Title: Quantum Theory Lecture - 100223

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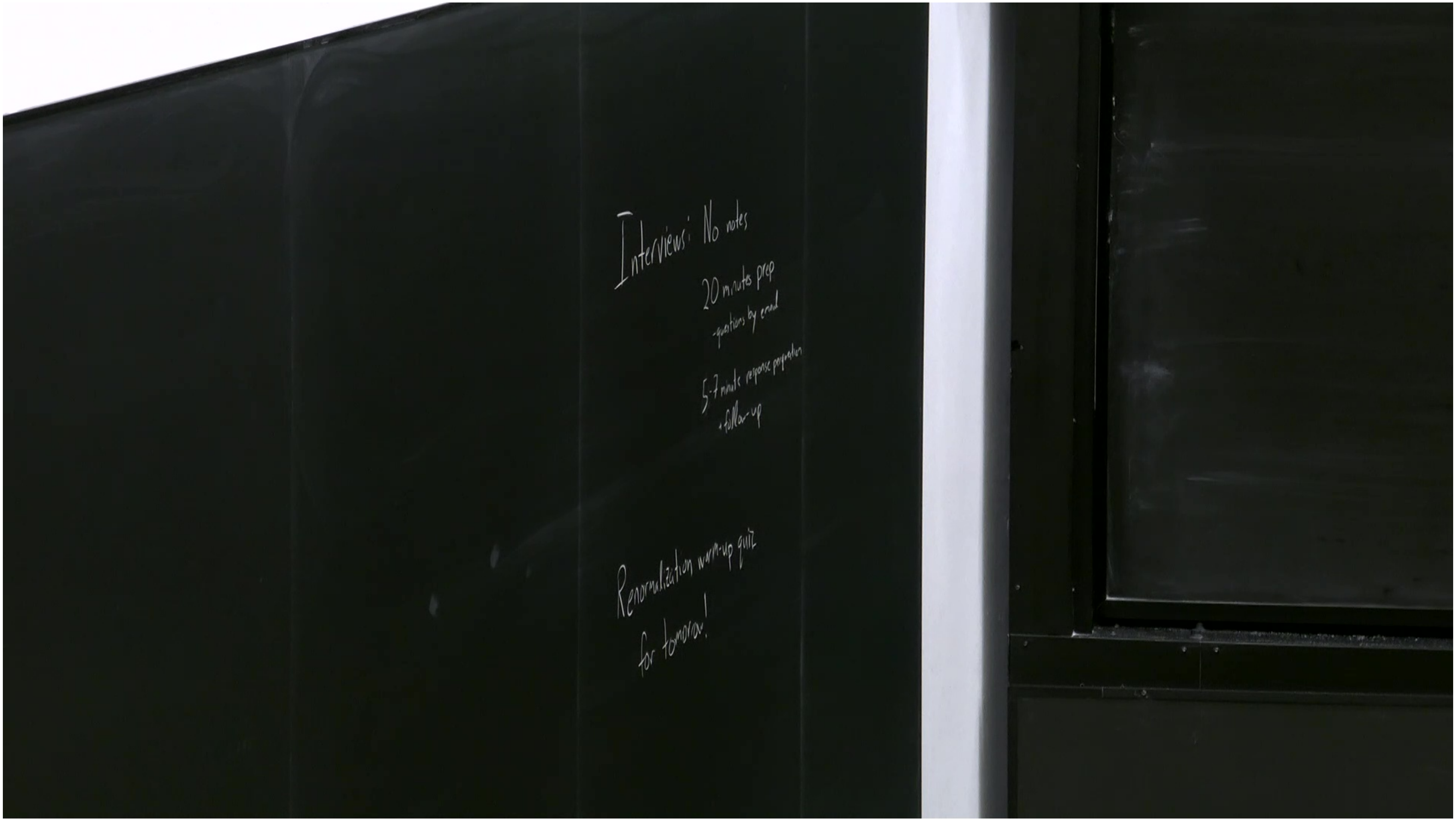
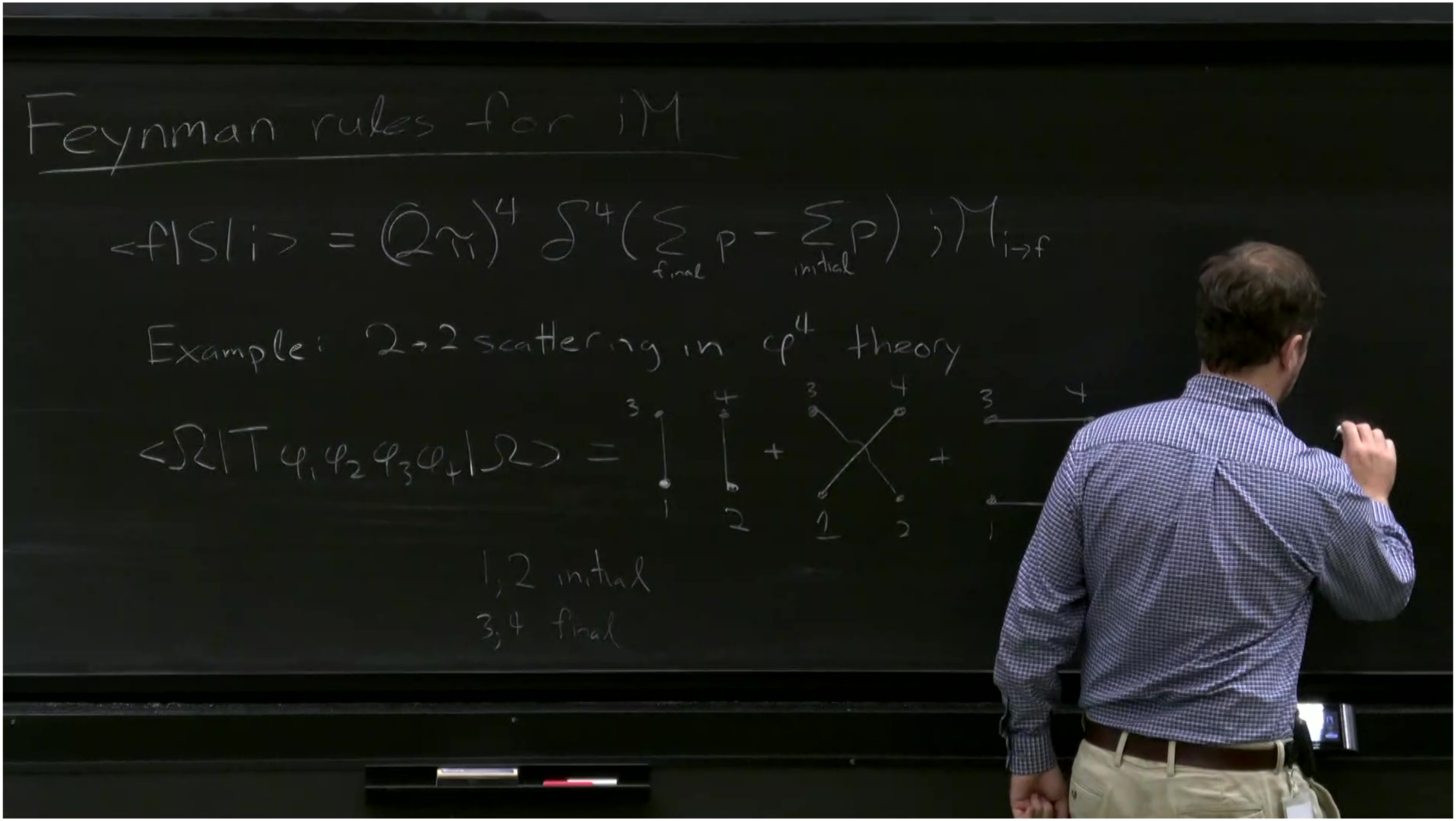
Collection: Quantum Theory 2023/24

