Weyl fermion $m=0$

① $m\bar{\psi}_+\psi_+ = 0$

② $m\bar{\psi}_-\psi_- > \bar{\psi}_-\psi_-$

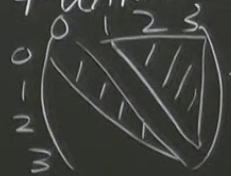
③ $\begin{cases} (i\gamma^\mu \partial_\mu - m)\psi = 0 \\ \{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} \\ (\gamma^\mu)^\dagger = \gamma^0 \gamma^\mu \gamma^0 \end{cases}$

→ if γ^μ is ^{one} #
4 unknowns
 = 10 equations

$$(\gamma^\mu)_{ab} (\gamma^\nu)_{bc} + (\gamma^\nu)_{ab} (\gamma^\mu)_{bc} = 2\eta^{\mu\nu} \mathbb{1}_{ac}$$

$$(i(\gamma^\mu)_{ab} \partial_\mu - m \mathbb{1}_{ab}) \psi_b = 0$$

if γ^μ is ^{one} #
4 unknowns

 = 10 equations

$$g_{bc} + (\gamma^\mu)_{ab} (\gamma^\mu)_{bc} = 2\eta^{\mu\nu} \mathbb{1}_{ac}$$

$$(\partial_\mu - m \mathbb{1}_{ab}) \psi_b = 0$$

$$(\gamma^\mu)_{ab} \quad (\mathcal{J}^{\mu\nu})_{ab}$$

$$D(\Lambda)_{ab} = e^{i\omega_{\mu\nu} (\mathcal{J}^{\mu\nu})_{ab}}$$

$$\psi'_a = D(\Lambda)_{ab} \psi_b$$

$$\psi_a(x,t) = \int dV_p \underbrace{b^s(p)}_{\text{coefficient}} \underbrace{u_a^s(p) e^{+ip \cdot x}}_{\text{solution}} + \underbrace{c^{*s}(p)}_{\text{coefficient}} \underbrace{v_a^s(p) e^{-ip \cdot x}}_{\text{solution}}$$

$$\psi_a(x,t) = \int_{\mathbb{R}^3} \underbrace{b^s(\vec{p})}_{\text{coefficient}} \underbrace{u_a^s(\vec{p}) e^{+i\vec{p}\cdot\vec{x}}}_{\text{solution}} + \underbrace{c^{*s}(\vec{p})}_{\text{coefficient}} \underbrace{v_a^s(\vec{p}) e^{-i\vec{p}\cdot\vec{x}}}_{\text{solution}}$$

$$\text{d.o.f} = \frac{\dim(\text{phase space})}{2} \quad \pi: i\psi^\dagger$$

$$\text{for Dirac} = \frac{\dim(\psi)}{2} = 4 \quad \pi: \psi \text{ and } \psi^\dagger$$

$$\psi = \begin{pmatrix} a_1 + ib_1 \\ a_2 + ib_2 \\ a_3 + ib_3 \\ a_4 + ib_4 \end{pmatrix} \quad \psi^\dagger = \begin{pmatrix} b_1 - ia_1 \\ b_2 - ia_2 \end{pmatrix}$$

Quantization of scalar field

step 1 $\mathcal{L}_G = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2$ $\lambda \phi^4$

step 2 $H = \int d^3x (\pi \dot{\phi} - \mathcal{L}_G)$

step 2.5 $H = \frac{1}{2} \int dV_p E_p (a_p^* a_p + a_p a_p^*)$

of Dirac field

$$\mathcal{L} = \bar{\psi} (i \gamma^\mu \partial_\mu - m) \psi$$

$$H = \int d^3x \psi^\dagger (-i \gamma^0 \gamma^i \partial_i + m \gamma^0) \psi$$

$$(\pi \dot{\psi} - \mathcal{L})$$

d
 $\ln \phi^n$

of Dirac field

$$\mathcal{L} = \bar{\psi} (i \gamma^\mu \partial_\mu - m) \psi$$

$$H = \int d^3x \psi^\dagger (-i \gamma^0 \gamma^i \partial_i + m \gamma^0) \psi$$

$$= \int d^3x (\pi \dot{\psi} - \mathcal{L})$$

$$H = \int dV_p E_p (b^{s*}(p) b^s(p) - c^s(p) c^{s*}(p))$$

Quantization of scalar field

step 1 $\mathcal{L}_{KG} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2$ $\lambda \phi^n$

