Title: Relativity Lecture - 101123

Speakers: David Kubiznak

Collection: Relativity 2023/24

Date: October 11, 2023 - 9:00 AM

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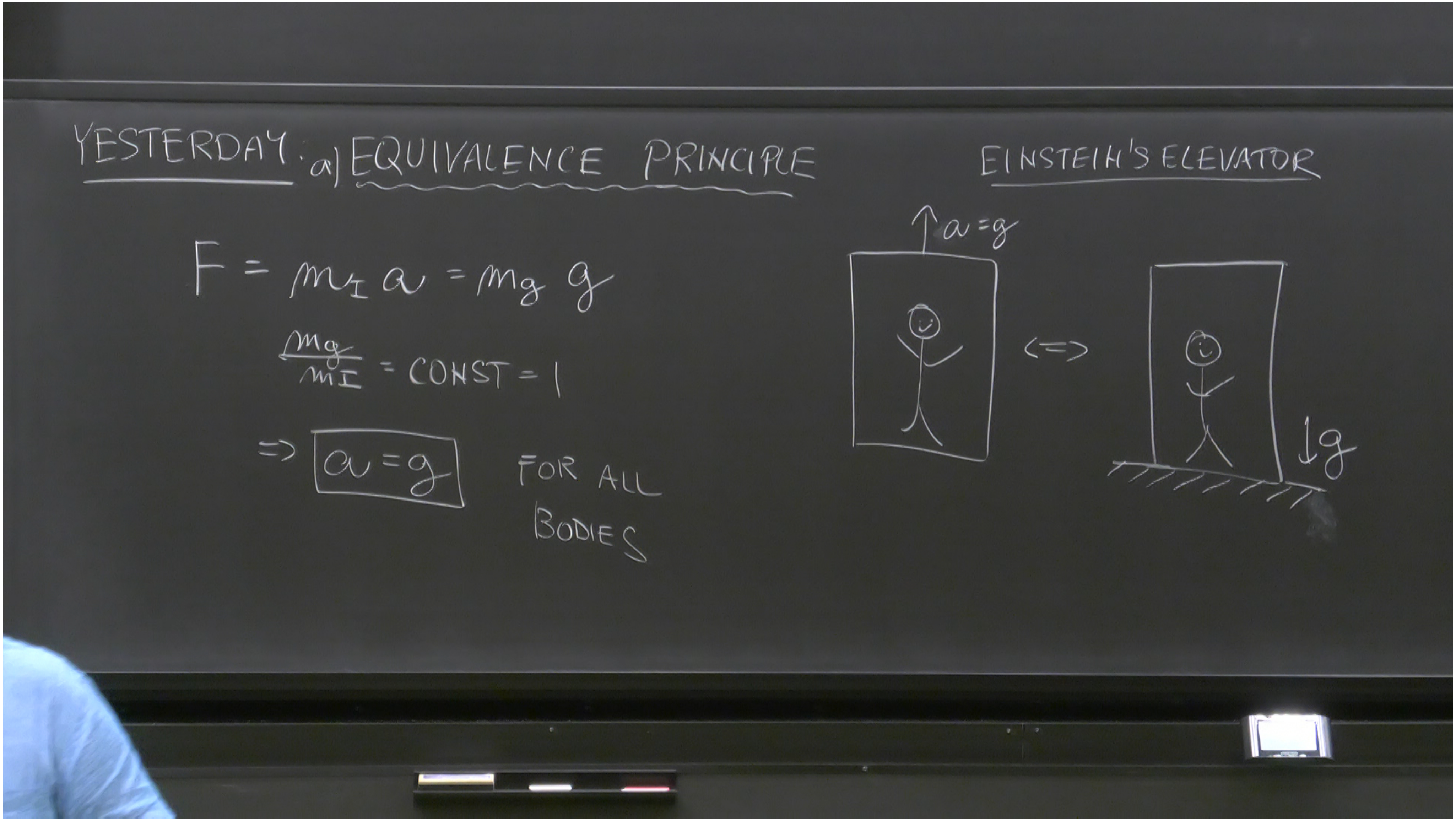
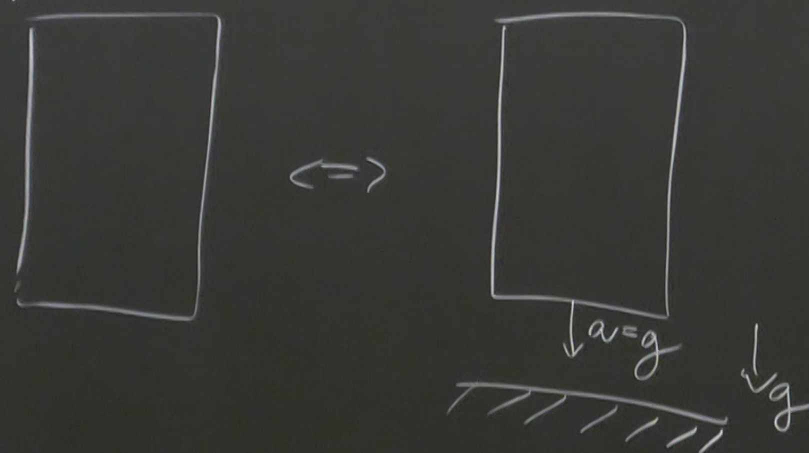

