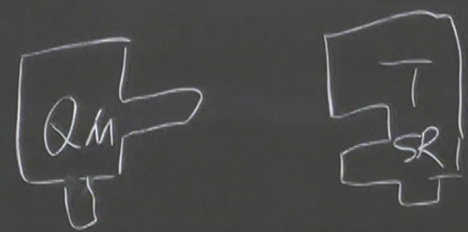
Title: QFT1 Lecture - 101023

Speakers: Gang Xu

Collection: Quantum Field Theory 1 2023/24

Date: October 10, 2023 - 10:45 AM

URL: https://pirsa.org/23100045



When QM meets SR Lorentz

$\downarrow$

inertial observator (Alice Bob)

state $|\psi\rangle$

Alice	Bob (rocket)
$|\psi\rangle$	$|\psi'\rangle$

big question. $|\psi'\rangle = O_{?=\text{Lorentz}}|\psi\rangle$

QM

$|\psi$

n QM meets SR Lorentz

$\downarrow$

inertial observator (Alice Bob)

tate $|\psi\rangle$

Alice $|\psi\rangle$

Bob (rocket) $|\psi'\rangle$

g question. $|\psi'\rangle = O_{?=\text{Lorentz}}|\psi\rangle$

QM

$|\psi\rangle \leftrightarrow |\psi'\rangle \qquad |\Phi\rangle \leftrightarrow |\Phi'\rangle$

probability

$$|\langle\psi|\Phi\rangle|^2 = |\langle\psi'|\Phi'\rangle|^2$$

$$\langle\psi'|\Phi'\rangle = \langle\psi|\Phi\rangle \qquad \langle\psi'|\Phi'\rangle = \langle\psi|\Phi\rangle^*$$

$$|\Phi'\rangle = U|\Phi\rangle$$

$$\langle\psi'|\Phi'\rangle = \langle\psi|U^\dagger U|\Phi\rangle$$

$$U^\dagger U = \mathbb{1}$$

QM

$|\psi\rangle \xrightarrow{S} |\psi'\rangle \qquad |\Phi\rangle \xrightarrow{S} |\Phi'\rangle$

probability

$$|\langle\psi|\Phi\rangle|^2 = |\langle\psi'|\Phi'\rangle|^2 \longrightarrow \text{Wigner theoren}$$

$\langle\psi'|\Phi'\rangle = \langle\psi|\Phi\rangle \qquad \langle\psi'|\Phi'\rangle = \langle\psi|\Phi\rangle^*$

$|\Phi'\rangle = U|\Phi\rangle$

$\langle\psi'|\Phi'\rangle = \langle\psi|U^\dagger U|\Phi\rangle$

$U^\dagger U = \mathbb{1}$

Alice Bob Charlie

$$|\psi\rangle \xrightarrow{U_1} |\psi'\rangle \xrightarrow{U_2} |\psi''\rangle$$

