Title: QFT1 Lecture - 101623

Speakers: Gang Xu
Date: October 16, 2023 - 10:45 AM

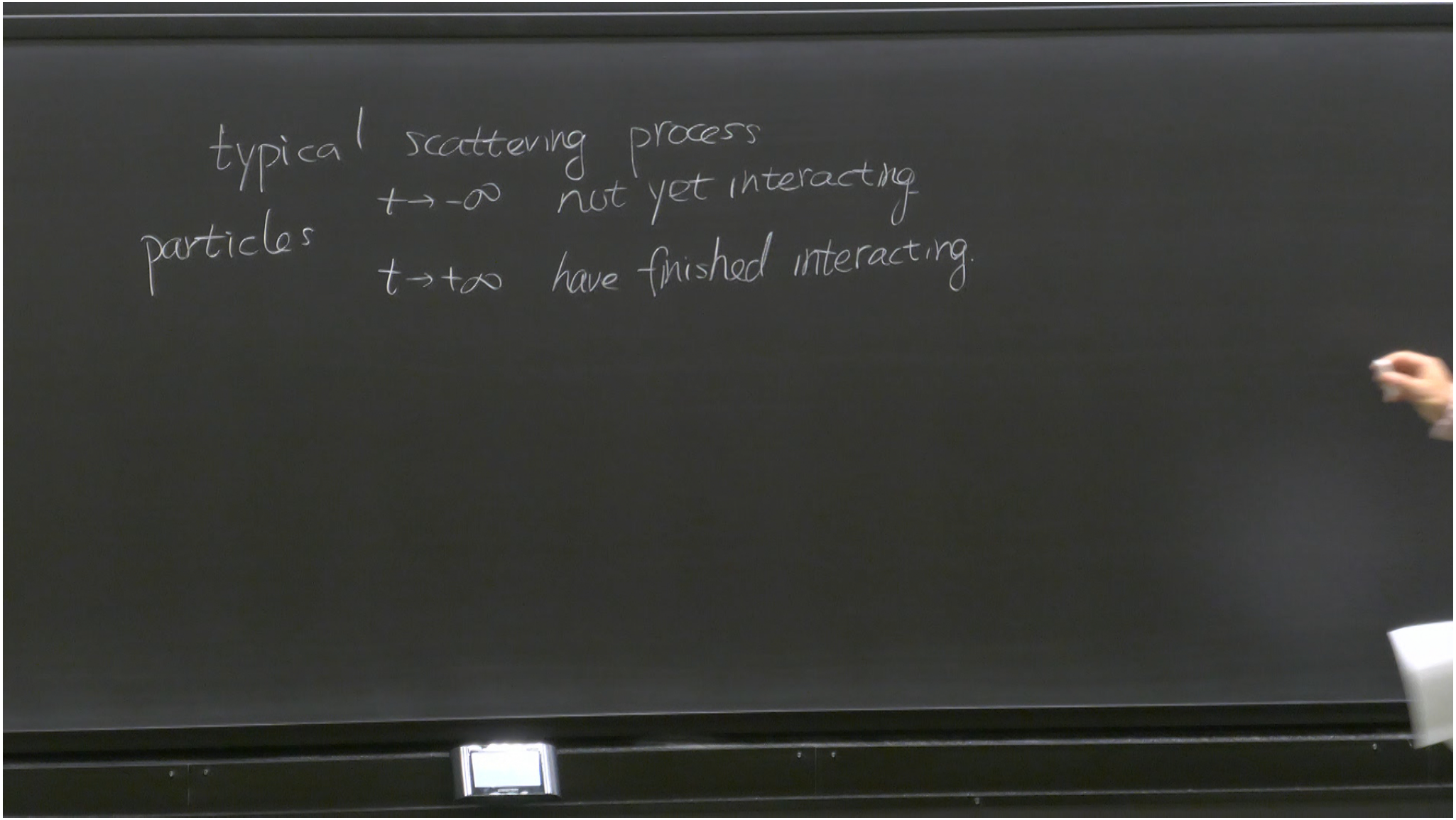
URL: https://pirsa.org/23100048

Scattering amplitude

- Asymtotic states:

free one-particle state $|p, s, n\rangle \equiv |q\rangle$

free multi-state $|q_1, q_2\rangle$


typical scattering process
particles
$t \to -\infty$ not yet interacting
$t \to +\infty$ have finished interacting.

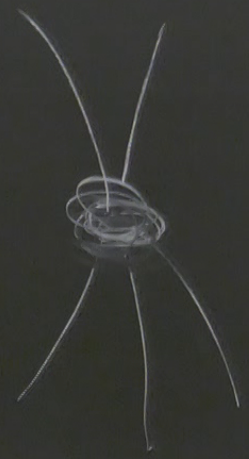
typical scattering process

particles

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$= |q\rangle$

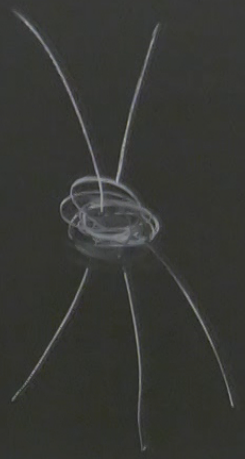


typical scattering process

particles $t \to -\infty$ not yet interacting

$t \to +\infty$ have finished interacting.

