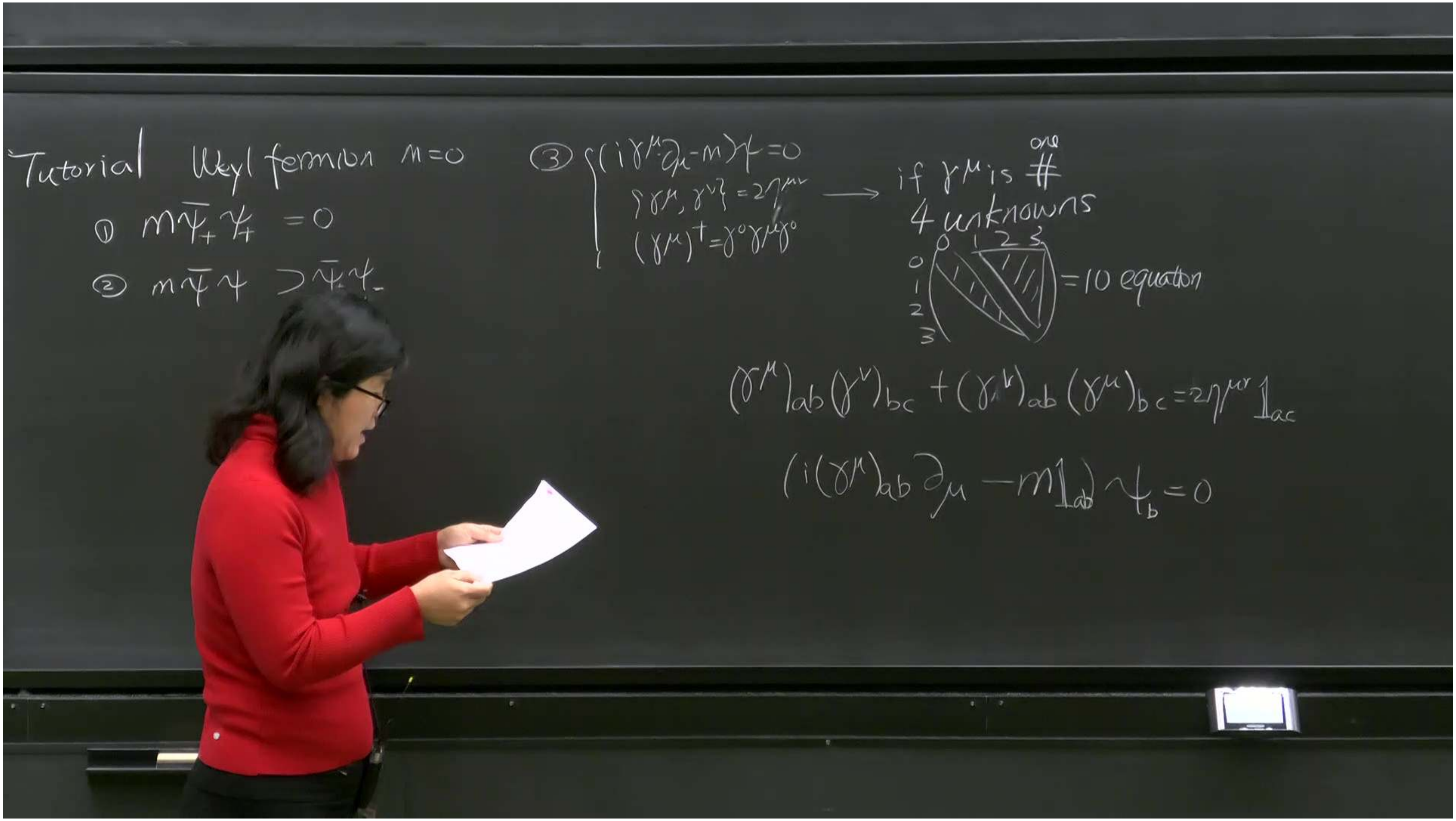
Title: QFT1 Lecture - 102523

Speakers: Gang Xu

Collection: Quantum Field Theory 1 2023/24

Date: October 25, 2023 - 10:45 AM

URL: https://pirsa.org/23100052


Tutorial Weyl fermion $m=0$
① $m\bar{\Psi}_+\psi_+ = 0$
② $m\bar{\Psi}\psi \supset \bar{\Psi}_+\psi_-$
③ $(i\gamma^\mu\partial_\mu - m)\psi = 0$
$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$
$(\gamma^\mu)^\dagger = \gamma^0\gamma^\mu\gamma^0$
$\longrightarrow$ if γ^μ is one #
4 unknowns
0 1 2 3
0 1 2 3
= 10 equation
$(\gamma^\mu)_{ab}(\gamma^\nu)_{bc} + (\gamma^\nu)_{ab}(\gamma^\mu)_{bc} = 2\eta^{\mu\nu}\mathbb{1}_{ac}$
$(i(\gamma^\mu)_{ab}\partial_\mu - m\mathbb{1}_{ab})\psi_b = 0$

if γ^μ is $\overset{\text{one}}{\#}$

4 unknowns

$$\begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & & & & \\ 1 & & & & \\ 2 & & & & \\ 3 & & & & \end{array} = 10 \text{ equation}$$