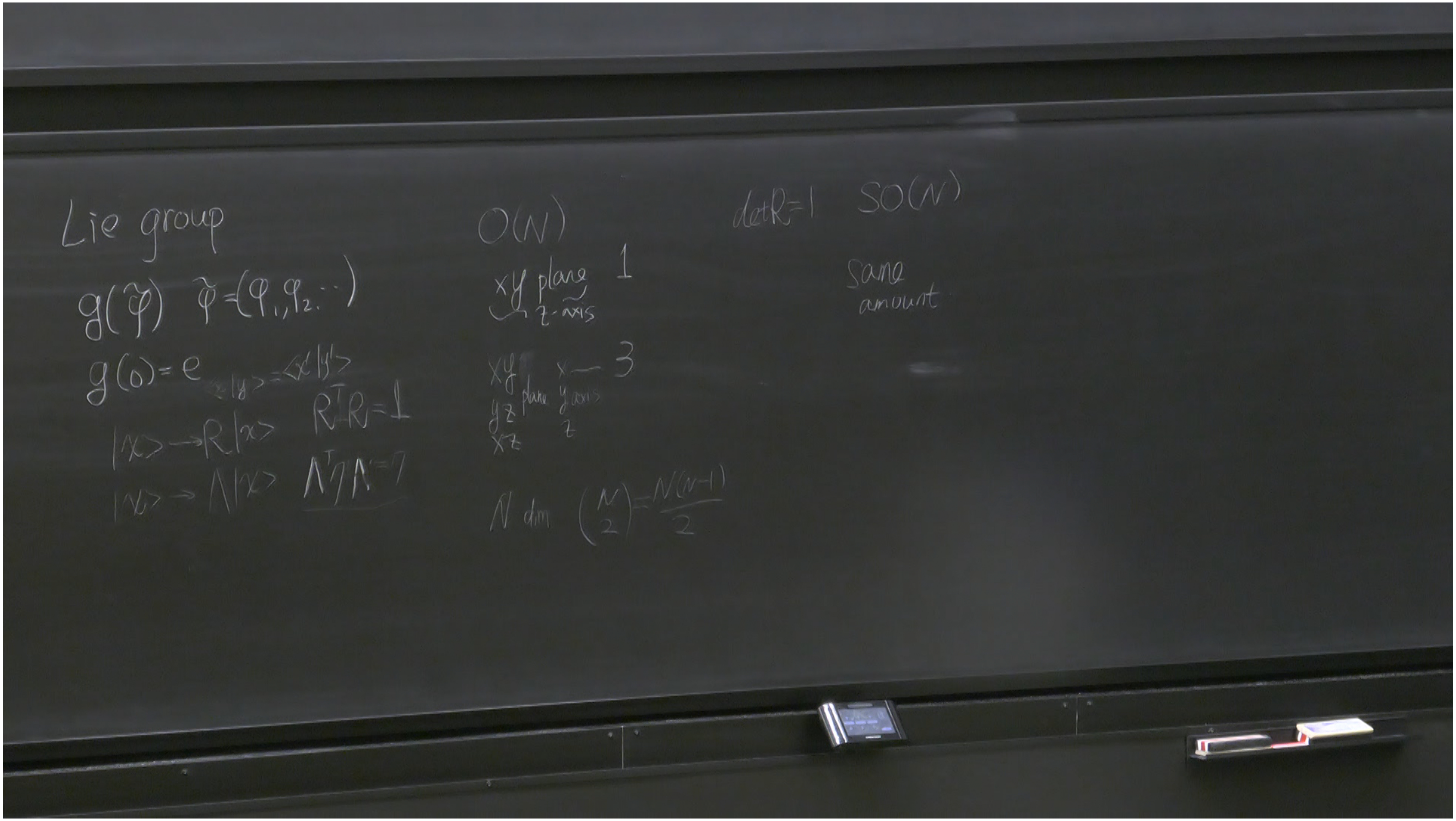
$$|\psi\rangle \xrightarrow{U_3} |\psi''\rangle$$

$|\psi'\rangle = U_1|\psi\rangle$

$|\psi''\rangle = U_2|\psi'\rangle$

$= U_2U_1|\psi\rangle$

$|\psi''\rangle = U_3|\psi\rangle$


Lie group
g(φ) φ=(φ1, φ2 ...)
g(0)=e
<x|y> = <x'|y'>
|x> → R|x>
R^T R = 1
|x> → Λ|x>
Λ^T η Λ = η
O(N)
xy plane 1
z-axis
3
xy plane
yz
xz
x axis
y axis
z
N dim
(N 2) = N(N-1)/2
detR=1
SO(N)
same
amount