YESTERDAY: a) EQUIVALENCE PRINCIPLE
$F = m_I a = m_g g$
$\frac{m_g}{m_I} = \text{CONST} = 1$
$\Rightarrow \boxed{a = g}$ FOR ALL BODIES
EINSTEIN'S ELEVATOR
$\uparrow a = g$
$\Leftrightarrow$
$\downarrow g$

OR <u>LOCAL INERTIAL FRAME</u>

NO GRAVITY



<u>IMMEDIATE CONSEQUENCES</u>

- LIGHT BENDING (EDDINGTON) — $\boxed{\text{1ST}}$
- GRAV. REDSHIFT (POUND & REBKA)

GEODESIC EQUATION:

$$\boxed{\frac{d^2x^\mu}{d\tau^2} + \underbrace{\Gamma^\mu{}_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau}}_{\text{GRAV \& INERTIAL FORCES}} = 0}$$

GRAV & INERTIAL FORCES
(TREATED ON THE SAME FOOTING)

WHERE

$$\Gamma^\mu{}_{\alpha\beta} = \frac{1}{2} g^{\mu\nu}\left(g_{\nu\alpha,\beta} + g_{\nu\beta,\alpha} - g_{\alpha\beta,\nu}\right)$$

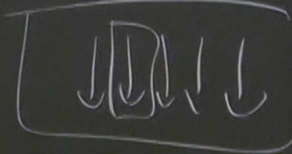
$ds^2 = g_{\mu\nu}\, d$

$g_{\alpha\beta}\, g^{\beta}$

$A A^{-1} =$

YESTERDAY: $g_{\mu\nu} = \eta_{\alpha\beta} \frac{\partial \xi^\alpha}{\partial x^\mu} \frac{\partial \xi^\beta}{\partial x^\nu}$

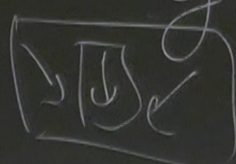
$x^\mu = x^\mu(\xi)$ ... COORD. TRANSF.



INERTIAL $\quad g \leftrightarrow \eta \rightarrow$ TIDAL $= 0$

(GLOBALLY... CAN GET RID OF "GRAVITY")

"TRUE" $\quad g \not\leftrightarrow \eta \rightarrow \boxed{\text{TIDAL}}$ (ONLY LOCALLY)



b) NEWTONIAN LIMIT (OF GEOD. EQ.)

• SLOW MOTION: $\left|\underbrace{\frac{dx^i}{d\tau}}_{v^i}\right| \ll \left|\frac{dt}{d\tau}\right|$

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{tt}\frac{dt}{d\tau}\frac{dt}{d\tau} + O(v^i) = 0$$

- STATIC GRAV. FIELD $\quad g_{\mu\nu,t} = \emptyset$

$$\Gamma^{\mu}{}_{tt} = \frac{1}{2} g^{\mu\nu}\left(\cancel{g_{\nu t,t}} + \cancel{g_{\nu t,t}} - g_{tt,\nu} \right) = -\frac{1}{2} g^{\mu\nu} g_{tt,\nu}$$

- WEAK GRAV. FIELD

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + O(h^2)$$

$\emptyset$

ATIC GRAV. FIELD $g_{\mu\nu,t} = \emptyset$

$$\Gamma^{\mu}{}_{tt} = \frac{1}{2} g^{\mu\nu}\left(\cancel{g_{\nu t,t}} + \cancel{g_{\nu t,t}} - g_{tt,\nu}\right) = -\frac{1}{2} g^{\mu\nu} g_{tt,\nu}$$

EAK GRAV. FIELD

$$g_{\mu\nu} = \eta_{\mu\nu} + \underbrace{h_{\mu\nu}}_{\text{SMALL PERTURBATION}} + O(h^2)$$

$$\boxed{\Gamma^{\mu}{}_{tt} = -\frac{1}{2} \eta^{\mu\nu} h_{tt,\nu}}$$

$$g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu}$$

$$g_{\mu\nu} g^{\nu\varkappa} = \delta_{\mu}{}^{\varkappa} = (\eta + h)(\eta - h)$$
$$= 1 + h - h + O(h^2)$$

• SLOW MOTION: $\left|\underbrace{\frac{dx^i}{d\tau}}_{v^i}\right| \ll \left|\frac{dt}{d\tau}\right|$

$$\boxed{\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu{}_{tt}\frac{dt}{d\tau}\frac{dt}{d\tau} + O(v^i) = 0} \quad (EQ)$$

