Title: Relativity Lecture - 101323

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Collection: Relativity 2023/24

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HEORY FOR GRAVITY

CLASSICAL FIELD THEORIES

	MAXWELL	NEWTON	EINSTEIN
FIELD	$A^\mu \Leftrightarrow F^{\mu\nu}$	ϕ	$g_{\mu\nu}$
EQ. OF MOTION FOR MATTER	$\frac{dp^\mu}{d\tau} = eF^{\mu\nu}u_\nu$	$\frac{d\vec{p}}{dt} = -m\vec{\nabla}\phi$	$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = 0$
FIELD EQUATIONS	$\Box A^\mu = -4\pi J^\mu$ $\partial_\mu A^\mu = \emptyset$	$\nabla^2\phi = 4\pi G\rho$	???
DOF	2	$\emptyset$	2

WHAT ARE THE FIELD EQS?

(EDUCATED GUESS)

NEWTONIAN LIMIT: $g_{tt} = -1 - 2\phi$

FIELD EQS BETTER REDUCE TO

$$\nabla^2 \phi = 4\pi G \rho$$

$$-\nabla^2 g_{tt} = 8\pi G \rho = 8\pi G T_{tt}$$

GUESS: $\boxed{8\pi G T^{\mu\nu} = G^{\mu\nu}(\partial\partial g, \partial g, g)}$

WE "KNOW" FROM SR

$$\nabla_\mu T^{\mu\nu} = 0$$

SLIGHTLY MORE GENERAL DERIVATIVE.

CONSISTENCY =>

$$\boxed{\nabla_\mu G^{\mu\nu} = 0}$$

... DETERMINES $G^{\mu\nu}$ "UNIQUELY"

$G^{\mu\nu}$ ... EINSTEIN TENSOR (4D)

$$G^{\mu\nu} + \alpha g^{\mu\nu}$$

$$\boxed{G^{\mu\nu} + \Lambda g^{\mu\nu} = 8\pi G T^{\mu\nu}}$$

COSMOLOGICAL CONSTANT

EINSTEIN EQUATIONS

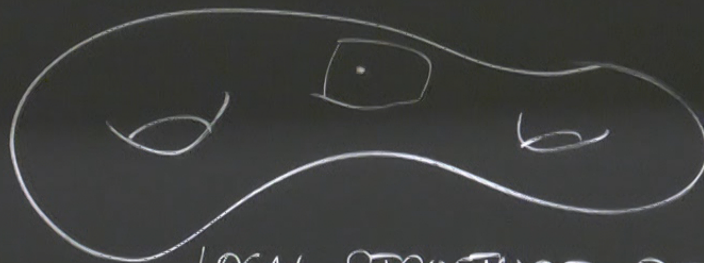
2) GEOMETRIC TOOLS

a) MANIFOLDS

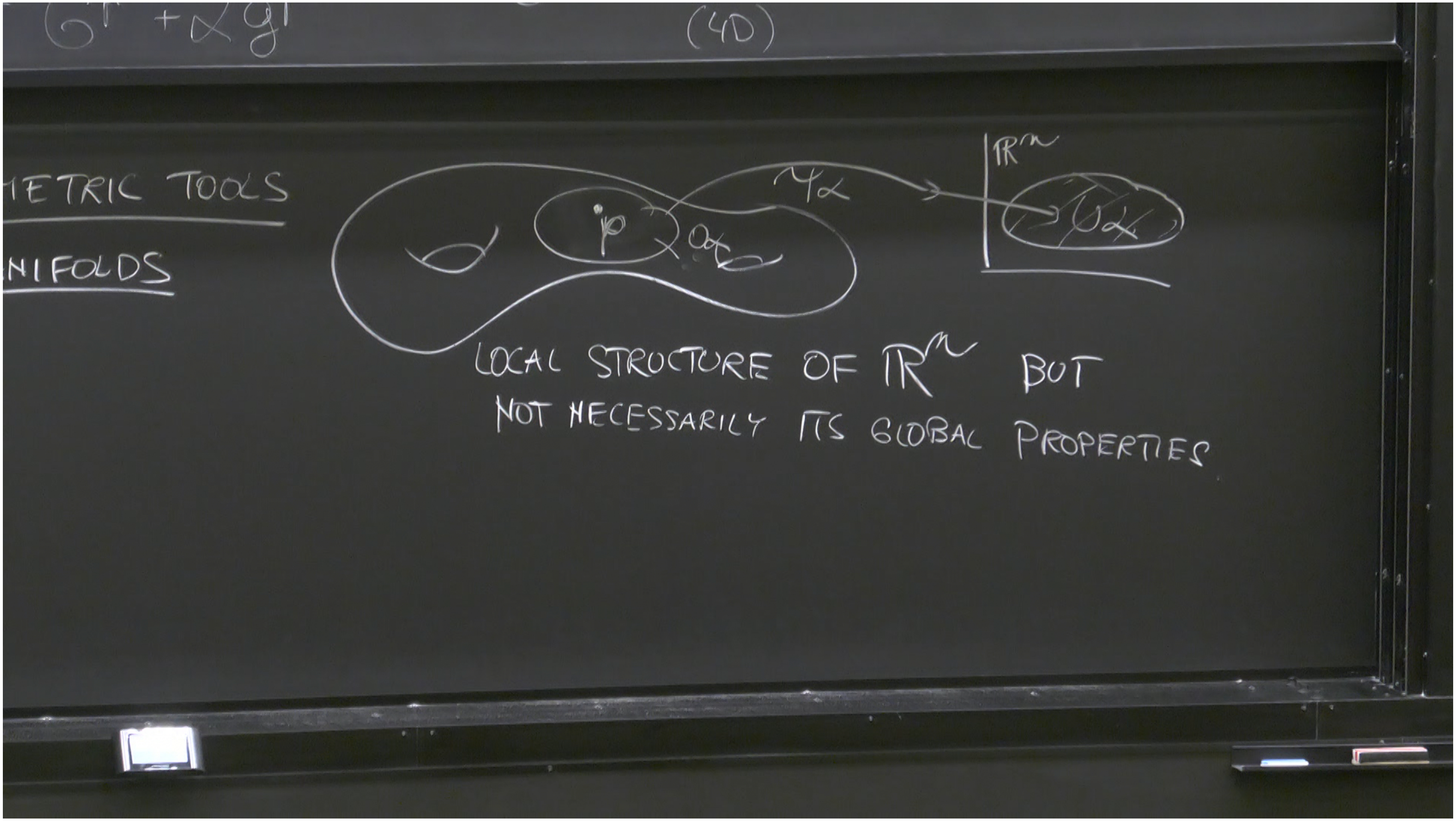
Let $G^{\mu\nu} = G^{\mu\nu} + \alpha g^{\mu\nu}$ (4D)

