Title: Relativity Lecture - 101023

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Collection: Relativity 2023/24

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APPROXIMATE PLAN

WEEK 1: GR WITHOUT DIFFERENTIAL GEOMETRY

-11- 2: DIFF. GEOMETRY

3: FOUNDATIONS OF GR

4&5: FUN TOPICS: APPLICATIONS TO BHS & GWS & .. ?

GENERAL RELATIVITY

1) FIRST LOOK AT GR

a) PRINCIPLE OF EQUIVALENCE

= ALL BODIES BEHAVE THE SAME WAY IN GRAVITATIONAL FIELD

(EXPERIENCE THE SAME ACCELARATION)

EX: $F = m_g g = m_I a$

GRAVITATIONAL MASS

INERTIAL MASS

$$\boxed{\frac{m_g}{m_I} = \text{CONST}}$$

THE SAME FOR ALL BODIES.

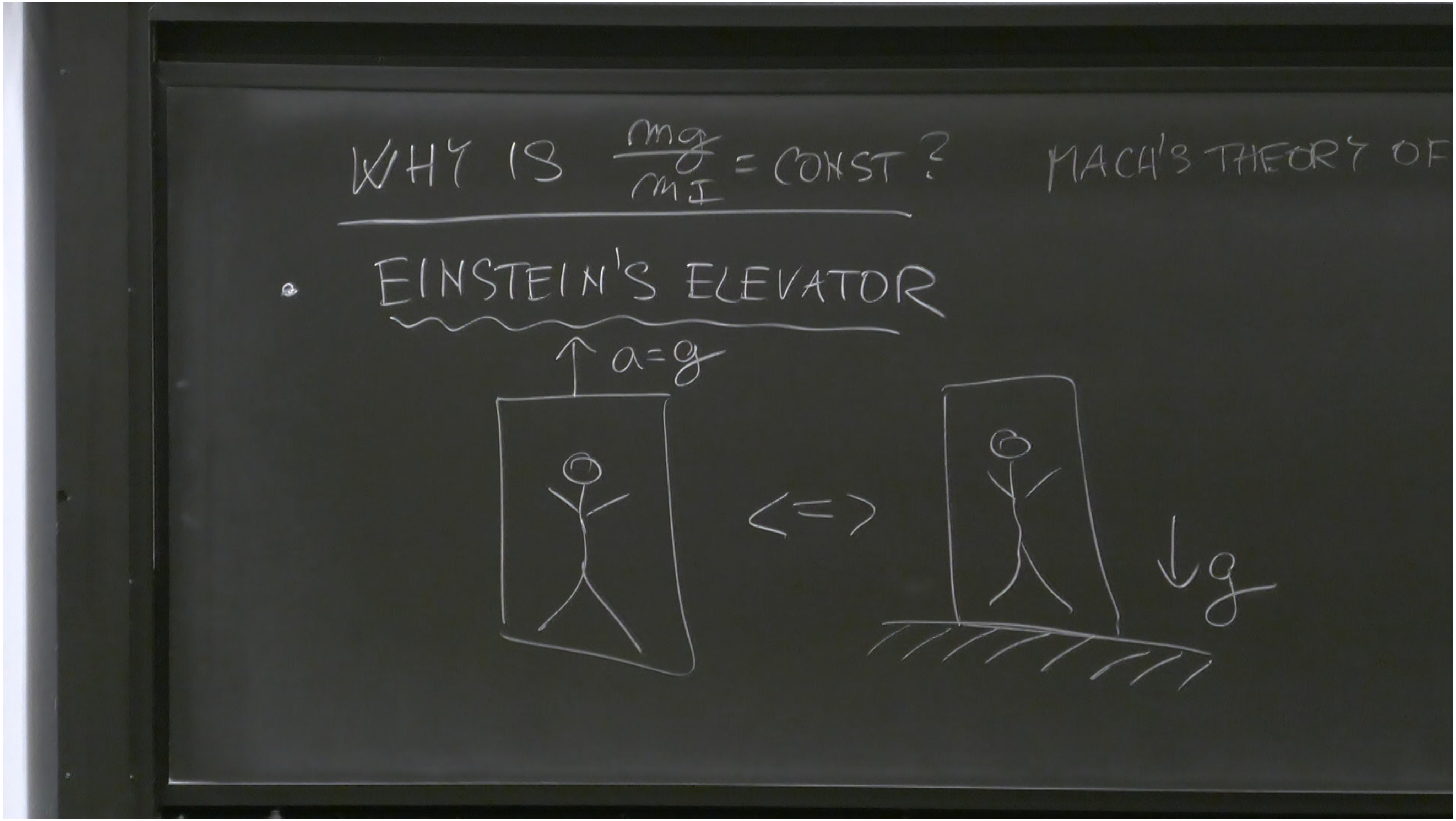
GRAVITY CAN BE GEOMETRIZED!

