Title: QFT1 Lecture - 102023

Speakers: Gang Xu

Collection: Quantum Field Theory 1 2023/24

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Build $|\frac{1}{2}$ fields

$$\chi_\ell^-(x) = \sum_s \int dV_p \, u_\ell^s(x,p)\, a_p^s$$

$$\sum_{s'} u_\ell^{s'}(\Lambda x$$

$\Uparrow$

dream

transform

Build $1\frac{1}{2}$ fields

$$\chi_\ell^-(x) = \sum_s \int dV_p \, u_\ell^s(x,p) \, a_p^s$$

$\downarrow$ dream

$\downarrow$ transform

- find equation for u

$$\sum_{s'} \underbrace{u_\ell^{s'}(\Lambda x + b, \Lambda p)}_{\Uparrow} D_{s's}^{\,j}(W($$

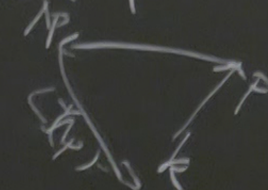
$W(\Lambda,$

equation for u

$$\sum_{s'} \underbrace{u_{\ell}^{s'i}(\Lambda x+b,\ \Lambda p)}_{\Uparrow} D_{s's}^{\ i}(W(\Lambda, p)) = \sum_{\ell} \underbrace{D_{\ell'\ell}(\Lambda)}_{\text{field transform}}\ \underbrace{u_{\ell}^{s}(x, p)}_{\Uparrow}\, e^{-ib\cdot\Lambda p}$$

a transform

$$W(\Lambda, p) = L^{-1}(\Lambda p)\,\Lambda\, L(p)$$



$\underbrace{u_\ell^s(x,p)}_{\Uparrow} e^{-ib\cdot\Lambda p}$

orm

p)

steps translation, boost, rotation

step 1 translation $T(\mathbb{1}, b)$

$\underbrace{u_\ell^s(x,p)}\, e^{-ib\cdot\Lambda p}$

...form

...p)

steps translation, boost, rotation

step 1 translation $T(\mathbb{1}, b)$

$$W(\mathbb{1}, p) = L^{-1}(\mathbb{1} p)\,\mathbb{1}\,L(p) = \mathbb{1}$$

$k \Longleftarrow p$

$$u_\ell^s(x+b, p) = u_\ell^s(x, \vec{p})\, e^{-ib\cdot p}$$

$\Downarrow \ \Downarrow x \qquad \Downarrow 0$

$$u_\ell^s(x, p) = u_\ell^s(\vec{p})\, e^{-ix\cdot p}$$

$\ldots e(\Lambda) u_\ell^s(x,p) e^{-ib\cdot\Lambda p}$

$\ldots$d transform

$\ldots$sform

$\ldots) \Lambda L(p)$

steps translation, boost, rotation.

step 1 translation $T(\mathbb{1}, b)$

$$W(\mathbb{1}\cdot p) = L^{-1}(\mathbb{1}\cdot p)\,\mathbb{1}\,L(p) = \mathbb{1}$$

$p^2 = m^2$

$k \longleftrightarrow p$

$$u_\ell^s(x+b, \vec{p}) = u_\ell^s(x, \vec{p})\, e^{-ib\cdot p}$$

$$u_\ell^s(x, p) = u_\ell^s(\vec{p})\, e^{-ix\cdot p}$$

quation for u

$$\sum_{s'} u_\ell^{s'}(\Lambda x+b, \Lambda p)\, D_{s's}{}^{j}(W(\Lambda, p)) = \sum_{\ell'} D_{\ell'\ell}(\Lambda)\, u_\ell^s(x, p)\, e^{-ib\cdot\Lambda p}$$

field transform

$u_\ell^{s'}(\Lambda p)\, e^{i(\Lambda x+b)\cdot\Lambda p}$

a transform

$u_\ell^s(p)\, e^{-ix\cdot p}$

$$W(\Lambda, p) = L^{-1}(\Lambda p)\,\Lambda\, L(p)$$

$$\Lambda p \leftarrow p,\quad k \rightarrow p,\quad k \xrightarrow{L(\Lambda p)} \Lambda p$$

steps t

step 1 t

$W(\mathbb{1}, p$

$u_\ell^s(x+$

u_ℓ

$$\sum_{s'} u_\ell^{s'}(\Lambda p)\, D^j_{s's}(W(\Lambda,p)) = \sum_{\ell} D_{\ell\bar\ell}(\Lambda)\, u_{\bar\ell}^s(p) \quad (*)$$