2) GEOMETRIC TOOLS

a) MANIFOLDS



LOCAL STRUCTURE OF $\mathbb{R}^n$ BUT NOT NECESSARILY ITS GLOBAL PROPERTIES


(4D)
ETRIC TOOLS
NIFOLDS
ℝⁿ
φα
p
Oα
LOCAL STRUCTURE OF ℝⁿ BUT
NOT NECESSARILY ITS GLOBAL PROPERTIES

DEF: AN n-DIM. MANIFOLD M IS A 'SET OF POINTS' TOGETHER WITH A COLLECTION OF SUBSETS $\{O_\alpha\}$ SATISFYING:

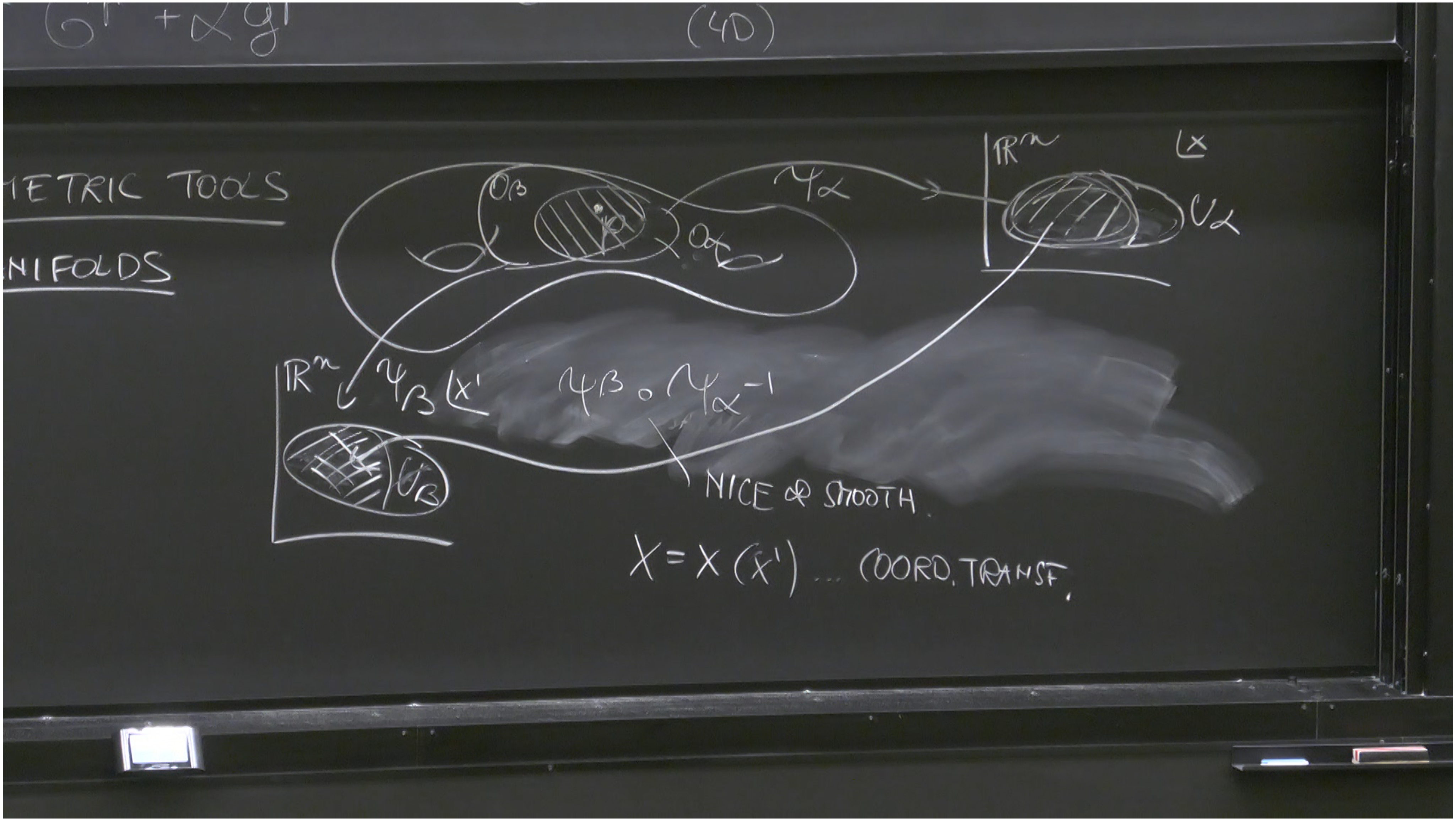
i) EACH $p \in M$ LIES IN AT LEAST ONE O_α

