Title: QFT1 Lecture - 101823

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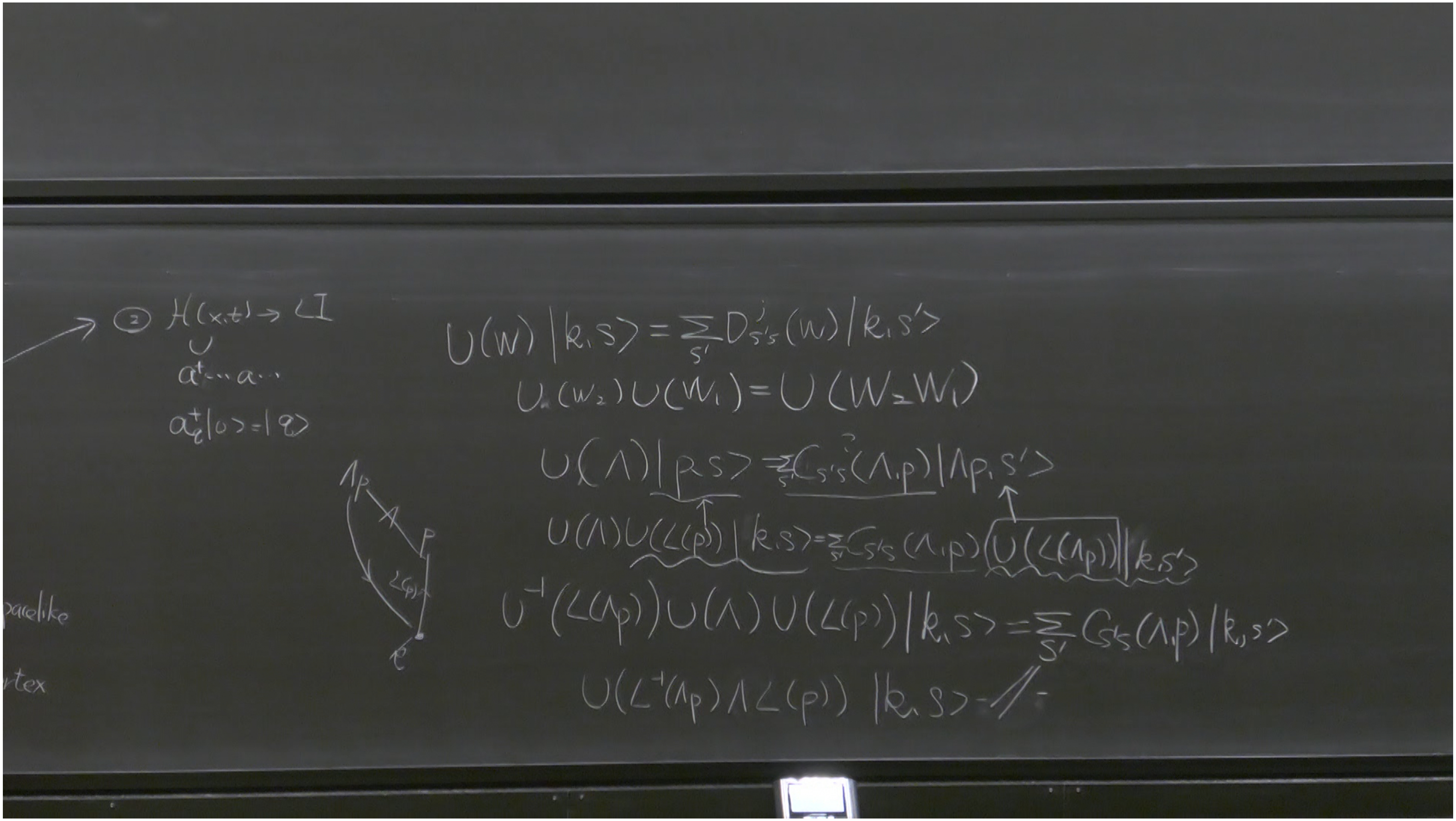
$$U(W)\,|k, s\rangle = \sum_{s'} D^{j}_{s's}(W)\,|k, s'\rangle$$

$$U(W_2)\,U(W_1) = U(W_2 W_1)$$

$$U(\Lambda)\,|p, s\rangle = C^{?}_{s's}(\Lambda, p)\,|\Lambda p, s'\rangle$$

$$U(\Lambda)\,U(L(p))\,|k, s\rangle = C_{s's}(\Lambda, p)\;U(L(\Lambda p))\,|k, s'\rangle$$