$$\ldots_{bc} + (\gamma^\nu)_{ab}(\gamma^\mu)_{bc} = 2\eta^{\mu\nu}\mathbb{1}_{ac}$$

$$\ldots_{b}\,\partial_\mu - m\mathbb{1}_{ab})\,\psi_b = 0$$

$$(\gamma^\mu)_{ab} \qquad (J^{\mu\nu})_{ab}$$

$$D(\Lambda)_{ab} = e^{i\omega_{\mu\nu}(J^{\mu\nu})_{ab}}$$

$$\psi_a' = D(\Lambda)_{ab}\,\psi_b$$

$$\psi_a(x,t) = \int dV_p \; \underbrace{b^s(\bar{p})}_{\text{coefficient}} \; \underbrace{u_a^s(\bar{p})\, e^{+i\bar{p}\cdot\bar{x}}}_{\text{solution}} \; + \; \underbrace{c^{*s}(\bar{p})}_{\text{coefficient}} \, \underbrace{v_a^s(\bar{p})\, e^{-i\bar{p}\cdot\bar{x}}}_{\text{solution}}$$

$$\psi_a(x,t) = \sum_s \int d\tilde{V}_p \; \underbrace{b^s(\vec p)}_{\text{coefficient}} \; \underbrace{u^s_a(\vec p)\, e^{+i\vec p\cdot\vec x}}_{\text{solution}} \;+\; \underbrace{c^{*s}(\vec p)}_{\text{coefficient}} \; \underbrace{v^s_a(\vec p)\, e^{-i\vec p\cdot\vec x}}_{\text{solution}}$$