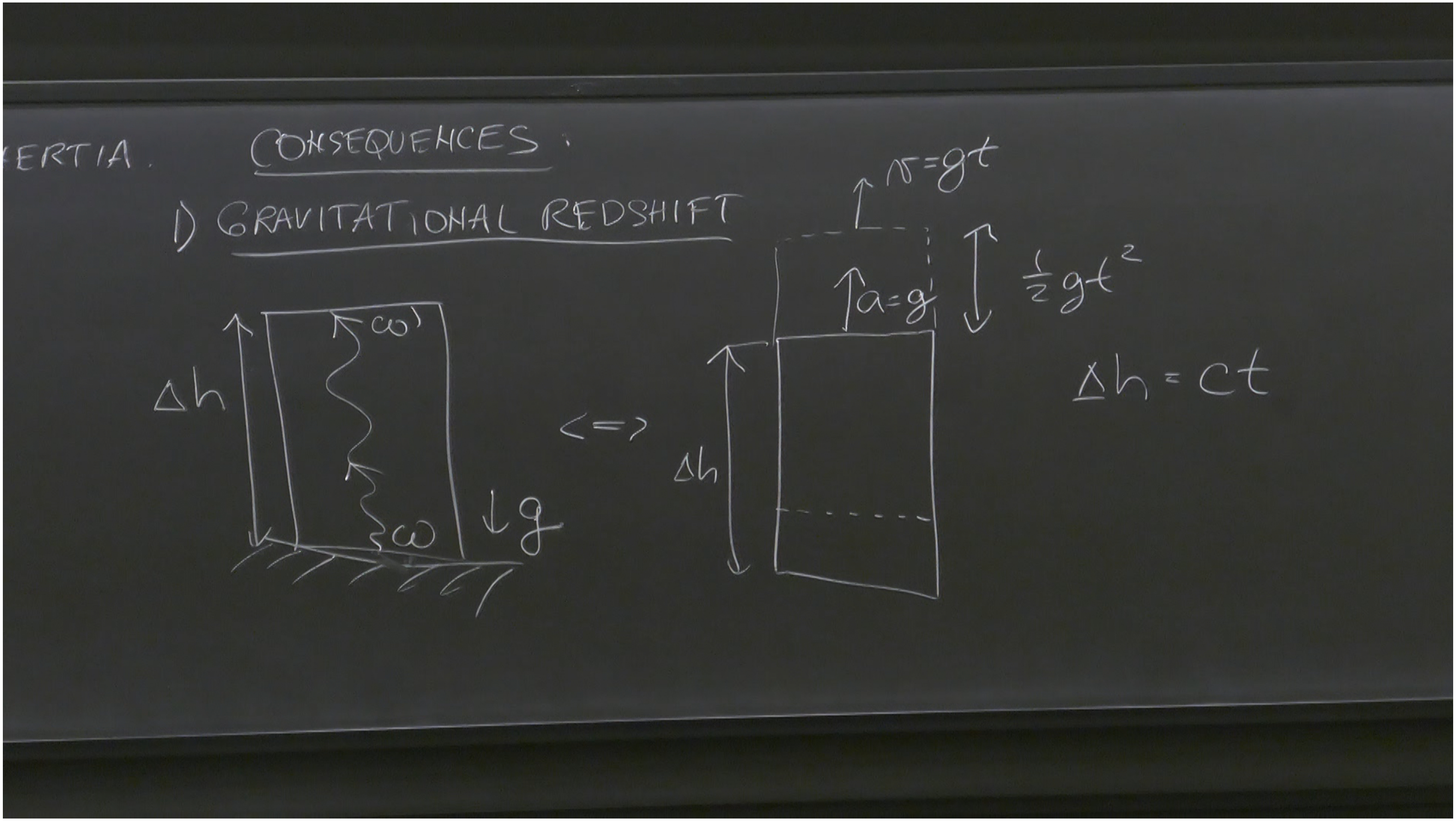
C.F: ELECTROMAGNETISM

$F = m_I a = q E$

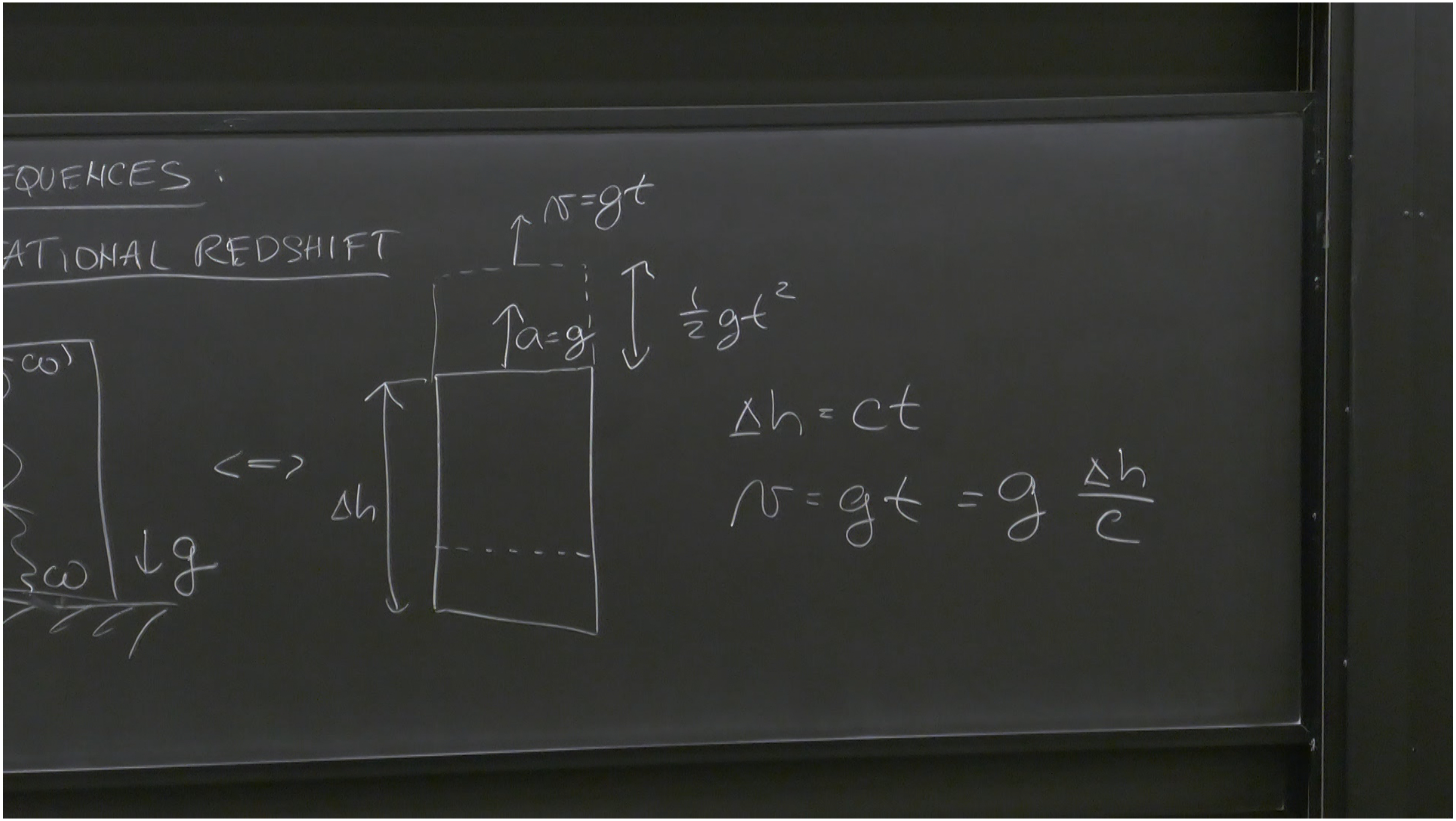
"ELECTROMAGNETIC MASS"

$\cancel{\frac{q}{m_I} = \text{CONST}}$??


WHY IS $\frac{m_g}{m_I}$ = CONST?
MACH'S THEORY OF
EINSTEIN'S ELEVATOR
a=g
<=>
g



ERTIA.
CONSEQUENCES:
1) GRAVITATIONAL REDSHIFT
$v = gt$
$a = g$
$\frac{1}{2}gt^2$
Δh
ω'
ω
g
<=>
$\Delta h = ct$


EQUENCES :
ATIONAL REDSHIFT
v = gt
a = g
½gt²
ω'
ω
g
<=>
Δh
Δh = ct
v = gt = g Δh/c

BOTTOM: $k^{\mu} = \frac{\omega}{c}(1,0,0,1)$

TOP: $k'^{\mu} = \frac{\omega'}{c}(1,0,0,1) = \gamma \frac{\omega}{c}(1-\beta)(1,0,0,1)$

DOPPLER SHIFT (TUTORIAL)

$$\simeq \underbrace{\frac{\omega}{c}\left(1-\frac{v}{c}\right)}_{\frac{\omega'}{c}}(1,0,0,1)$$

$$\omega' = \omega\left(1-\frac{v}{c}\right) = \omega\left(1-g\frac{\Delta h}{c^2}\right)$$

$$z = \frac{\omega' - \omega}{\omega} = -\frac{g \Delta h}{c^2}$$

PHOTONS ARE "REDDER" UP THERE.

