Title: Quantum Theory Lecture - 100223

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Collection: Quantum Theory 2023/24

Date: October 02, 2023 - 10:45 AM

URL: https://pirsa.org/23100034

Interviews: No notes

20 minutes prep

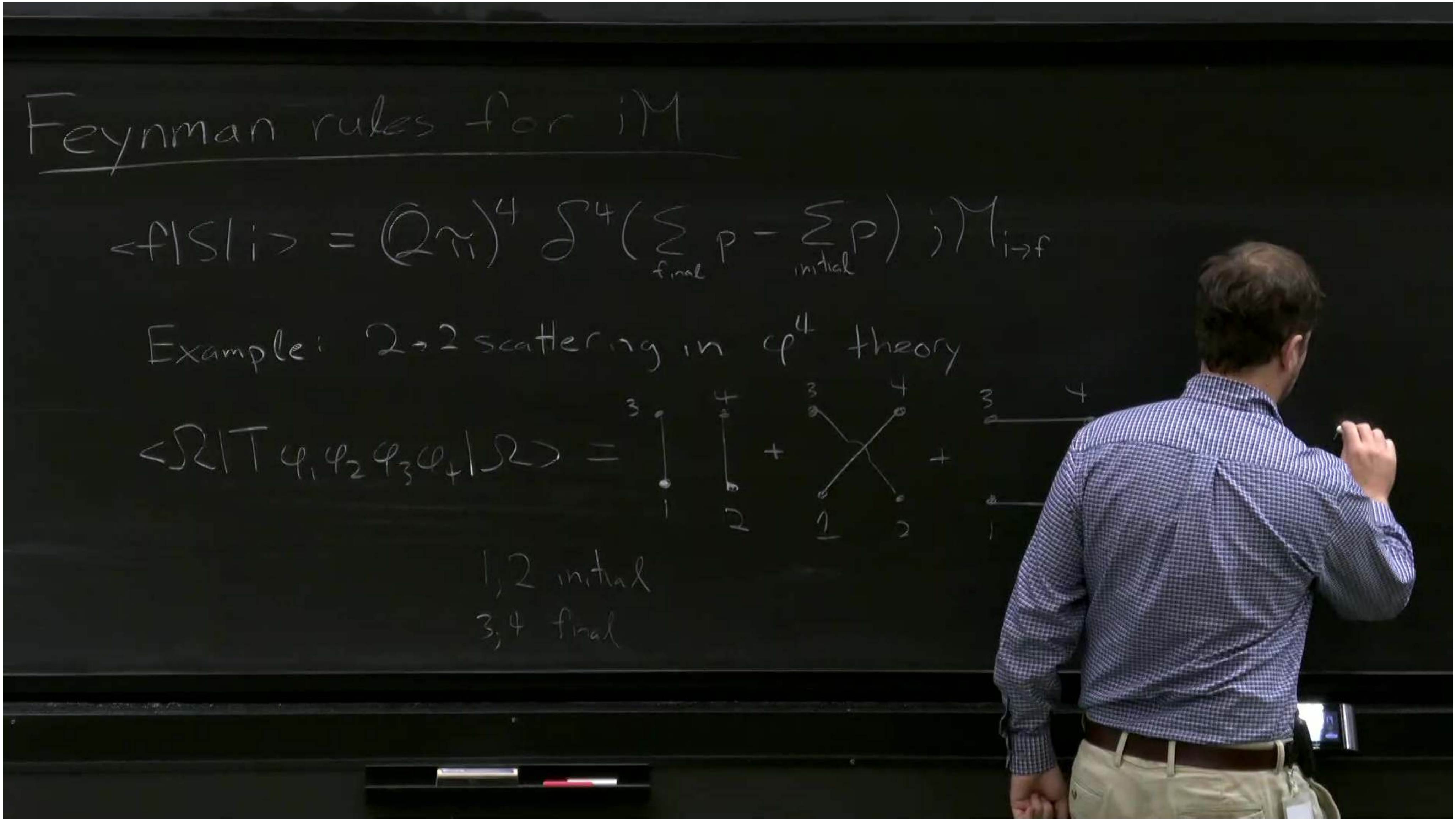
-questions by email

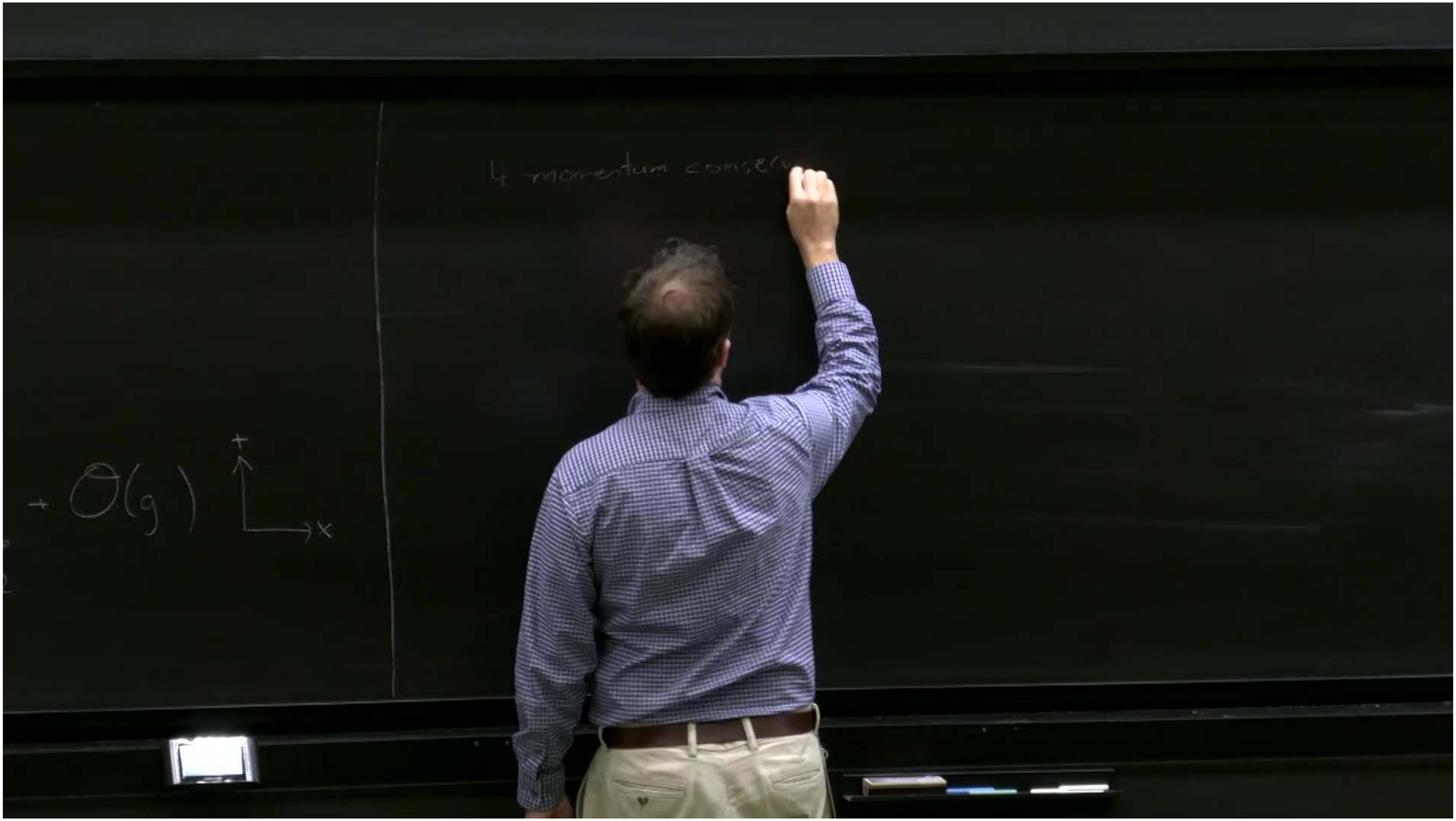
5-7 minute response preparation

+follow-up

Renormalization warm-up quiz

for tomorrow!


Feynman rules for iM
$\langle f|S|i\rangle = (2\pi)^4 \delta^4\left(\sum_{\text{final}} p - \sum_{\text{initial}} p\right) i\mathcal{M}_{i\to f}$
Example: 2→2 scattering in φ^4 theory
$\langle \Omega | T \varphi_1 \varphi_2 \varphi_3 \varphi_4 | \Omega \rangle =$
+
+
3
4
1
2
1,2 initial
3,4 final


4-momentum conse
+ O(g)
t
x

4-momentum conserving delta per connected component!

(last time missed $(\partial^2 + m^2)$)

in $\langle f|S|i\rangle$

$$\langle f|S|i\rangle_{||} \propto \delta^4(p_1 - p_3)\, \delta^4(p_2 - p_4)$$

no scattering!

not interesting

$$\langle f|S|i\rangle_{=} \propto \delta^4(p_1 + p_2)\, \delta^{(4)}(p_3 + p_4)$$

if $m > 0$

g)

4-momentum conserving delta per connected component!