$= |q\rangle$

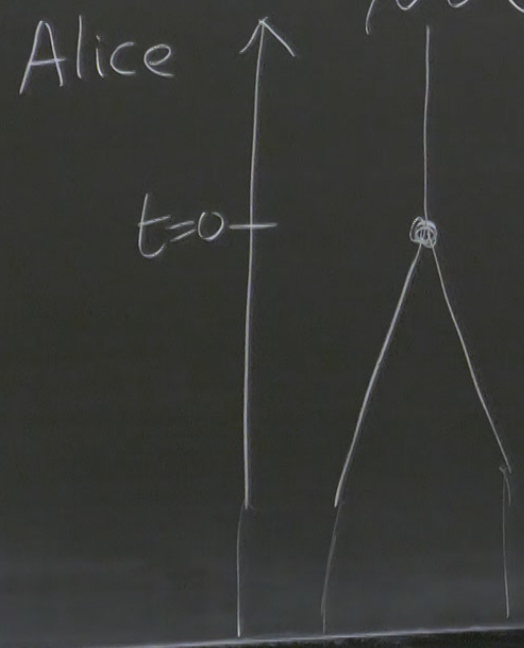


ng process
not yet interacting
have finished interacting.

observe at $t \to -\infty$ contain particles $(q_1, q_2 \ldots)$
$+\infty$ $(q_1', q_2', \ldots)$

SR: Heisenberg picture

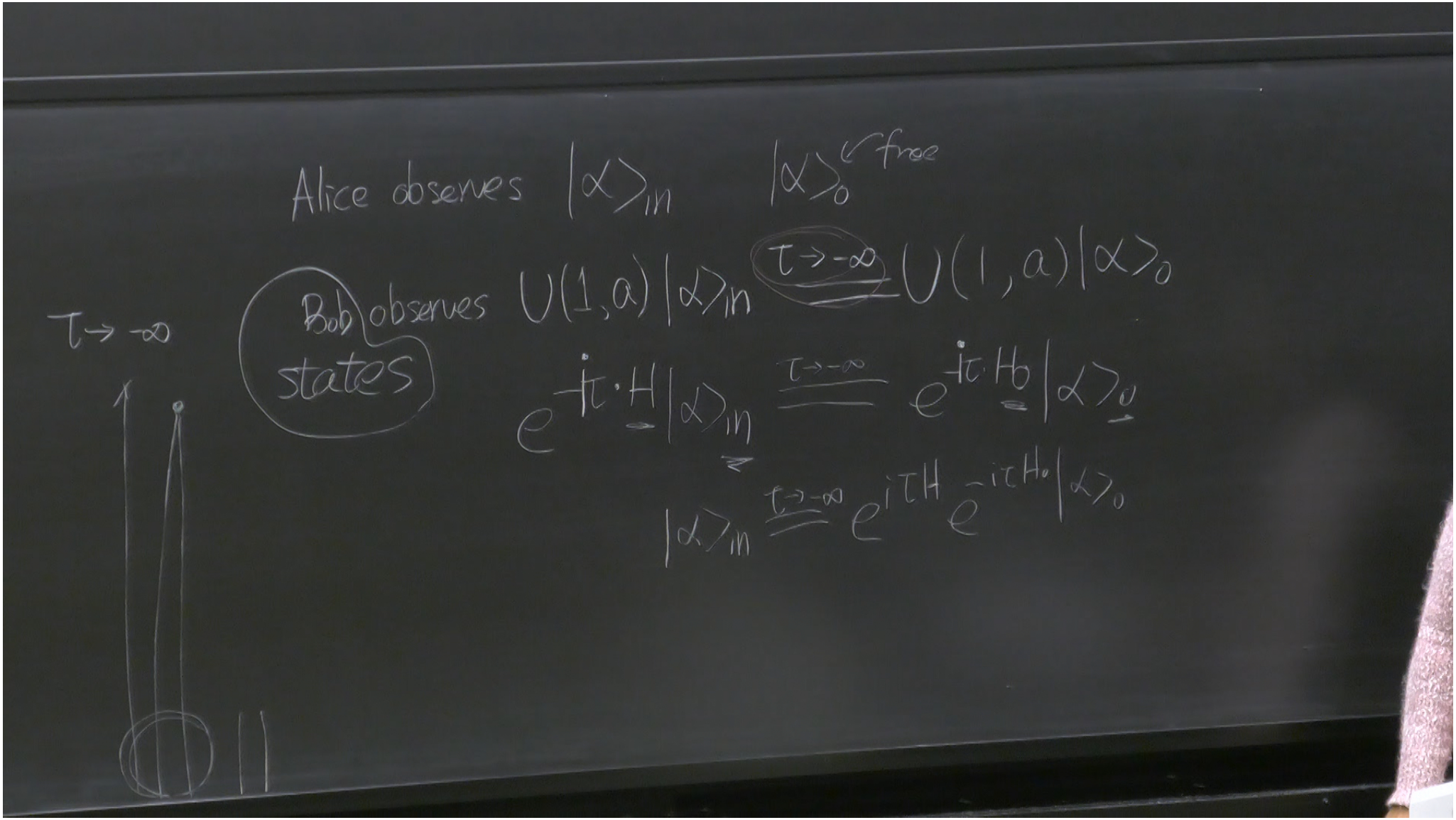
$S = {}_{out}\langle \beta | \alpha \rangle_{in}$ $\alpha, \beta = \{q_1, q_2, \ldots\}$