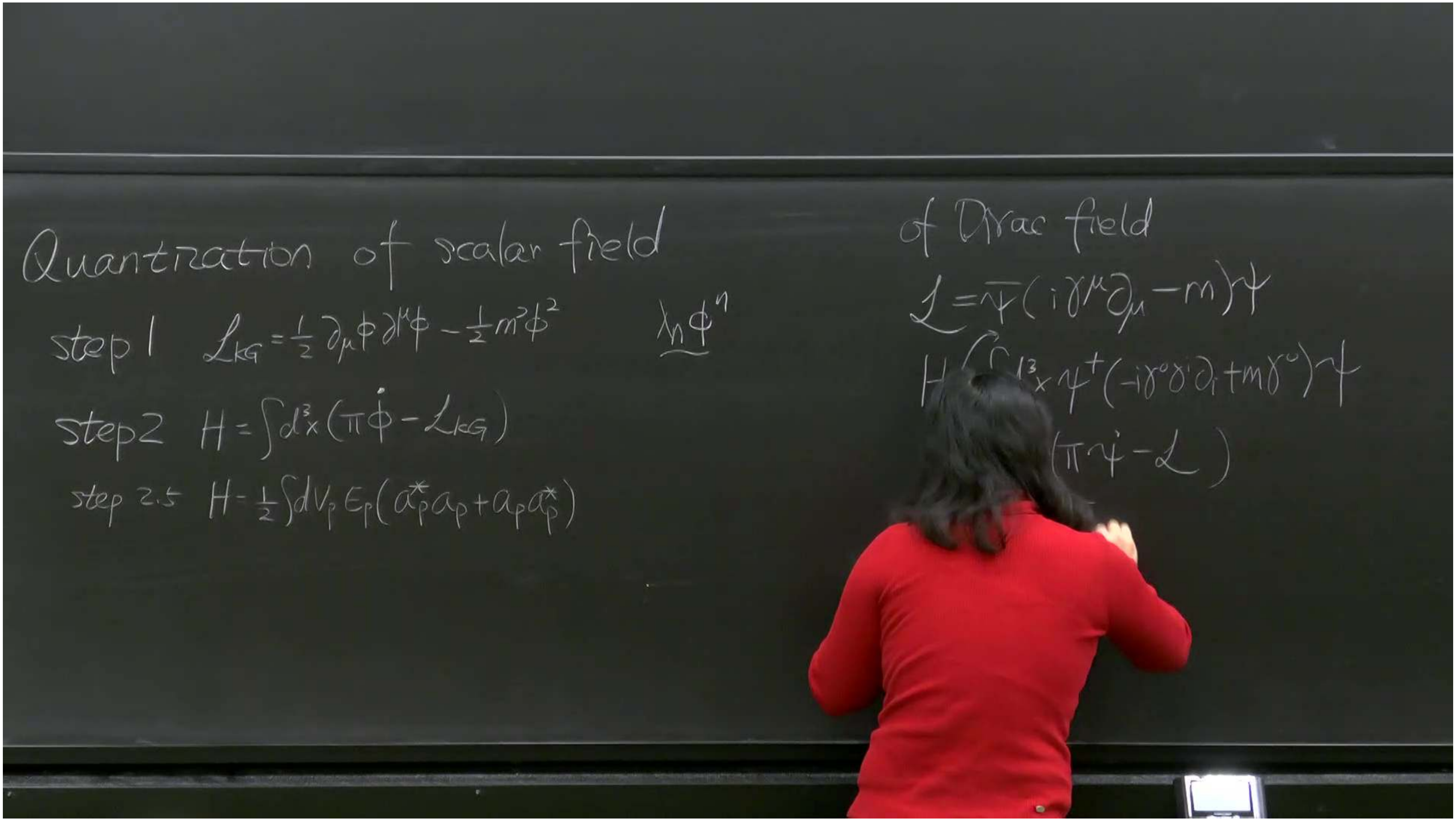
$$\text{d.o.f} = \frac{\dim(\text{phase space})}{2}$$

$$\pi : i\psi^\dagger$$

$$\psi = \begin{pmatrix} a_1 + i b_1 \\ a_2 + i b_2 \\ a_3 + i b_3 \\ a_4 + i b_4 \end{pmatrix} \qquad \psi^\dagger = \begin{pmatrix} b_1 - i a_1 \\ b_2 - i a_2 \\ \\ \end{pmatrix}$$

$$\underline{\text{for Dirac}} = \frac{\dim(\psi)}{2} = 4$$

$$\dim(\psi \text{ and } \overset{\pi}{\psi^\dagger})$$


Quantization of scalar field
step 1 $\mathcal{L}_{KG} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2$
$\lambda_n\phi^n$
step2 $H = \int d^3x\,(\pi\dot{\phi} - \mathcal{L}_{KG})$
step 2.5 $H = \frac{1}{2}\int dV_p\, E_p(a_p^* a_p + a_p a_p^*)$
of Dirac field
$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi$
$\psi^\dagger(-i\gamma^0\gamma^i\partial_i + m\gamma^0)\psi$
$(\pi\dot{\psi} - \mathcal{L})$