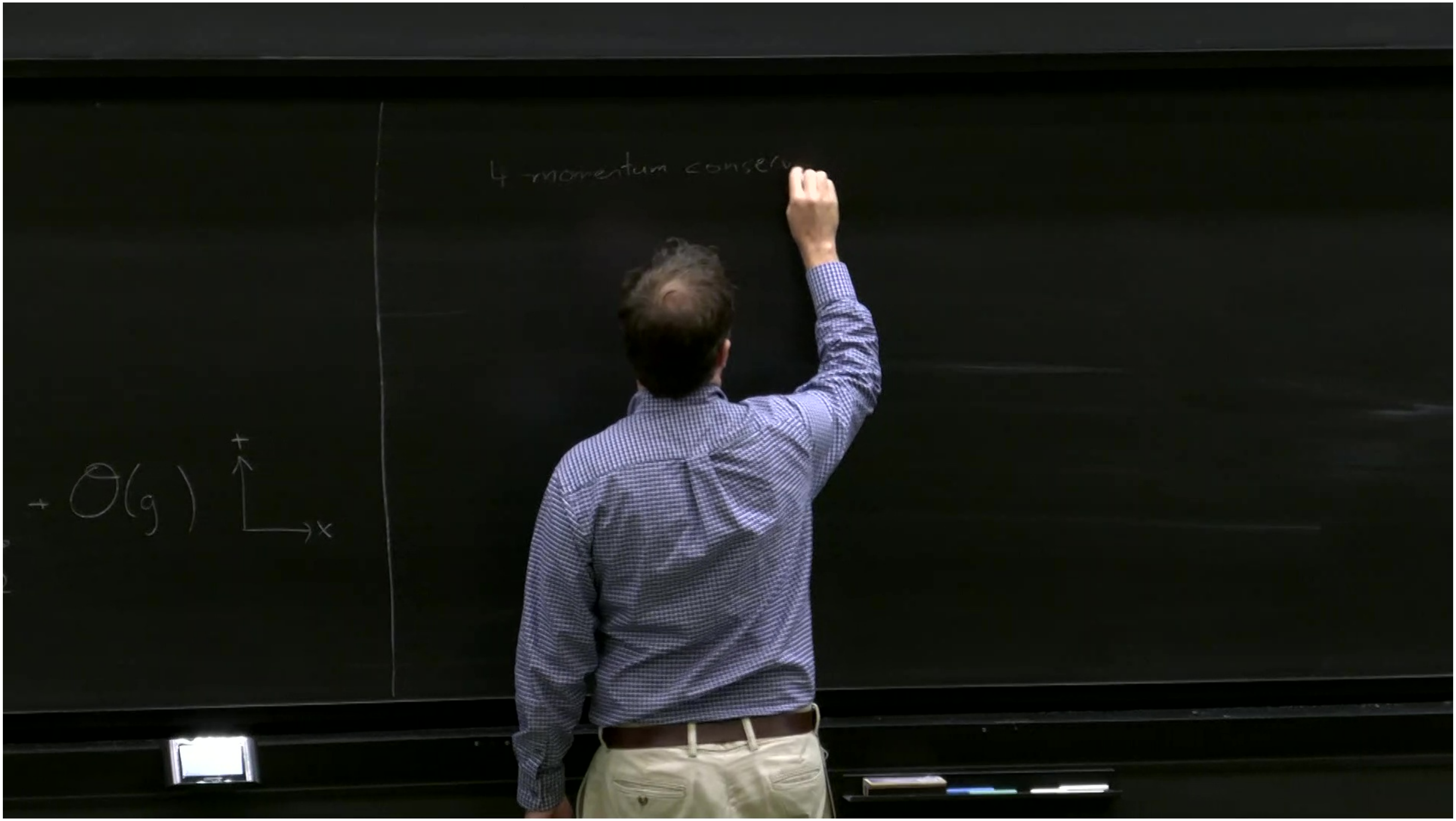
Date: October 02, 2023 - 10:45 AM

URL: https://pirsa.org/23100034



Interviews: No notes
20 minutes prep
-questions by email
5-7 minutes response preparation
+follow-up
Renormalization warm-up quiz
for tomorrow!


