Title: QFT1 Lecture - 101323

Speakers: Gang Xu

Collection: Quantum Field Theory 1 2023/24

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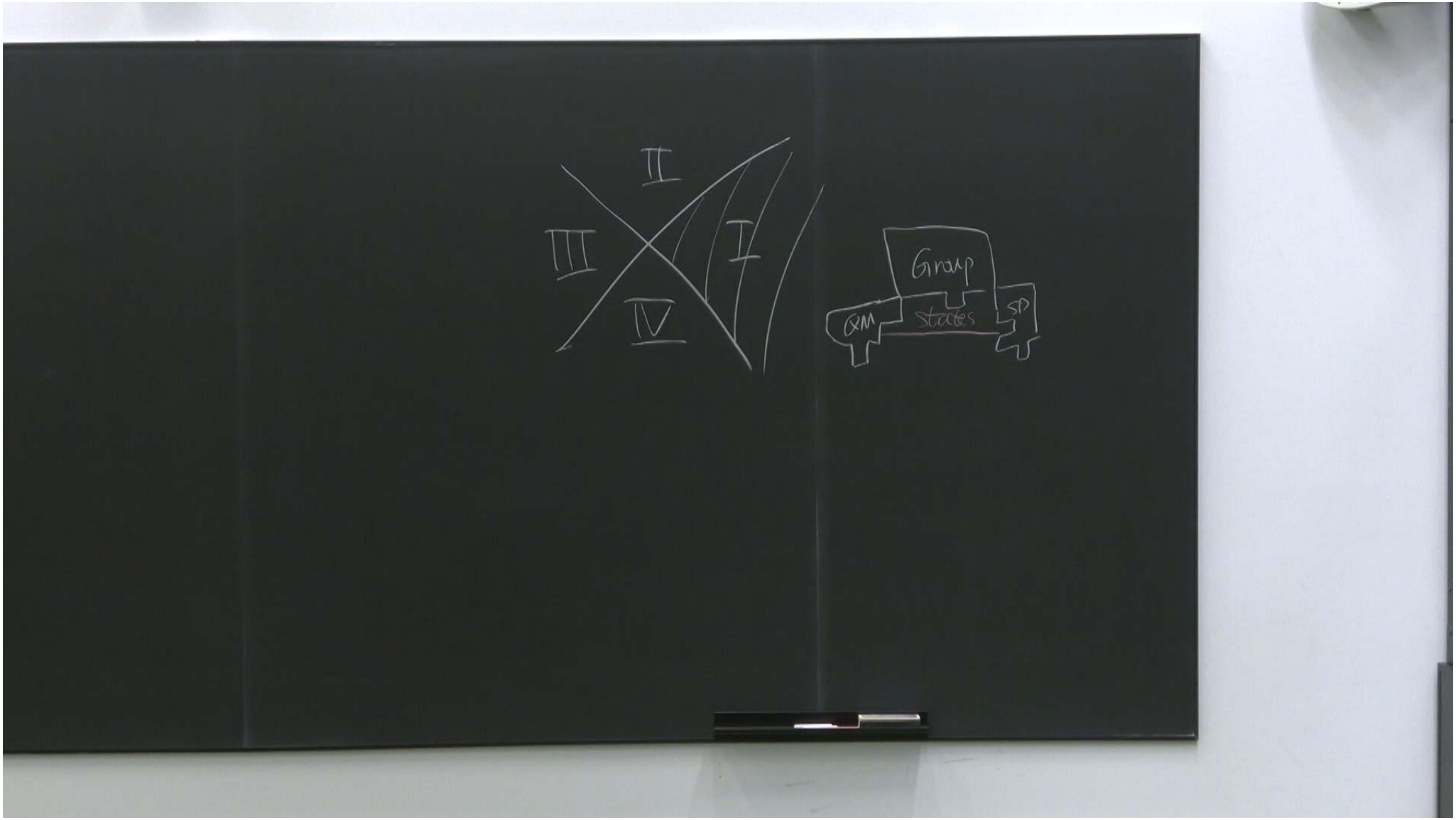
II
III
I
IV
Group
QM
states
SP

$O(B_1)$ is our only hope. Lie

$R^2 \otimes D_2$ is our only hope

$SU(2)$ is our only hope.

