Title: QFT1 Lecture - 101123

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Collection: Quantum Field Theory 1 2023/24

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$= U|\psi\rangle$

$\hookrightarrow$ irrep $\rightarrow$ states

ition

cture

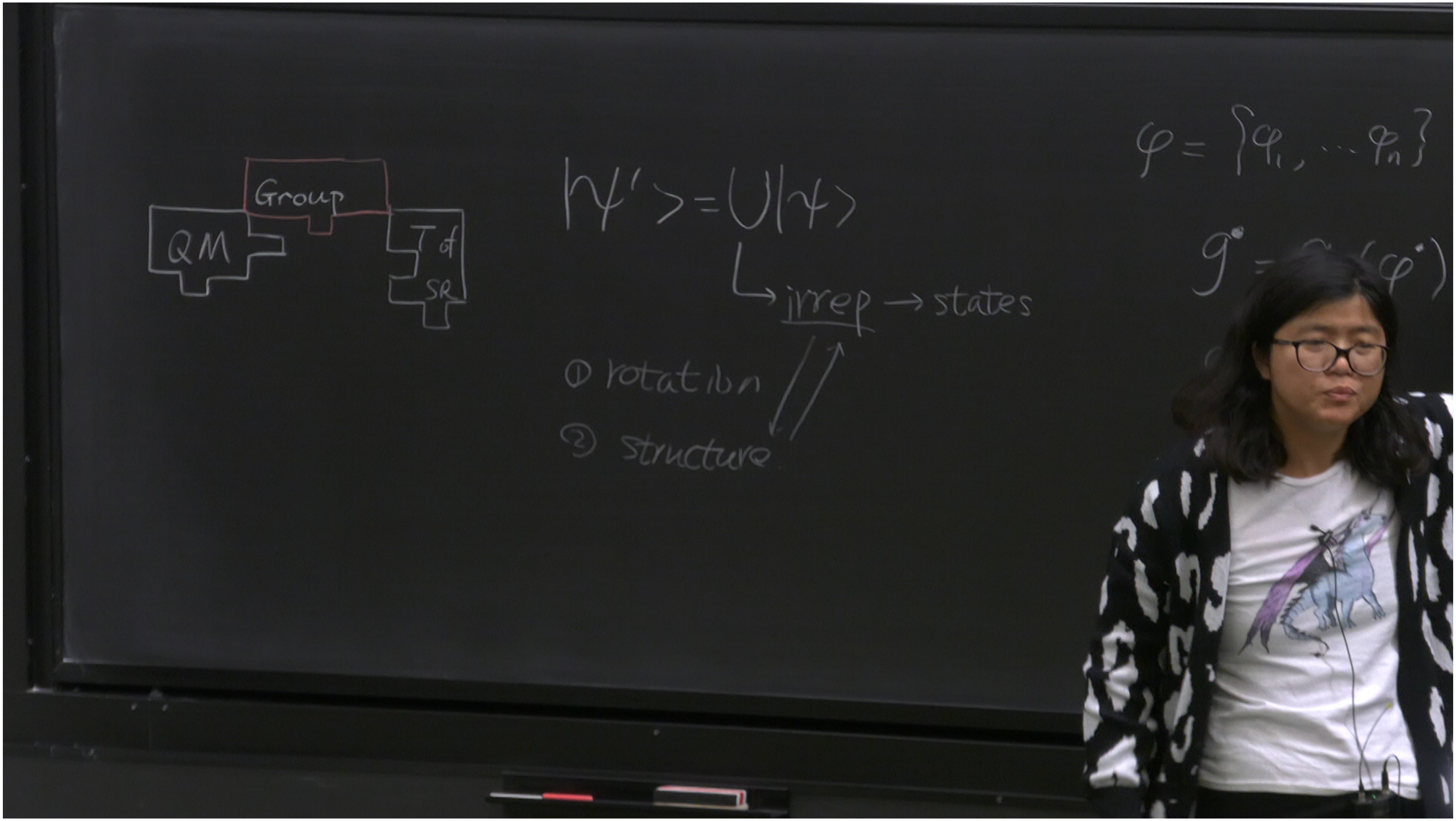
$$\varphi = \{\varphi_1, \dots \varphi_n\}$$