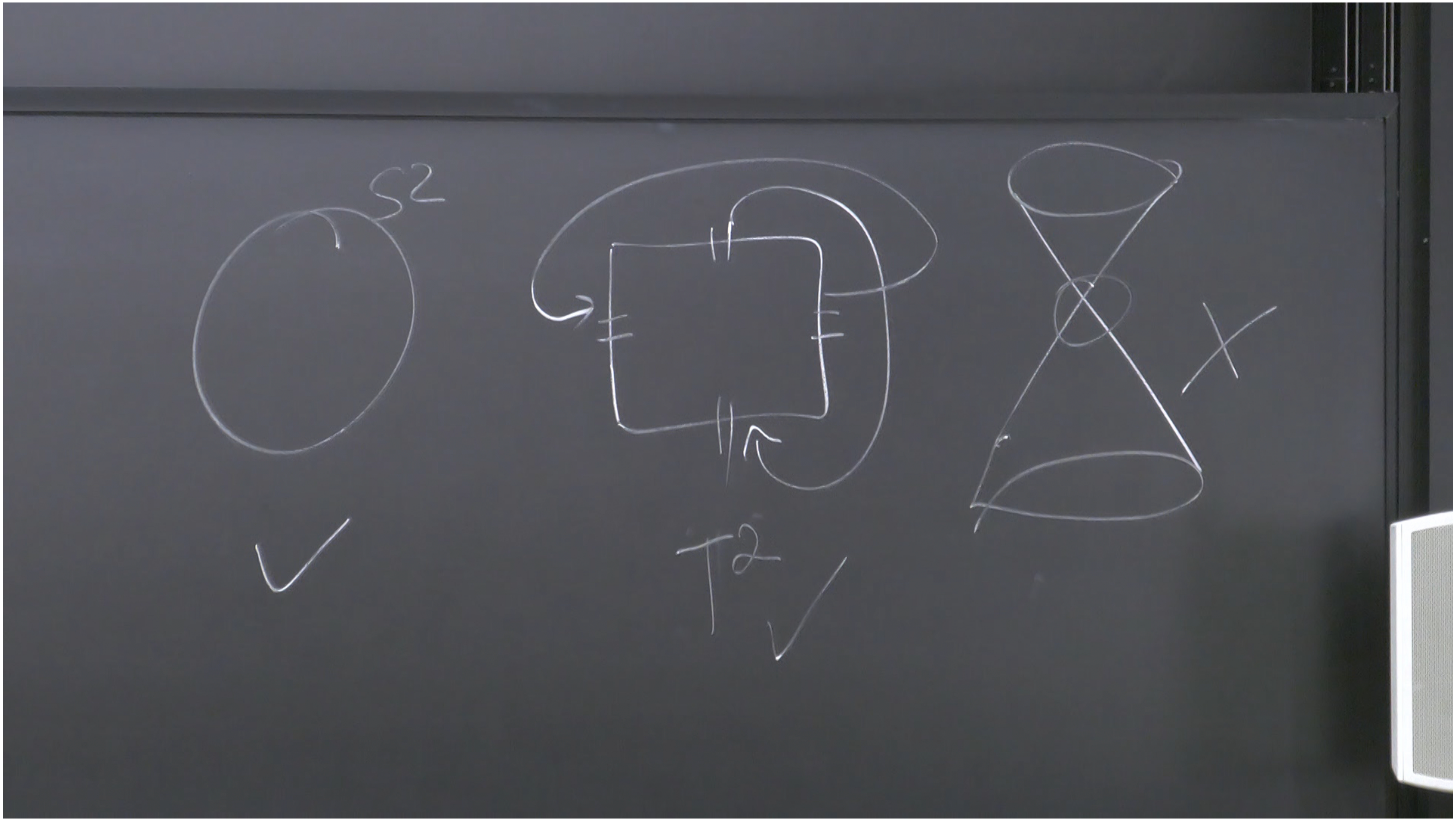
$\{O_\alpha\}$ COVER M.