Lie

$SU(2) \overset{\downarrow}{\sim} SO(3) \underset{\Delta}{\subset} SO(1,3) \subset SO(1,3) \overset{\text{semi}}{\text{product}} \mathbb{R}^{1,3}$

$U(\Lambda, a) = U(1,a)\, U(\Lambda$

$[T_a, T_b] = i f_{abc} T_c$

real

completely

anti-symmetric

ε_{abc}

SU(2) – unitary

compact

simple

$m=2$ defining rep

$m=3$

Lie

$SU(2) \subset SO(3) \subset SO(1,3) \subset SO(1,3)$ semi product $R^{1,3}$

$U(\Lambda, a) = U(1, a) U(\Lambda$

$[T_a, T_b] = i f_{abc} T_c$

$SU(2)$ – unitary
compact
simple

real
completely
anti-symmetric

ε_{abc}

$m = 2$ defining rep

$m = 3$ rotation

$T_1 = \begin{pmatrix} & & \\ & & -i \\ & i & \end{pmatrix}$

$(T_1)_{23} = -i\varepsilon_{123}$

Recipe

SU(2)

1 maximally commuting J_3 special

2 eigenvector of special $[J_3, \cdot] = \text{const} \cdot$

$\cdot = J_1 + a J_2$

3 Highest weight method

$J_\pm = \frac{1}{\sqrt{2}}(J_1 \pm i J_2)$

$[J_3, J_\pm] = \pm J_\pm$

eigenvalue

$O(3) \subset SO(1,3) \subset SO(1,3) \overset{\text{semi product}}{\ltimes} R^{1,3}$