step 2 $H = \int d^3x (\pi \dot{\phi} - \mathcal{L}_{KG})$

step 2.5 $H = \frac{1}{2} \int dV_p E_p (a_p^* a_p + a_p a_p^*)$

↓ promote to operators

step 3+4 $[x, p] = i \rightarrow [\phi(x), \pi_\phi(y)] = i \delta(x-y)$

↓ $[a_p, a_q^\dagger] = N_0 \delta(p-q)$

of Dirac field

$$\mathcal{L} = \bar{\psi} (i \gamma^\mu \partial_\mu - m) \psi$$

$$H = \int d^3x \psi^\dagger (-i \gamma^0 \gamma^i \partial_i + m \gamma^0) \psi$$
$$= \int d^3x (\pi \dot{\psi} - \mathcal{L})$$

$$H = \int dV_p E_p (b^{s*}(p) b^s(p) - c(p) c^s)$$

Quantization of scalar field

step 1 $\mathcal{L}_{KG} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2$ $\lambda \phi^4$

step 2 $H = \int d^3x (\pi \dot{\phi} - \mathcal{L}_{KG})$

step 2.5 $H = \frac{1}{2} \int dV_p E_p (a_p^* a_p + a_p a_p^*)$

↓ promote to operators

step 3+4 $[x, p] = i \rightarrow [\phi(x), \pi_\phi(y)] = i \delta(x-y)$

↓ $[a_p, a_q^*] = N_0 \delta(p-q)$

of Dirac field

$$\mathcal{L} = \bar{\psi} (i \gamma^\mu \partial_\mu - m) \psi$$

$$H = \int d^3x \psi^\dagger (-i \gamma^0 \gamma^i \partial_i + m \gamma^0) \psi$$

$$= \int d^3x (\pi \dot{\psi} - \mathcal{L})$$

$$H = \int dV_p E_p (b^{s*}(p) b^s(p) - C(p) C^s)$$

normal ordering

$$: a a^\dagger : = a^\dagger a$$

$$: H : = \int dV_p E_p (a_p^\dagger a_p)$$

of scalar field

$$\int d^3x \left(\frac{1}{2} \dot{\phi}^2 - \frac{1}{2} m^2 \phi^2 \right)$$

$$\dot{\phi} - \mathcal{L}(\phi, \dot{\phi})$$

$$\int d^3x (a_p^* a_p + a_p a_p^*)$$

to operators

$$[\phi(x), \pi_\phi(y)] = i \delta(x-y)$$

$$\Downarrow [a_p, a_q^\dagger] = N_0 \delta(p-q)$$

of Dirac field

$$\mathcal{L} = \bar{\psi} (i \gamma^\mu \partial_\mu - m) \psi$$

$$H = \int d^3x \psi^\dagger (-i \gamma^0 \partial_i + m \gamma^0) \psi$$
$$= \int d^3x (\pi \dot{\psi} - \mathcal{L})$$

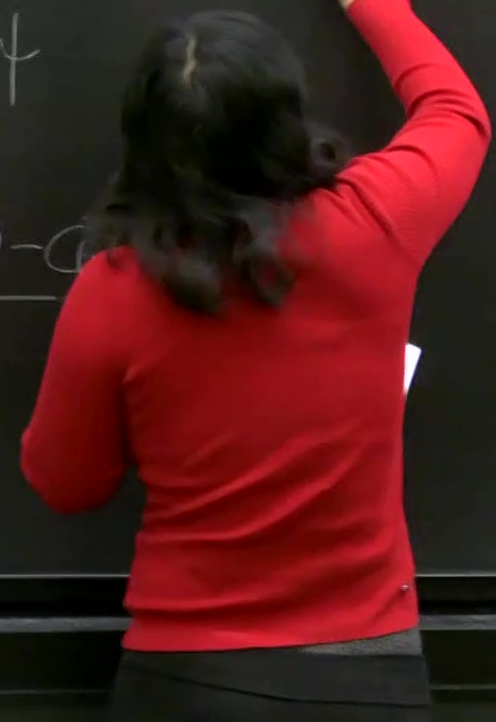
$$H = \int dV_p E_p (b^{s*}(p) b^s(p) - c^{s*}(p) c^s(p))$$