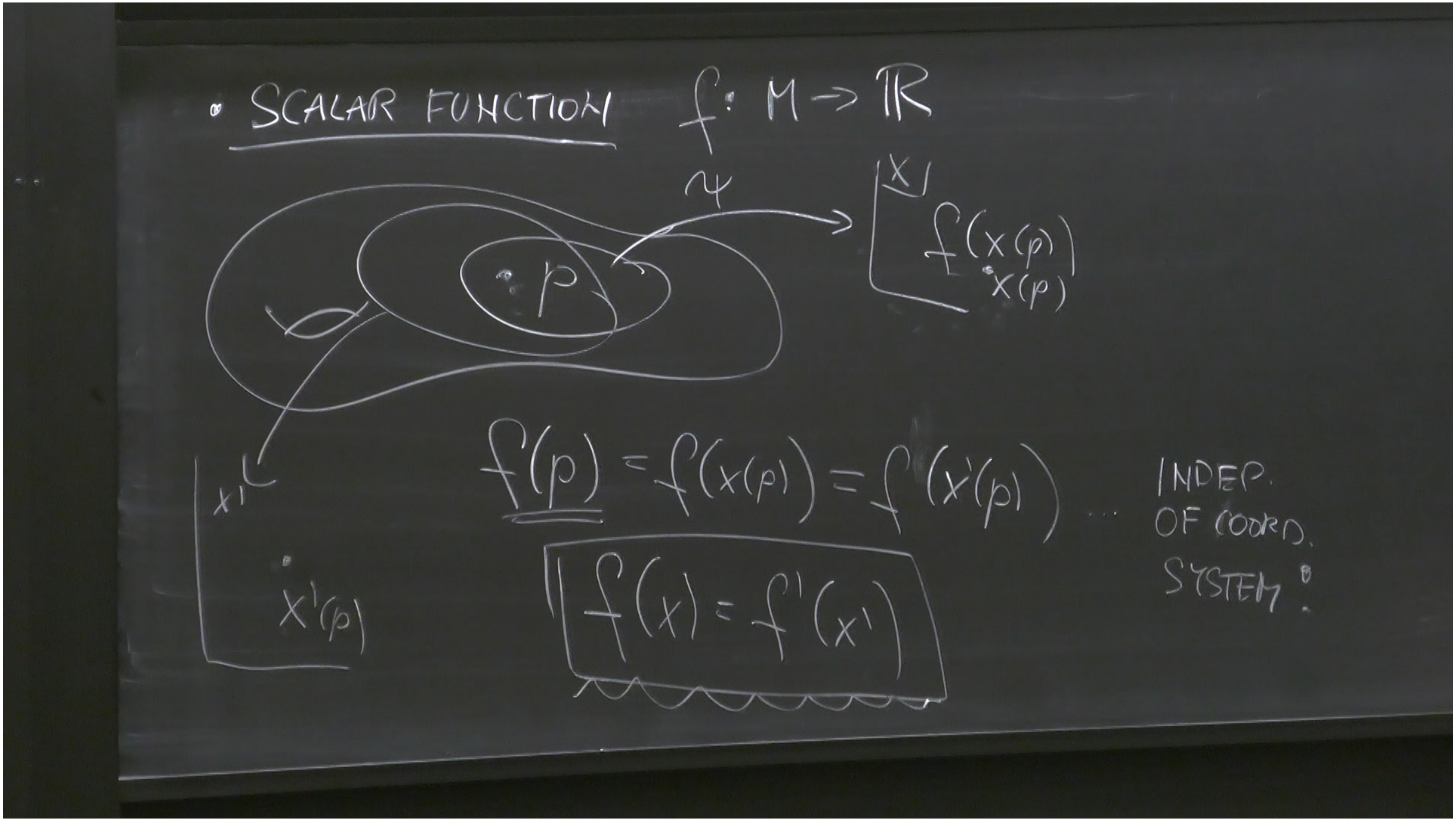
ii) FOR EACH α, THERE IS 1-1, ONTO, MAP $\psi_\alpha : O_\alpha \to U_\alpha$, WHERE U_α IS AN OPEN SUBSET OF $\mathbb{R}^n$