$\Gamma^\mu{}_{tt} = \frac{1}{2} g^{\mu\nu}($

• WEAK GRAV. FIELD

$g_{\mu\nu}$

$g^{\mu\nu}$

i) $\underline{\mu = t}$: $\quad \dfrac{d^2 t}{d\tau^2} + 0 = \emptyset$

$$\frac{dt}{d\tau} = C = \text{CONST}$$

ii) $\mu = i$ $\quad \dfrac{d^2 x^i}{d\tau^2} + \Gamma^i{}_{tt} \underbrace{\dfrac{dt}{d\tau}}_{C} \underbrace{\dfrac{dt}{d\tau}}_{C} = \emptyset$

$$\frac{d^2 x^i}{dt^2}\left(\frac{dt}{d\tau}\right)^2 = \frac{d^2 x^i}{dt^2} C^2$$

$$\frac{d^2 x^i}{dt^2} + \Gamma^i{}_{tt} = \emptyset$$

$$\boxed{\frac{d^2 x^i}{dt^2} = \frac{1}{2} h_{tt,}{}^{i}}$$

$$\ldots + 0 = \emptyset$$

$$= c = \mathrm{CONST}$$

$$\frac{\ldots x^i}{\ldots \tau^2} + \Gamma^i_{tt} \underbrace{\frac{dt}{d\tau}}_{c} \underbrace{\frac{dt}{d\tau}}_{c} = \emptyset$$

$$\left(\frac{dt}{d\tau}\right)^2 = \frac{d^2x^i}{dt^2} c^2$$

$$\ldots + \Gamma^i_{tt} = \emptyset$$

$$\boxed{\frac{d^2x^i}{dt^2} = \frac{1}{2} h_{tt,}{}^{i}}$$

NEWTON: $\dfrac{d^2x^i}{dt^2} = g^i = -\nabla^i \phi = -\phi_,{}^{i}$

GRAV. POTENTIAL

$$\boxed{\phi = -\frac{1}{2} h_{tt}}$$

$$\boxed{g_{tt} = -\left(1 + \frac{2\phi}{c^2}\right)}$$

TIC GRAV. FIELD $\quad g_{\mu\nu,t} = 0$

$$\Gamma^{\mu}{}_{tt} = \frac{1}{2} g^{\mu\nu}\left(\cancel{g_{\nu t,t}} + \cancel{g_{\nu t,t}} - g_{tt,\nu}\right) = -\frac{1}{2} g^{\mu\nu} g_{tt,\nu}$$

AK GRAV. FIELD

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + O(h^2)$$

SMALL PERTURBATION

$$\boxed{\Gamma^{\mu}{}_{tt} = -\frac{1}{2}\eta^{\mu\nu} h_{tt,\nu}}$$

$$g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu}$$

$$g_{\mu\nu} g^{\nu\alpha} = \delta_\mu{}^\alpha = (\eta + h)(\eta - h)$$
$$= 1 + h - h + O(h^2)$$

i

$\nabla^i \phi = -\phi_{,}{}^{i}$

...AV. POTENTIAL

$\frac{\phi}{c^2}$

VALIDITY: $\left|\frac{\Phi}{c^2}\right| \simeq \frac{GM}{c^2 r} \ll 1$

PROTON: 10^{-39}

NEUTRON STAR: $10^{-2} - 10^{-1}$

EARTH: 10^{-9}

BH: $10^{-1} - 1$

SUN: 10^{-6}

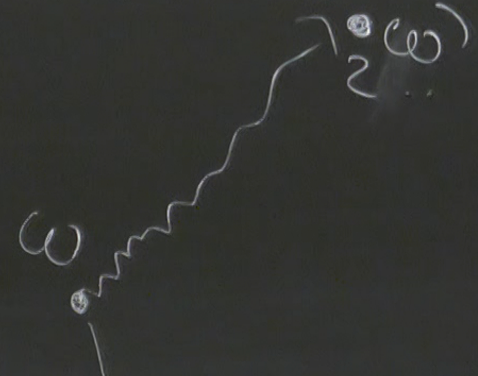
REMARK: WEAK & SLOWLY MOVING GRAV. SOURCES

$$\boxed{ds^2 = -\left(1 + \frac{2\Phi}{c^2}\right) dt^2 c^2 + \left(1 - \frac{2\Phi}{c^2}\right) \delta_{ij}\, dx^i dx^j}$$

C) RETURN TO GRAV. REDSHIFT

$$ds^2 = -d\tau^2 = g_{\mu\nu}\, dx^\mu dx^\nu$$

$$d\tau = \sqrt{-g_{\mu\nu}\frac{dx^\mu}{dt}\frac{dx^\nu}{dt}}\; dt$$



$$\frac{\omega'}{\omega} = \frac{d\tau}{d\tau'} = \frac{\sqrt{\quad}_{\,1}\, dt}{\sqrt{\quad}_{\,2}\, dt}$$

$$\frac{\omega'}{\omega} = \frac{d\tau}{d\tau'} = \frac{\sqrt{\quad}_1\, dt}{\sqrt{\quad}_2\, dt} = \frac{\sqrt{-g_{tt}|_1}}{\sqrt{-g_{tt}|_2}}$$

$$\boxed{z = \frac{\omega' - \omega}{\omega} = \phi - \phi'}$$

N NEWTONIAN REGIME.

$$\frac{\omega'}{\omega} = \frac{\sqrt{1+2\phi}}{\sqrt{1+2\phi'}} \approx \frac{1+\phi}{1+\phi'} \approx 1 + \phi - \phi'$$

$$\frac{\gamma}{\gamma'} = \frac{\sqrt{}\,{}_1\, dt}{\sqrt{}\,{}_2\, dt} = \frac{\sqrt{-g_{tt}|_1}}{\sqrt{-g_{tt}|_2}}$$