(last time missed (∂^2+m^2))

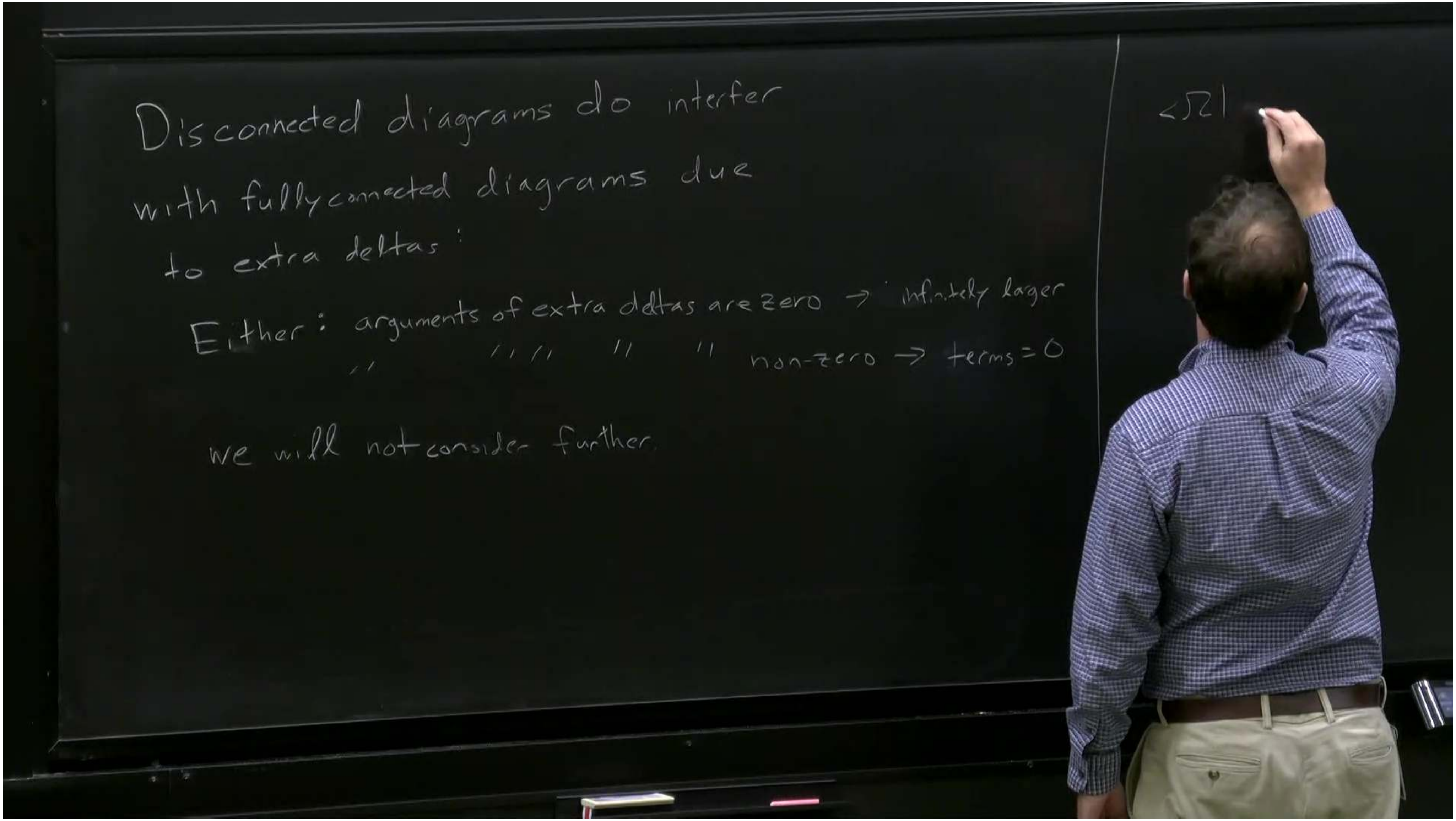
in $\langle f|S|i\rangle$

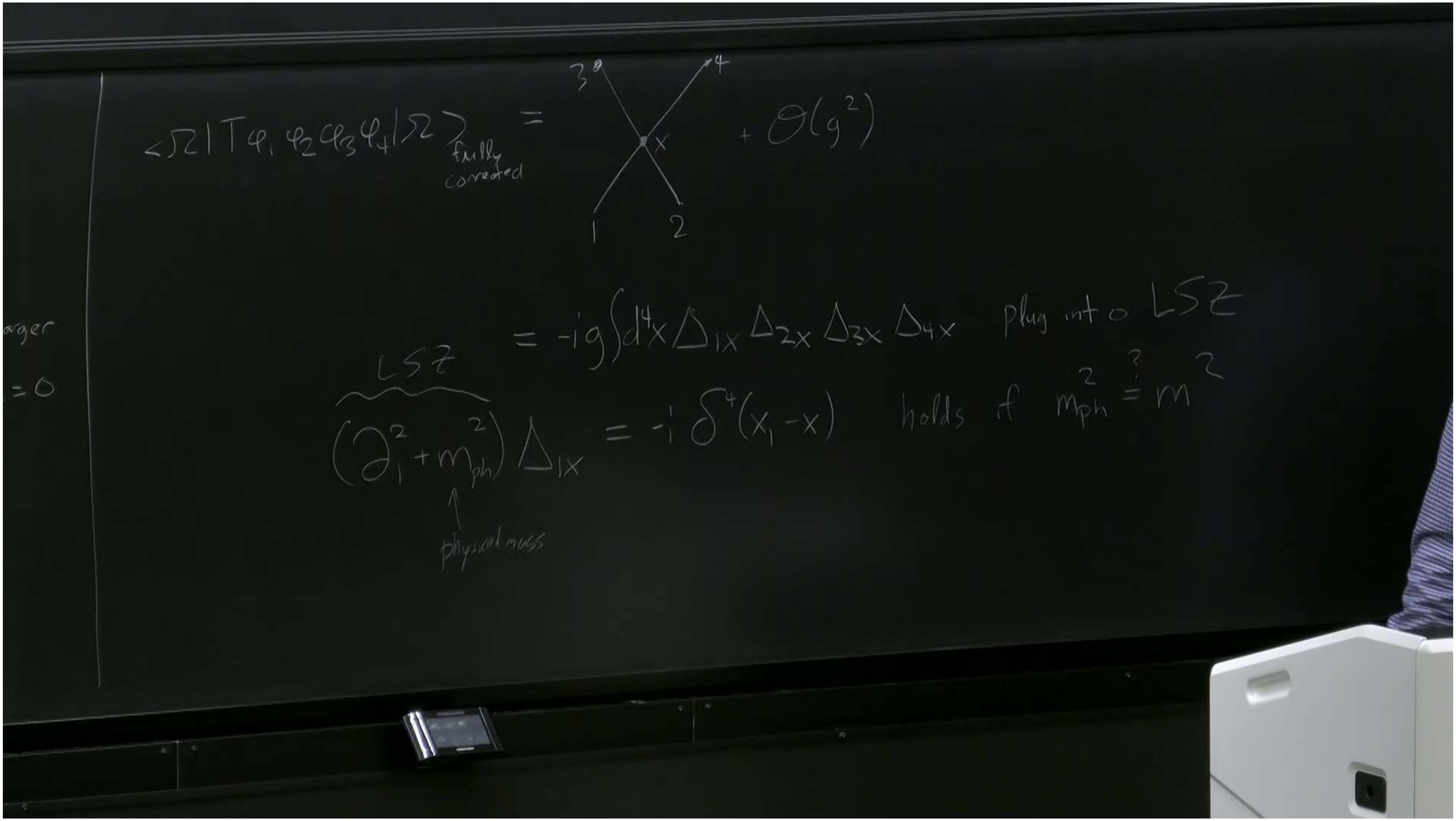
$$\langle f|S|i\rangle_{||} \propto \delta^4(p_1-p_3)\,\delta^4(p_2-p_4)$$