iii) IF ANY TWO SETS O_α AND O_β OVERLAP, $O_\alpha \cap O_\beta \neq 0$

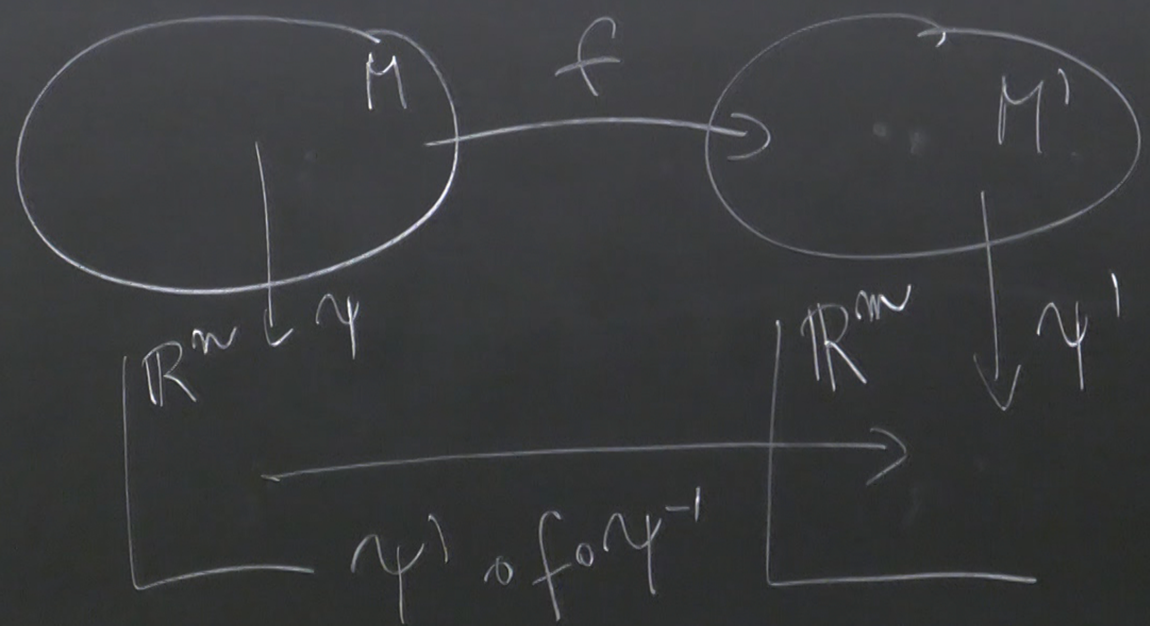
$\Rightarrow$ MAPS $\psi_\beta \circ \psi_\alpha^{-1}$ IS C^∞.


(4D)
ETRIC TOOLS
NIFOLDS
O_β
O_α
ψ_α
$\mathbb{R}^n$
x
U_α
$\mathbb{R}^n$
ψ_β
x'
$\psi_\beta \circ \psi_\alpha^{-1}$
U_β
NICE & SMOOTH
$X = X(X')$ ... COORD. TRANSF.