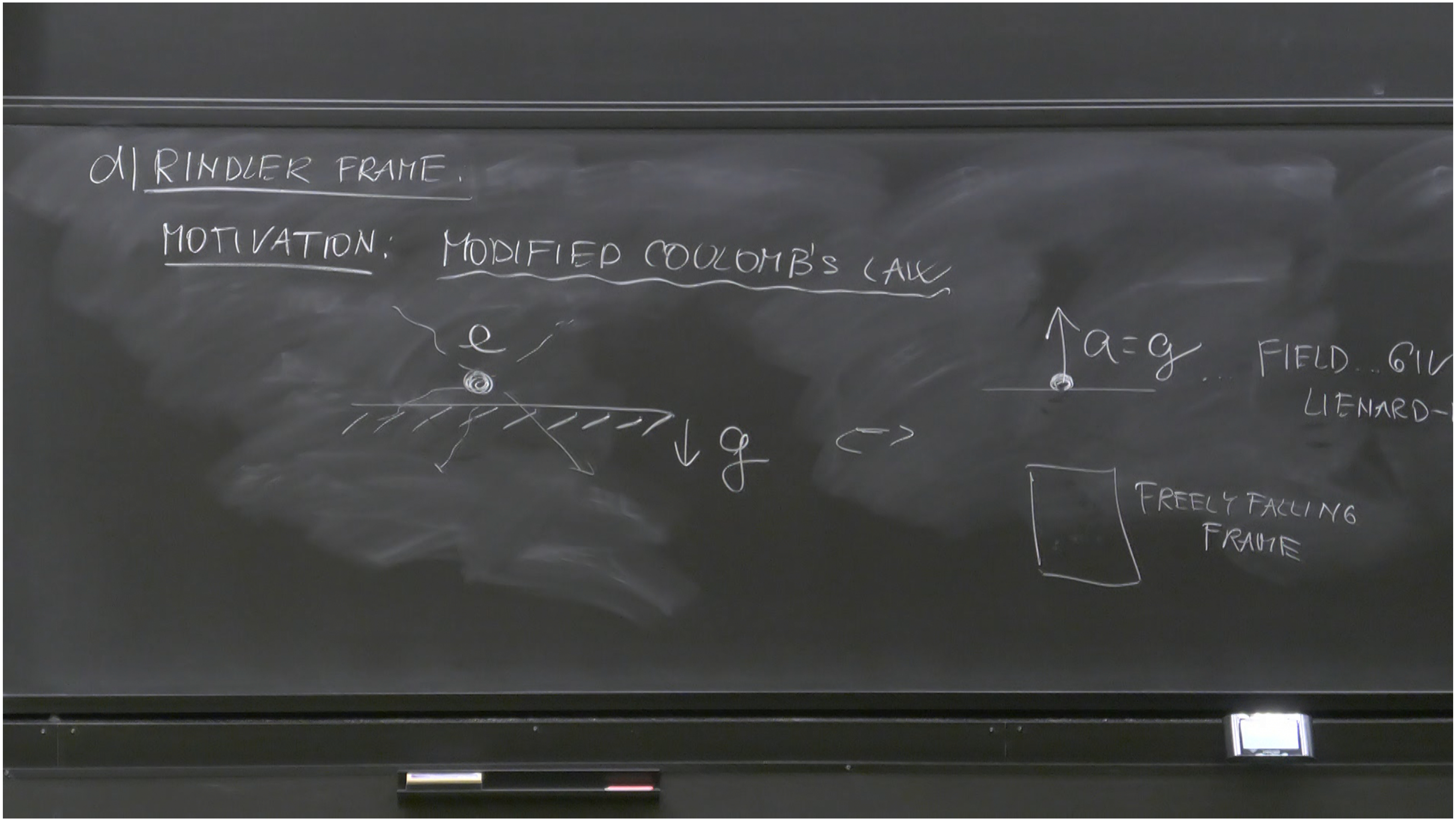
$$\boxed{z = \frac{\omega' - \omega}{\omega} = \phi - \phi'}$$

IAN REGIME.

$$\frac{\sqrt{1+2\phi}}{\sqrt{1+2\phi'}} \approx \frac{1+\phi}{1+\phi'} \approx 1 + \phi - \phi'$$

$$\boxed{z = -\frac{g\Delta h}{c^2}}$$

HOM. GRAV. FIELD.