d

$\lambda_n \phi^n$

of Dirac field

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi$$

$$H = \int d^3x\, \psi^\dagger(-i\gamma^0\gamma^i\partial_i + m\gamma^0)\psi$$

$$= \int d^3x(\pi\dot{\psi} - \mathcal{L})$$

$$H = \int dV_{\vec{p}}\, E_{\vec{p}}\left(b^{s*}(p)\, b^s(p) - c^s(p)\, c^{s*}(p)\right)$$

Quantization of scalar field

step 1 $\mathcal{L}_{KG} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2$ $\quad \lambda_n\phi^n$

step 2 $H = \int d^3x\,(\pi\dot{\phi} - \mathcal{L}_{KG})$

step 2.5 $H = \frac{1}{2}\int dV_p\, E_p(a_p^* a_p + a_p a_p^*)$

$\downarrow$ promote to operators

step 3+4 $[x, p] = i \rightarrow [\phi(x), \pi_\phi(y)] = i\delta(x-y)$

$\Downarrow$ $[a_p, a_q^\dagger] = N_D\,\delta(p-q)$

of Dirac field

$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi$

$H = \int d^3x\,\psi^\dagger(-i\gamma^0\gamma^i\partial_i + m\gamma^0)\psi$

$= \int d^3x\,(\pi\dot{\psi} - \mathcal{L})$

$H = \int dV_{\vec{p}}\, E_{\vec{p}}\,(b^{s*}(p)\,b^s(p) - c^s(p)\,c^s$

Quantization of scalar field

step 1 $\mathcal{L}_{KG} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2$ $\quad \underline{\lambda_n \phi^n}$

step 2 $H = \int d^3x\,(\pi\dot{\phi} - \mathcal{L}_{KG})$

step 2.5 $H = \frac{1}{2}\int dV_p\, \epsilon_p (a_p^* a_p + a_p a_p^*)$

$\downarrow$ promote to operators

step 3+4 $[x, p] = i \rightarrow \underline{[\phi(x), \pi_\phi(y)] = i\delta(x-y)}$