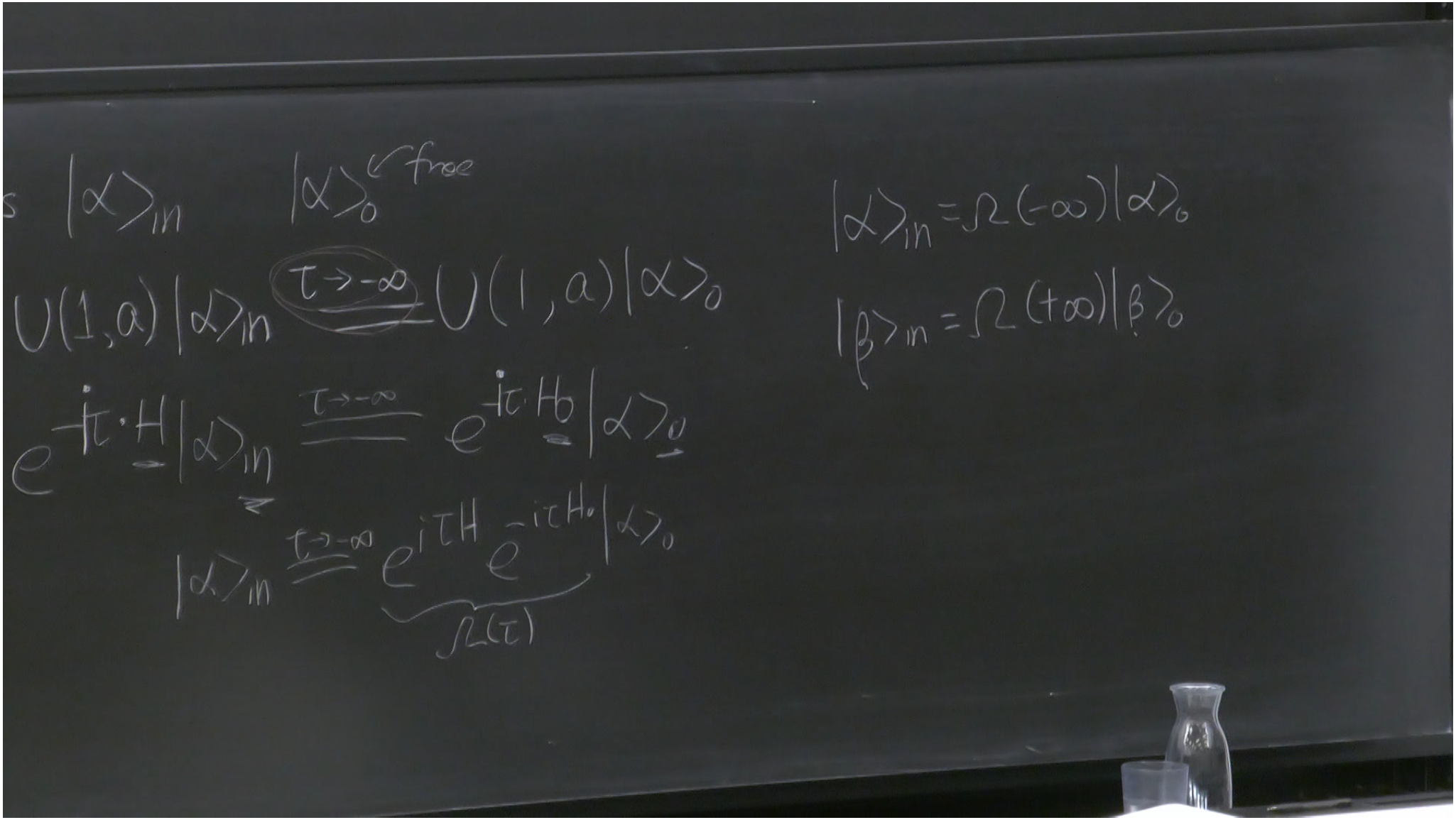


Bob $t' = t - \tau$

$a^\mu = (-\tau, 0, 0, 0)$

Bob sees interaction $t' = -\tau$


Alice observes $|\alpha\rangle_{in}$ $|\alpha\rangle_0$ free
Bob observes states
$U(1,a)|\alpha\rangle_{in}$ $\tau \to -\infty$ $U(1,a)|\alpha\rangle_0$
$e^{-it\cdot H}|\alpha\rangle_{in} \overset{\tau\to-\infty}{=} e^{-it\cdot H_0}|\alpha\rangle_0$
$|\alpha\rangle_{in} \overset{\tau\to-\infty}{=} e^{iTH}e^{-itH_0}|\alpha\rangle_0$
$\tau \to -\infty$


$|\alpha\rangle_{in}$ $|\alpha\rangle_0$ free
$U(1,a)|\alpha\rangle_{in}$ $\tau\to-\infty$ $U(1,a)|\alpha\rangle_0$
$e^{-i\tau\cdot\underline{H}}|\alpha\rangle_{in}$ $\tau\to-\infty$ $e^{-i\tau\cdot\underline{H_0}}|\alpha\rangle_0$
$|\alpha\rangle_{in}$ $\tau\to-\infty$ $e^{iTH}e^{-iTH_0}|\alpha\rangle_0$
$\Omega(T)$
$|\alpha\rangle_{in}=\Omega(-\infty)|\alpha\rangle_0$
$|\beta\rangle_{in}=\Omega(+\infty)|\beta\rangle_0$