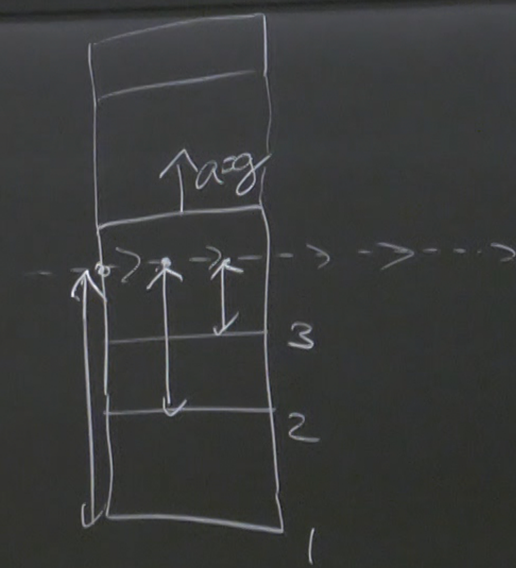
MEASURED BY PAUND & REBKA (1960)

$$\Delta h \simeq 23\,m$$

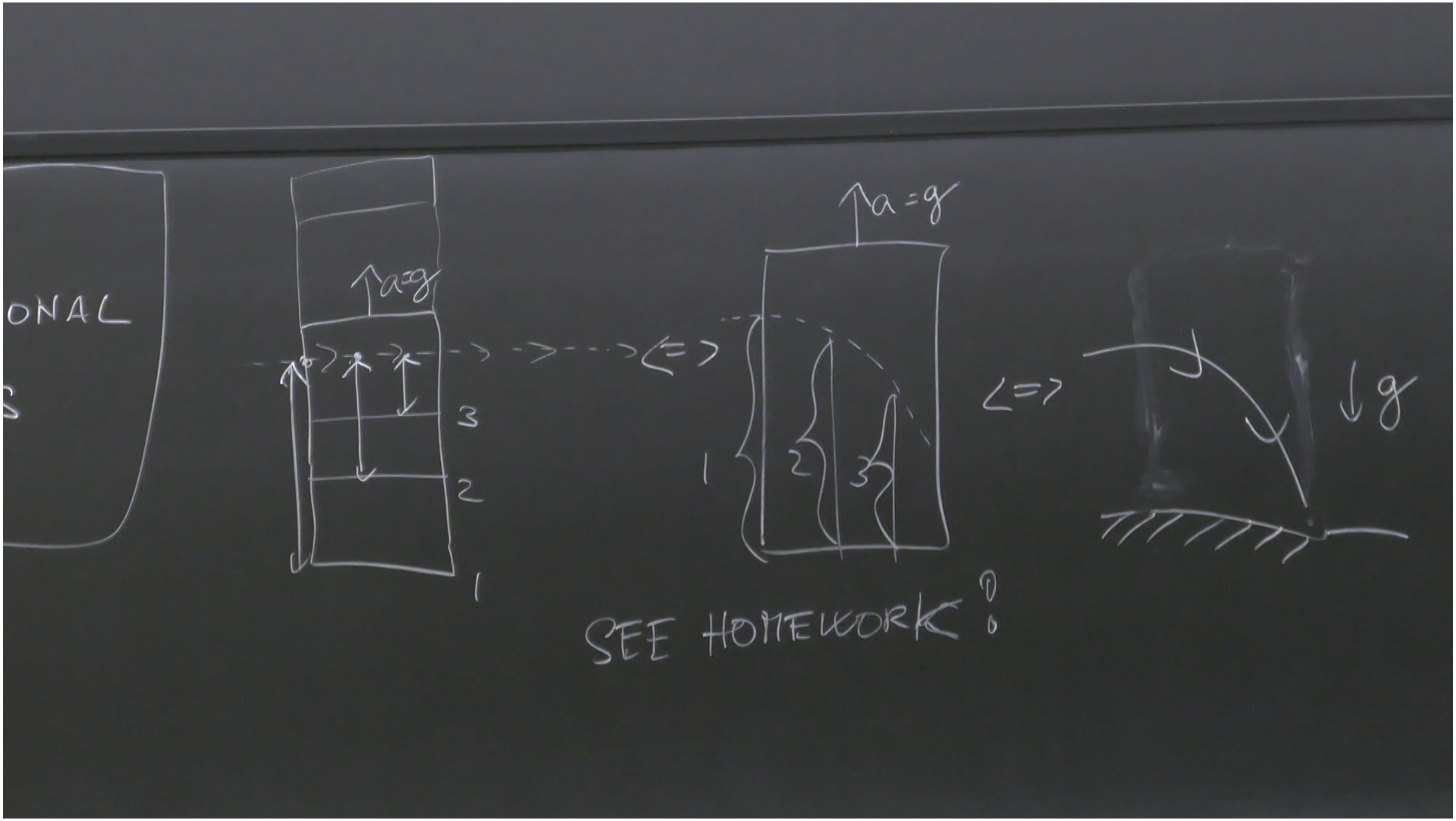
$$\boxed{z \simeq 2.5 \times 10^{-15}}$$