② $H(x,t) \to \langle I$
$a^{+} \cdots a \cdots$
$a^{+}_{q}|0\rangle = |q\rangle$
pacelike
rtex
$U(W)|k,s\rangle = \sum_{s'} D^{j}_{s's}(W)|k,s'\rangle$
$U(W_2)U(W_1) = U(W_2 W_1)$
$U(\Lambda)|p,s\rangle = \sum_{s'} C^{j}_{s's}(\Lambda,p)|\Lambda p,s'\rangle$
$U(\Lambda)U(L(p))|k,s\rangle = \sum_{s'} C_{s's}(\Lambda,p)\,U(L(\Lambda p))|k,s'\rangle$
$U^{-1}(L(\Lambda p))U(\Lambda)U(L(p))|k,s\rangle = \sum_{s'} C_{s's}(\Lambda,p)|k,s'\rangle$
$U(L^{-1}(\Lambda p)\Lambda L(p))|k,s\rangle$
$\Lambda p$
$\Lambda$
$p$
$L(p)$
$k$

$$(W)\,|k,s\rangle = \sum_{s'} D^{j}_{s's}(W)\,|k,s'\rangle$$

$$U(W_2)\,U(W_1) = U(W_2 W_1)$$

$$U(\Lambda)\,|p,s\rangle \overset{?}{=} \sum_{s'} C_{s's}(\Lambda,p)\,|\Lambda p,s'\rangle$$

$$U(\Lambda)\,U(L(p))\,|k,s\rangle = \sum_{s'} C_{s's}(\Lambda,p)\,\boxed{U(L(\Lambda p))}\,|k,s'\rangle$$

$$U^{-1}(L(\Lambda p))\,U(\Lambda)\,U(L(p))\,|k,s\rangle = \sum_{s'} C_{s's}(\Lambda,p)\,|k,s'\rangle$$

$$U(L^{-1}(\Lambda p)\,\Lambda\,L(p))\,|k,s\rangle = -//-$$

$$W = L^{-1}(\Lambda p)\,\Lambda\,L(p)$$

$$|p,s\rangle = \sum_{s'} D_{s's}(W(\Lambda,p))\,|\Lambda p,s'\rangle$$

Why fields?

• why not fields? → ② $H(x,t) \to LI$

$\cup$

$a^\dagger \cdots a \cdots$

$a_q^\dagger|0\rangle = |q\rangle$

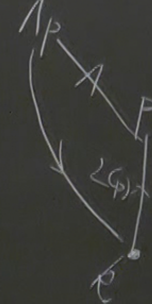
$D(W_2 W_1)_{s s_3} = \sum_{s_2} D(W_2)_{s_1 s_2} D(W_1)_{s_2 s_3}$

① $|\vec{p}, s, n\rangle$

② $H(x,t)$ LI

③ $[H(x,t), H(x',t')] = 0$ spacelike

④ cdp $\delta^4(\Sigma p)$ each vertex




$$U(W)|k,s\rangle = \sum_{s'} D^j_{s's}(W)|k,s'\rangle$$

$$U(W_2)U(W_1) = U(W_2 W_1)$$

$$U(\Lambda)|p,s\rangle = \sum_{s'} C^j_{s's}(\Lambda,p)|\Lambda p$$

$$U(\Lambda)U(L(p))|k,s\rangle = \sum_{s'} C_{s's}(\Lambda,p)$$

$$U^{-1}(L(\Lambda p))U(\Lambda)U(L(p))|k,s\rangle$$

$$U(L^{-1}(\Lambda p)\Lambda L(p))|k,s\rangle$$

$$\ldots D^{j}_{s's}(W_1)\,|k,s'\rangle$$

$$\ldots) = U(W_2 W_1)$$

$$\ldots s\rangle = \sum_{s'} C^{j}_{s's}(\Lambda,p)\,|\Lambda p, s'\rangle$$

$$\ldots |k,s\rangle = \sum_{s'} C_{s's}(\Lambda,p)\,\boxed{U(L(\Lambda p))}\,|k,s'\rangle$$