no scattering!
not interesting

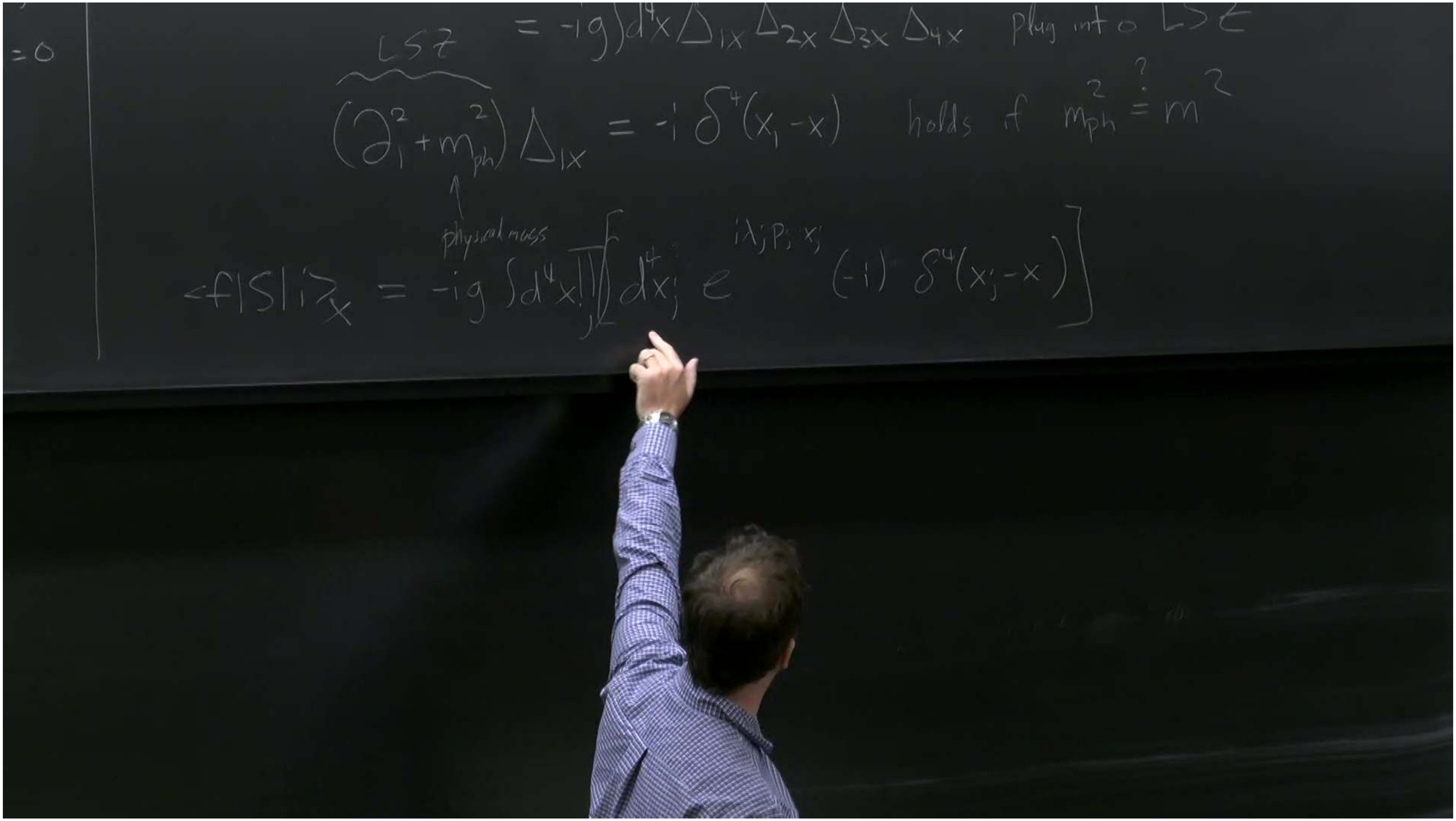
$$\langle f|S|i\rangle_{=} \propto \delta^4(p_1+p_2)\,\delta^{(4)}(p_3+p_4) = 0$$

if $m>0$, $p_1^0+p_2^0>0$


Disconnected diagrams do interfer
with fullyconnected diagrams due
to extra deltas:
Either: arguments of extra deltas are zero → infinitely larger
" " " " " non-zero → terms = 0
we will not consider further.
<Ω|



⟨Ω| T φ₁ φ₂ φ₃ φ₄ |Ω⟩ fully corrected = + O(g²)
3
4
x
1
2
arger
= 0
= -ig ∫d⁴x Δ1x Δ2x Δ3x Δ4x plug into LSZ
LSZ
(∂₁² + m²ph) Δ1x = -i δ⁴(x₁ - x) holds if m²ph ≟ m²
physical mass



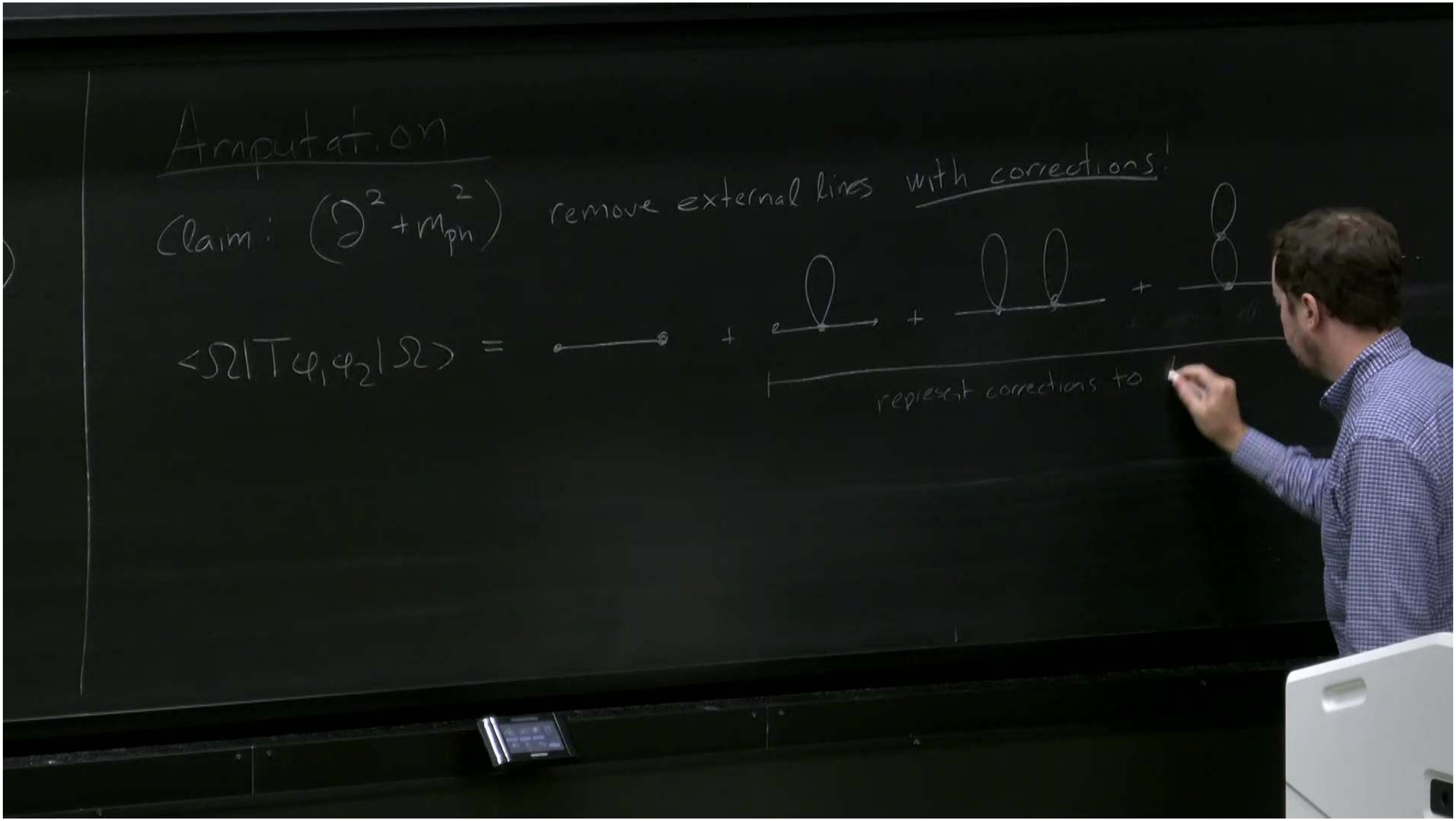
$= 0$
LSZ
$= -ig\int d^4x\, \Delta_{1x}\, \Delta_{2x}\, \Delta_{3x}\, \Delta_{4x}$ plug into LSZ
$(\partial_1^2 + m_{ph}^2)\, \Delta_{1x} = -i\, \delta^4(x_1 - x)$ holds if $m_{ph}^2 \overset{?}{=} m^2$
physical mass
$\langle f|S|i\rangle_x = -ig\int d^4x \prod_j \left[\int d^4x_j\, e^{i\lambda_j p_j x_j}\, (-i)\, \delta^4(x_j - x)\right]$

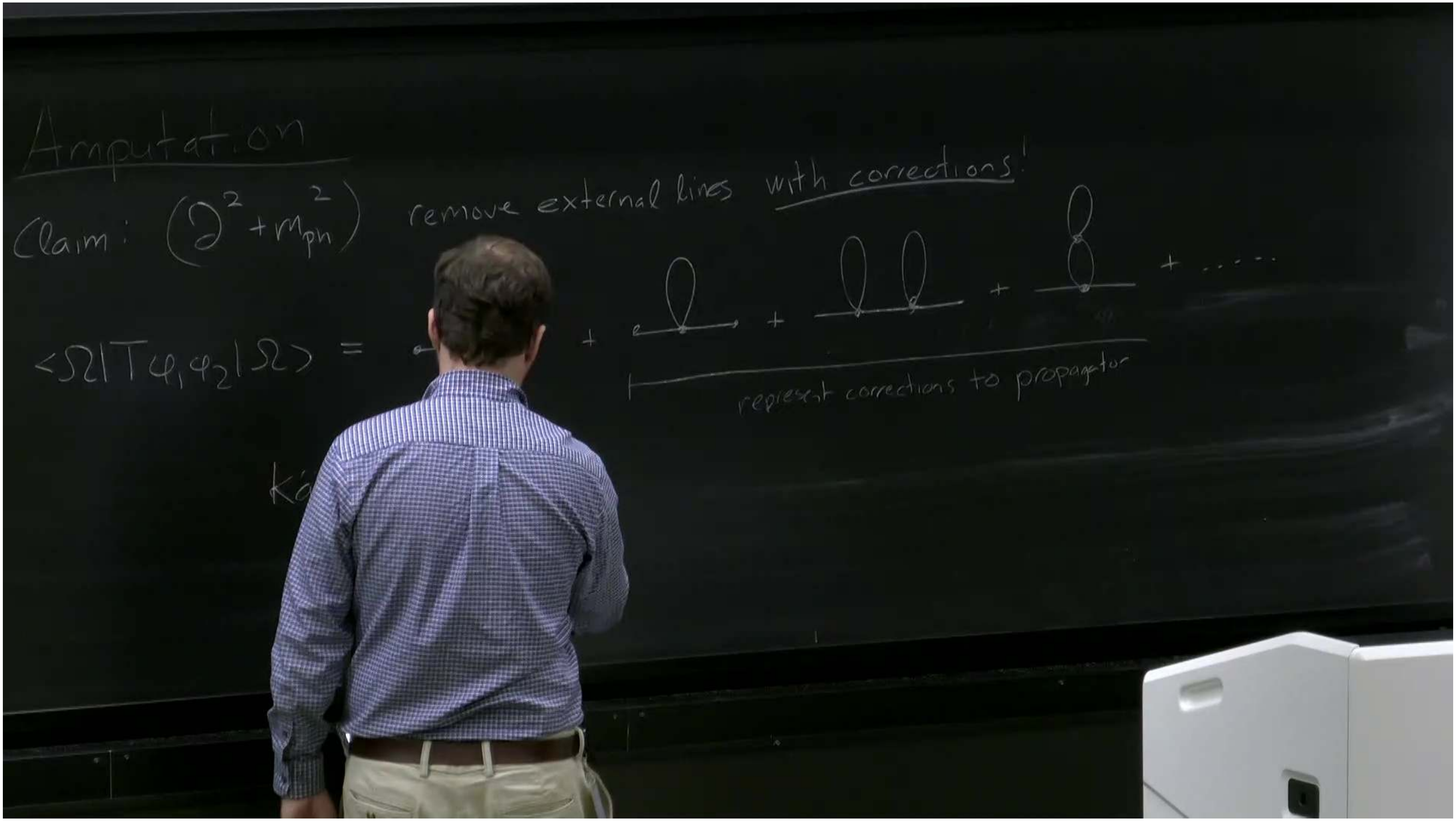
$$\langle f|S|i\rangle_x = -ig\int d^4x\, e^{i(p_1+p_2-p_3-p_4)\cdot x}$$