rotate in complex plane $U(N)$

special $SU(N)$

$$U^{\dagger}U = 1 \qquad (U^{\dagger}U)^{\dagger} = U^{\dagger}U$$

real parameters

$$(\quad)^{\dagger}(\quad) = 1$$

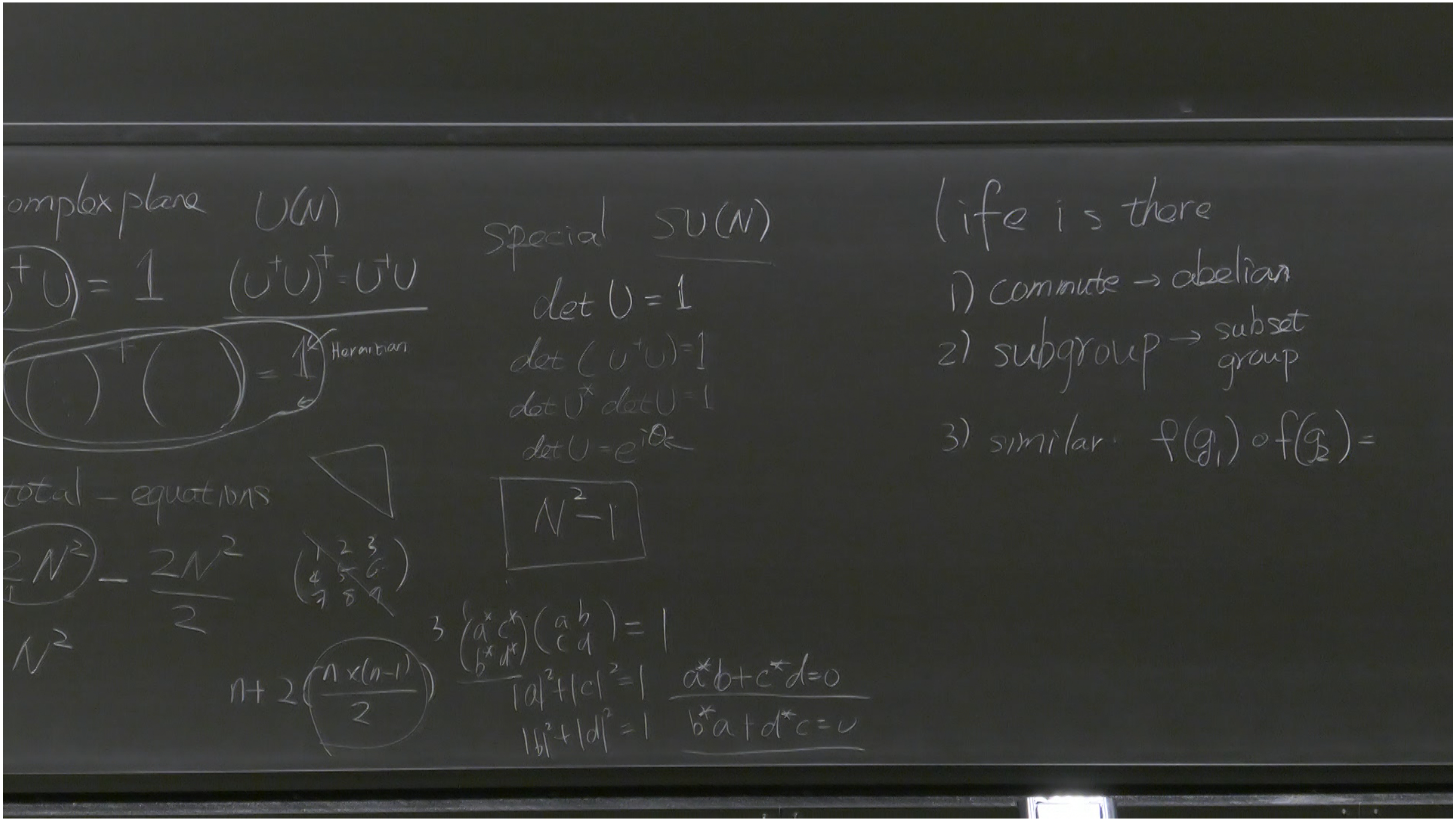
$$= \text{total} - \text{equations}$$

$$= 2N^2 - \frac{2N^2}{2}$$

$$= N^2$$

total

hermitian


omplex plane
U(N)
$U^\dagger U = 1$
$(U^\dagger U)^\dagger = U^\dagger U$
Hermitian
total - equations
$2N^2 - \frac{2N^2}{2}$
N^2
$n + 2\left(\frac{n \times (n-1)}{2}\right)$
Special SU(N)
$\det U = 1$
$\det(U^\dagger U) = 1$
$\det U^* \det U = 1$
$\det U = e^{i\theta}$
$\boxed{N^2 - 1}$
$\begin{pmatrix} a^* & c^* \\ b^* & d^* \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = 1$
$|a|^2 + |c|^2 = 1$
$|b|^2 + |d|^2 = 1$
$a^* b + c^* d = 0$
$b^* a + d^* c = 0$
life is there
1) commute → abelian
2) subgroup → subset group
3) similar $f(g_1) \circ f(g_2) =$