Feynman rules for iM
$\langle f|S|i\rangle = (2\pi)^4 \delta^4\left(\sum_{\text{final}} p - \sum_{\text{initial}} p\right) iM_{i\to f}$
Example: 2→2 scattering in φ^4 theory
$\langle\Omega|T\varphi_1\varphi_2\varphi_3\varphi_4|\Omega\rangle =$ + +
1,2 initial
3,4 final


4-momentum conse
+ O(g)
t
x

4-momentum conserving delta per connected component!

(last time missed (∂^2+m^2))

in $\langle f|S|i\rangle$

$$\langle f|S|i\rangle_{||} \propto \delta^4(p_1-p_3)\,\delta^4(p_2-p_4)$$

no scattering!

not interesting

$$\langle f|S|i\rangle_{=} \propto \delta^4(p_1+p_2)\,\delta^{(4)}(p_3+p_4)$$

if $m>0$

g)

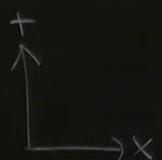
4-momentum conserving delta per connected component!

(last time missed (∂^2+m^2))

in $\langle f|S|i\rangle$

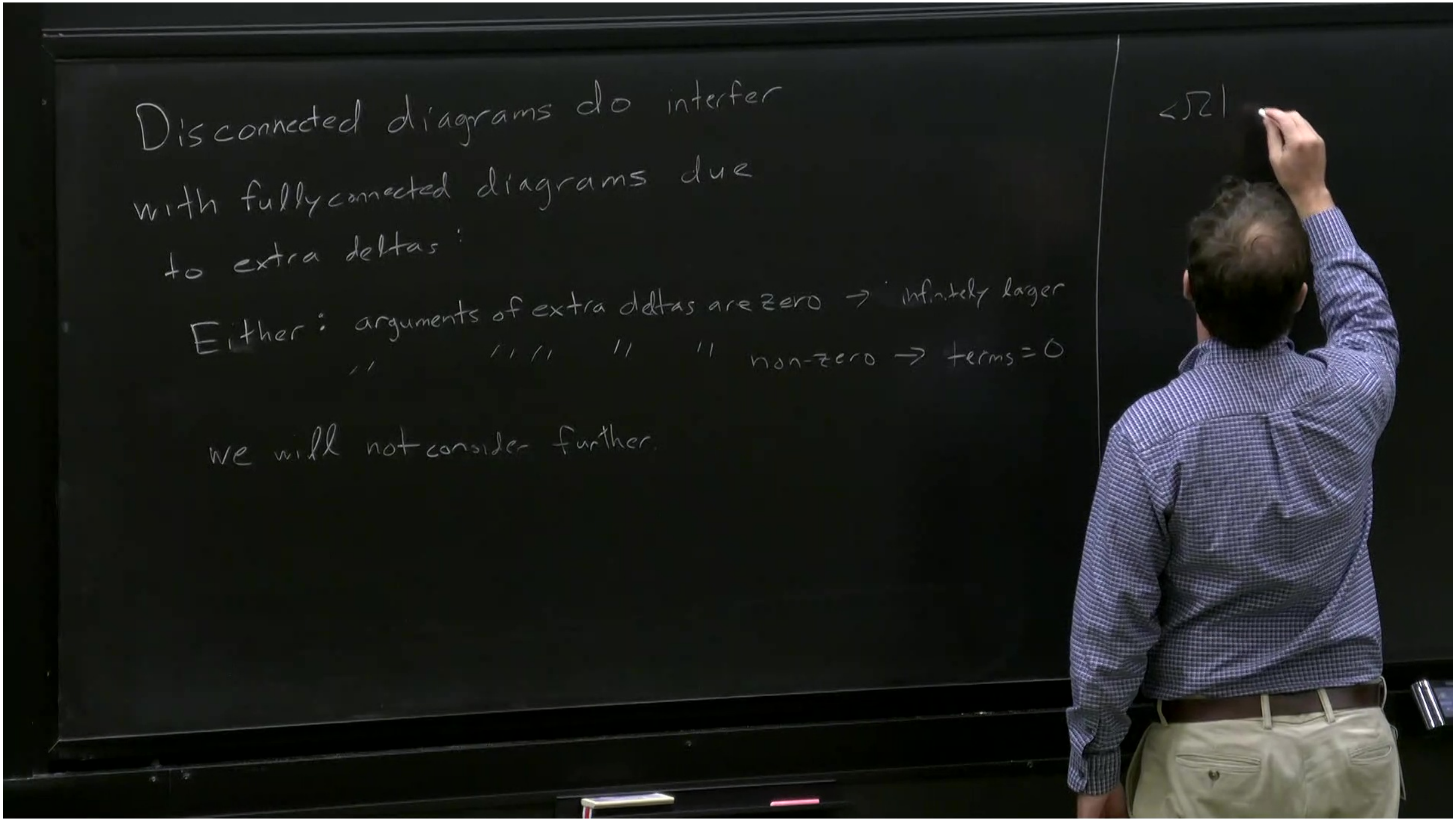
$$\langle f|S|i\rangle_{||} \propto \delta^4(p_1-p_3)\,\delta^4(p_2-p_4)$$