$U(\Lambda, a) = U(1, a)\, U(\Lambda, 0)$

$U(\Lambda, a)\,|\psi\rangle = |\psi'\rangle$

$[J_a, J_b] = i f_{abc} J_c$

real

completely anti-symmetric

ϵ_{abc}

$m = 2$ defining rep

$m = 3$ rotation

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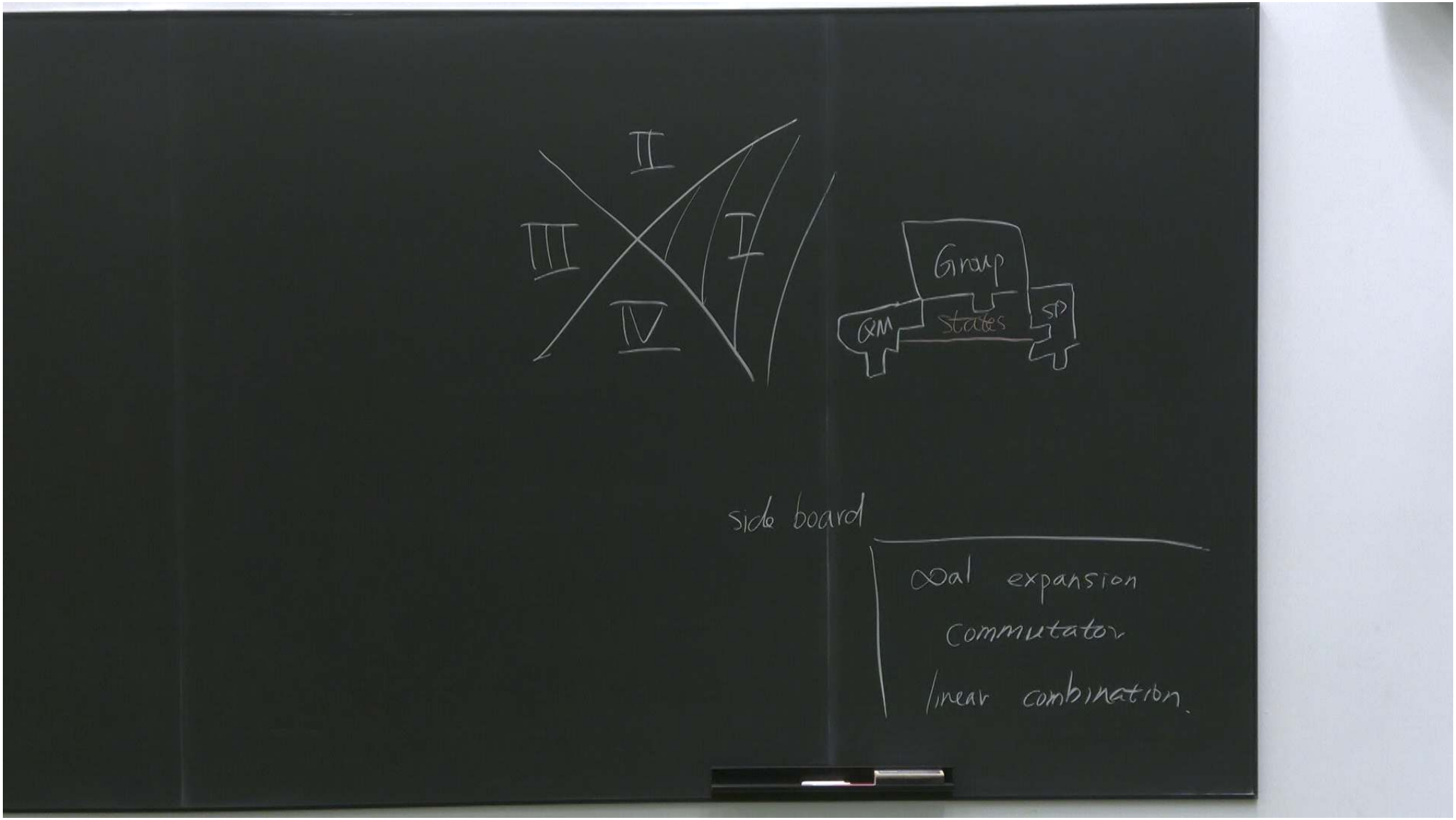
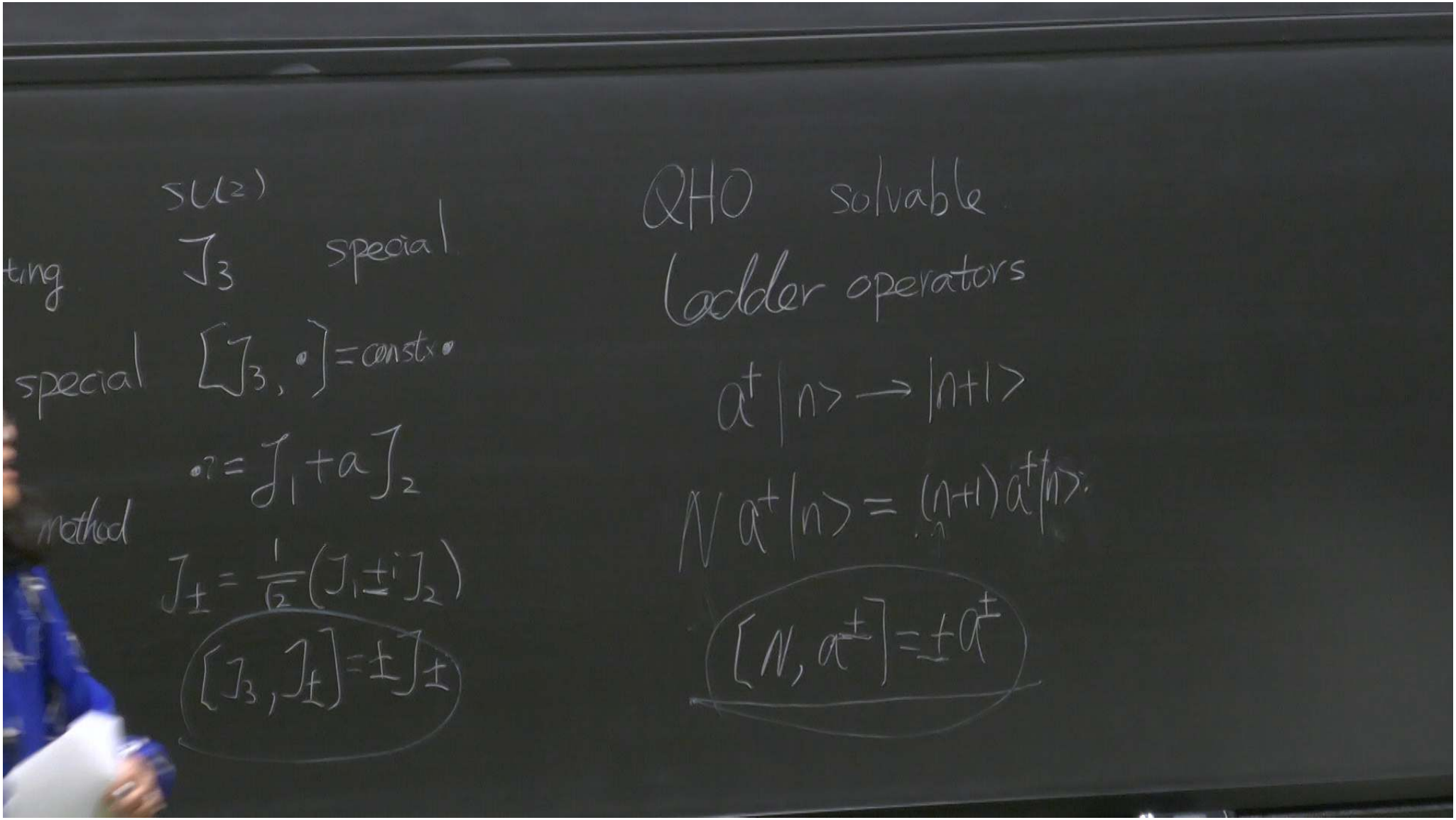
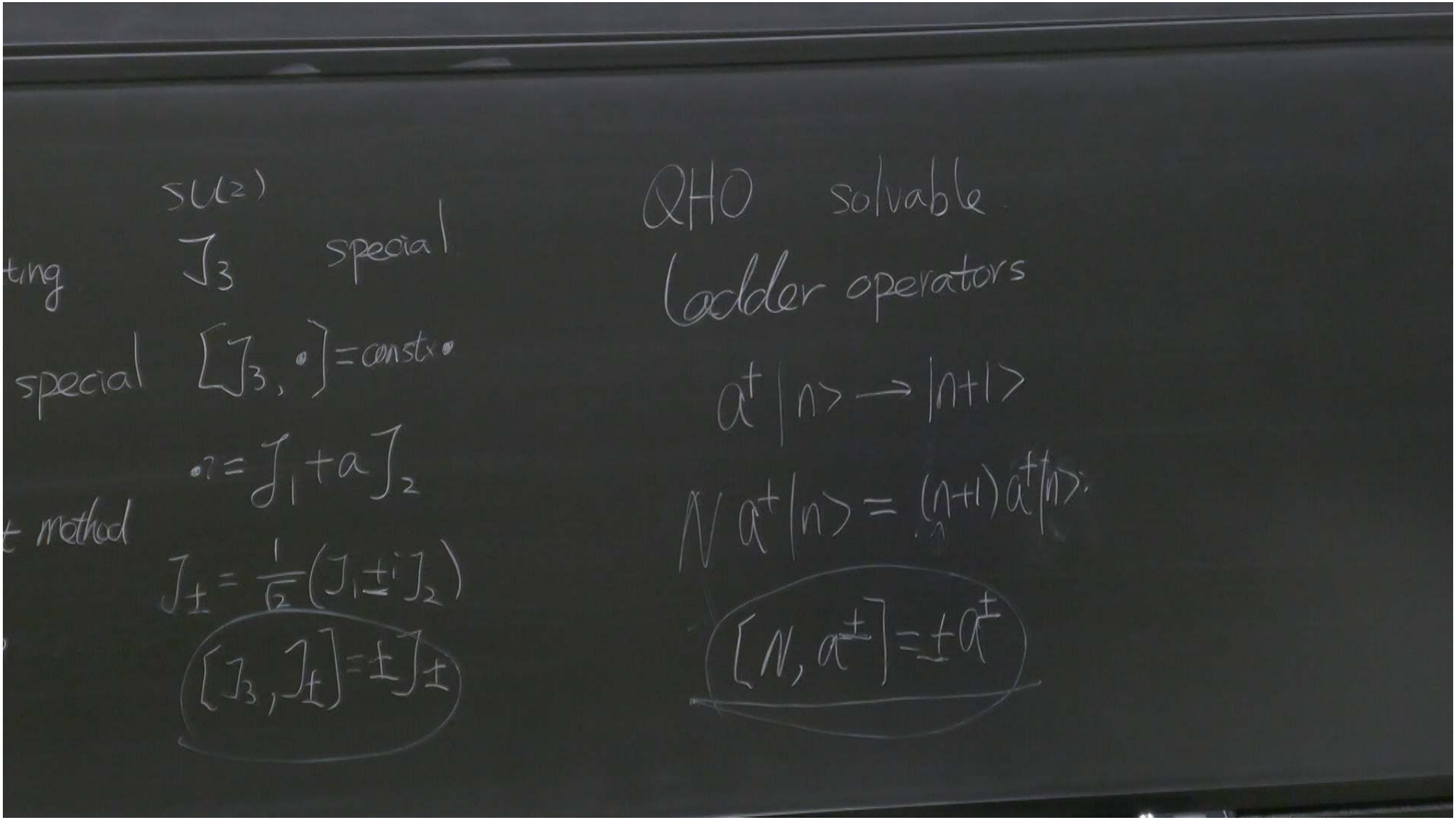
$[J_a, J_b] = i\epsilon_{abc} J_c$