$$g^{\bullet} = g(\varphi^{\bullet})$$

$$g^{\text{✡}}$$

$$g^{\bullet} \circ g^{\text{✡}} = g^{\triangle}$$

$$\varphi^{\bullet} \cdot \varphi^{\text{✡}} \qquad \varphi^{\triangle} =$$


Group
QM
T of SR
|ψ'> = U|ψ>
irrep → states
① rotation
② structure
φ = {φ1, ... φn}

states

$$\varphi = \{\varphi_1, \dots \varphi_n\}$$

$$g^{\bullet} = g(\varphi^{\bullet})$$

$$g^{\star}$$

$$g^{\bullet} \circ g^{\star} = g^{\triangle}$$

$$\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ \varphi^{\bullet} \, . & \varphi^{\star} & \varphi^{\triangle} = f(\varphi^{\bullet}, \varphi^{\star}) \end{array}$$

$$g^{\circ} \circ g^{*} = g^{\varphi}$$

$$\uparrow \qquad \uparrow \qquad \uparrow$$

$$\varphi^{\circ} \cdot \varphi^{*} \qquad \varphi^{\varphi} = f(\varphi^{\circ}, \varphi^{*})$$

$$f(0,0) = 0$$

$$f(\varphi^{\circ}, 0) = f(\varphi^{\circ}) = \varphi^{\circ}$$

$$f(0, \varphi^{*}) = \varphi^{*}$$

unitary rep:

$$U(g^{\circ})\, U(g^{*}) = U(g^{\varphi})$$

$$U(\varphi^{\circ})\, U(\varphi^{*}) = U(f(\varphi^{\circ}, \varphi^{*}))$$

$$U_{m\times m}(\epsilon) = \mathbb{1}_{m\times m} + \sum_a i\epsilon_a (T_a)_{m\times m}$$

$$e^{x} = \lim_{N\to\infty}\left(1 + \frac{x}{N}\right)^{N}$$

$$U(\varphi) \equiv \lim_{N\to\infty}\left(1 + \sum_a \frac{\varphi_a T_a}{N}\right)^{N} \equiv e^{\sum_a i \varphi_a T_a}$$

$$U(\varepsilon)U^\dagger(\varepsilon)=1$$

$$T_a$$

$$(\mathbb{1}+i\epsilon_a T_a)(\mathbb{1}-i\epsilon_a T_a^\dagger)=\mathbb{1}$$

1st order $\quad i\epsilon_a T_a - i\epsilon_a (T_a)^\dagger = 0$

$$U(\varepsilon)\,U^{\dagger}(\varepsilon)=1$$

2nd Order

$$U(\epsilon)=1+i\epsilon_a T_a+\frac{1}{2}\epsilon_a\epsilon_b l_{ab}$$

$$T_a$$

$$(\mathbb{1}+i\epsilon_a T_a)(\mathbb{1}-i\epsilon_a T_a^{\dagger})=\mathbb{1}$$

1st order $i\epsilon_a T_a - i\epsilon_a (T_a)^{\dagger}=0$

$$f_a(\varepsilon', \varepsilon^{*}) = \cancel{A_a}^{\,0} + B_{ba}\epsilon_b' + C_{ba}\epsilon_b^{*}$$
$$+ D_{bca}\,\epsilon_b'\epsilon_c^{*}$$
$$+ E_{bca}\epsilon_b'\epsilon_c' + F_{bca}\epsilon_b^{*}\epsilon_c^{*}$$

$$+\tfrac{1}{2}\epsilon_a\epsilon_b l_{ab}$$

$$\overset{\to 0}{\cancel{A_a}} + \overset{\to \delta_{ba}}{B_{ba}}\,\epsilon_b^{\circ} + \overset{\to \delta_{ba}}{C_{ba}}\,\epsilon_b^{*}$$