normal ordering

$$: a a^\dagger : = a^\dagger a$$

$$: H : = \int N_p E_p (a_p^\dagger a_p)$$

les



eld

$$\partial_\mu - m) \psi$$

$$(-i \gamma^0 \partial_t + m \gamma^0) \psi$$

$$i - \mathcal{L}$$

$$= (b^{s*}(\mathbf{p}) b^s(\mathbf{p}) - c^s(\mathbf{p}) c^{s*}(\mathbf{p}))$$

ring

a

$$(a_p^\dagger a_p)$$

lesson from scalar field
assume commutator

$$[c_{\vec{p}}, c_{\vec{p}}^\dagger] = N_D \delta(\vec{p} - \vec{p})$$

$$= N_D \delta(0)$$

$$b^\dagger b - c c^\dagger$$

commutator

$$= b^\dagger b - (c c^\dagger - c^\dagger c) - c^\dagger c$$

$$= b^\dagger b - c^\dagger c$$

$$= N_b - N_c$$

$$H = \int \frac{d^3 p}{(2\pi)^3} E_{\vec{p}} (N_b - N_c) \text{ problematic}$$

$$\{C_{\vec{p}}, C_{\vec{q}}^{\dagger}\} = N_0 \delta^3(\vec{p} - \vec{q})$$

commutator

$$\begin{aligned} & b^{\dagger}b - c c^{\dagger} \\ &= b^{\dagger}b - (c c^{\dagger} + c^{\dagger}c) + c^{\dagger}c \\ &= b^{\dagger}b + c^{\dagger}c \\ &= N_b + N_c \end{aligned}$$

normal ordering
◦ $c c^{\dagger} = -c^{\dagger}c$

$$\{\gamma_a, \gamma_b^{\dagger}\} = i \delta^3(x-y) \delta_{ab}$$

$$\{\gamma_a, \gamma_b\} = \{\gamma_a^{\dagger}, \gamma_b^{\dagger}\} = 0$$

ng

$(-\gamma) \delta_{ab}$

$\begin{bmatrix} + \\ b \end{bmatrix} = 0$

states

vacuum:

$$\left\{ \begin{array}{l} \langle 0|0\rangle = 1 \\ H|0\rangle = 0 \\ b_p^2|0\rangle = c_p^2|0\rangle = 0 \end{array} \right.$$

1-particle

$$\begin{aligned} |p,s\rangle &= b_p^{st} |0\rangle \\ |\bar{p},s\rangle &= c_p^{st} |0\rangle \end{aligned}$$

$$\langle p,s | q,r \rangle = N_0 \delta^{st} (\bar{p}-q) \delta_{rs}$$

$$\frac{|p_1, s_1, p_2, s_2\rangle}{\begin{matrix} +s_1 & +s_2 \\ b_{p_1} & b_{p_2} \end{matrix}} = \overset{\text{fermions}}{-} \frac{|p_2, s_2, p_1, s_1\rangle}{\begin{matrix} +s_2 & +s_1 \\ b_{p_2} & b_{p_1} \end{matrix}}$$

$$|p,s; p,s\rangle = -|p,s, p,s\rangle \Rightarrow |p,s, p,s\rangle = 0$$

$$\underbrace{\{b_p^s, b_q^r\}} = 0 = \underbrace{\{b_q^{st}, b_q^{rt}\}} = 0$$

anti-particle

$$\mathcal{L} = \bar{\psi} (i\gamma^\mu \partial_\mu - m) \psi$$

charge \rightarrow current

\rightarrow Noether theorem

\rightarrow symmetry

$$\psi = e^{-ie\alpha} \psi$$

\mathcal{L} does not change

$$J^\mu = \frac{\partial \mathcal{L}}{\partial (e\psi)} \frac{\delta \psi}{\delta x}$$

$$= i\bar{\psi} \gamma^\mu (-ie) \psi$$

$$= e\bar{\psi} \gamma^\mu \psi$$

charge

$$Q_\psi = \int d^3x J^0 = \int d^3x e\bar{\psi} \gamma^0 \psi \\ = e \int d^3x \psi^\dagger \psi$$

$$v_p \sim p(u_p u_p)$$

$$d^3x J^0 = \int d^3x e^{-i\vec{p}\cdot\vec{x}} \psi^\dagger \psi$$

$$= e \int d^3x \psi^\dagger \psi$$

$$= \int d^3x \int dV_p \frac{u^+ b^+ e^{-i\vec{p}\cdot\vec{x}} + v^+ c^+ e^{+i\vec{p}\cdot\vec{x}}}{\textcircled{1}} + \int dV_q \frac{u^+ b^+ e^{+i\vec{q}\cdot\vec{x}} + v^+ c^+ e^{-i\vec{q}\cdot\vec{x}}}{\textcircled{2}}$$

1st integral of d^3x term

$$\int d^3x \int dV_p dV_q u_p^+ b_p^+ u_q b_q e^{-i\vec{p}\cdot\vec{x}} e^{+i\vec{q}\cdot\vec{x}}$$

$$= \int dV_p u_p^+ u_p b_p^+ b_p + \text{c.c.}$$

$$:Q: = \int dV_p b_p^+ b_p - c_p^+ c_p = e \int dV_p (W_b - N_c)$$