$\Downarrow$ $[a_p, a_q^\dagger] = N_p\,\delta(p-q)$

of Dirac field

$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi$

$H = \int d^3x\, \psi^\dagger(-i\gamma^0\gamma^i\partial_i + m\gamma^0)\psi$

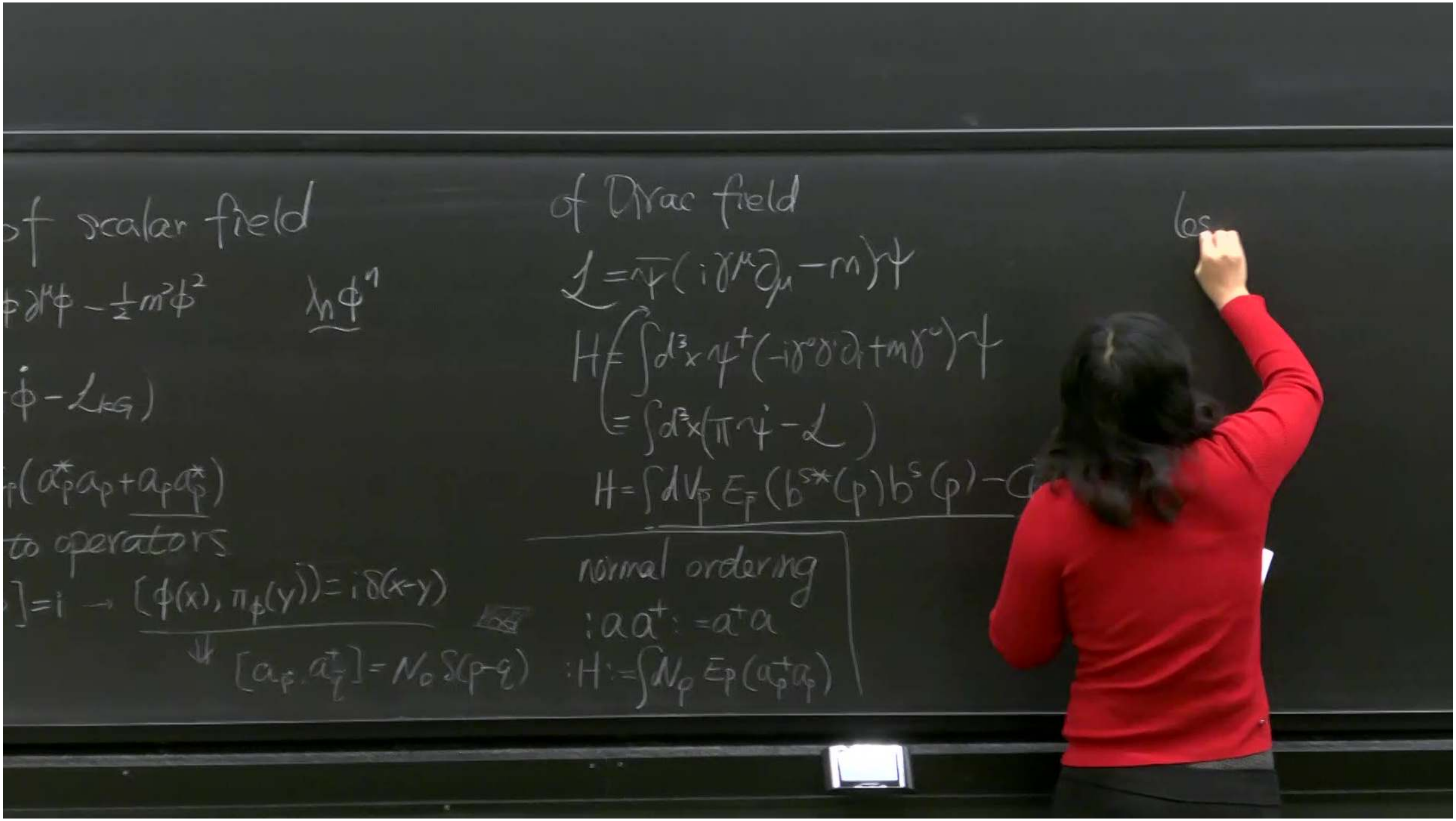
$= \int d^3x\,(\pi\dot{\psi} - \mathcal{L})$

$\underline{H = \int dV_p\, E_p\,(b^{s*}(p)\,b^s(p) - c^s(p)c^{s}}$

normal ordering

$:aa^\dagger: = a^\dagger a$

$:H: = \int dV_p\, E_p\,(a_p^\dagger a_p)$


of scalar field
$\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2$
$\lambda_n\phi^n$
$\dot\phi - \mathcal{L}_{KG})$
$(a_p^* a_p + a_p a_p^*)$
to operators
$]=i \rightarrow [\phi(x), \pi_\phi(y)] = i\delta(x-y)$
$[a_p, a_q^\dagger] = N_0\,\delta(p-q)$
of Dirac field
$\mathcal{L} = \bar\psi(i\gamma^\mu\partial_\mu - m)\psi$
$H = \int d^3x\, \psi^\dagger(-i\gamma^0\gamma^i\partial_i + m\gamma^0)\psi$
$= \int d^3x(\pi\dot\psi - \mathcal{L})$
$H = \int dV_p\, E_p\,(b^{s*}(p)\,b^s(p) - c$
normal ordering
$:aa^\dagger: = a^\dagger a$
$:H: = \int dV_p\, E_p\,(a_p^\dagger a_p)$
los

eld

$\partial_\mu - m)\psi$

$(-i\gamma^0\gamma^i\partial_i + m\gamma^0)\psi$

$\dot\psi - \mathcal{L})$

$(b^{s*}(p)b^s(p) - c^s(p)c^{s*}(p))$

ering

a

$(a_p^\dagger a_p)$

lesson from scalar field

assume commutator.