$u_\ell^s(p)$ to $u_\ell^s(\overset{k}{0})$

$\Lambda = L(p)$ starting (RHS) rest frame

$k \to \Lambda k = p$

$W(\Lambda,p) = L^{-1}(\Lambda p)\Lambda L(p)$

$$W(\Lambda,p) = W(L(p),k) = L^{-1}(L(p)k)L(p)L(k)$$

$$\sum_{s'} u_\ell^{s'}(\Lambda p)\, D^{j}_{s's}(W(\Lambda,p)) = \sum_{\bar\ell} D_{\ell\bar\ell}(\Lambda)\, u_{\bar\ell}^{s}(p) \quad (*) \quad \Rightarrow \quad u^{s}_{\ell'}(\bar p) = \sum_{\ell}$$

$u_\ell^s(p)$ to $u_\ell^s(\overset{k}{0})$

$\Lambda : L(p)$ starting (RHS) rest frame

$k \to \Lambda k = p$

$$W(\Lambda,p) = L^{-1}(\Lambda p)\,\Lambda\, L(p)$$

$$W(\Lambda,p) = W(L(p),k) = L^{-1}(\underbrace{L(p)k}_{p})\,L(p)\,\underset{?}{L(k)} = \mathbb{1}$$

$$\sum_{s'} u_\ell^{s'}(\Lambda p)\, D^{j}_{s's}(W(\Lambda, p)) = \sum_{\ell'} D_{\ell\ell'}(\Lambda)\, u_{\ell'}^{s}(p) \quad (*)$$

$\Rightarrow \quad u_{\ell'}^{s}(\bar{p})$

step2 boost

$u_\ell^s(p)$ to $u_\ell^s\begin{pmatrix} k \\ 0 \end{pmatrix}$

ste

$\Lambda : L(p)$ starting (RHS) rest frame

$k \to \Lambda k = p$

$$W(\Lambda, p) = L^{-1}(\Lambda p)\, \Lambda\, L(p)$$

$$W(\Lambda, p) = W(L(p), k) = L^{-1}(\underbrace{L(p)k}_{p})\, L(p)\, L(k) \overset{?}{=} \mathbb{1}$$

$$u^{s}_{\ell'}(\bar{p}) = \underbrace{\sum_{\ell} D_{\ell'\ell}(L(\varphi))}_{\text{my choice}} u^{s}_{\ell}(0)$$

step 3 rotation

$$W(\Lambda, p) = L^{-1}(\Lambda p)\,\Lambda\,L(\varphi) = R$$

$$\sum_{s} u^{s'}_{\ell'}(0)\, D^{j}_{s's}(R) = \sum_{\ell} D_{\ell'\ell}(R)\, u^{s}_{\ell}(0) \qquad \text{equation for } u$$

$$\rightarrow \sum_{s} u^{s'}_{\ell'}(0)\, J^{j}_{s's}(R) = \sum_{\ell} J_{\ell'\ell}(R)\, u^{s}_{\ell}(0) \qquad (**)$$

how a transform what particle

rep we pick for field

$(L(p)) u_\ell^s(0)$

$$\sum_s u_{\ell'}^{s'}(0) D^j_{s's}(R) = \sum_\ell D_{\ell'\ell}(R) u_\ell^s(0)$$ equation for u

ice

$$\rightarrow \sum_s u_{\ell'}^{s'}(0)\, J^i_{s's}(R) = \sum_\ell J_{\ell'\ell}(R) u_\ell^s(0) \quad (**)$$

$D = 1 + \omega_{ij} J^{ij}$

how a transform what particle rep of SU(2)

rep we pick for field → rep of SO(1,3)

tion

R R R

$+(\Lambda p)\Lambda L(p)$

R_1

$$\sum_{u} u_{\ell'}^{s'}(0)\, \vec{J}^{\,j}_{s's}(R) = \sum_{\ell} \vec{\mathcal{J}}_{\ell'\ell}(R)\, u_{\ell}^{s}(0)$$

general recipe

① pick a rep for Lorentz (field)

② what is the spin of the particle

③ can I make causality work.