life is there

1) commute → abelian

2) subgroup → subset group

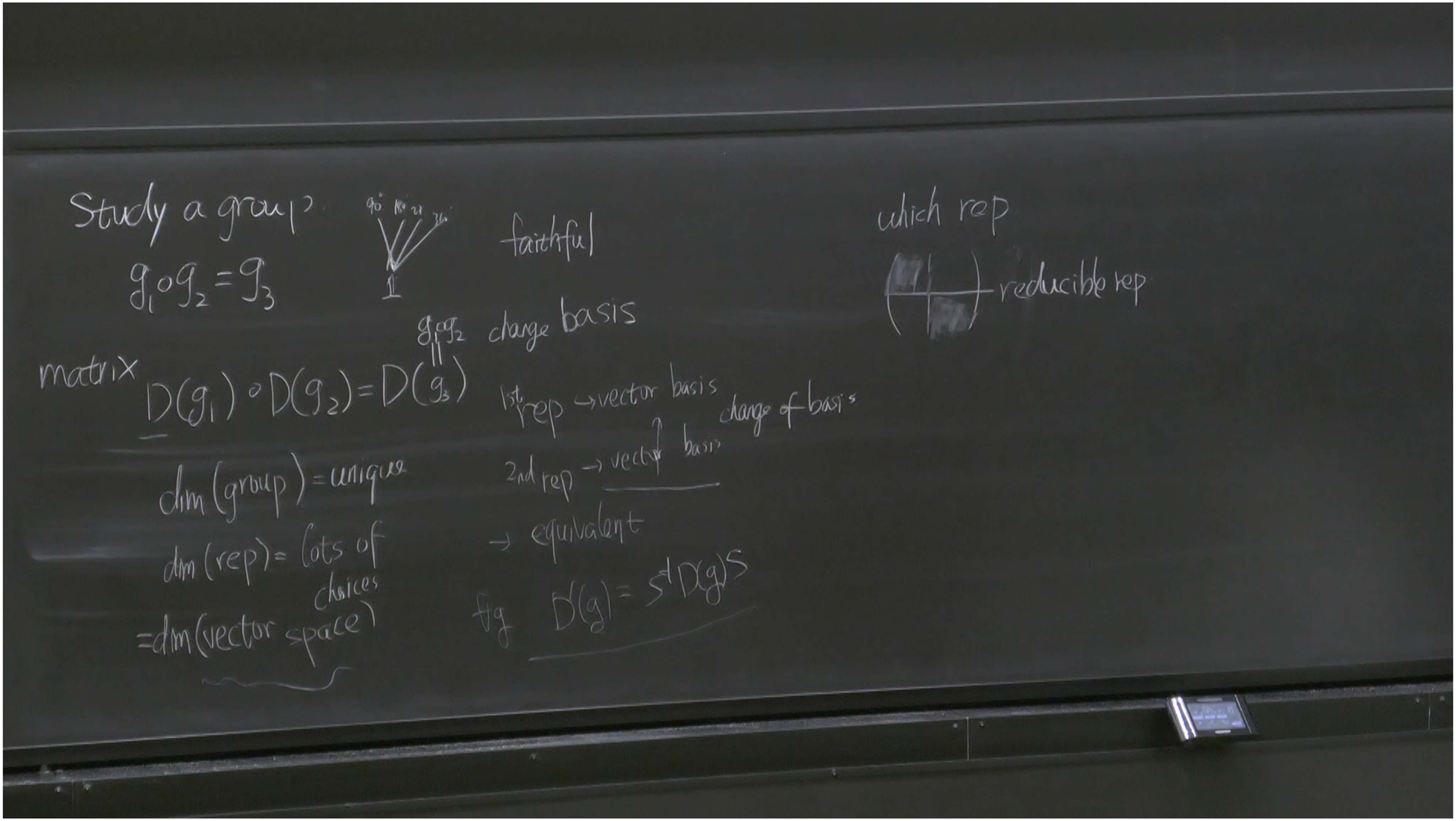
$SO(3)$ $SU(2)$

3) similar: $f(g_1) \circ f(g_2) = f(g_1 \circ g_2)$

4) same f: one to one

5) bigger $G \otimes G'$ (g, g')

$(g_1, g_1') \circ (g_2, g_2') = (g_1 \circ g_2, g_1' \circ g_2')$


Study a group?
$g_1 \circ g_2 = g_3$
faithful
which rep
reducible rep
$g_1 \circ g_2$ change basis
matrix $D(g_1) \circ D(g_2) = D(g_3)$
1st rep → vector basis
change of basis
2nd rep → vector basis
dim (group) = unique
dim (rep) = lots of choices
= dim (vector space)
→ equivalent
$\forall g$ $D'(g) = S^{-1} D(g) S$