$$b^\dagger b - cc^\dagger$$

$\delta(0) \to \infty$

commutator $= b^\dagger b - (\overbrace{cc^\dagger - c^\dagger c}^{[c,\,c^\dagger]}) - c^\dagger c$

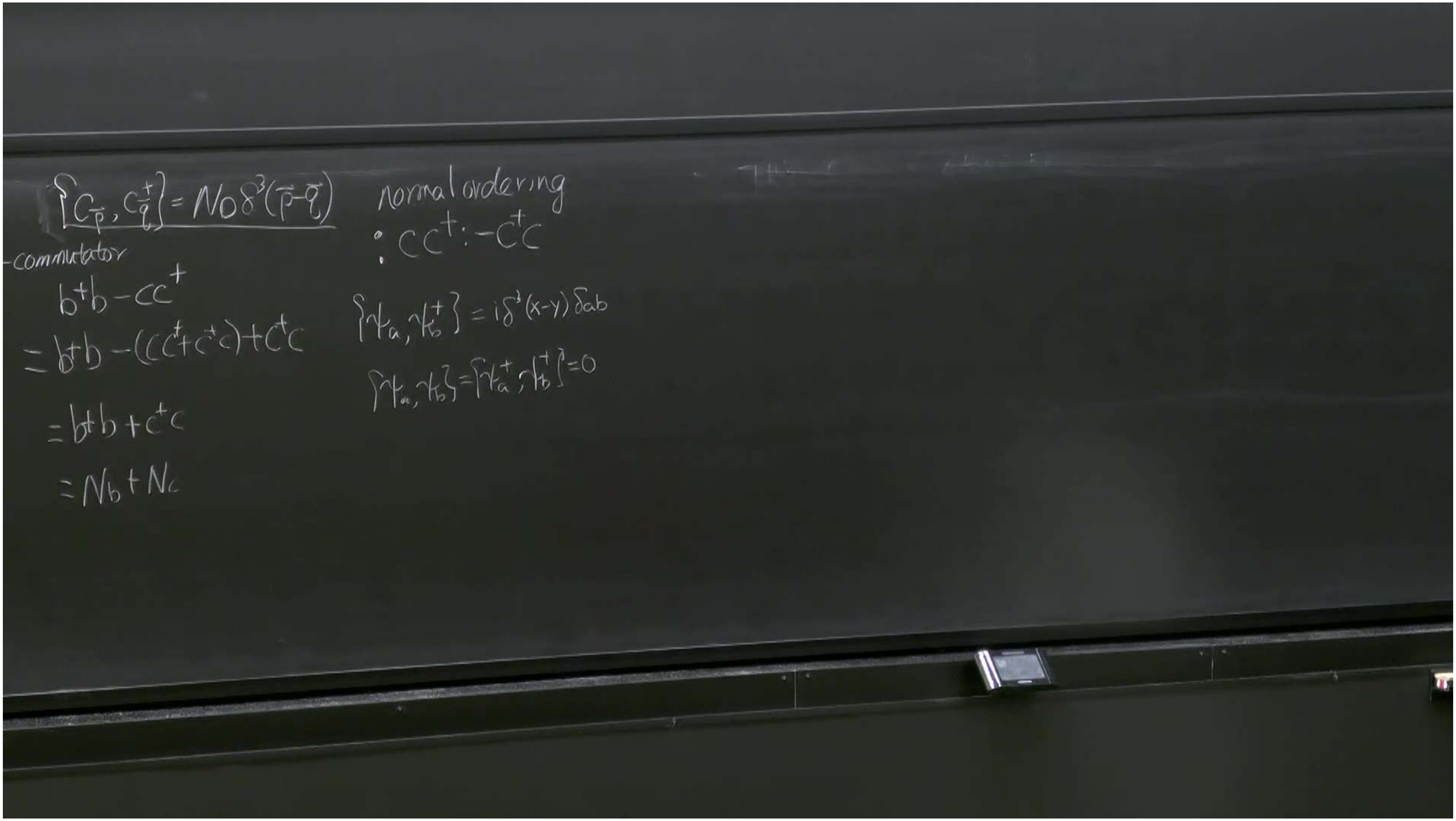
$= b^\dagger b - c^\dagger c$

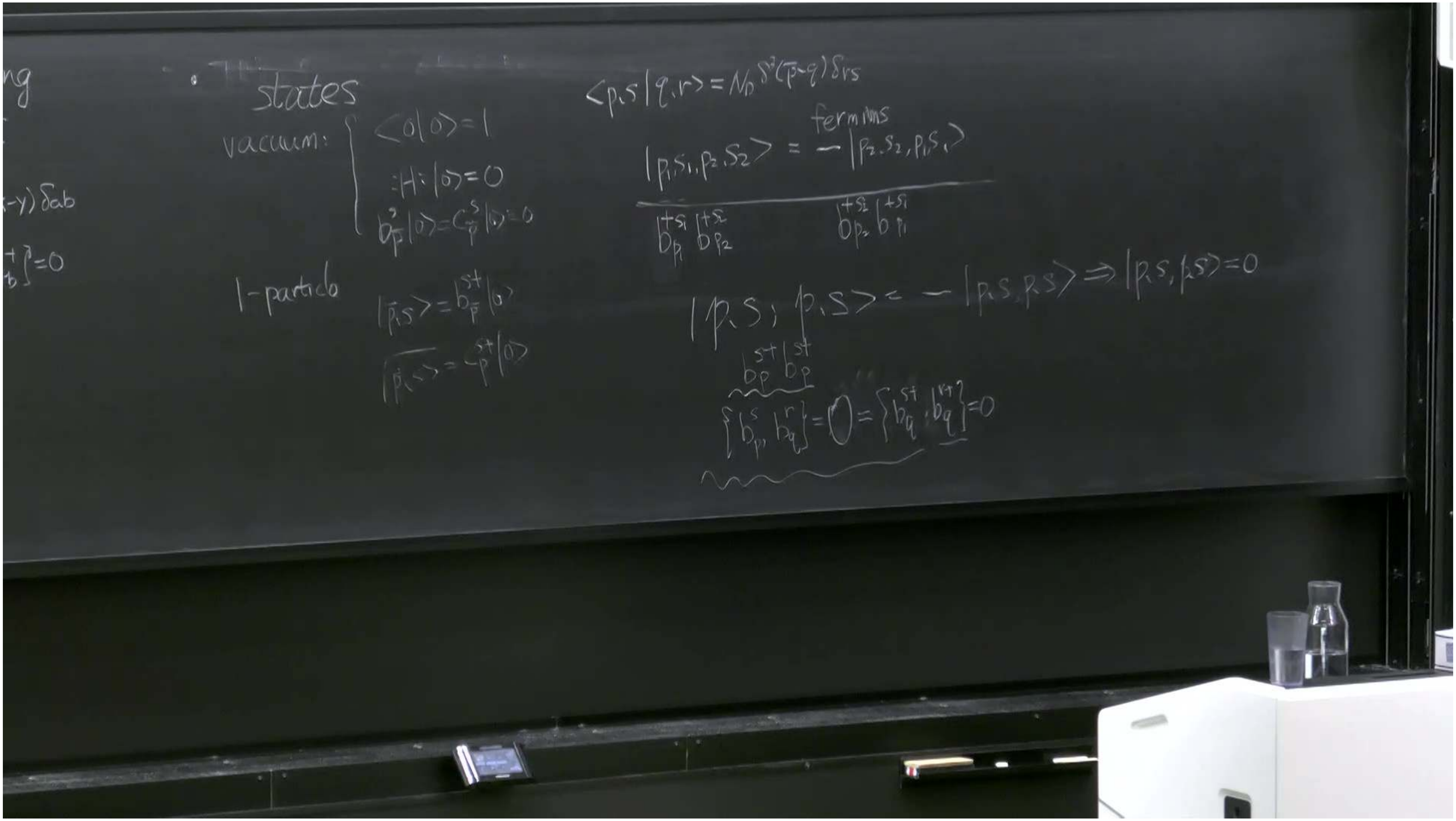
$= N_b - N_c$

$H = \sum_s \int d\!\!\!/ \, \vec{p}\, E_{\vec p}\,(N_b - N_c)$ problematic

$[c_{\vec p}, c^\dagger_{\vec p}] = N_D\,\delta(\vec p - \vec p)$

$= N_D\,\delta(0)$


$\{C_{\vec p}, C^\dagger_{\vec q}\} = N_D \delta^3(\vec p - \vec q)$
normal ordering
$:CC^\dagger: = -C^\dagger C$
-commutator
$b^\dagger b - cc^\dagger$
$= b^\dagger b - (cc^\dagger + c^\dagger c) + c^\dagger c$
$= b^\dagger b + c^\dagger c$
$= N_b + N_c$
$\{\psi_a, \psi_b^\dagger\} = i\delta^3(x-y)\delta_{ab}$