$J_1\ J_2\ J_3$


II
III
I
IV
Group
QM
states
SP
side board
∞al expansion
Commutator.
linear combination.


SU(2)
J_3 special
$[J_3, \cdot] = \text{const} \cdot$
$\cdot = J_1 + aJ_2$
$J_\pm = \frac{1}{\sqrt{2}}(J_1 \pm iJ_2)$
$[J_3, J_\pm] = \pm J_\pm$
QHO solvable
ladder operators
$a^\dagger |n\rangle \to |n+1\rangle$
$N a^\dagger |n\rangle = (n+1) a^\dagger |n\rangle$
$[N, a^\pm] = \pm a^\pm$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J_3 special
$[J_3, \cdot] = \text{const} \cdot \cdot$
$\cdot = J_1 + aJ_2$
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Recipe

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2 eigenvector of special

3 Highest weight method

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$SU(2)$

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$[N, a^\pm] = \pm a^\pm$

$\delta_{nn'}$

① $\hat{N}|n\rangle = n|n\rangle$

② $a^{\pm}|n\rangle \propto |n\pm1\rangle$

③ $a|0\rangle = 0$ (bc)

SU(2)

①′ $J_3|m\rangle = m|m\rangle$ $\quad \langle m'|m\rangle = \delta_{m'm}$

②′ $J_\pm|m\rangle \longrightarrow |m\pm1\rangle$

③ $J_+|j\rangle = 0$ (bc)

$J_-|m\rangle = N(m)|m-1\rangle$

$\langle m-1|J_-|m\rangle = N(m)$

$J_+|m\rangle = N(m+1)|m+1\rangle$

$\delta_{nn'}$

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$J_-|m\rangle = N(m)|m-1\rangle$