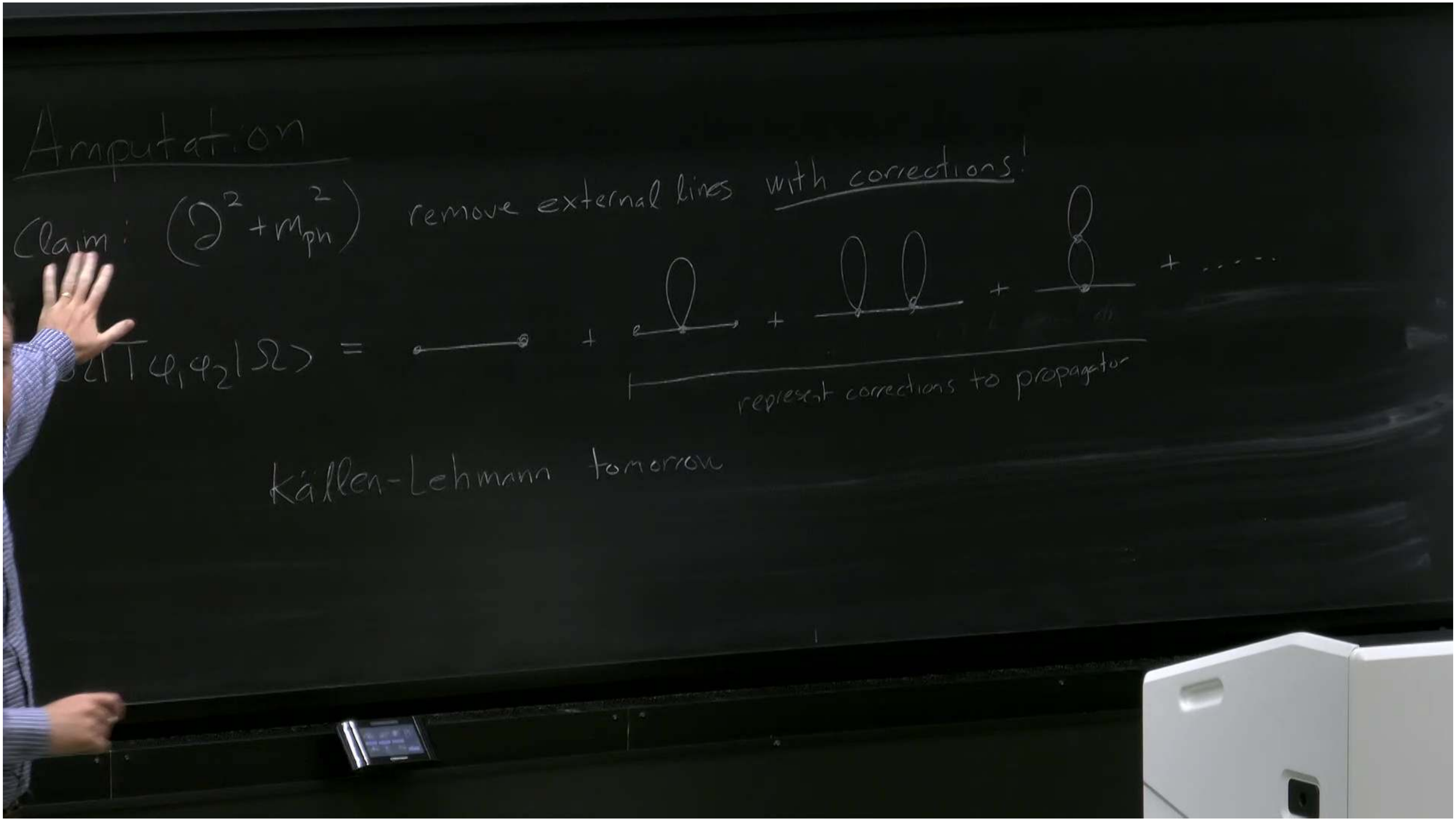
$$= \underbrace{-ig}_{iM_{i\to f}}\,(2\pi)^4\,\delta^4(p_1+p_2-p_3-p_4)$$