$$\ldots \Lambda)\,U(L(p))\,|k,s\rangle = \sum_{s'} C_{s's}(\Lambda,p)\,|k,s'\rangle$$

$$\ldots \Lambda L(p))\,|k,s\rangle = \ldots$$

$$W = L^{-1}(\Lambda p)\,\Lambda\, L(p)$$

$$U(\Lambda)\,|p,s\rangle = \sum_{s'} D_{s's}(W(\Lambda,p))\,|\Lambda p, s'\rangle$$

$$U(\Lambda,b)\,|p,s\rangle = e^{ib\cdot\Lambda p}\sum_{s'} D^{j}_{s's}(W(\Lambda,p))\,|\Lambda p, s'\rangle$$

$$U(\Lambda,b)\,a^{\dagger s}_{p}|0\rangle = e^{ib\cdot\Lambda p}\sum_{s'} D^{j}_{s's}(W(\Lambda,p))\,a^{\dagger s}_{\Lambda p}|0\rangle$$

$$U(\Lambda,b)\,a^{\dagger s}_{p}\,U^{-1}(\Lambda,b) = e^{ib\cdot\Lambda p}\sum_{s'} D^{j}_{s's}(W(\Lambda,p))\,a^{\dagger s}_{\Lambda p}$$

$H \supset a^\dagger a^\dagger a$

$$\sum_{s_1 s_2 s_3} \iiint d^3p_1\, d^3p_2\, d^3p_3\; \underbrace{a^{\dagger s_1}_{p_1} a^{\dagger s_2}_{p_2} a^{\dagger s_3}_{p_3}}_{\text{junk}}\; \underbrace{C_{2,1}(p_1 p_2 p_3\, s_1 s_2 s_3)}_{\text{junk}^{-1}}$$

life is too hard

· motivation for fields

suppose $\exists$

$$U(\Lambda, a)\, \chi^{\pm}_{\ell}(x)\, U^{-1}(\Lambda, a)$$

$$= \sum_{\ell'} D_{\ell \ell'}(\Lambda^{-1})\, \chi^{\pm}_{\ell'}(\Lambda x + a)$$

$$D(\Lambda^{-1}) D(\Lambda'^{-1}) = D(\Lambda^{-1} \Lambda'^{-1})$$

elds

$(\Lambda, a)$

$\Lambda^{-1})\, \chi^{\pm}_{\ell'}(\Lambda x + a)$

$) = D(\Lambda^{-1}\Lambda'^{-1})$

$$H = \sum_{NM} \sum_{\{\ell', \ell\}} g_{\{\ell', \ell\}}\, \chi^\dagger_{\ell'_1} \cdots \chi^\dagger_{\ell'_N}\, \chi_{\ell_1} \cdots \chi_{\ell_M}$$

- how to make field.

$$\chi^-_\ell(x) = \sum_s \int dV_p\, u^s_\ell(x, \vec{p})\, a^s_{\vec{p}}$$

$$dV_p \equiv \frac{d^3p}{(2\pi)^3\, 2E_p}$$

• success of field

a) Need to CDP ( $V$ has $1\ \delta^4(\Sigma p)$ )

$$u_e^s(p;x) = e^{ipx}\, u_e^s(p)$$

$$V = \int d^3x\ \mathcal{H} = \text{coefficient}\ \chi^+\chi^+\chi^+$$

$$= \int d^3x\ \text{coeff} \sum\!\!\!\!\int d\nu\ u_{e_1}^{s_1}(p_1)\, e^{ip_1x} \cdots e^{ip_2x} \cdots e^{ip_3x}\, a^+a^+a^+$$

$$\supset \int d^3x\ e^{i(p_1x + p_2x + p_3x)}$$

$$= \delta^4(p_1+p_2+p_3)$$

• success of field

a) Nod to CDP ( $V$ has 1 $\delta^4(\Sigma p)$ )

b)