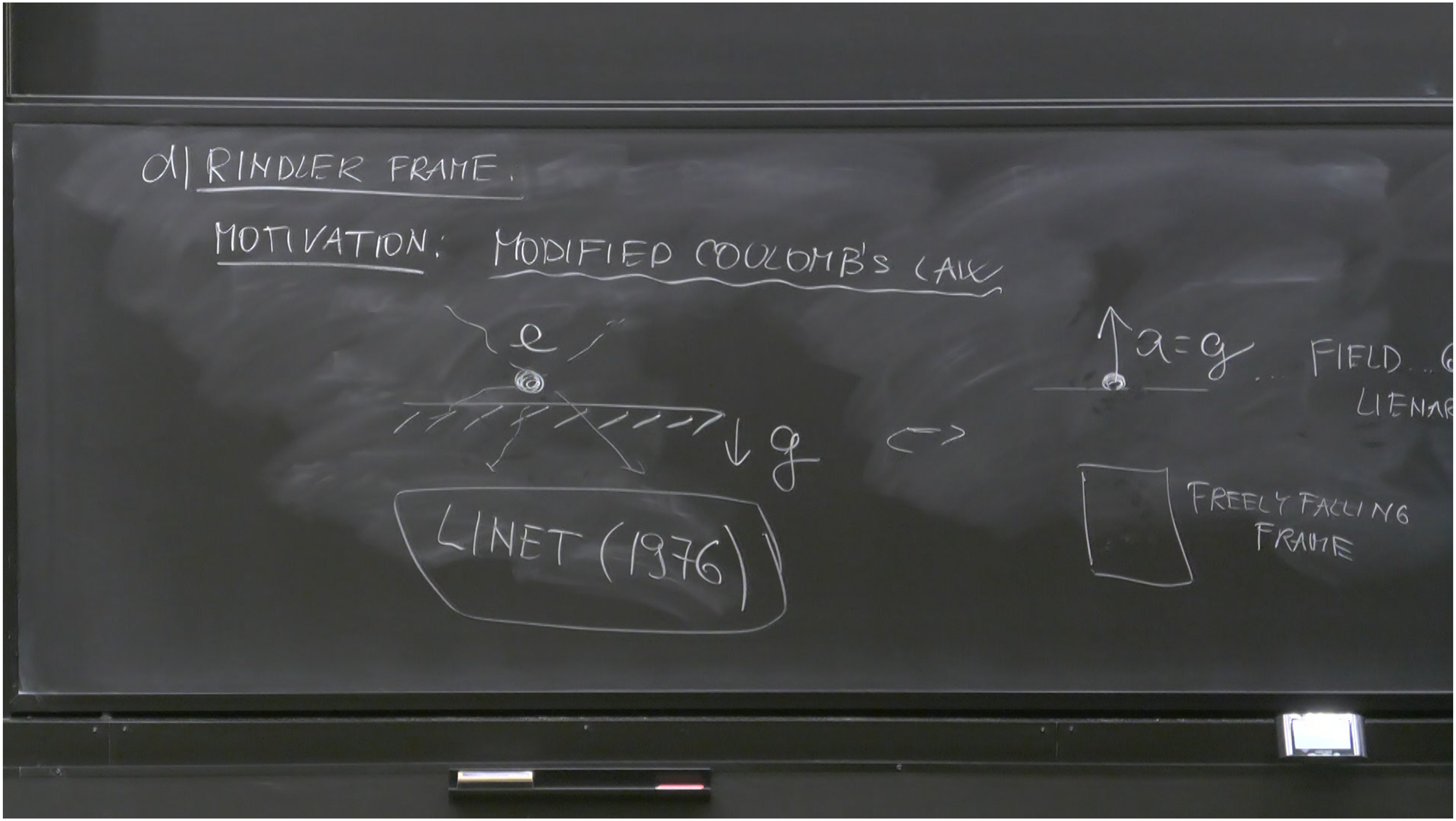

d) RINDLER FRAME.
MOTIVATION: MODIFIED COULOMB'S LAW
e
g
a = g
FIELD ... GIV
LIENARD-
FREELY FALLING FRAME

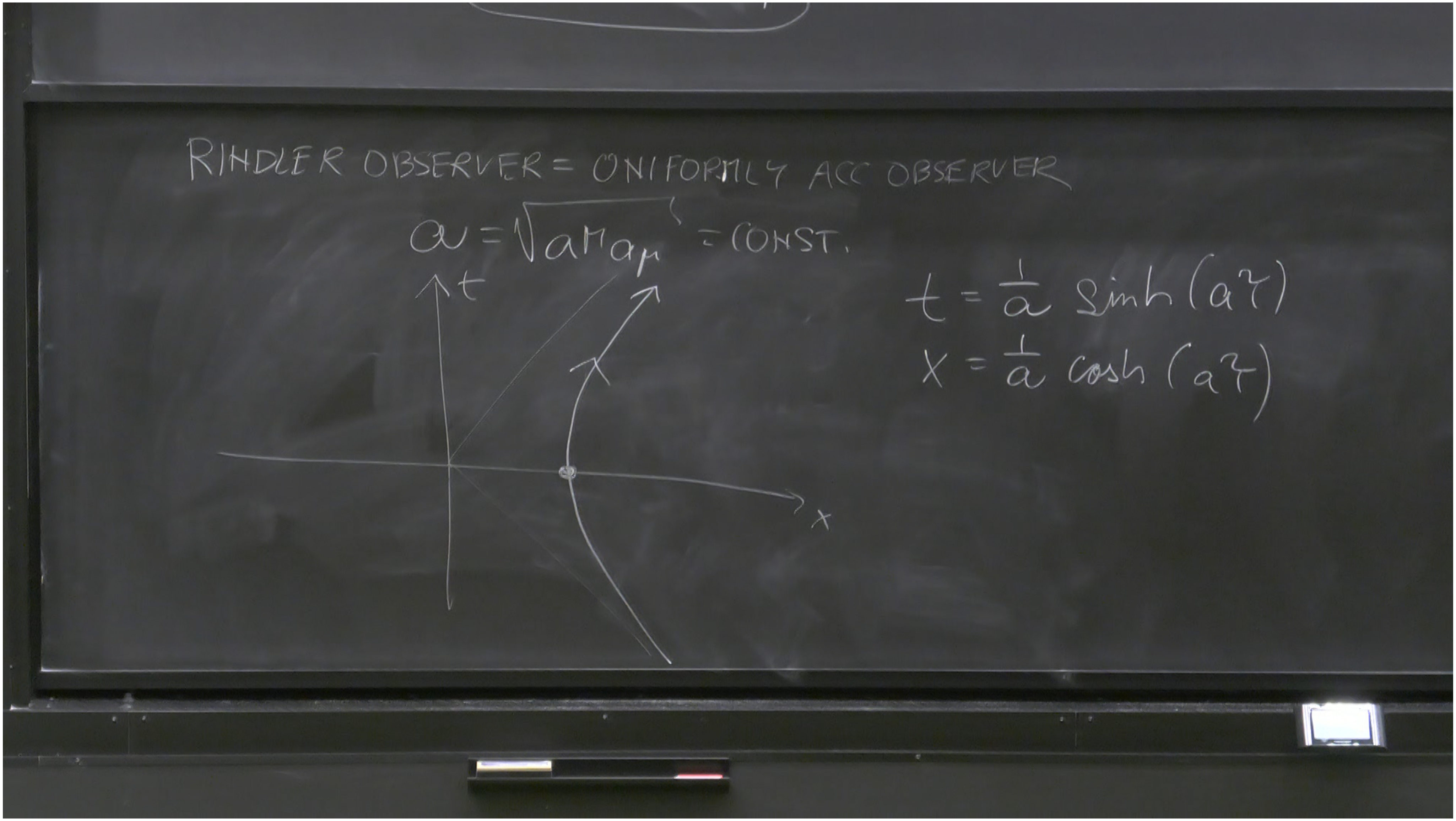
LOOK AT THIS FROM STATIC FRAME
ON EARTH
(UNIFORMLY ACCELERATED
W.R.T. FREELY FALLING FRAME)