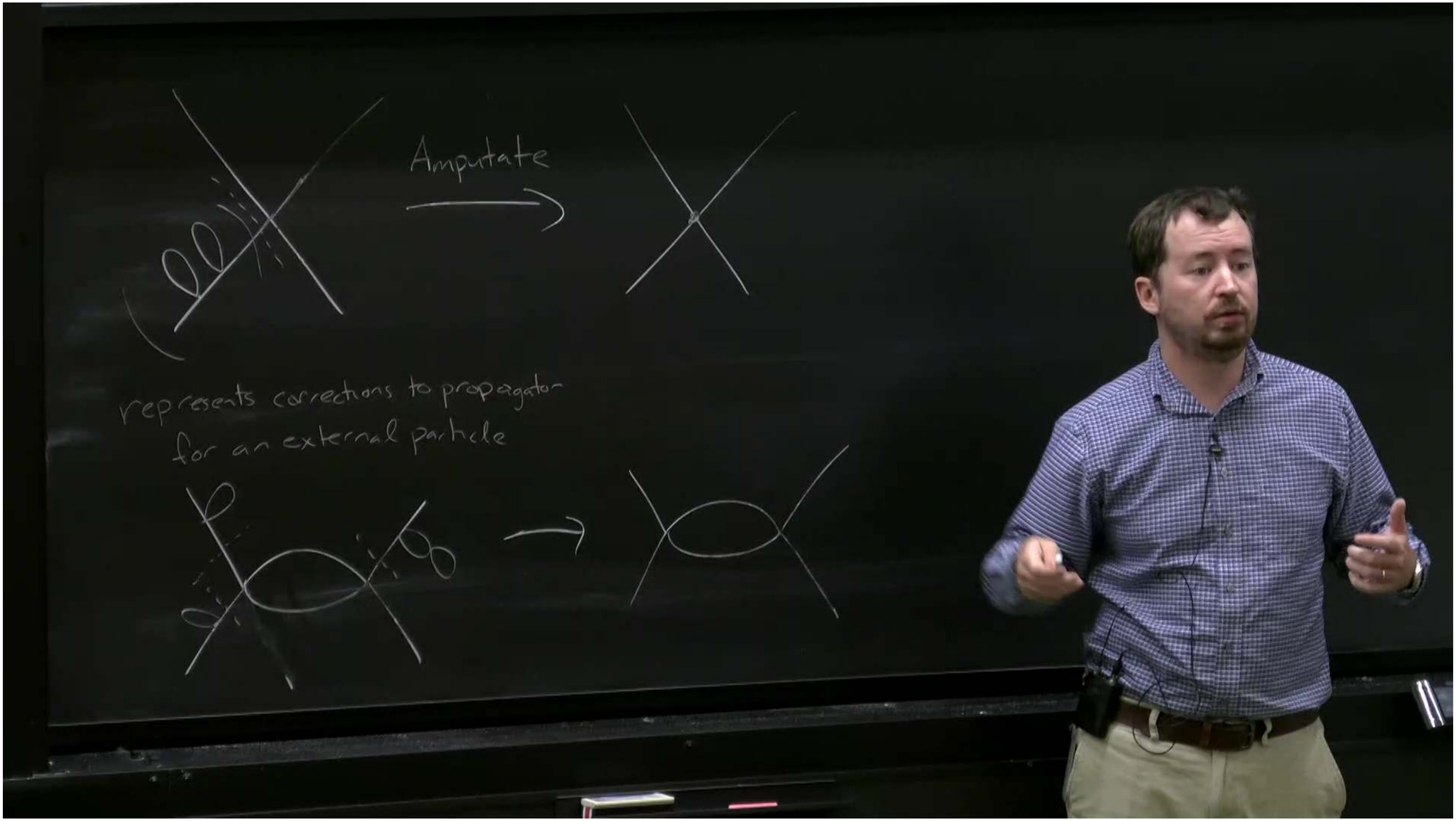
$$iM = -ig + O(g^2)$$

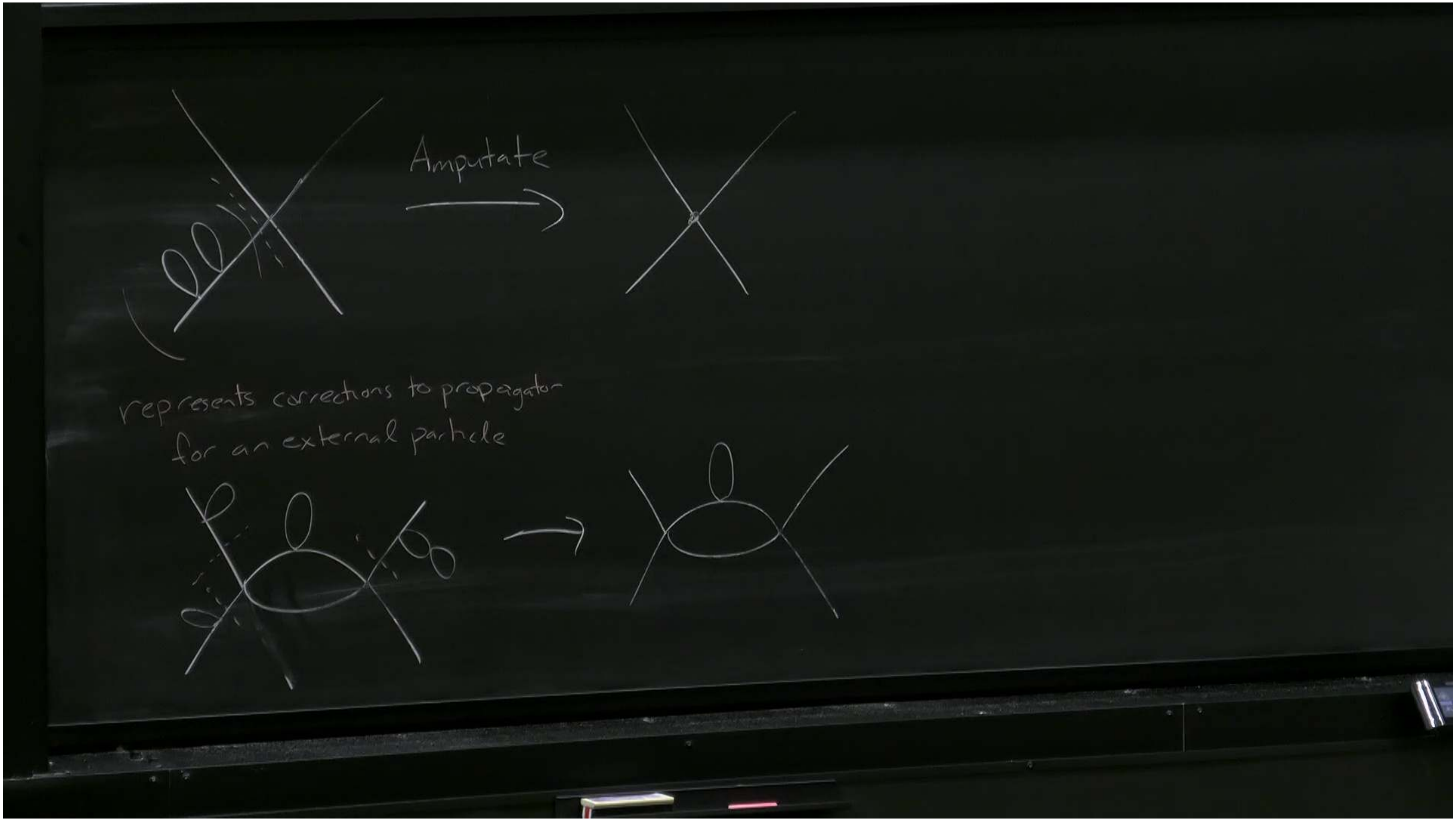
no propagators for external lines!


Amputation
Claim: (∂² + m_ph²) remove external lines with corrections!
<Ω|Tφ₁φ₂|Ω> = + + +
represent corrections to


Amputation
Claim: $(\partial^2 + m_{ph}^2)$ remove external lines with corrections!
$\langle \Omega | T \varphi_1 \varphi_2 | \Omega \rangle =$
+ + + + ...
represent corrections to propagator


Amputation
Claim: $(\partial^2 + m_{ph}^2)$ remove external lines with corrections!
$\langle\Omega|T\varphi_1\varphi_2|\Omega\rangle$ =
+ + + + ...
represent corrections to propagator
Källen-Lehmann tomorrow


Amputate
represents corrections to propagator
for an external particle


Amputate
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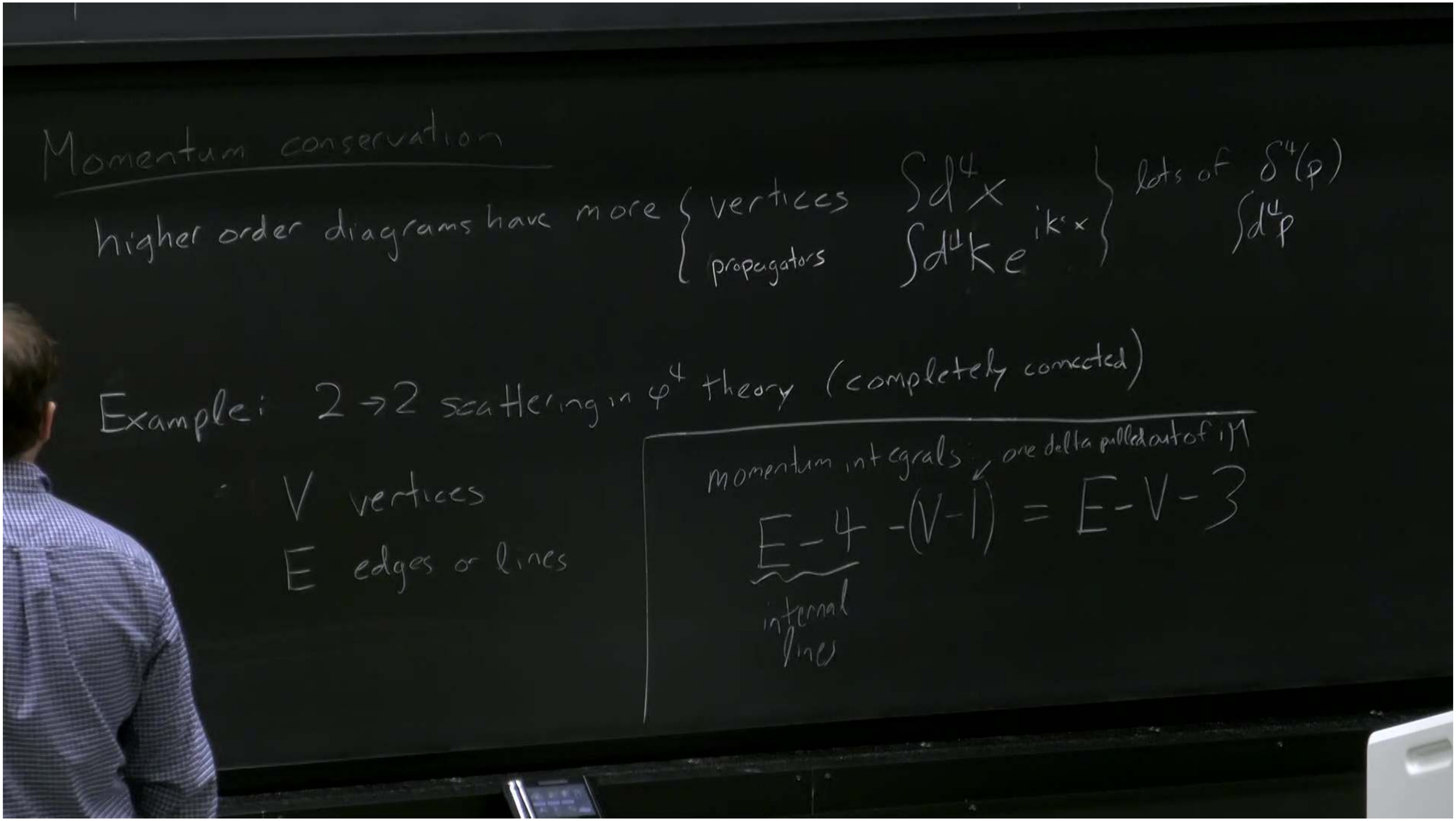
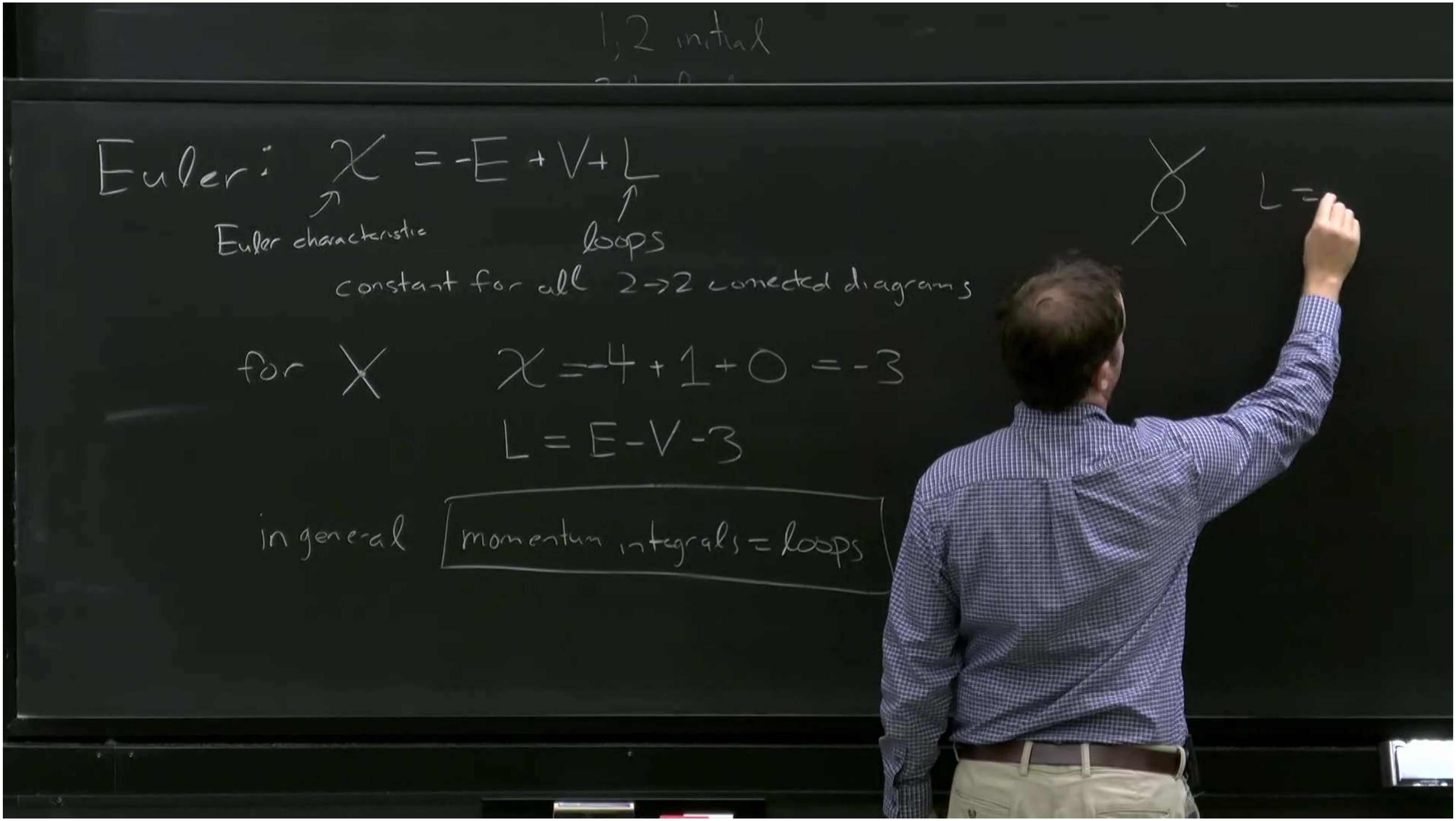