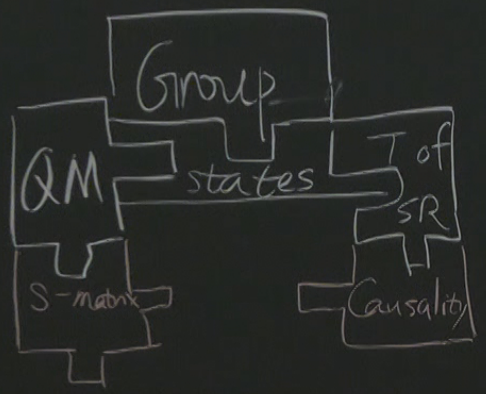
$$S = {}_{out}\langle \beta | \alpha \rangle_{in} = {}_{0}\langle \beta | \underbrace{\Omega^{\dagger}(+\infty)\,\Omega(-\infty)}_{S(+\infty,-\infty)} | \alpha \rangle_{0}$$

SR

(1)

$$S(+\infty,-\infty) = T e^{i\int_{-\infty}^{\infty} dt\, V(t)}$$

$V(t)$: $H - H_0$



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HW due

$\langle \beta | \alpha \rangle_{in} = \langle \beta | \underbrace{\Omega^{\dagger}(+\infty)\Omega(-\infty)}_{S(+\infty,-\infty)} | \alpha \rangle$

SR Implication

① $V(t) = \int d^3x \, \mathcal{H}(x,t)$

$S(+\infty,-\infty) = T e^{i\int_{-\infty}^{\infty} dt \, V(t)}$ ($V(t)$: $H - H_0$)

$T[H(t_1)H(t_2)] = \begin{cases} H(t_1)H(t_2) & t_1 > t_2 \\ H(t_2)H(t_1) & t_2 > t_1 \end{cases}$

$|2(-\infty)|\alpha\rangle_0$ SR Implication

$0,-\infty)$ ① $V(t) = \int d^3x\, \mathcal{H}(x,t)$

$e^{i\int_{-\infty}^{\infty} dt\, V(t)}\, T_{H(t_1)H(t_2)} = \begin{cases} H(t_1)\,H(t_2) & t_1 > t_2 \\ H(t_2)\,H(t_1) & t_2 > t_1 \end{cases}$

$\uparrow$

$H - H_0$

spacelike separation

$[H(x,t), H(x',t')] = 0$

olication

$t) = \int d^3x\, H(x,t)$

$(t_1)\, H(t_2) \quad t_1 > t_2$

$(t_2)\, H(t_1) \quad t_2 > t_1$

spacelike separation

$[H(x,t), H(x',t')] = 0$

Computation

$|q\rangle \equiv a_q^\dagger |0\rangle$

$a_{q_1}^\dagger |q_2\rangle = |q_1, q_2\rangle$

$a \equiv (a^\dagger)^\dagger$

tion

$\int d^3x\, H(x,t)$

(t_2) $t_1 > t_2$

(t_1) $t_2 > t_1$

spacelike separation

$[H(x,t), H(x',t')] = 0$

Computation

$|q\rangle \equiv a_q^\dagger |0\rangle$

$a_{q_1}^\dagger |q_2\rangle = |q_1, q_2\rangle$

$a \equiv (a^\dagger)^\dagger$

$[a, a^\dagger]$

Important conjecture

Any operato

$$O = \sum_{N=0}^{\infty} \sum_{M=0}^{\infty} \int dq_1' \cdots dq_N' \int dq_1 \cdots dq_M$$

$$a^\dagger(q_1') \cdots a^\dagger(q_N') a(q_M) \cdots a(q_1))$$

$$\cdot C_{NM}(q_1' \cdots q_N', q_1 \cdots q_M)$$ — Nst operator.

Important conjecture

Any operato

$$O = \sum_{N=0}^{\infty} \sum_{M=0}^{\infty} \int dq_1' \cdots dq_N' \int dq_1 \cdots dq_M$$

$$a^\dagger(q_1') \cdots a^\dagger(q_N') a(q_M) \cdots a(q_1)$$

$$\cdot C_{NM}(q_1' \cdots q_N'; q_1 \cdots q_M)$$ — nst operator.