... HOMEWORK

g ... FIELD ... GIVEN BY
LIENARD-WIECHERT
POTENTIALS

FREELY FALLING
FRAME


d) RINDLER FRAME.
MOTIVATION: MODIFIED COULOMB'S LAW
e
g
a=g
FIELD
LIENAR
FREELY FALLING FRAME
LINET (1976)


RINDLER OBSERVER = UNIFORMLY ACC OBSERVER
$a = \sqrt{a^\mu a_\mu} = \text{CONST.}$
$t = \frac{1}{a} \sinh(a\tau)$
$x = \frac{1}{a} \cosh(a\tau)$
t
x

...nh $(a\tau)$

...sh $(a\tau)$

RINDLER FRAME. $(t,x) \to (T,X)$ .. RINDLER

$$t = \left(\frac{1}{a} + X\right) \sinh(aT)$$

$$x = \left(\frac{1}{a} + X\right) \cosh(aT)$$

TRAJECTORY OF THE OBJECT: $X = 0,\ T = \tau$

$$\boxed{ds^2 = -dt^2 + dx^2 + \ldots = -(1+aX)^2 dT^2 + dX^2 + \ldots}$$

RINDLER FRAME: $(t,x) \longrightarrow (T,X)$ ... RINDLER

$$t = \left(\frac{1}{a} + X\right) \sinh(aT)$$

$$x = \left(\frac{1}{a} + X\right) \cosh(aT)$$

TRAJECTORY OF THE OBJECT: $X = 0\,,\; T = \tau$

$$T \in (-\infty, \infty)$$

$$X \in \left(-\frac{1}{a}, \infty\right)$$

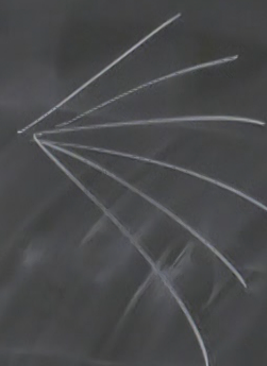
$$\boxed{ds^2 = -dt^2 + dx^2 + \dots = -(1+aX)^2\, dT^2 + dX^2 + \dots}$$

HOW DO LINES OF CONSTANT

T & X LOOK LIKE ?