Momentum conservation
higher order diagrams have more { vertices $\int d^4x$; propagators $\int d^4k\, e^{ik\cdot x}$ } lots of $\delta^4(p)$, $\int d^4p$
Example: $2\to 2$ scattering in φ^4 theory (completely connected)
V vertices
E edges or lines
momentum integrals ; one delta pulled out of $i\mathcal{M}$
$\underbrace{E-4}_{\text{internal lines}} - (V-1) = E-V-3$


1,2 initial
Euler: $\chi = -E + V + L$
Euler characteristic
loops
constant for all 2→2 conected diagrams
for X $\chi = -4 + 1 + 0 = -3$
$L = E - V - 3$
in general
momentum integrals = loops
L =

1,2 initial

$\chi = -E + V + L$

characteristic

loops

constant for all $2 \to 2$ connected diagrams

$\chi = -4 + 1 + 0 = -3$

$L = E - V - 3$

general $\boxed{\text{momentum integrals} = \text{loops}}$

$L = 1$
$E = 6$
$V = 2$

$\chi = -6 + 1 + 2 = -3$

$L = 2$
$E = 8$
$V = 3$

$\chi = -8 + 2 + 3 = -3$

1, 2 initial

3, 4 final

nman rules for $i\mathcal{M}$

$\mathcal{M}$ = sum of all completely connected, amputated diagrams

(nteresting part)

internal line $\overrightarrow{p}$ $= \dfrac{i}{p^2 - m^2 + i\varepsilon}$

arrow next to line

external lines $\overrightarrow{p}$ $= 1$

grams

3.  $= -ig$ (depends on theory)

4. Impose (by hand) momentum conservation at each vertex

grams

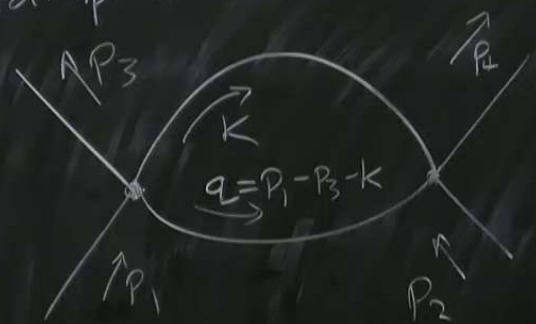
3.  $= -ig$ (depends on theory)

4. Impose (by hand) momentum conservation at each vertex (No δ!)

5. Integrate over undetermined momenta $\int \frac{d^4 k}{(2\pi)^4}$ (1 per loop)

6. Divide by symmetry factor

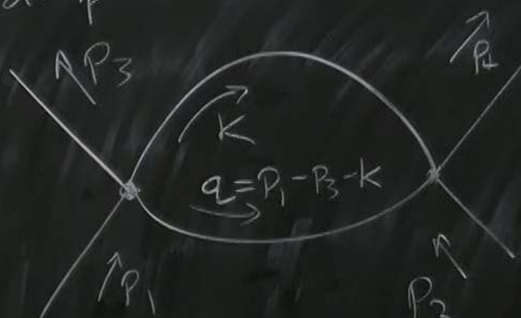
Example:



$$= \frac{i}{k^2 - m^2 + i\epsilon} \quad \frac{i}{(p_1 - p_3 - k)^2 - m^2 + i\epsilon}$$

$p_1 = p_3 + k + q$ $\qquad$ $p_2 + \cancel{k} + p_1 - p_3 - \cancel{k} = p_4$

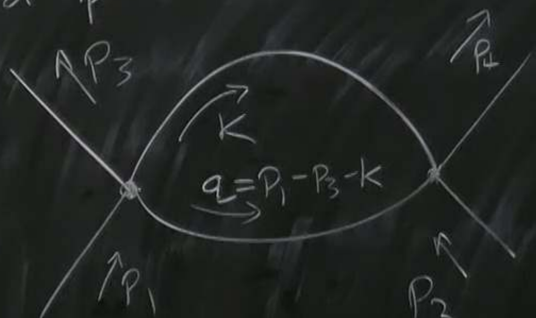
Example:



$$= (-i\lambda)^2 \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m^2 + i\varepsilon} \frac{i}{(p_1 - p_3 - k)^2 - m^2 + i\varepsilon}$$

$$p_1 = p_3 + k + q \qquad p_2 + \cancel{k} + p_1 - p_3 - \cancel{k} = p_4$$

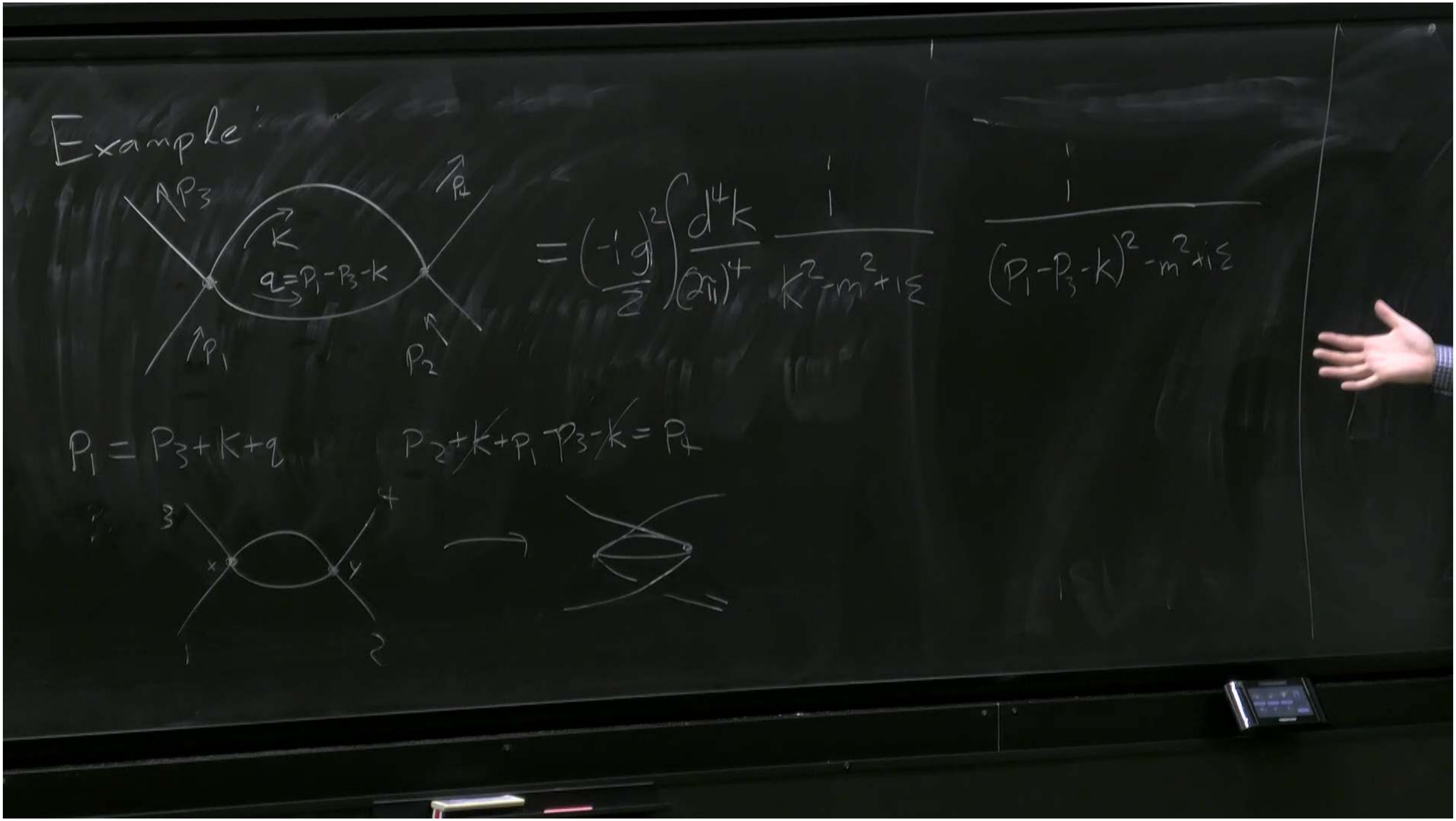
Example:

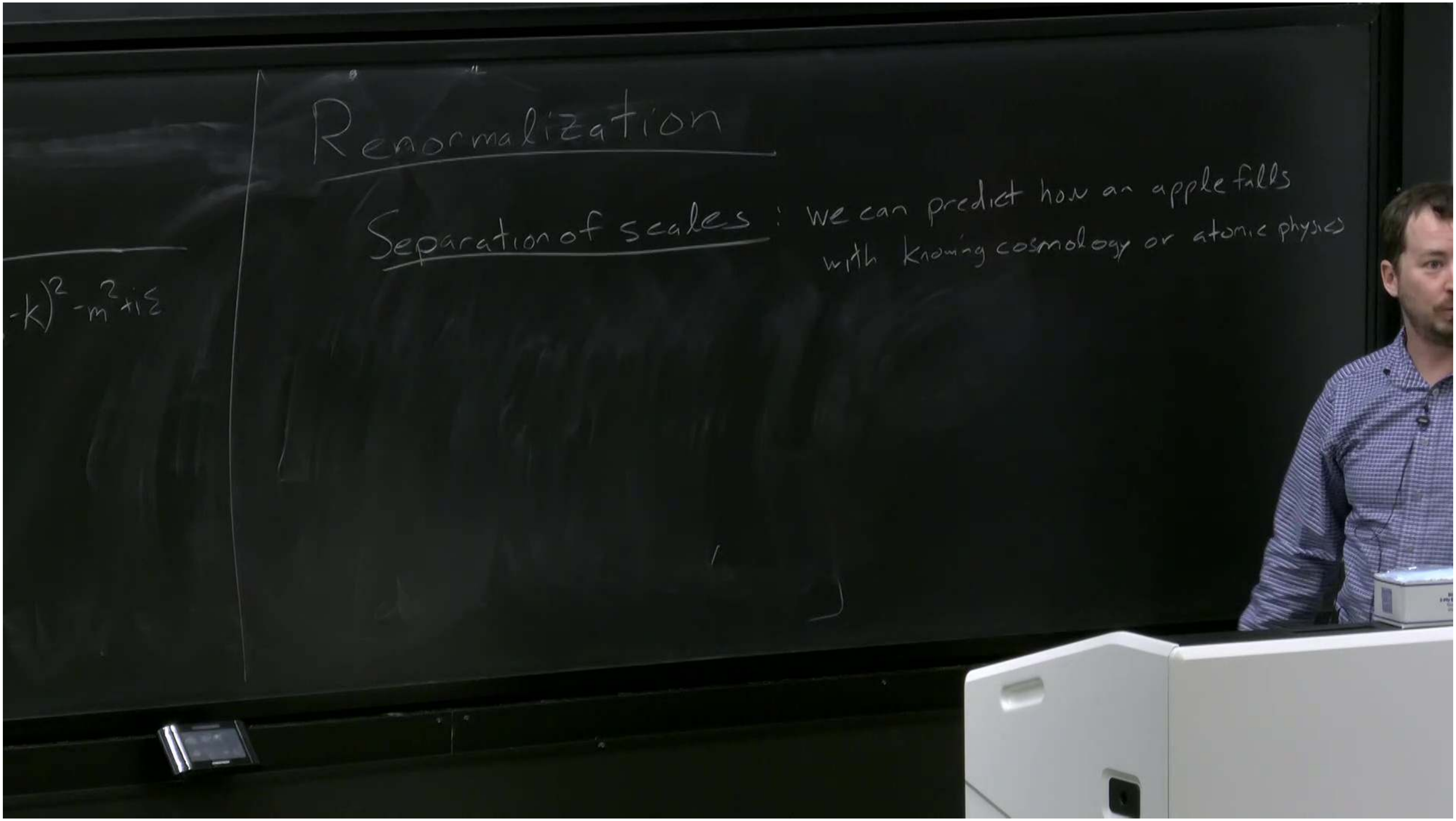


$$= \left(\frac{-ig}{2}\right)^2 \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m^2 + i\varepsilon} \frac{i}{(p_1 - p_3 - k)^2 - m^2 + i\varepsilon}$$

$p_1 = p_3 + k + q$

$p_2 + \cancel{k} + p_1 - p_3 - \cancel{k} = p_4$


Example:
p_3
p_4
k
$q = p_1 - p_3 - k$
p_1
p_2
$= \frac{(-ig)^2}{2}\int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2-m^2+i\epsilon} \frac{i}{(p_1-p_3-k)^2-m^2+i\epsilon}$
$p_1 = p_3 + k + q$
$p_2 + \cancel{k} + p_1 - p_3 - \cancel{k} = p_4$
3
4
x
y
1
2


-k)^2 -m^2+iε
Renormalization
Separation of scales : we can predict how an apple falls
with knowing cosmology or atomic physics