$$+ D_{bca}\,\epsilon_b^{\circ}\epsilon_c^{*}$$

$$+ \overset{\to 0}{\cancel{E_{bca}\epsilon_b^{\circ}\epsilon_c^{\circ}}} + \overset{\to 0}{\cancel{F_{bca}\epsilon_b^{*}\epsilon_c^{*}}}$$

$$f_a(\epsilon^{\circ}, \epsilon^{*}) = \varepsilon_a^{\circ} + \varepsilon_a^{*} + D_{bca}\,\varepsilon_b^{\circ}\varepsilon_c^{*}$$

$$U(f(\varepsilon', \varepsilon^*)) = \qquad = U(\varepsilon')\,U(\varepsilon^*)$$

$$1 + i(\epsilon_a + \epsilon_a^*)T_a + iD_{bca}\varepsilon_b'\varepsilon_c^* T_a = \left(1 + i\epsilon_a' T_a + \tfrac{1}{2}\cancel{\epsilon_a \epsilon_b}\, l_{ab}\right)$$

$$+ \tfrac{1}{2}(\varepsilon_a' + \varepsilon_a^*)(\epsilon_b' + \epsilon_b^*)\, l_{ab} \qquad \times \left(1 + i\epsilon_c^* T_c + \tfrac{1}{2}\cancel{\epsilon_c^* \epsilon_d^*}\, l_{cd}\right)$$

$$\Sigma\, \epsilon_a' \epsilon_b^*\, l_{ab} + \sum_a iD_{bca}\epsilon_b'\epsilon_c^* T_a = -\epsilon_a'\epsilon_c^* T_a T_c$$

$$\boxed{[T_a, T_b] = \sum_c i f_{abc} T_c}$$

$$f_{abc} = D_{abc} - D_{bac}$$

$$e^A e^B = e^{A+B+\frac{1}{2}[A,B] + \#[A,[A,B]]}$$

$$g^\circ \cdot g^*$$

Adjoint rep: $\underset{\text{dim(rep)}}{m} = \underset{\#\text{ of gen}}{n}$ dim(group)