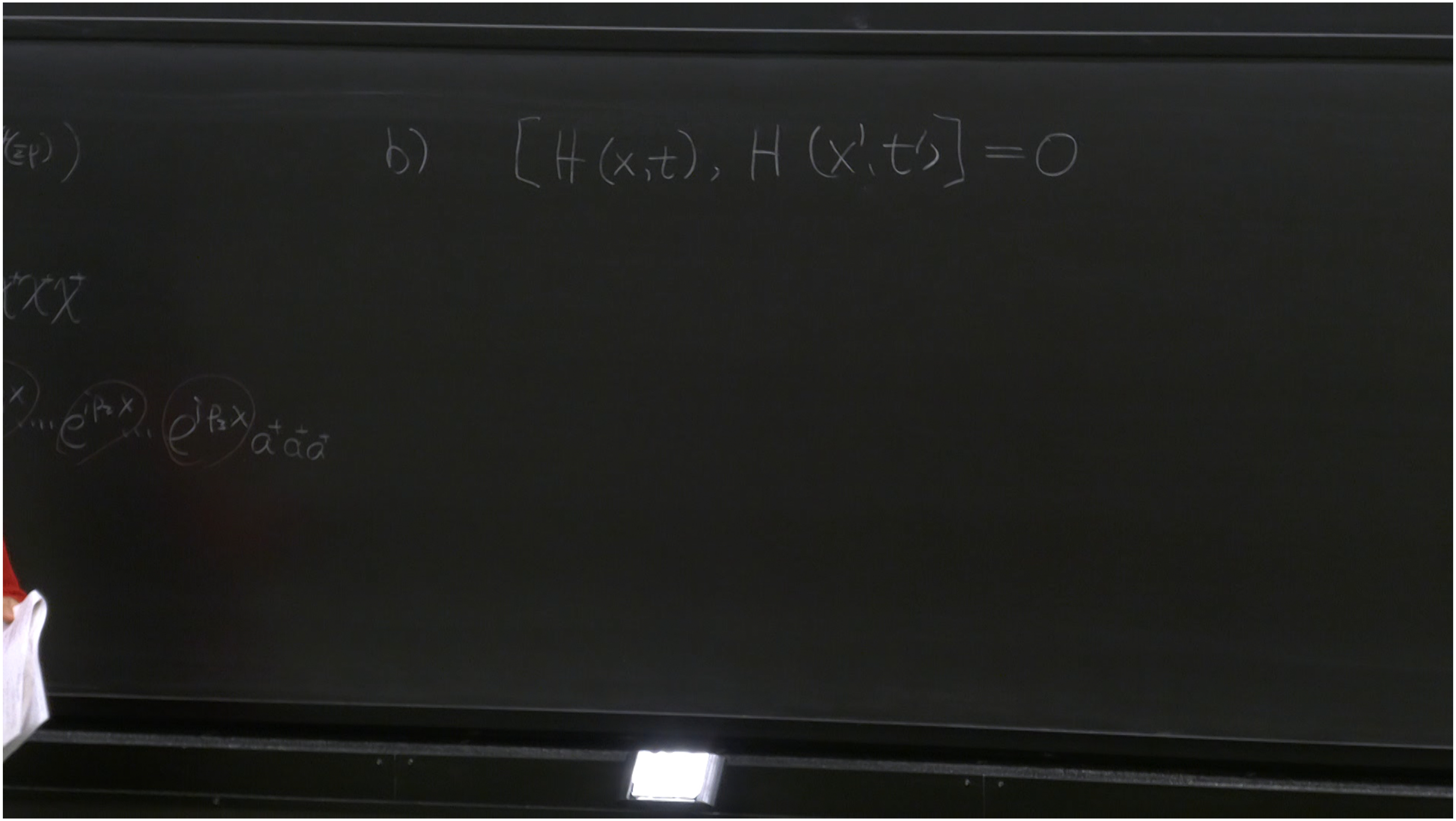
$$u_\ell^s(p;x) = e^{ipx}\, u_\ell^s(p)$$

$$V = \int d^3x \, \mathcal{H} \quad = \text{coefficient } \chi^\dagger \chi^\dagger \chi^\dagger$$

$$= \int d^3x \, \text{coeff} \sum \int d\nu \, u_{\ell_1}^{s_1}(p_1)\, e^{ip_1x} \cdots e^{ip_2x} \cdots e^{ip_3x}\, a^\dagger a^\dagger a^\dagger$$