$\langle m-1|J_-|m\rangle = N(m)$

$J_+|m\rangle = N(m+1)|m+1\rangle$

$$[J_+, J_-] = J_3$$

$$\langle m | [J_+, J_-] | m \rangle = \langle m | J_3 | m \rangle$$

$$J_+ J_- - J_- J_+$$

$$|J_- |m\rangle|^2 - |J_+ |m\rangle|^2 = m$$

$$\begin{cases} N^2(m) - N^2(m+1) = m \\ N(j+1) = 0 \end{cases}$$

$$N(m) = \sqrt{(j+m)(j-m+1)}$$

$$[J_+, J_-] = J_3$$

$$\langle m | [J_+, J_-] | m \rangle = \langle m | J_3 | m \rangle$$

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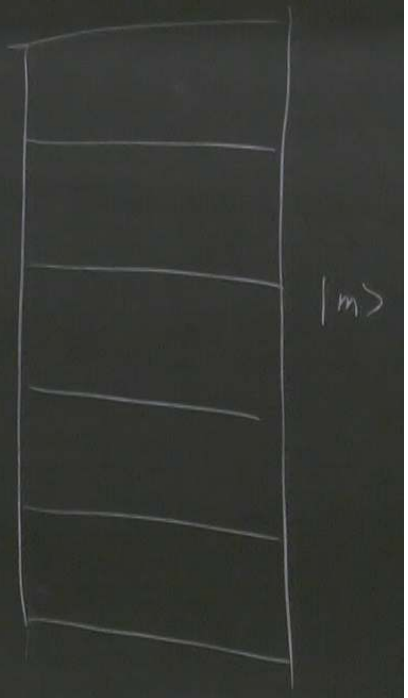
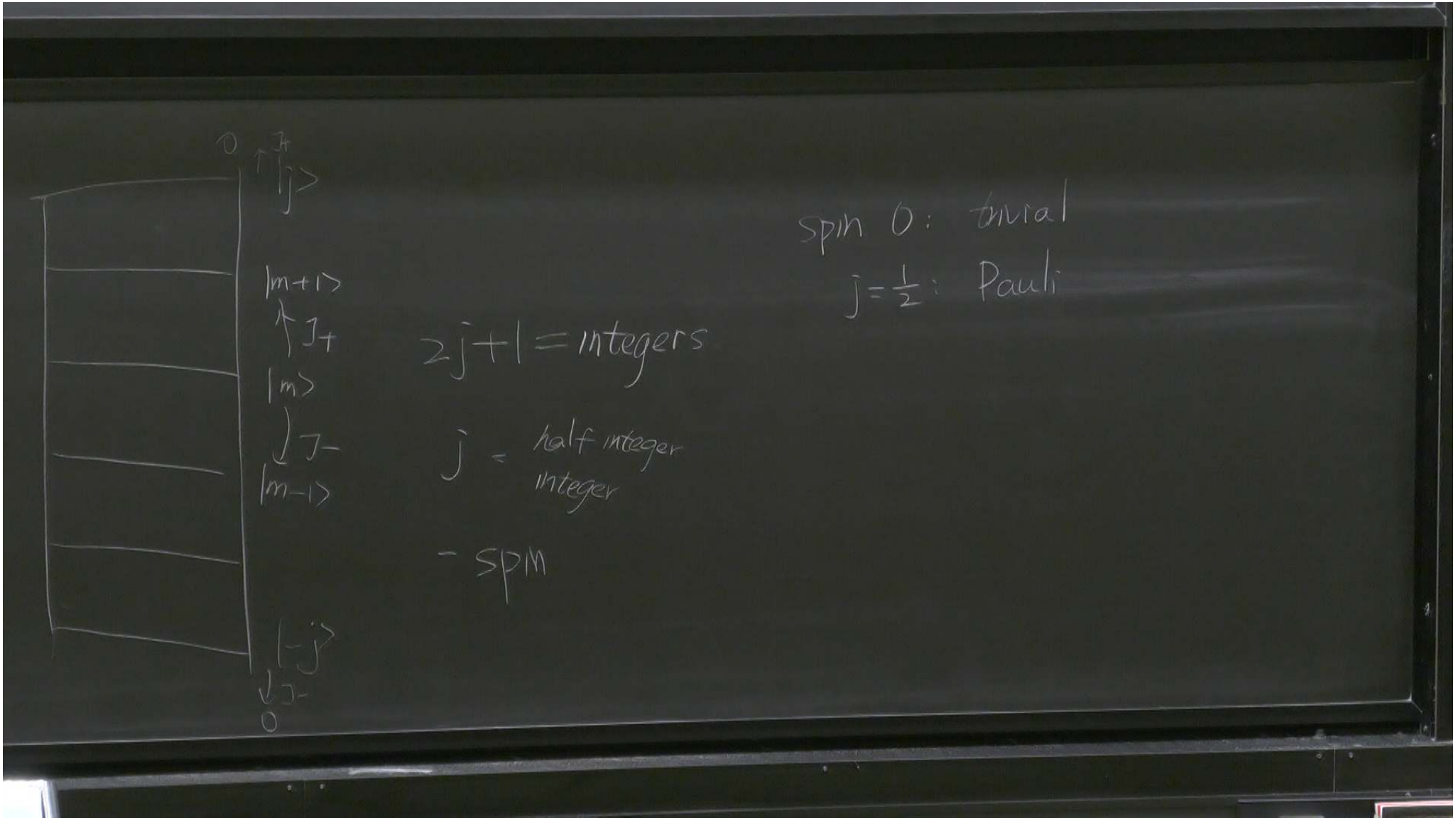
$m = -j$ $\quad$ $m = j+1$

$$J_- |-j\rangle = 0$$




|m+1>
J+
|m>
J-
|m-1>
|-j>
J-
spin 0: trivial
j = 1/2: Pauli
2j+1 = integers
j = half integer
integer
- spin