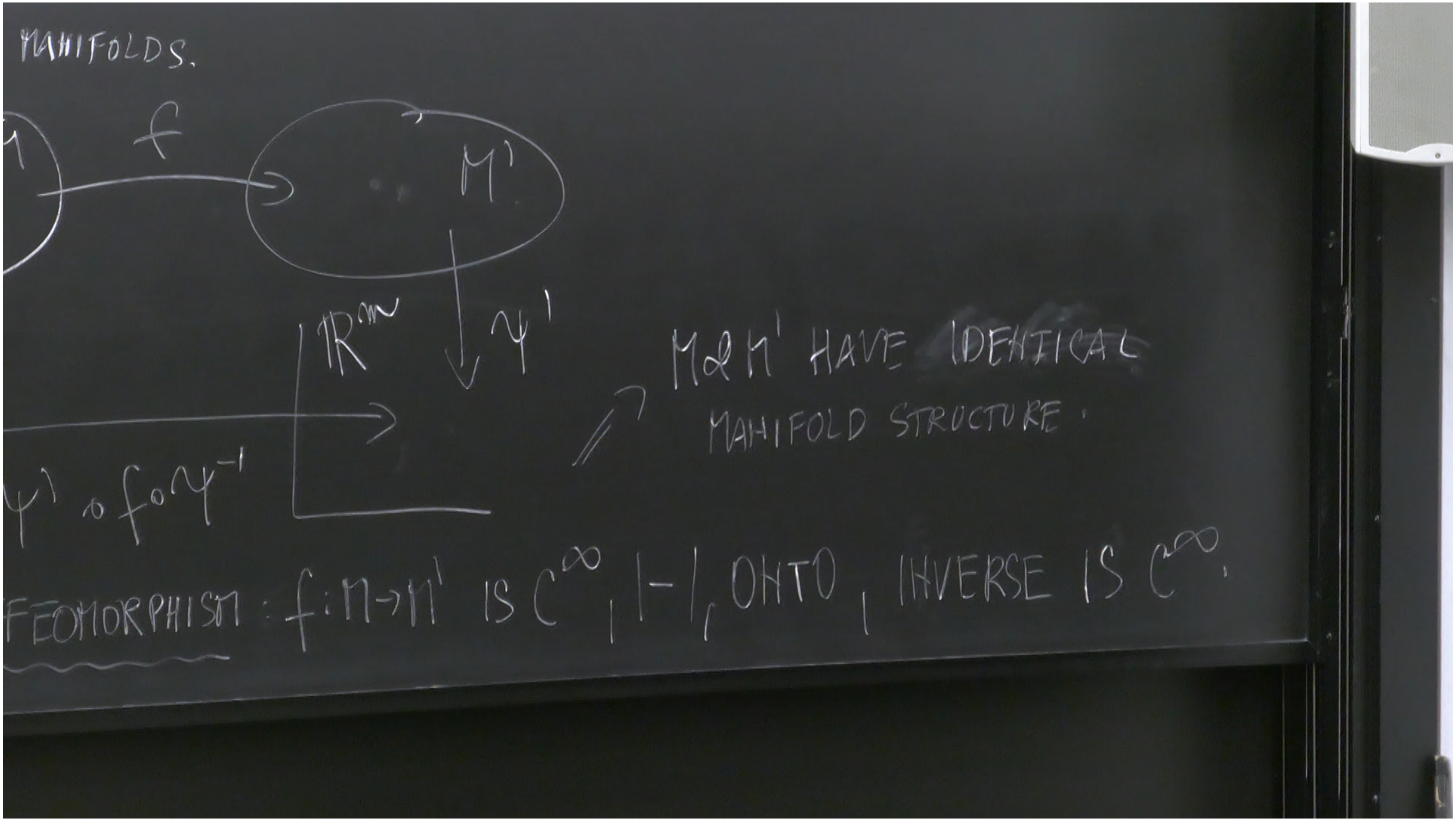

S^2
T^2


• SCALAR FUNCTION $f: M \to \mathbb{R}$
φ
x
$f(x(p))$
$x(p)$
p
x'
$x'(p)$
$f(p) = f(x(p)) = f'(x'(p))$ … INDEP. OF COORD. SYSTEM:
$f(x) = f'(x')$

• MAPS BETWEEN MANIFOLDS.