$$\supset \int d^3x \, e^{i(p_1x + p_2x + p_3x)}$$

$$= \delta^4(p_1+p_2+p_3)$$


b) $[H(x,t), H(x',t')] = 0$

b) $[H(x,t), H(x',t')] = 0$ spacelike

$$\chi_e^-(x) = \sum_s \int dV_{p_1}\, u_e^s(p_1)\, a_{p_1}^s\, e^{-ip_1 x}$$

$$\chi_{e'}^+(y) = \sum_s \int dV_{p_2}\, v_{e'}^s(p_2)\, a_{p_2}^{\dagger s}\, e^{+ip_2 y}$$

b) $[H(x,t), H(x',t')] = 0$ spacelike

$$\chi_\ell^-(x) = \sum \int dV_{p_1}\, u_\ell^s(p_1)\, a_{p_1}^s\, e^{-ip_1 x}$$

$$\chi_{\ell'}^+(y) = \sum \int dV_{p_2}\, v_{\ell'}^s(p_2)\, a_{p_2}^{+s}\, e^{+ip_2 y}$$

$$[\chi_\ell^-(x), \chi_{\ell'}^+(y)] = \sum \int dV_{p_1}\, e^{-p_1(x-y)}\, u_\ell^s(p_1) v_{\ell'}^s(p_1)$$

$$\chi_\ell = k\chi_\ell^-(x) + \lambda \chi_\ell^+(x)$$

$$[\chi_\ell(x), \chi_\ell(y)]_{\mp} = 0 \quad \text{spacelike}$$

$= 0$ spacelike

$e^{-ip_1 x}$

$a^{+s}_{p_2} e^{+ip_2 y}$

$\sum \int dV_{p_1} e^{-p_1(x-y)} u^s_\ell(p_1) V^s_{\ell'}(p_1)$

$(x)$

$= 0$ spacelike

c) reason for anti-particles

$$[H, Q] = 0$$

$a^+$ carry charge $q$

$$Q a^+|0\rangle = q\, a^+|0\rangle$$

$$Q|\alpha\rangle = q_\alpha |\alpha\rangle$$

$$Q(a^+|\alpha\rangle) = (q + q_\alpha) a^+|\alpha\rangle$$

$$[Q, a^+]\,|\alpha\rangle = Q a^+|\alpha\rangle - a^+ Q|\alpha\rangle = q\, a^+|\alpha\rangle$$

$$[Q, a^+] = q a^+$$

$$[Q, a] = -q a$$

$$[Q, \chi_\ell] = q\chi_\ell$$

$$[H, Q] = 0$$

$$[\chi_+ \chi_-, Q] = \chi_+[\chi_-, Q] + [\chi_+, Q]\chi_-$$

$$[Q, a^\dagger]|\alpha\rangle = Qa^\dagger|\alpha\rangle - a^\dagger Q|\alpha\rangle = q\, a^\dagger|\alpha\rangle$$

$$[Q, a^\dagger] = q a^\dagger$$

$$[Q, a] = -q a$$

$$\boxed{[Q, \chi_\ell] = q\chi_\ell}$$

$$[H, Q] = 0$$

$$[\chi_+\chi_-, Q] = \chi_+[\chi_-, Q] + [\chi_+, Q]\chi_-$$

$$\chi_\ell = \chi_\ell^+ + \chi_\ell^-$$

$$[Q, \chi_\ell] = q\chi_\ell^+ - q\chi_\ell^-$$

$$[Q, \chi_\ell] = q\chi_\ell$$

$$[H, Q] = 0$$

$$[\chi_+\chi_-, Q]$$
$$= \chi_+[\chi_-, Q] + [\chi_+, Q]\chi_-$$

a type particle

$$\chi_\ell = \chi_\ell^+ + \chi_\ell^-$$

b type particle

$$[Q, \chi_\ell]$$
$$= q\chi_\ell^+ - q\chi_\ell^-$$

$a^\dagger \leftarrow$ creates charge $q$

$a \leftarrow$ annililates charge $q$

$a^+$ $b$

$$[Q, \chi_\ell]$$
$$= q\chi_\ell^+ - (-q)\chi_\ell^-$$

$a^\dagger$ created charge $q$

$b^\dagger$ created charge $-q$

$b$ create charge $q$