$$[J_+, J_-] = J_3$$

$$\langle m | [J_+, J_-] | m \rangle = \langle m | J_3 | m \rangle$$

$$J_+ J_- - J_- J_+$$

$$|J_- |m\rangle|^2 - |J_+ |m\rangle|^2 = m$$

$$\begin{cases} N^2(m) - N^2(m+1) = m \\ N(j+1) = 0 \end{cases}$$

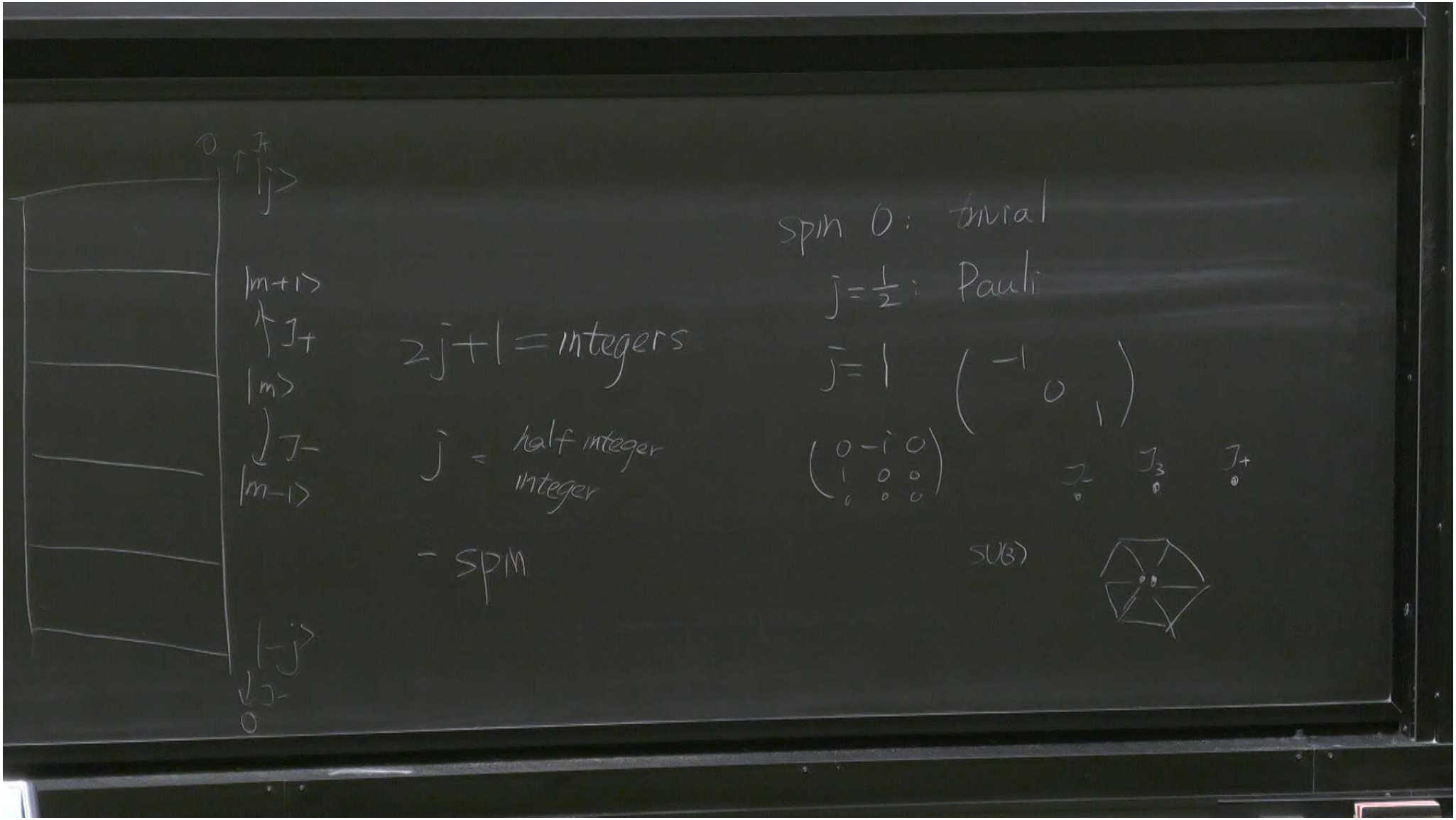
$$N(m) = \sqrt{(j+m)(j-m+1)}$$

$m = -j$ $\quad$ $m = j+1$

$$J_- |-j\rangle = 0$$

$$(J_+)_{m'm} = \langle m' | J_+ | m \rangle$$

$$= N(m+1)\, \delta_{m', m+1}$$


0
J+
|j>
|m+1>
J+
|m>
J-
|m-1>
|-j>
J-
0
2j+1 = integers
j = half integer
integer
- spin
spin 0: trivial
j = 1/2: Pauli
j = 1
J-
J3
J+
SU(3)

$U(\varepsilon,\omega)|\psi\rangle = |\psi'\rangle$

$U(\varepsilon,\omega) = e^{i[\varepsilon_\mu P^\mu + \frac{1}{2}\omega_{\mu\nu} J^{\mu\nu}]}$

non-abelian

non-compact

not-simple

not-semi-simple

oh crap!