NOTE: $\omega \sim \frac{1}{\gamma}$

IN STRONG GRAVITATIONAL FIELD TIME GOES MORE SLOWLY !



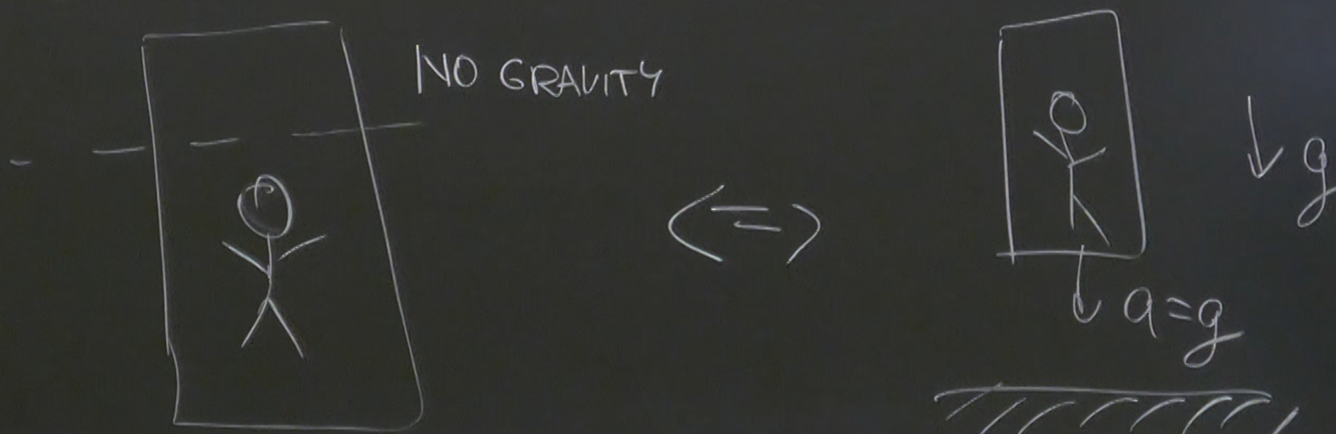
- BENDING OF LIGHT


ONAL
a=g
3
2
1
a=g
1
2
3
g
SEE HOMEWORK!

LOCAL INERTIAL FRAME: WE CAN (LOCALLY)

GET RID OF THE GRAV. FIELD

BY GOING TO THE "FREELY FALLING FRAME"



IN THIS FRAME, WE CAN USE KNOWN PHYSICS (SPECIAL RELATIVITY)

APPLY: MOTION OF PARTICLES ... GEODESIC EQ.

- IN FREELY FALLING FRAME ... STRAIGHT LINE
  (ξ^μ .. COORDINATES)

$$\frac{d^2 \xi^\mu}{d\tau^2} = 0$$

- WHAT HAPPENS IN SOME OTHER FRAME ... x^α .. COORDS

$$x^\alpha = x^\alpha(\xi^\mu)$$

$$\frac{d^2\xi^\mu}{d\tau^2} = \frac{d}{d\tau}\left(\frac{\partial \xi^\mu}{\partial x^\alpha}\frac{dx^\alpha}{d\tau}\right) = \frac{\partial^2 \xi^\mu}{\partial x^\beta \partial x^\alpha}\frac{dx^\beta}{d\tau}\frac{dx^\alpha}{d\tau} + \frac{\partial \xi^\mu}{\partial x^\alpha}\frac{d^2x^\alpha}{d\tau^2} = 0$$

$$\frac{d^2x^\nu}{d\tau^2} + \underbrace{\frac{\partial x^\nu}{\partial \xi^\mu}\frac{\partial^2 \xi^\mu}{\partial x^\beta \partial x^\alpha}}_{\Gamma^\nu_{(\beta\alpha)}}\frac{dx^\beta}{d\tau}\frac{dx^\alpha}{d\tau} = 0$$