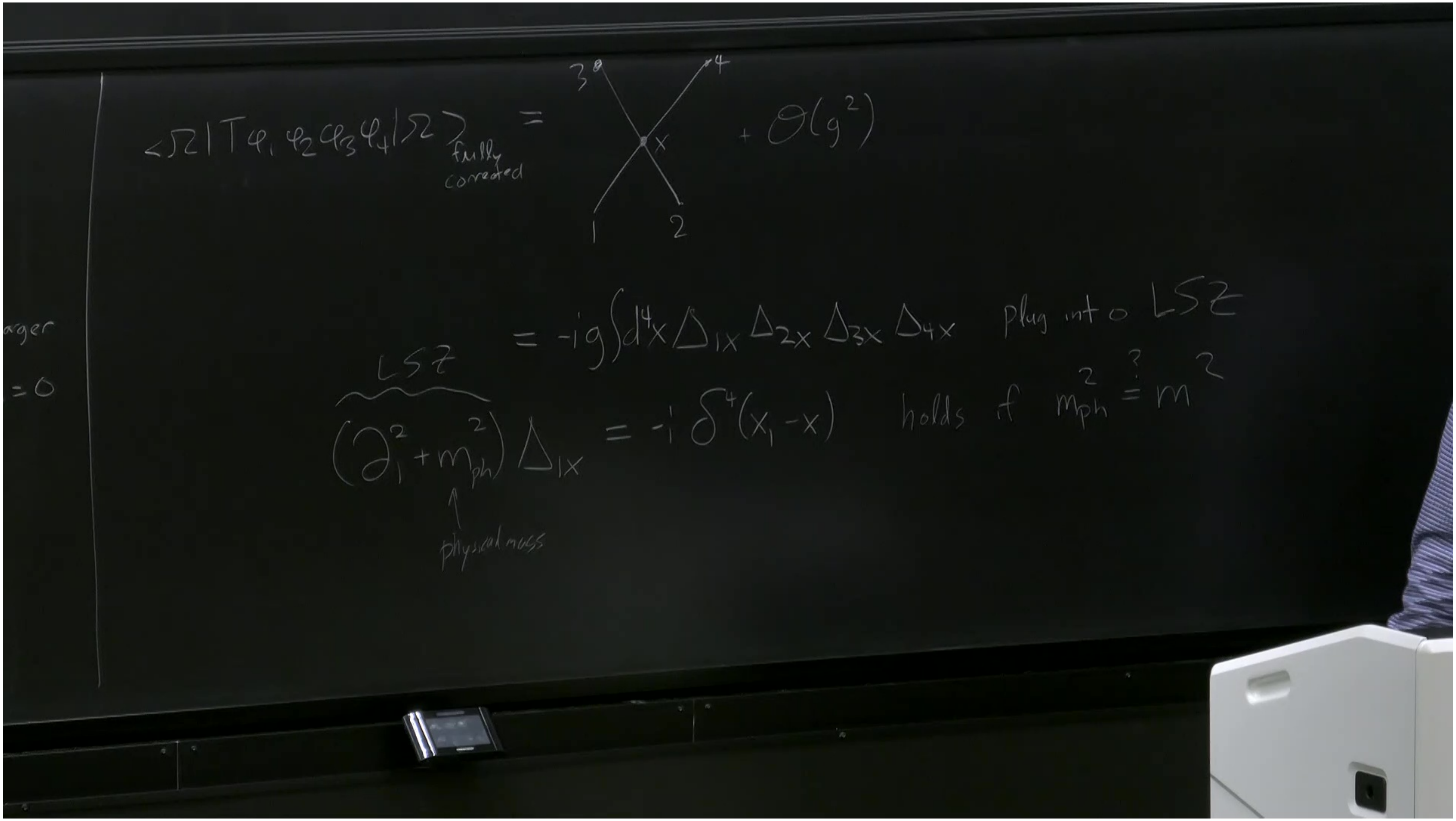
no scattering! not interesting



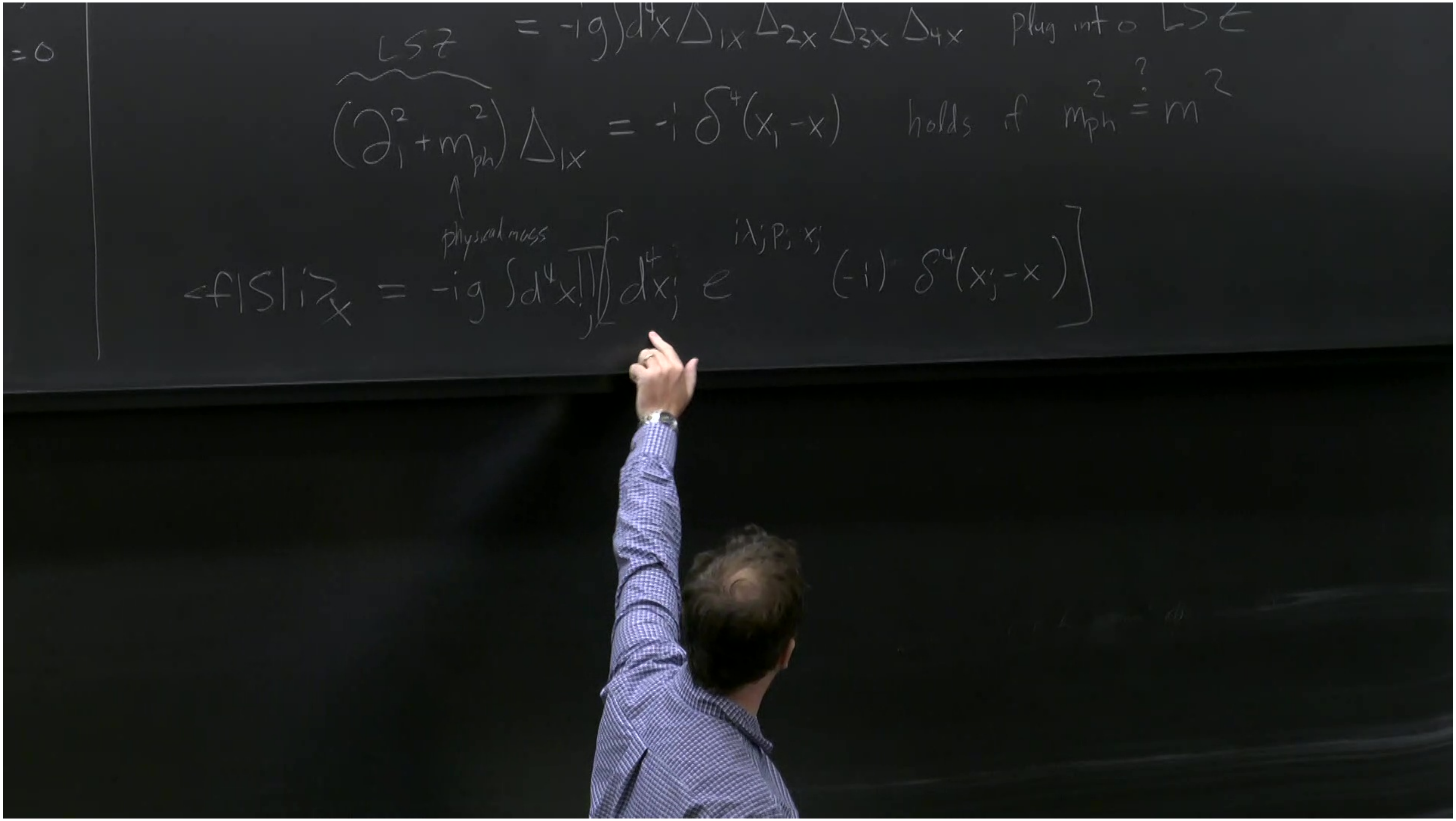
$$\langle f|S|i\rangle_{=} \propto \delta^4(p_1+p_2)\,\delta^{(4)}(p_3+p_4) = 0$$