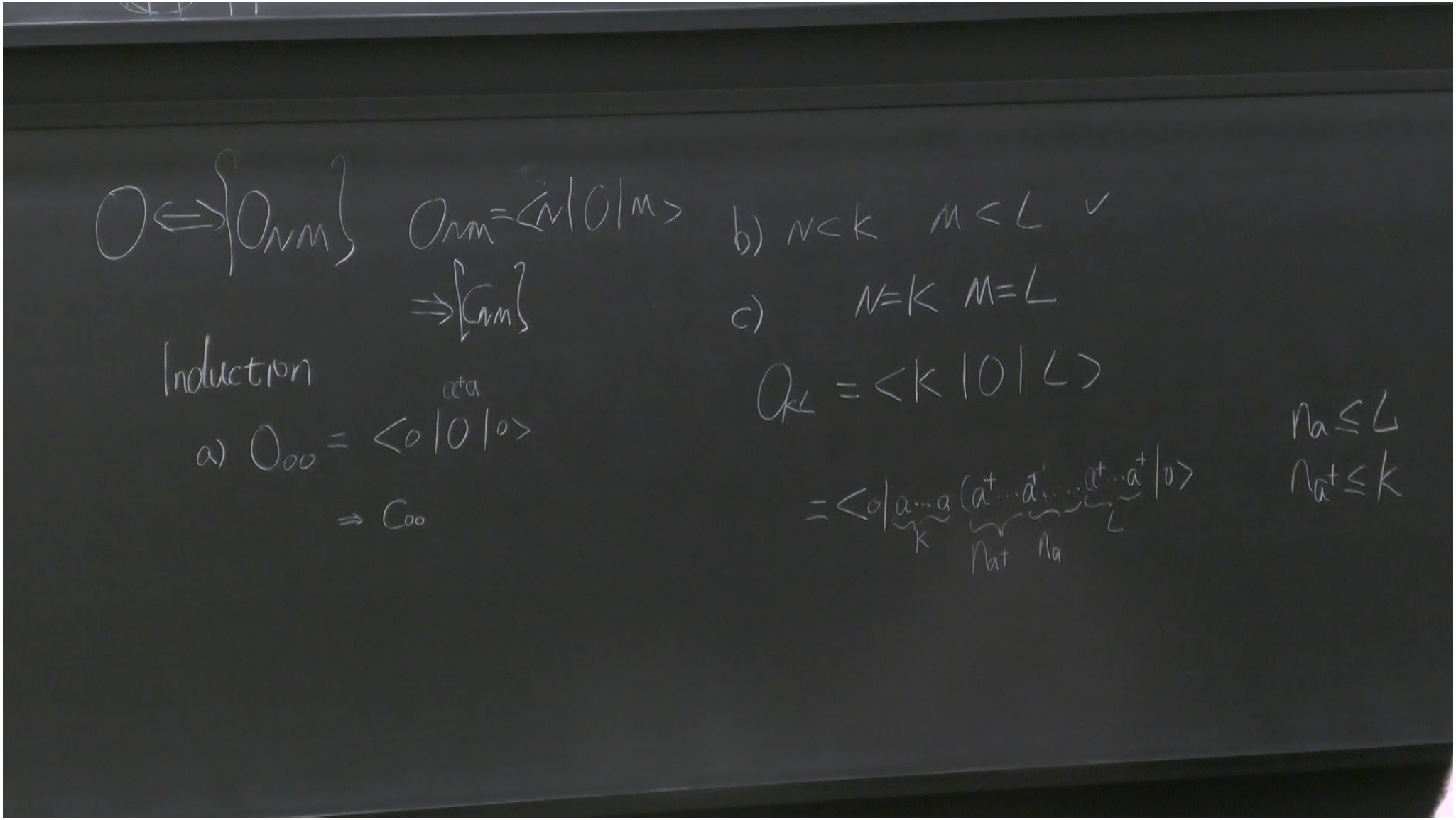
$O \Leftrightarrow \{O_{NM}\}$

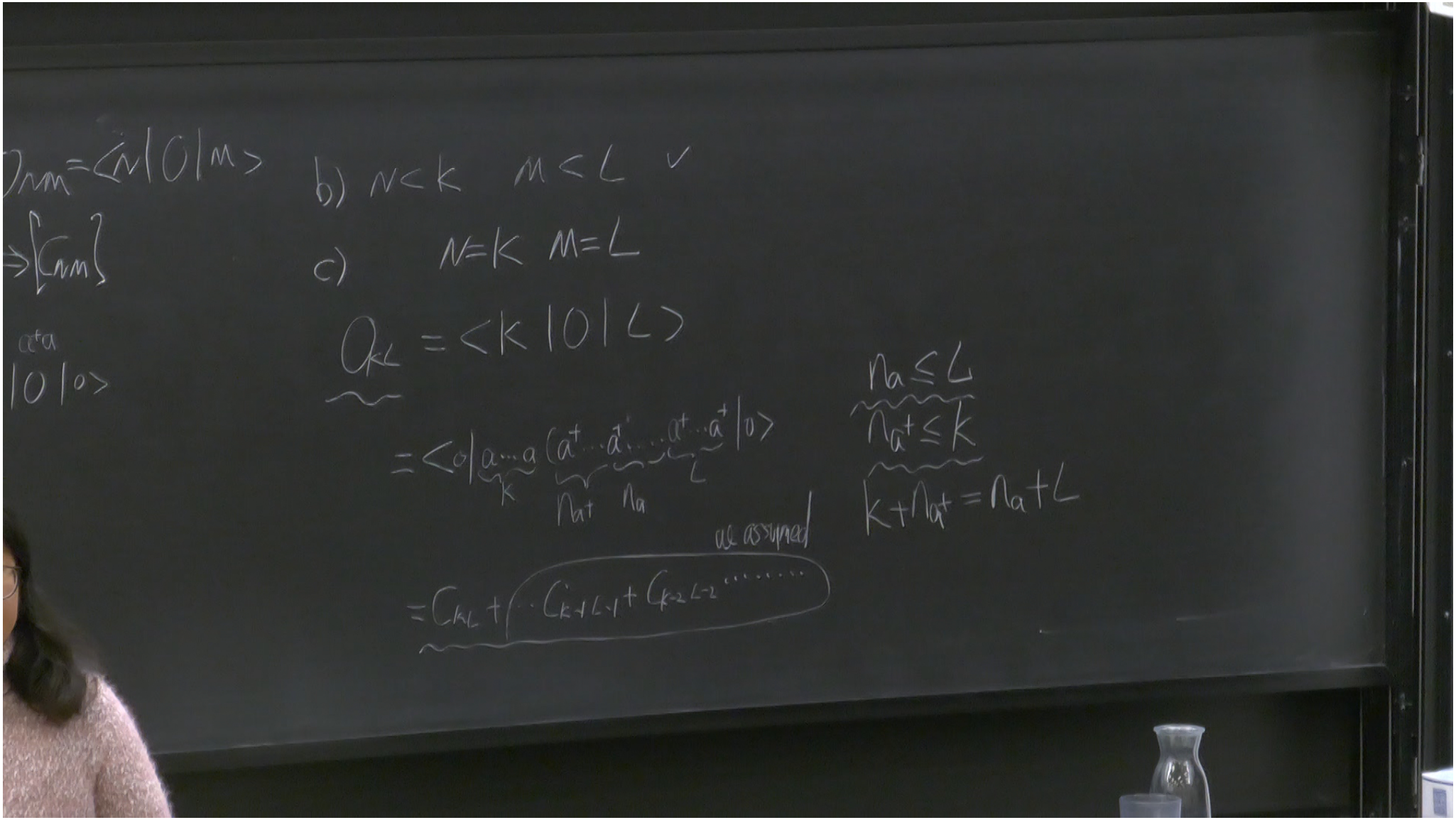
$\Rightarrow$

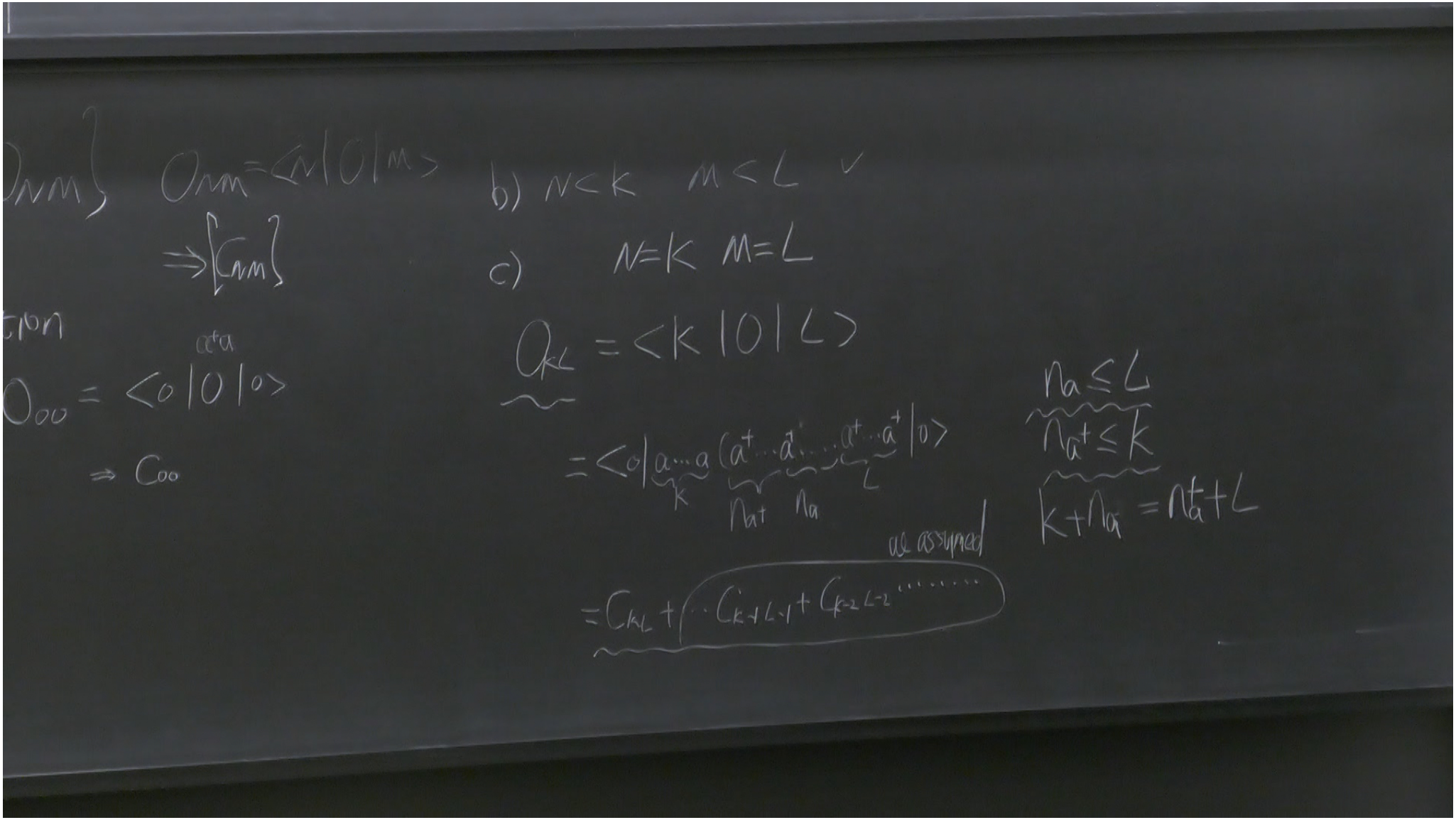
O

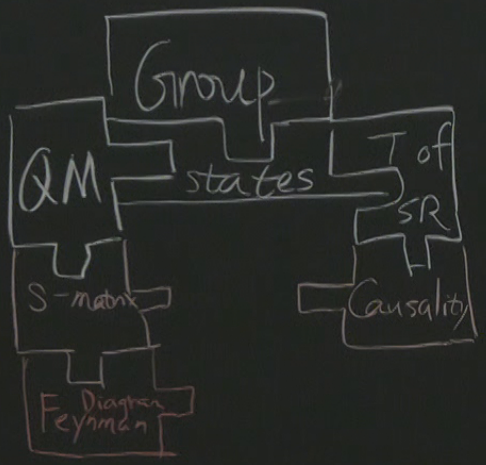
$\iint a^\dagger a \, C_1(\ )$

$t \iint a^\dagger a^\dagger a a \, C_2(\ )$


$O \Leftrightarrow \{O_{NM}\}$
$O_{NM} = \langle N|O|M\rangle$
$\Rightarrow \{C_{NM}\}$
Induction
a) $O_{00} = \langle 0|O|0\rangle$
$\Rightarrow C_{00}$
b) $N<k$ $M<L$ ✓
c) $N=K$ $M=L$
$O_{KL} = \langle K|O|L\rangle$
$= \langle 0|\underbrace{a\cdots a}_{k}\underbrace{a^\dagger\cdots a^\dagger}_{n_{a^\dagger}}\underbrace{a\cdots a}_{n_a}\underbrace{a^\dagger\cdots a^\dagger}_{L}|0\rangle$
$n_a \leq L$
$n_{a^\dagger} \leq k$