$\Gamma^\nu_{(\beta\alpha)}$ ... CHRISTOFFEL SYMBOL

($4 \times 10 = 40$ COMPONENTS)

$$\boxed{\frac{d^2x^\nu}{d\tau^2} + \Gamma^\nu_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = 0}$$

GEODESIC EQ.

$$\frac{dx^\alpha}{d\tau} + \frac{\partial \xi^\mu}{\partial x^\alpha}\frac{d^2x^\alpha}{d\tau^2} = 0$$

$$\frac{\partial x^\nu}{\partial x^\alpha} = \delta^\nu_\alpha = \frac{\partial x^\nu}{\partial \xi^\mu}\frac{\partial \xi^\mu}{\partial x^\alpha}$$

MULTIPLY BY $\frac{\partial x^\nu}{\partial \xi^\mu}$

$$\frac{\beta}{\tau}\frac{dx^\alpha}{d\tau} = 0$$

NEXT TIME: STATIC & WEAK GRAV. FIELD

$$\boxed{\frac{d^2x}{dt^2} \simeq mg = -m\nabla\phi}$$

RISTOFFEL SYMBOL

+10 = 40 COMPONENTS)

- <u>CURVED METRIC</u>

IN INERTIAL FRAME $\mathrm{DIAG}(-1,1,1,1)$

$$ds^2 = -d\tau^2 = \eta_{\mu\nu}\, d\xi^\mu d\xi^\nu$$

$$= \eta_{\mu\nu} \frac{\partial \xi^\mu}{\partial x^\alpha} \frac{\partial \xi^\nu}{\partial x^\beta} dx^\alpha dx^\beta$$

$$ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$$

$$\boxed{g_{\alpha\beta} \equiv \eta_{\mu\nu} \frac{\partial \xi^\mu}{\partial x^\alpha} \frac{\partial \xi^\nu}{\partial x^\beta} = g_{\alpha\beta}(x)}$$

$$\Gamma_{\mu\alpha\beta} = g_{\mu\nu} \Gamma^\nu_{\ \alpha\beta}$$ ... CHRISTOFFEL ... 2ND KIND

$(\Gamma \sim \partial g)$

$$\Gamma_{\mu\alpha\beta} = \frac{1}{2}\left(g_{\mu\alpha,\beta} \right.$$

$$ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$$

$$\boxed{g_{\alpha\beta} \equiv \eta_{\mu\nu} \frac{\partial \xi^\mu}{\partial x^\alpha} \frac{\partial \xi^\nu}{\partial x^\beta}} = g_{\alpha\beta}(x)$$

$$\Gamma_{\mu\alpha\beta} = g_{\mu\nu} \Gamma^\nu{}_{\alpha\beta} \quad \ldots$$

CHRISTOFFEL ... 2ND KIND

$(\Gamma \sim \partial g)$

$$\boxed{\Gamma_{\mu\alpha\beta} = \frac{1}{2}\left(g_{\mu\alpha,\beta} + g_{\mu\beta,\alpha} - g_{\alpha\beta,\mu}\right)}$$

$\alpha\beta$

$$\left(\boxed{\;{}_{,\alpha} = \frac{\partial}{\partial x^\alpha}\;}\right)$$

$$\Gamma^{\cdot}{}_{\cdot\alpha} = \frac{\cdots}{\partial x^\alpha}$$

PROOF OF THE FORMULA:

$$g_{\mu\alpha,\beta} = \frac{\partial}{\partial x^\beta}\left(\eta_{\varkappa\sigma}\frac{\partial\xi^\varkappa}{\partial x^\mu}\frac{\partial\xi^\sigma}{\partial x^\alpha}\right) = \eta_{\varkappa\sigma}\left(\frac{\partial^2\xi^\varkappa}{\partial x^\beta\partial x^\mu}\frac{\partial\xi^\sigma}{\partial x^\alpha} + \right.$$

$$\frac{\partial^2\xi^\varkappa}{\partial x^\beta\partial x^\mu} = \frac{\partial\xi^\varkappa}{\partial x^\lambda}\Gamma^\lambda{}_{\beta\mu} =$$