$$\frac{t}{x} = \tanh(aT)$$

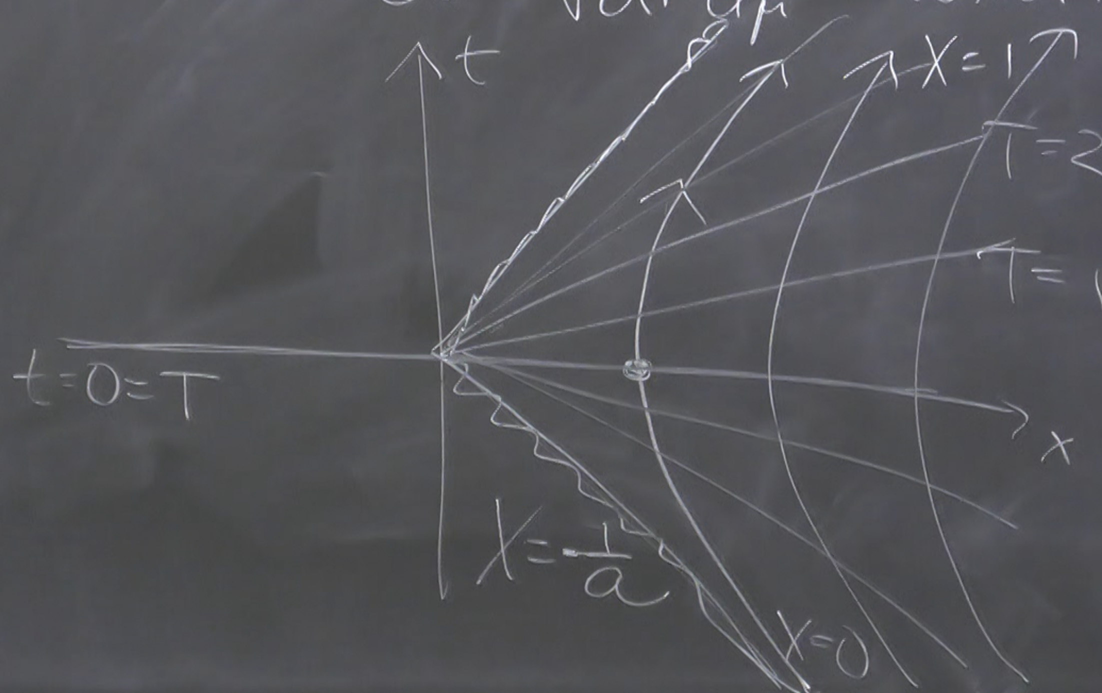
$\Rightarrow T = \text{CONST}$



$$\frac{x^2}{(\ )^2} - \frac{t^2}{(\ )^2} = \cosh^2 - \sinh^2 = 1$$

RINDLER OBSERVER = UNIFORMLY ACC OBSERVER

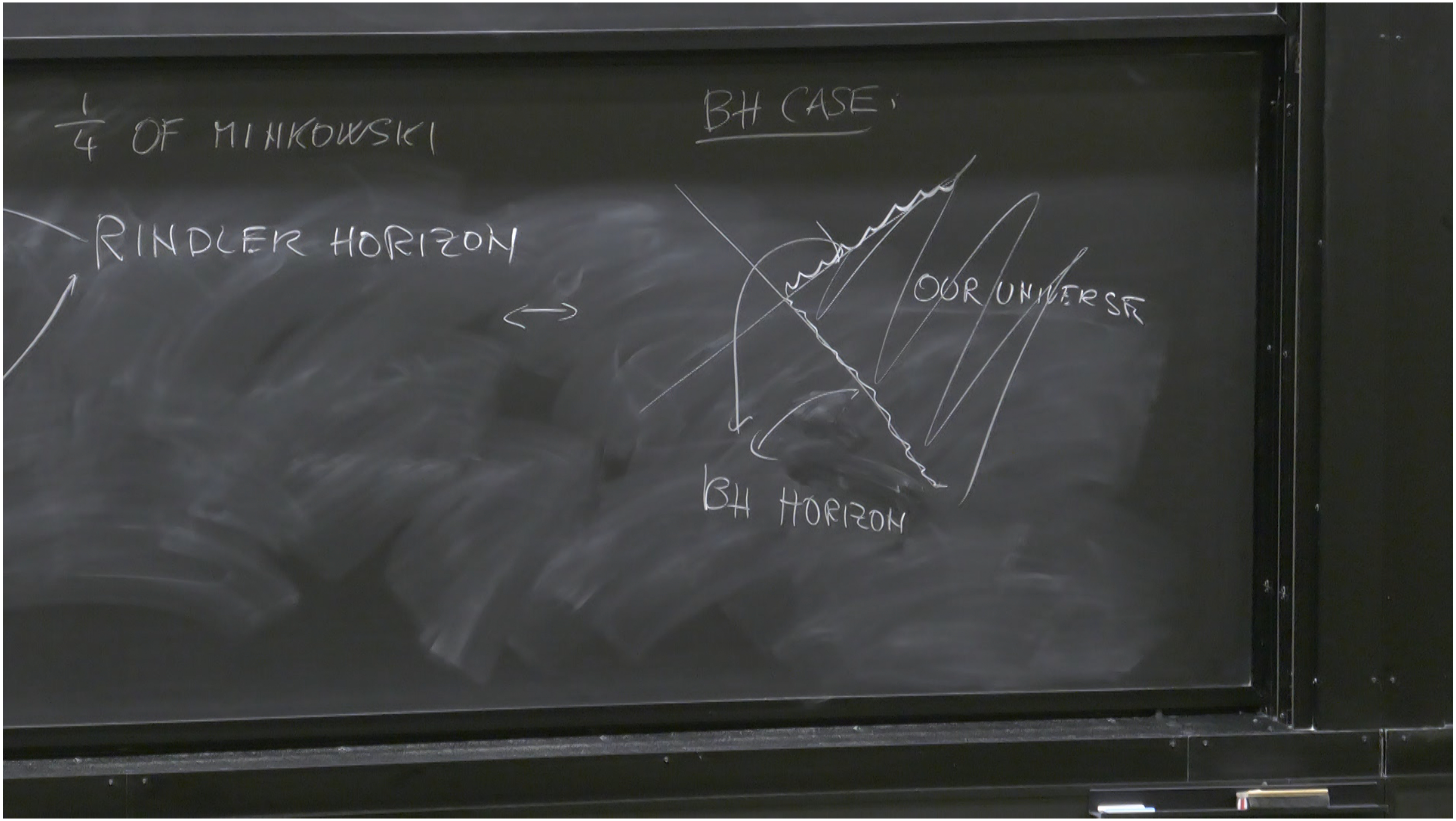
$$a = \sqrt{a^{\mu} a_{\mu}} = \text{CONST.}$$