$O_{nm} = \langle n|O|m\rangle$
$\Rightarrow \{C_{nm}\}$
$a^\dagger a$
$|0|0\rangle$
b) $n<k$ $m<L$ ✓
c) $n=k$ $m=L$
$O_{kL} = \langle k|O|L\rangle$
$= \langle 0|a\cdots a\,(a^\dagger\cdots a^\dagger\cdots a^\dagger)\,a^\dagger|0\rangle$
k
$n_{a^\dagger}$ n_a
L
use assumed
$= C_{kL} + \cdots C_{k+L-1} + C_{k-2\,L-2}\cdots\cdots$
$n_a \le L$
$n_{a^\dagger} \le k$
$k + n_{a^\dagger} = n_a + L$


$O_{NM}\}$ $O_{NM} = \langle N|O|M\rangle$
b) $N<K$ $M<L$ ✓
$\Rightarrow \{C_{NM}\}$
c) $N=K$ $M=L$
$O_{KL} = \langle K|O|L\rangle$
$O_{00} = \langle 0|O|0\rangle$
$\Rightarrow C_{00}$
$= \langle 0|a\cdots a\,(a^\dagger \cdots a^\dagger \cdots a^\dagger \cdots a^\dagger)|0\rangle$
$n_a \le L$
$n_{a^\dagger} \le K$
$K + n_a = n_{a^\dagger} + L$
$= C_{KL} + \cdots C_{K-1,L-1} + C_{K-2,L-2} \cdots$



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- HW due

c) $N=K \quad M=L$

$$O_{KL} = \langle K | O | L \rangle$$

$$= \langle 0 | \underbrace{a \cdots a}_{K} \underbrace{(a^\dagger \cdots \hat{a} \cdots a^\dagger \cdots a^\dagger}_{L} | 0 \rangle$$

$n_{a^\dagger}$ n_a

$n_a \leq L$

$n_{a^\dagger} \leq K$

$K + n_a = n_{a^\dagger} + L$

O_{32}

$= C_{32} + C_{21} + C_{10}$

$KL \quad NM$

we assumed

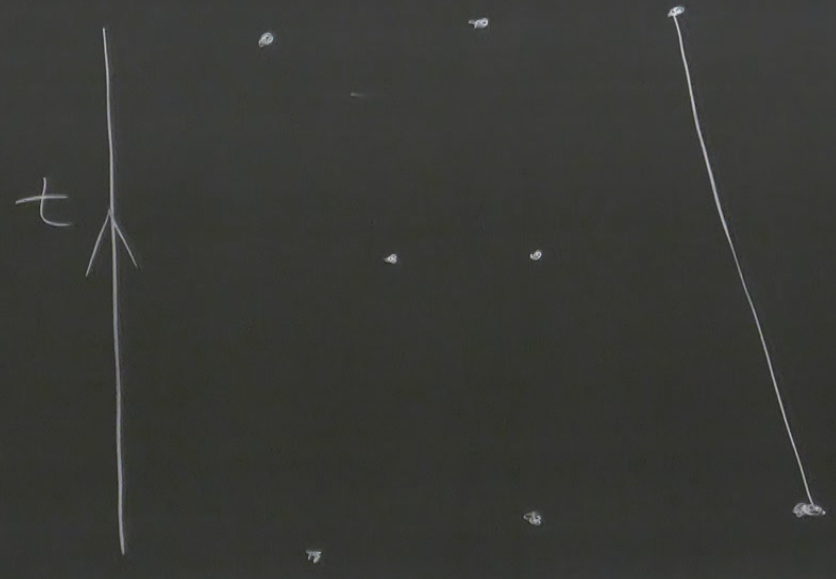
$$= C_{KL} + \cdots C_{K-1\,L-1} + C_{K-2\,L-2} \cdots\cdots$$