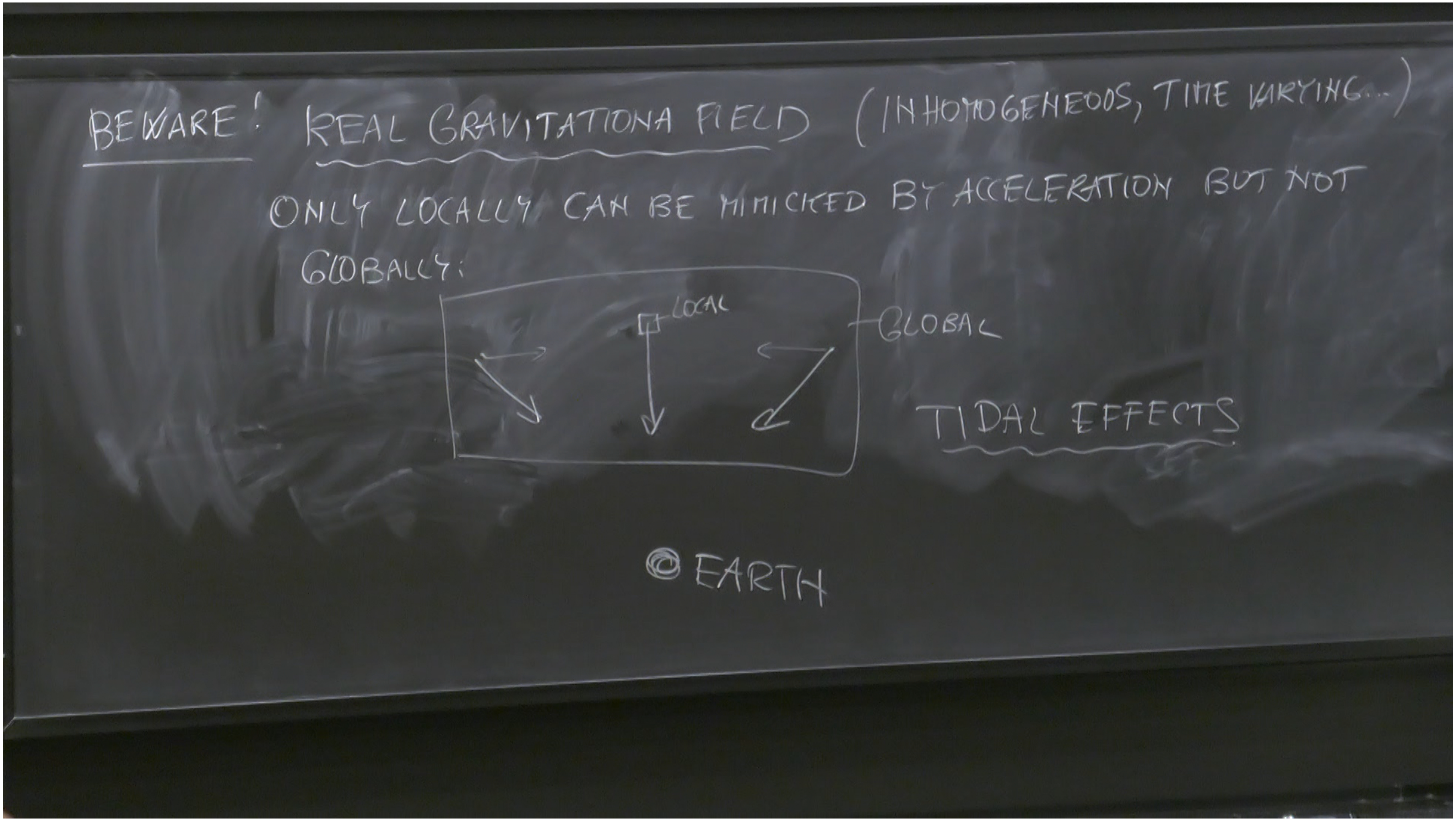
$$= \Gamma_{\alpha\beta\mu} + \Gamma_{\mu\alpha\beta}$$