- scalar field

$\vec{J}_{\ell'\ell} = 0$

assume $\begin{cases} \text{scalar} \\ \text{boson} \end{cases}$

$a^\dagger e^{+ipx} + a e^{-ipx}$

promote promote

$\vec{J}_{e'e} = 0$

$\vec{J}_{s's}(R) = 0$

$u(0) = \text{const}$

$u(x,p) = u(0)\, e^{-ixp}$

$v(x,p) = v(0)\, e^{+ix\cdot p}$

$\phi(x) = k\phi^-(x) + \lambda\phi^+(x)$

$\phi(y) = k\phi^-(y) + \lambda\phi^+(y)$

$[\phi^-(x), \phi^+(y)]_\mp = \int dv_p\, e^{-ip(x-y)} = D(x-y)$

$[\phi(x), \phi(y)]_\mp = k\lambda[\phi^-(x), \phi^+(y)]_\mp \quad = k\lambda D(x-y)$

$+ k\lambda[\phi^+(x), \phi^-(y)]_\mp \quad + k\lambda(-$

$= 0$

$) = 0$

onst

$u(0)\, e^{-ixp}$

$= v(0)\, e^{+ix\cdot p}$

$$\phi(x) = k\phi^-(x) + \lambda\phi^+(x)$$
$$\phi(y) = k\phi^-(y) + \lambda\phi^+(y)$$
$$[\phi^-(x), \phi^+(y)]_\mp = \int dV_p\, e^{-ip(x-y)} = D(x-y)$$

→ trivial rep
spin 0
boson

spacelike

$$[\phi(x), \phi(y)]_\mp = k\lambda[\phi^-(x), \phi^+(y)]_\mp + k\lambda[\phi^+(x), \phi^-(y)]_\mp$$
$$= k\lambda D(x-y) + k\lambda(\mp) D(y-x)$$
$$= (k\lambda \mp k\lambda) D(x-y)$$
$$= 0$$

$$[\phi(x), (\phi(y))^\dagger]$$

$$\overset{\text{spacelike}}{=} (|\kappa|^2 \mp |\lambda|^2)\, D(x-y)$$

Causal spinor field → spin 1/2

$$u^{s'}_{\ell'}(0)\, \vec{J}^{\,j}_{s's}(R) = \vec{\mathcal{J}}_{\ell'\ell}(R)\, u^{s}_{\ell}(0)$$

tutorial: $\vec{J} = \frac{1}{2}\begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix} \begin{matrix} + \\ - \end{matrix}$ $\quad \vec{\sigma}_{m'm}$

$\ell = 1 \quad 2 \quad 3 \quad 4$

trick: $\ell = m, +$ or $m, -$ $\qquad \ell = 1 = \frac{1}{2}, +$; $\frac{1}{2} = m$

Pauli $\quad \begin{array}{c|cc} & \frac{1}{2} & -\frac{1}{2} \\ \hline \frac{1}{2} & & \\ -\frac{1}{2} & & \end{array}$

$$\mathcal{J}_{\ell'\ell}(R) = \mathcal{J}_{(m',\pm),(m,\pm)}(R)$$

$$= \mathcal{J}_{m'm,\pm} = \frac{1}{2}\vec{\sigma}_{m'm}$$

$$u^{s'}_{m',\pm}(0)\, J^{j}_{s's}(R) = \frac{1}{2}\sigma_{m'm}\, u^{s}_{m,\pm}(0)$$

$$\left(U_\pm \vec{J}^{j}(R) = \frac{1}{2}\vec{\sigma}\, U_\pm\right)_{m's}$$

rep of SU(2)   rep of SU(2)

$U_\pm$ is a square matrix

$m's$

$\frac{1}{2}, -\frac{1}{2}$ $\frac{1}{2}, -\frac{1}{2}$ $j = \frac{1}{2}$

$U_\pm = \begin{cases} 0 \\ \text{non singular square matrix} \end{cases}$

$$\boxed{u^{s'}_{m'\pm} = \delta_{m's'}\, C_\pm}$$