$\{\psi_a, \psi_b\} = \{\psi_a^\dagger, \psi_b^\dagger\} = 0$


states
vacuum:
$\langle 0|0\rangle = 1$
$:H:|0\rangle = 0$
1-particle
$\langle p,s|q,r\rangle = N_p\,\delta^3(\vec{p}-\vec{q})\,\delta_{rs}$
fermions
$|p_1 s_1, p_2 s_2\rangle = -|p_2 s_2, p_1 s_1\rangle$
$|p,s;p,s\rangle = -|p,s,p,s\rangle \Rightarrow |p,s,p,s\rangle = 0$

anti-particle

$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi$

charge → current

→ Noether theorem

→ symmetry

$\psi = e^{-ie\alpha}\psi$

$\mathcal{L}$ does not change

$J^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu\psi)}\frac{\delta\psi}{\delta\alpha}$

$= i\bar{\psi}\gamma^\mu(-ie)\psi$

$= e\bar{\psi}\gamma^\mu\psi$

charge

$Q_\psi = \int d^3x\, J^0 = \int d^3x\, e\bar{\psi}\gamma^0\psi$

$= e\int d^3x\, \psi^\dagger\psi$

$$\int d^3x\, J^0 = \int d^3x\, e\bar{\psi}\gamma^0\psi$$
$$= e\int d^3x\, \psi^\dagger\psi$$

$$\int d^3x\, \psi^\dagger\psi$$
$$= \int d^3x \quad \int dV_p\, \underbrace{u^\dagger b^\dagger e^{-i\vec{p}\cdot\vec{x}}}_{1} + \underbrace{v^\dagger c\, e^{+i\vec{p}\cdot\vec{x}}}_{2}$$
$$\int dV_q\, \underbrace{u\, b\, e^{+i\vec{q}\cdot\vec{x}}}_{1} + \underbrace{v\, c^\dagger e^{-i\vec{q}\cdot\vec{x}}}_{2}$$

1st integral of d^3x
term

$$= \int d^3x \int dV_p\, dV_q\, u_p^\dagger b_p^\dagger u_q b_q\, e^{-i\vec{p}\cdot\vec{x}}\, e^{+i\vec{q}\cdot\vec{x}}$$

$$= \int dV_p\, \cancel{u_p^\dagger u_p}\, \underline{b_p^\dagger b_p} + c\, c^\dagger$$

$$:Q: = e\int dV_p\, b_p^\dagger b_p - c_p^\dagger c_p = e\int dV_p (N_b - N_c)$$