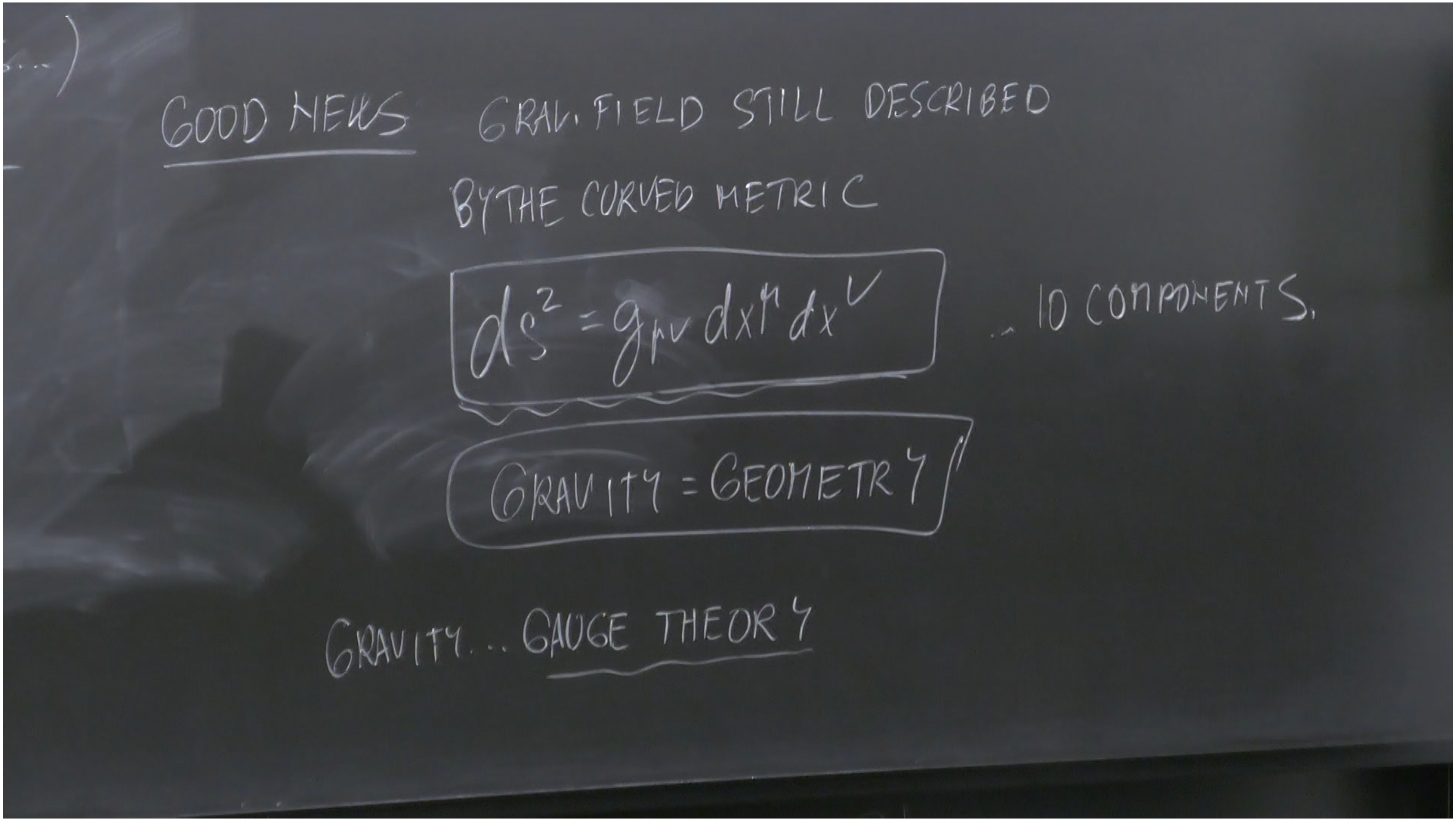
$$\ldots \frac{\partial \xi^\sigma}{\partial x^\alpha}\Big) = \eta_{\varkappa\sigma}\left(\frac{\partial^2\xi^\varkappa}{\partial x^\beta\partial x^\mu}\frac{\partial\xi^\sigma}{\partial x^\alpha} + \frac{\partial\xi^\varkappa}{\partial x^\mu}\frac{\partial^2\xi^\sigma}{\partial x^\beta\partial x^\alpha}\right)$$

$$\frac{\partial^2\xi^\varkappa}{\partial x^\beta\partial x^\mu} = \frac{\partial\xi^\varkappa}{\partial x^\lambda}\Gamma^\lambda_{\beta\mu} = \boxed{\frac{\partial\xi^\varkappa}{\partial x^\lambda}\frac{\partial x^\lambda}{\partial\xi^\delta}}\frac{\partial^2\xi^\delta}{\partial x^\beta\partial x^\mu} =$$

$$\delta^\varkappa_\delta$$

β


BEWARE! REAL GRAVITATIONA FIELD (INHOMOGENEOUS, TIME VARYING...)
ONLY LOCALLY CAN BE MIMICKED BY ACCELERATION BUT NOT
GLOBALLY:
LOCAL
GLOBAL
TIDAL EFFECTS
EARTH


GOOD NEWS GRAV. FIELD STILL DESCRIBED
BY THE CURVED METRIC
$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$
... 10 COMPONENTS.
GRAVITY = GEOMETRY
GRAVITY ... GAUGE THEORY