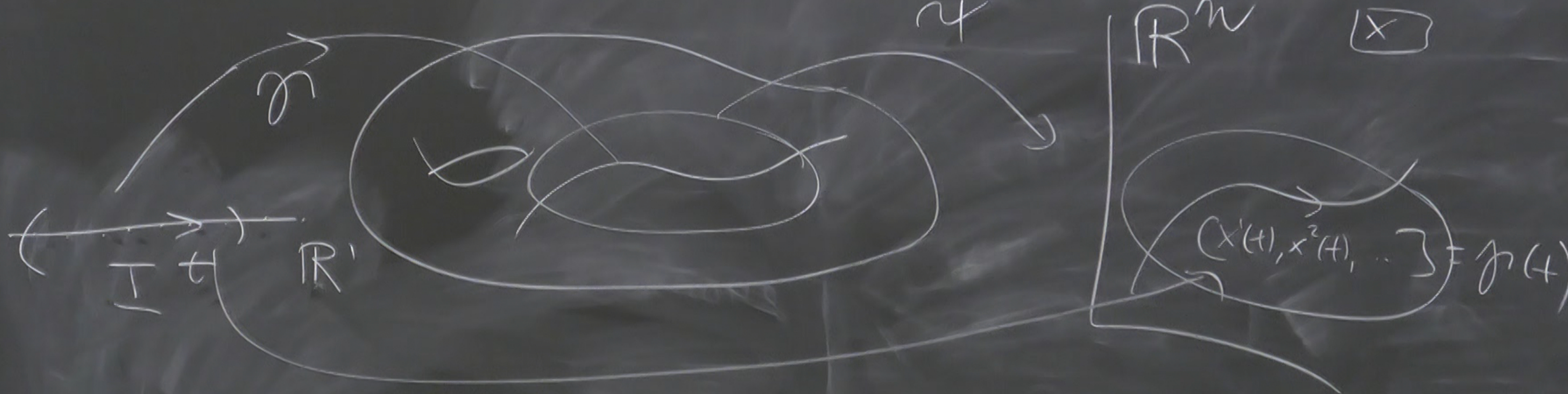


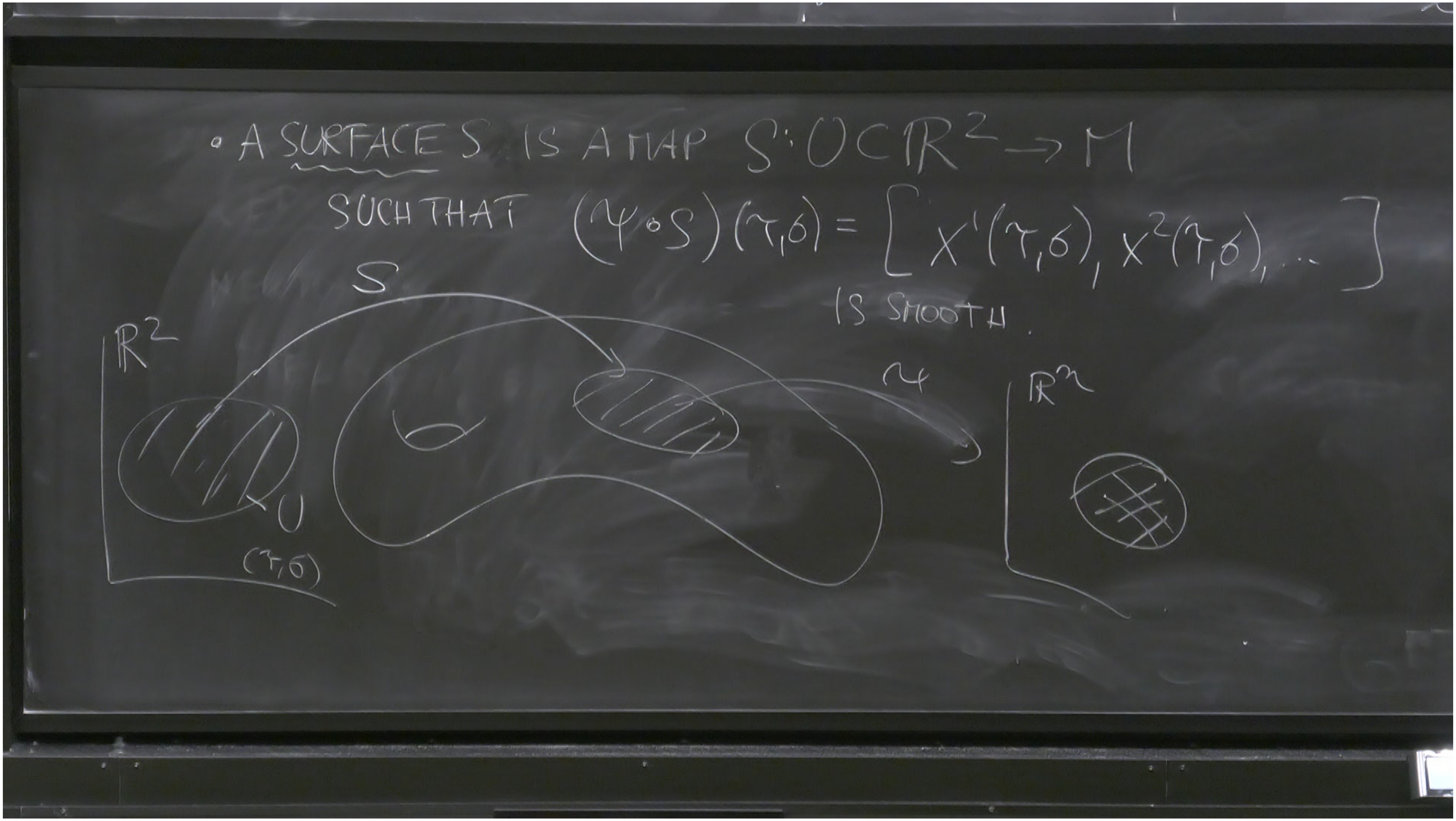
EXAMPLE: DIFFEOMORPHISM: $f: M \to M'$ IS C^∞, 1-1, ONTO, INVERSE