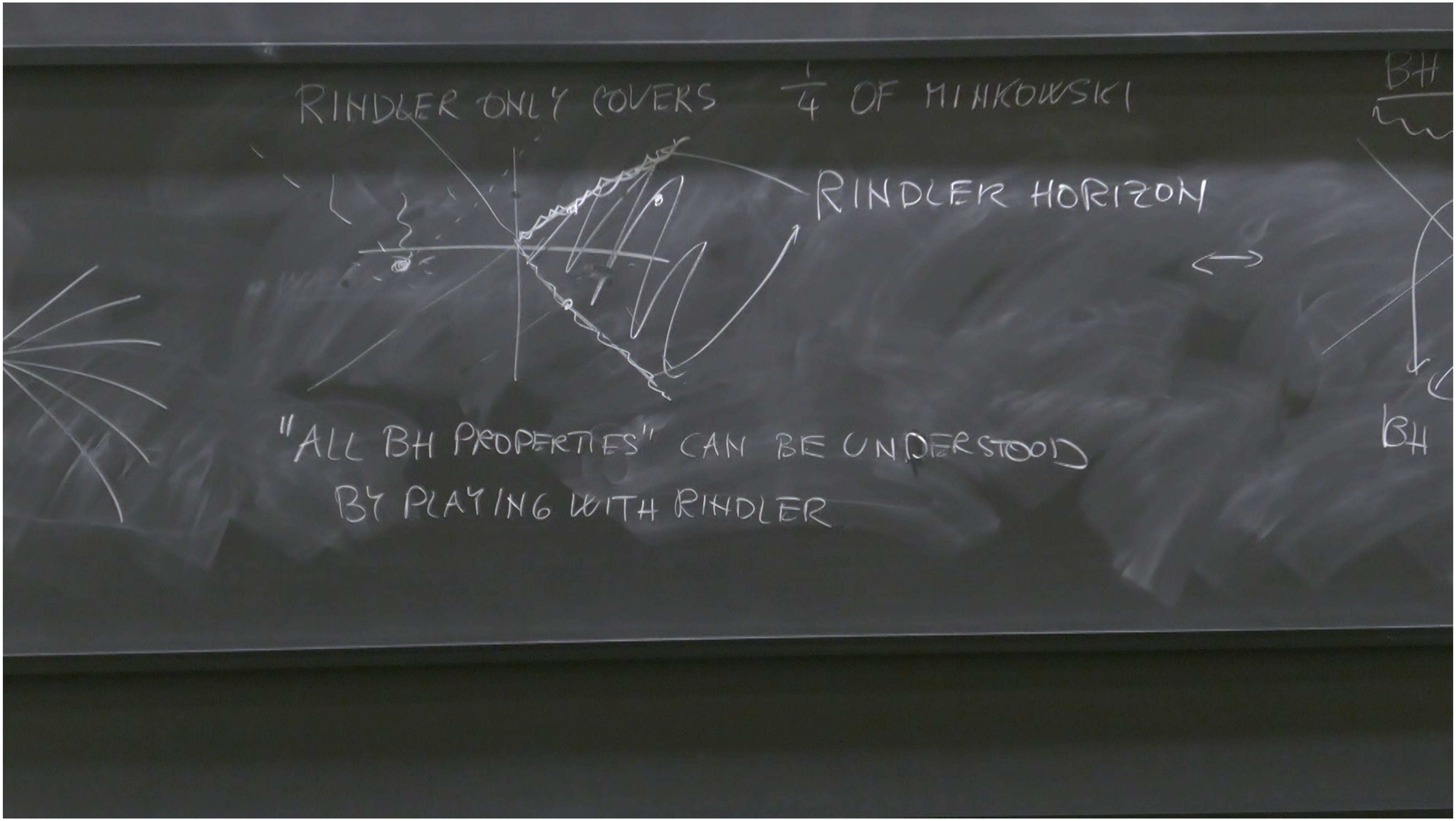
$$t = \frac{1}{a} \sinh(a\tau)$$

$$x = \frac{1}{a} \cosh(a\tau)$$

HOW DO LINES OF CONSTANT



1/4 OF MINKOWSKI
RINDLER HORIZON
BH CASE:
OUR UNIVERSE
BH HORIZON


RINDLER ONLY COVERS 1/4 OF MINKOWSKI
RINDLER HORIZON
BH
"ALL BH PROPERTIES" CAN BE UNDERSTOOD
BY PLAYING WITH RINDLER
BH

QM: BLACK HOLES RADIATE AS BLACK BODY.

$$\boxed{T = \frac{\kappa}{2\pi}}$$

SURFACE GRAVITY

HAWKING RAD.

FOR RINDLER... ACC. OBS. SEES

THERMAL BATH OF PARTICLES

$$\boxed{T = \frac{a}{2\pi}}$$

UNRUH RAD.