Goal: predict near a physical scale μ

$\mathcal{O}(|p|)$ for $|p| \approx \mu$

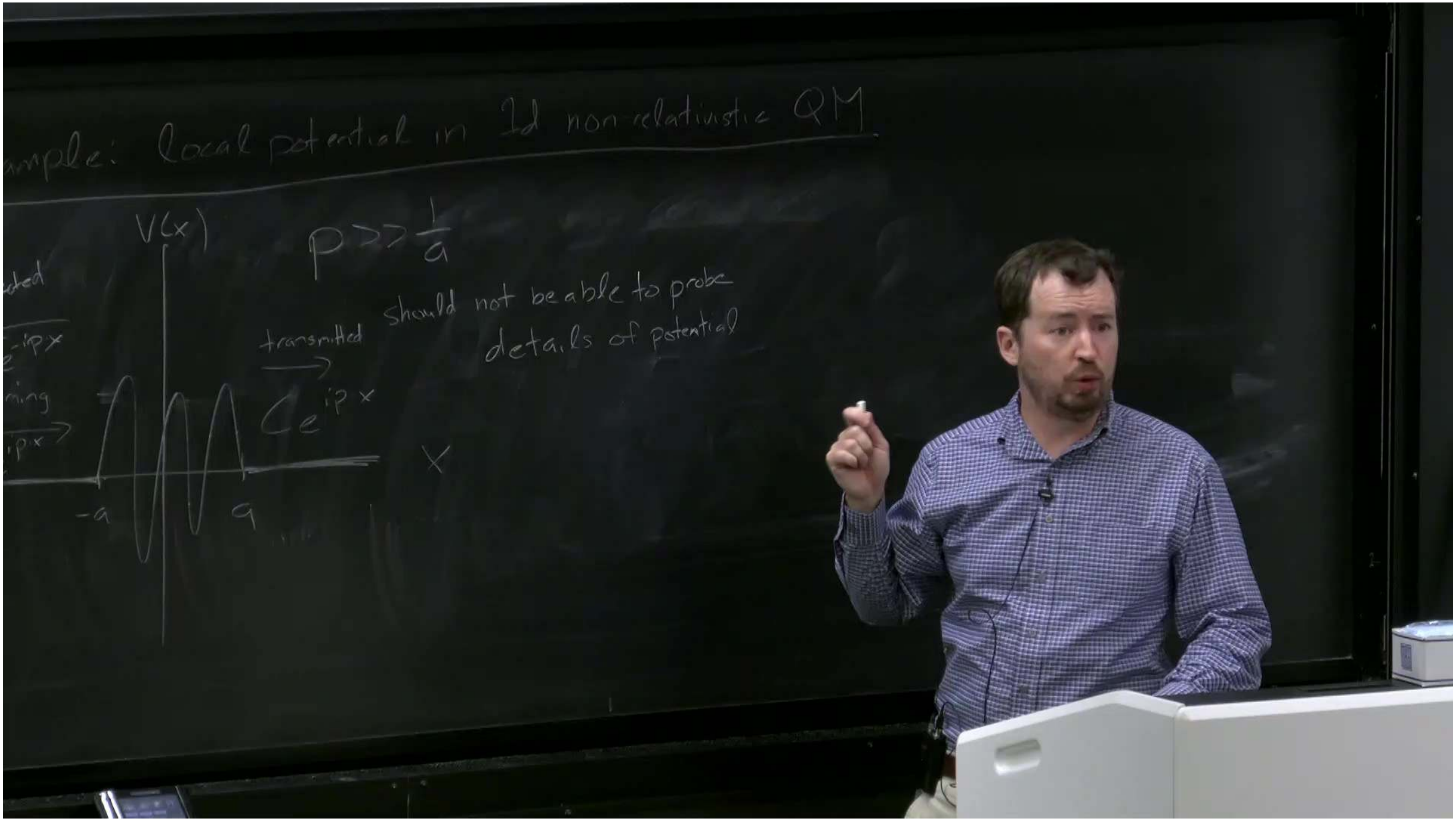
introduce unobservable quantity

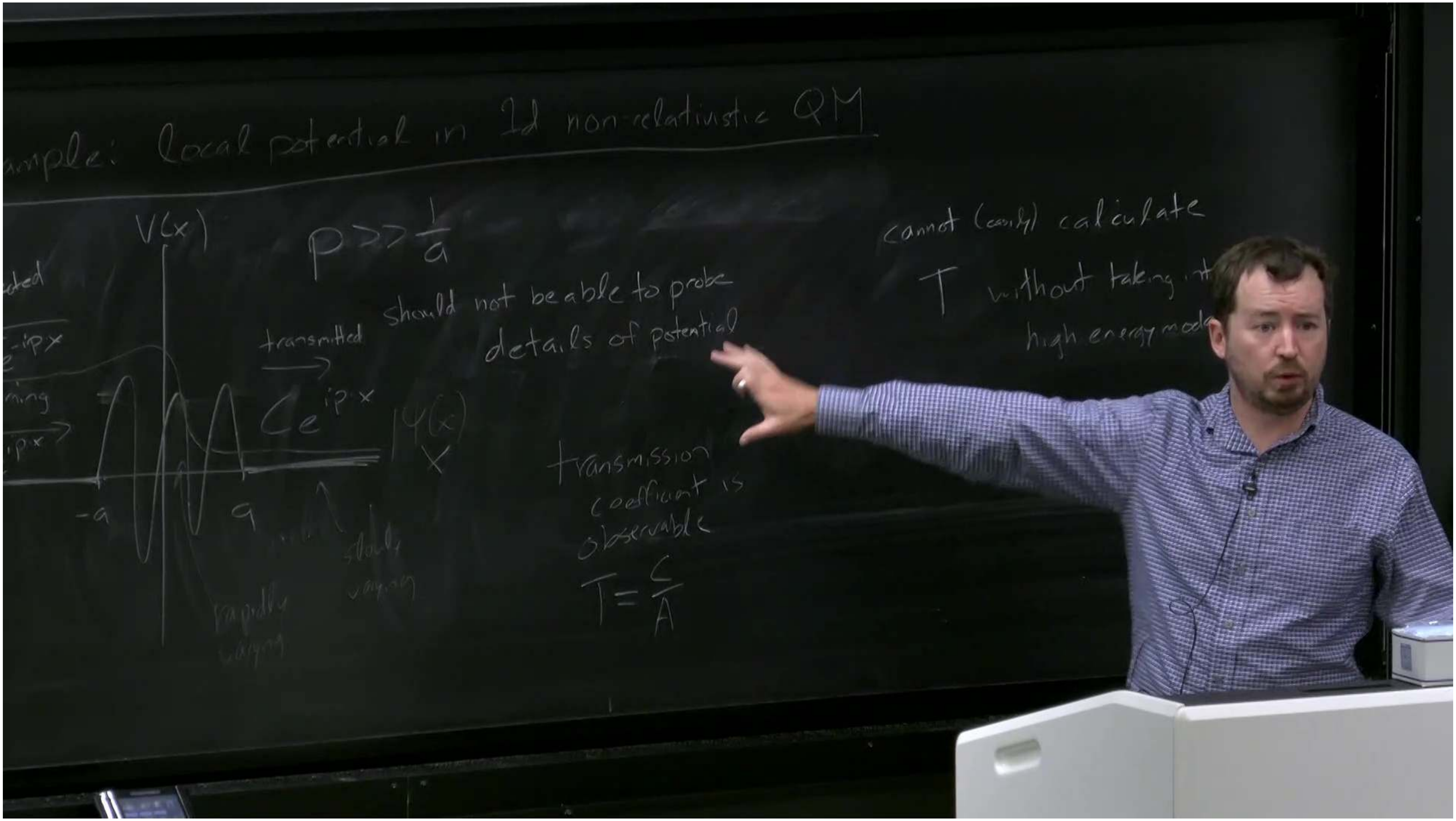
$$Q(\Lambda, \mu, \tfrac{1}{L}, P, c)$$

coupling constants

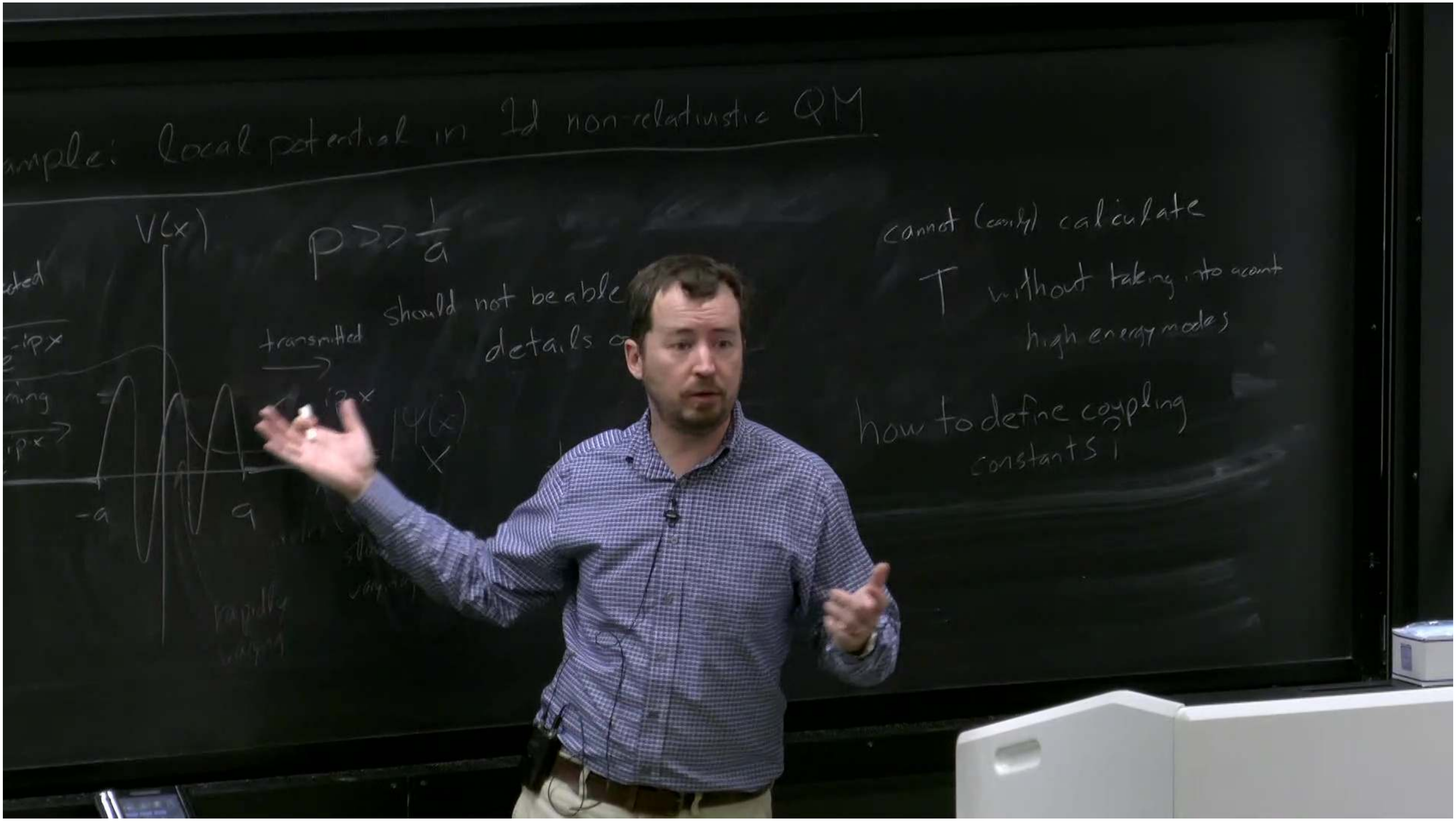
scale at which we are measuring

$$\mathcal{O}(p, c)$$


ample: local potential in 1d non-relativistic QM
V(x)
$p \gg \frac{1}{a}$
should not be able to probe
details of potential
transmitted
Ce^{ipx}
$-a$
a
x



ample: local potential in 1d non-relativistic QM
V(x)
p >> 1/a
should not be able to probe
details of potential
transmitted
Ce^{ip·x}
ψ(x)
x
-a
a
rapidly
varying
slowly
varying
transmission
coefficient is
observable
T = C/A
cannot (easily) calculate
T without taking into
high energy mode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V(x)
p >> 1/a
should not be able
details
transmitted
-a
a
rapidly
cannot (easily) calculate
T without taking into acount
high energy modes
how to define coupling
constants?