$[J_i, P_j] = \epsilon_{ijk} P_k$

$[P^\mu, P^\nu] = 0$

quantum state

$|p, s_{3m}\rangle$

$U(1,a)|p,s\rangle = e^{i a_\mu p^\mu}|p,s\rangle$

$U(\Lambda,a) = U(1,a)U(\Lambda,0)$

$U(\Lambda)|p,s\rangle$

$$U(\Lambda)|p,s\rangle = \sum_{s'} C_{s's}(\Lambda p)|\Lambda p, s'\rangle$$

$$\Lambda p = p$$

new name

$$\underbrace{W^{\mu}_{\ \nu}}_{\text{little group}} k^{\nu} = k^{\mu}$$

$$U(W)|k,s\rangle = \underbrace{D_{s's}(W)}|k,s'\rangle$$

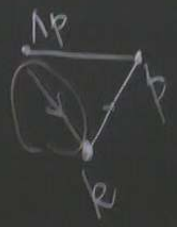
$$U(W_1)U(W_2) = U(W_1 W_2)$$

$\Downarrow$ $D_{s's}(W)$ is rep of W

$$k^{\mu} = \begin{pmatrix} m \\ 0 \\ 0 \\ 0 \end{pmatrix} \longrightarrow SO(3) \downarrow SU(2)$$

$|p,s\rangle$: $p^\mu = L^\mu_\nu(p) k^\nu$

$= U(L(\Lambda p))\, U(W(\Lambda,p))\, |k,s\rangle$



$|p,s\rangle \equiv U(L(p))\, |k,s\rangle$

$U(\Lambda)\, |p,s\rangle = U(\Lambda)\, U(L(p))\, |k,s\rangle$

$= U(L(\Lambda p))\, U(L^{-1}(\Lambda p))\, U(\Lambda)\, U(L(p))\, |k,s\rangle$

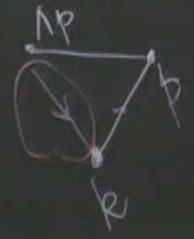
$p^\mu = L^\mu_\nu(p) k^\nu$

$|p,s\rangle \equiv U(L(p))|k,s\rangle$

$|\Lambda p, s\rangle = U(\Lambda) U(L(p))|k,s\rangle$

$= U(L(\Lambda p)) \underbrace{U(L^{-1}(\Lambda p)) U(\Lambda) U(L(p))}_{U(W(\Lambda,p))} |k,s\rangle$

$L^{-1}(\Lambda p)\Lambda L(p)$

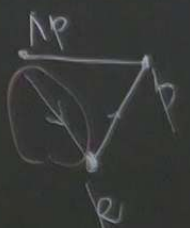


$= \sum_{s'} D_{s's}(W(\Lambda,p)) U(L(\Lambda p))|k,s'\rangle$

$U(\Lambda)|p,s\rangle = \sum_{s'} D_{s's}(W(\Lambda,p))|\Lambda p, s'\rangle$

$L^{-1}(\Lambda p) \Lambda L(p)$

$= U(L(\Lambda p)) \; U(W(\Lambda, p)) \; |k, s\rangle$



$= \sum_{s'} D_{s's}(W(\Lambda, p)) \; U(L(\Lambda p)) \; |k, s'\rangle$

$|k, s\rangle$

$(p)) |k, s\rangle$

$U(\Lambda) U(L(p)) |k, s\rangle \quad U(\Lambda) |p, s\rangle = \sum_{s'} D_{s's}(W(\Lambda, p)) \; |\Lambda p, s'\rangle$

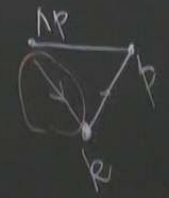
$|\psi'\rangle = U|\psi\rangle$

$|p,s\rangle$: $p^\mu = L^\mu_\nu(p) k^\nu$

$|p,s\rangle \equiv U(L(p))|k,s\rangle$

$U(\Lambda)|p,s\rangle = U(\Lambda) U(L(p))|k,s\rangle$

$= U(L(\Lambda p)) \underbrace{U(L^{-1}(\Lambda p)) U(\Lambda) U(L(p))}_{U(W(\Lambda,p))} |k,s\rangle$



$L^{-1}(\Lambda p)\Lambda L(p)$

$= U(L(\Lambda p)) U(W(\Lambda,p)) |k,s\rangle$

$= \sum_{s'} D_{s's}(W(\Lambda,p)) U(L(\Lambda p)) |k,s'\rangle$

$U(\Lambda)|p,s\rangle = \sum_{s'} D_{s's}(W(\Lambda,p)) |\Lambda p, s'\rangle$

$|p\rangle = U|+\rangle$