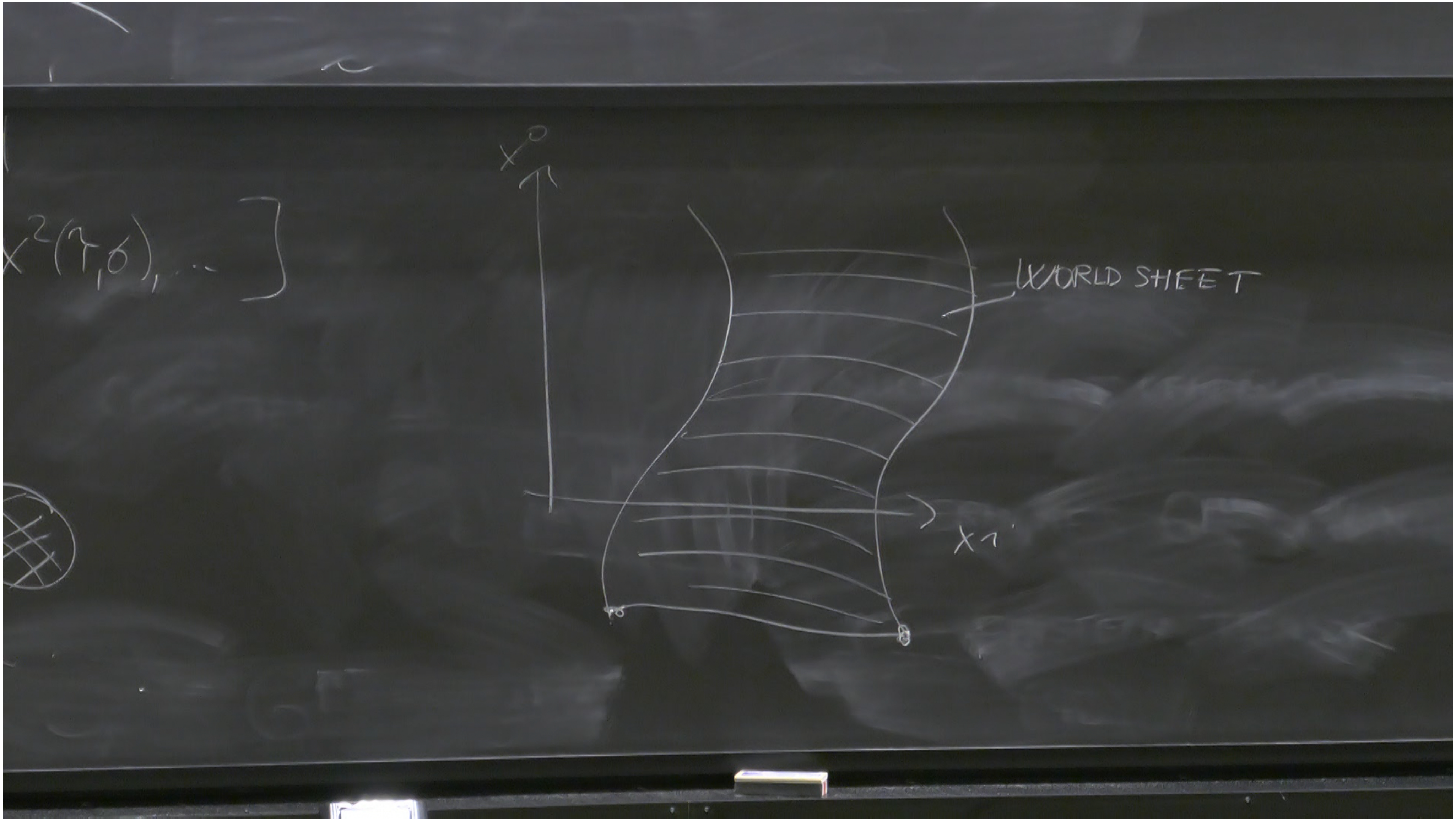


MANIFOLDS.
f
M'
$\mathbb{R}^m$
ψ'
$\psi' \circ f \circ \psi^{-1}$
M & M' HAVE IDENTICAL
MANIFOLD STRUCTURE.
FEOMORPHISM: $f: M \to M'$ IS C^∞, 1-1, ONTO, INVERSE IS C^∞.

• A CURVE γ ON M IS A MAP $\gamma : I \subset \mathbb{R}^1 \to M$
SUCH THAT $(\psi \circ \gamma)(t) = [\, x^1(t), x^2(t), x^3(t), \ldots \,]$
IS SMOOTH

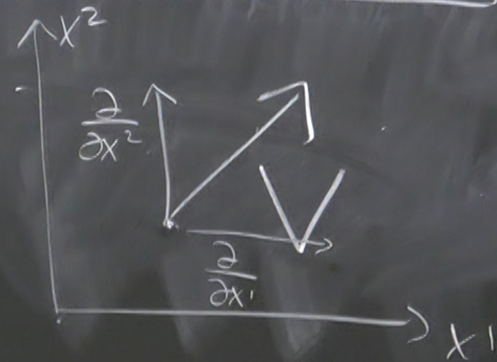



• A SURFACE S IS A MAP $S: U \subset \mathbb{R}^2 \to M$
SUCH THAT $(\psi \circ S)(\tau,\sigma) = [X^1(\tau,\sigma), X^2(\tau,\sigma), \dots]$
IS SMOOTH.
S
$\mathbb{R}^2$
U
(τ,σ)
ψ
$\mathbb{R}^n$


X²(τ,σ), ...
x⁰
WORLD SHEET
x¹

b) TENSORS = INVARIANT OBJECTS THAT CAN 'LIVE ON

- A TANGENT VECTOR:



COMPONENTS

$$V^\mu = (V^1, V^2, V^3)_{\text{CARTESIAN}}$$

IN CARTESIAN COORD. SYSTEM.

ANIFOLD'

$$V = V^1 \frac{\partial}{\partial x^1} + V^2 \frac{\partial}{\partial x^2} + V^3 \frac{\partial}{\partial x^3}$$

DIRECTIONAL
DERIVATIVE

$$= \underbrace{V^\mu}_{\text{COMPONENTS}} \frac{\partial}{\partial x^\mu}$$

— BASIS

ANY COORDS.

$$\longleftrightarrow \quad \hat{V} = V^\mu \frac{\partial}{\partial x^\mu}$$

DEF: LE $\mathcal{F}$ BE A COLLECTION OF C^∞ FUNCTIONS.

A TANGENT VECTOR $\underline{V}$ AT POINT $p \in M$

IS A MAP $V: \mathcal{F} \to \mathbb{R}$ THAT IS

i) LINEAR: $V(af+bg) = aV(f)+bV(g)$

ii) LEIBNITZ $V(fg) = V(f)\,g(p) + V(g)\,f(p)$