Compute $S_{\alpha\beta} = {}_{out}\langle\beta|\alpha\rangle_{in}$

$= {}_{0}\langle\beta|S|\alpha\rangle_{0}$

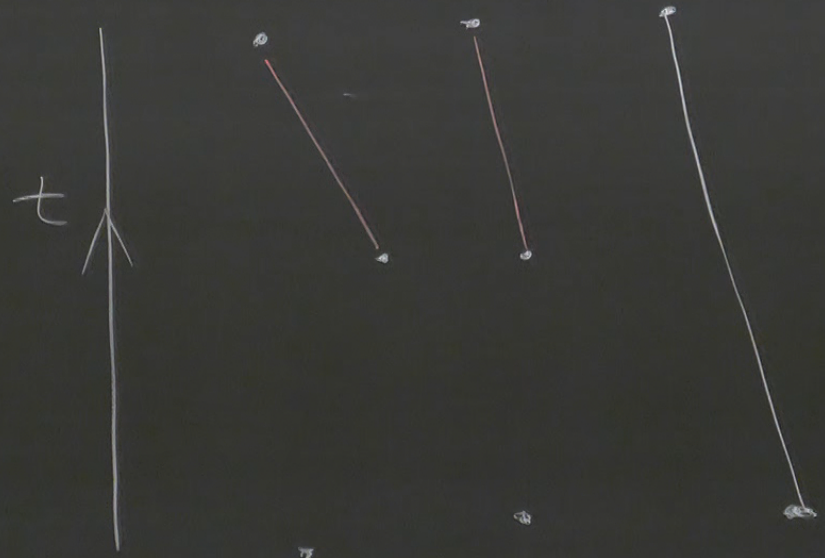
$= \not{\text{const}}\,\underbrace{{}_{0}\langle 0|a\cdots a}_{\beta}\underbrace{a^{+}\cdots a}_{V}\cdots\underbrace{a^{+}\cdots a}_{V}\cdots\underbrace{\cdots}_{\text{more } V}\underbrace{a^{+}\cdots a^{+}|0\rangle}_{\alpha}$

$$S_{\alpha\beta} = \langle 0 | aaa(a^\dagger a^\dagger aa)(a^\dagger a^\dagger aa)\, a^\dagger a^\dagger a^\dagger | 0 \rangle$$



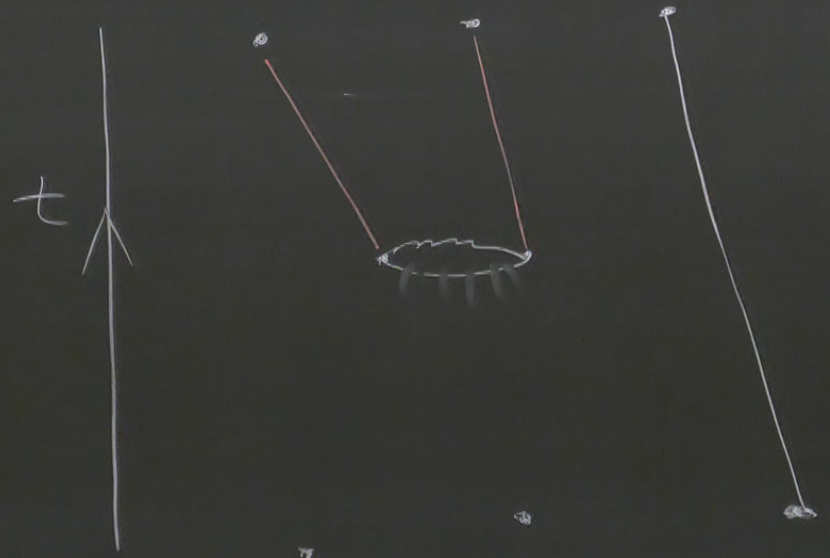
$$S_{\alpha\beta} = \langle 0 | a a a a (a^\dagger a^\dagger a a)(a^\dagger a^\dagger a a) a^\dagger a^\dagger a^\dagger | 0 \rangle$$

$$\underbrace{a^\dagger \cdots a^\dagger}_{\alpha} | 0 \rangle$$



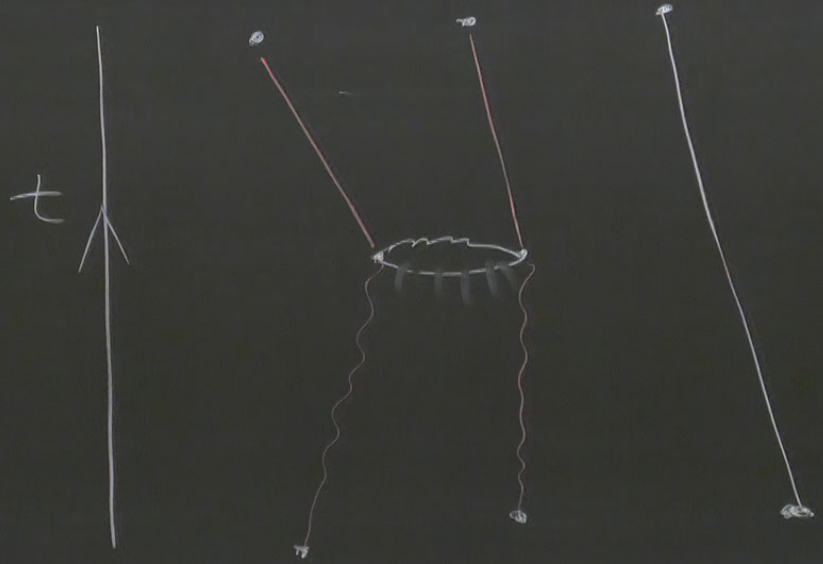
$$S_{\alpha\beta} = \langle 0 | a a a (a^\dagger a^\dagger a a)(a^\dagger a^\dagger a a) a^\dagger a^\dagger a^\dagger | 0 \rangle$$

$$\underbrace{a^\dagger \cdots a^\dagger}_{\alpha} | 0 \rangle$$



$$S_{\alpha\beta} = \langle 0 | a a a a (a^\dagger a^\dagger a a)(a^\dagger a^\dagger a a) a^\dagger a^\dagger a^\dagger | 0 \rangle$$

$\underbrace{\cdots \cdots}_{\text{make } V}$ $\underbrace{a^\dagger \cdots a^\dagger}_{\alpha} |0\rangle$



S^c contain only 1 δ.

$\chi = 1$

Connecte graph

$$(I = V - 1 + L)$$

$$V \cdot x - I + L = 1$$

$$V - I + L = 1.$$

CQP