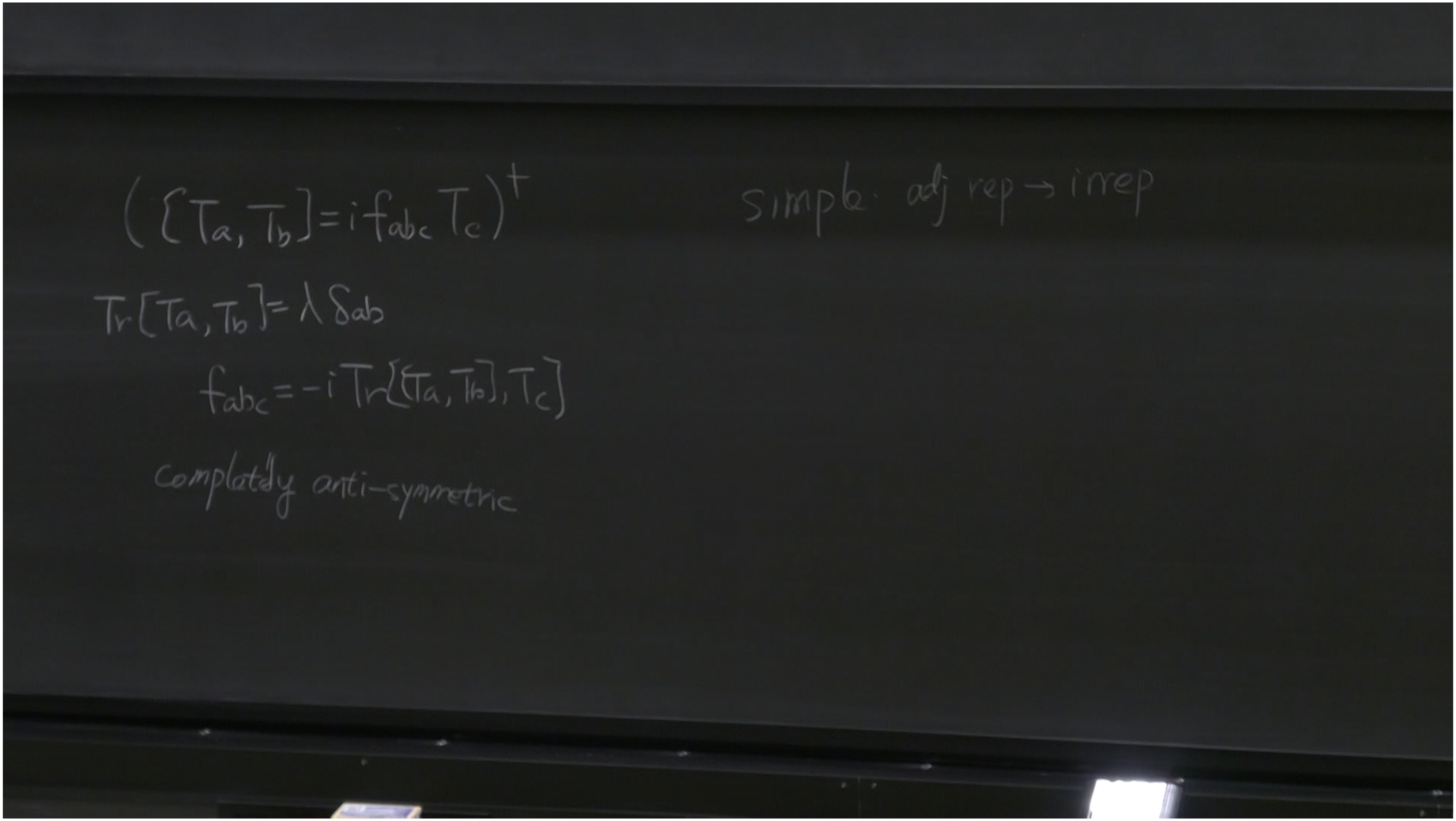
$$(T_a^{adj})_{bc} = -i f_{abc}$$

$$[T_a, T_b] = \sum_c i f_{abc} T_c$$

$$[[T_a, T_b], T_c] + \text{cyclic} = 0$$

inner product: $\mathrm{Tr}[T_a T_b] = \lambda \delta_{ab}$

$$T_a | T_b \rangle = | \quad \rangle$$


$\left([T_a, T_b] = i f_{abc} T_c \right)^\dagger$
simple: adj rep $\rightarrow$ irrep
$\mathrm{Tr}[T_a, T_b] = \lambda \delta_{ab}$
$f_{abc} = -i \, \mathrm{Tr}[[T_a, T_b], T_c]$
completely anti-symmetric

simple · adj rep → irrep

↓

simple – no nontrivial (identity) invariant subgroup

$\underbrace{\text{invariant subgroup}}_{\text{Normal}}$

commute $\quad g_1 g_2 = g_2 g_1$

somewhat commute. $\quad g h_1 = h_2 g$

$\forall g \in G$

$h_1\, h_2 \in N$

$$g \tilde{\oslash} = \tilde{\oslash} g$$

Mattress $\{e, H, V, HV\}$

find $\{e, H\}$

separate group into piles

$\{e, H\} = \{H, e\}$

$\{V, VH\} = \{HV, V\}$

coset

product = # × coset × size of blob

= # group

$\{e, V\}$ = quotient group

$\{e, H\} \otimes \{e, V\}$

$= \{e, H, V, HV\}$

Mattress $\{e, H, V, HV\}$

find $\{e, H\}$

separate group into piles

e : $\{e, H\} = \{H, e\}$

coset V : $\{V, VH\} = \{HV, V\}$

product = # x coset x size of blob

= # group

$\{e, V\}$ = quotient group

$\{e, H\} \otimes \{e, V\}$

$= \{e, H, V, HV\}$