if $m>0$, $p_1^0+p_2^0>0$


Disconnected diagrams do interfer
with fullyconnected diagrams due
to extra deltas:
Either: arguments of extra deltas are zero → infinitely larger
" " " " non-zero → terms = 0
we will not consider further.
<Ω|



arger
= 0
$\langle\Omega|T\varphi_1\varphi_2\varphi_3\varphi_4|\Omega\rangle_{\text{fully corrected}} =$
3
4
x
1
2
$+ \mathcal{O}(g^2)$
$= -ig\int d^4x\, \Delta_{1x}\Delta_{2x}\Delta_{3x}\Delta_{4x}$
plug into LSZ
LSZ
$(\partial_1^2 + m_{ph}^2)\Delta_{1x} = -i\delta^4(x_1 - x)$
holds if $m_{ph}^2 \overset{?}{=} m^2$
physical mass


$=0$
$= -ig \int d^4x \, \Delta_{1x} \Delta_{2x} \Delta_{3x} \Delta_{4x}$ plug into LSZ
LSZ
$(\partial_1^2 + m_{ph}^2) \Delta_{1x} = -i \delta^4(x_1 - x)$ holds if $m_{ph}^2 \overset{?}{=} m^2$
physical mass
$\langle f|S|i\rangle_x = -ig \int d^4x \prod_j \left[\int d^4x_j \, e^{i\lambda_j p_j x_j} (-i) \, \delta^4(x_j - x)\right]$

$$\langle f|S|i\rangle_x = -ig\int d^4x\, e^{i(p_1+p_2-p_3-p_4)\cdot x}$$

$$= \underbrace{-ig}_{iM_{i\to f}}\,(2\pi)^4\,\delta^4(p_1+p_2-p_3-p_4)$$

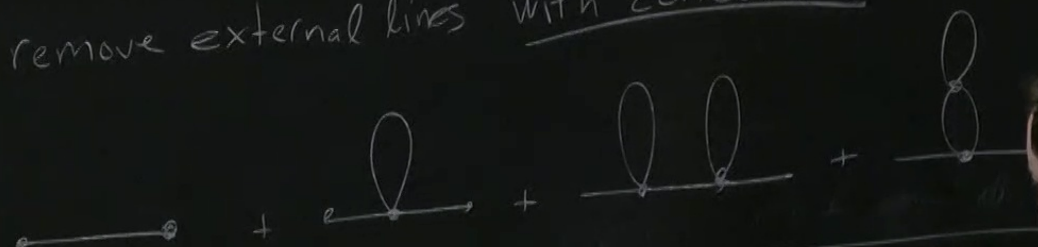
$$iM = -ig + O(g^2)$$

no propagators for external lines!

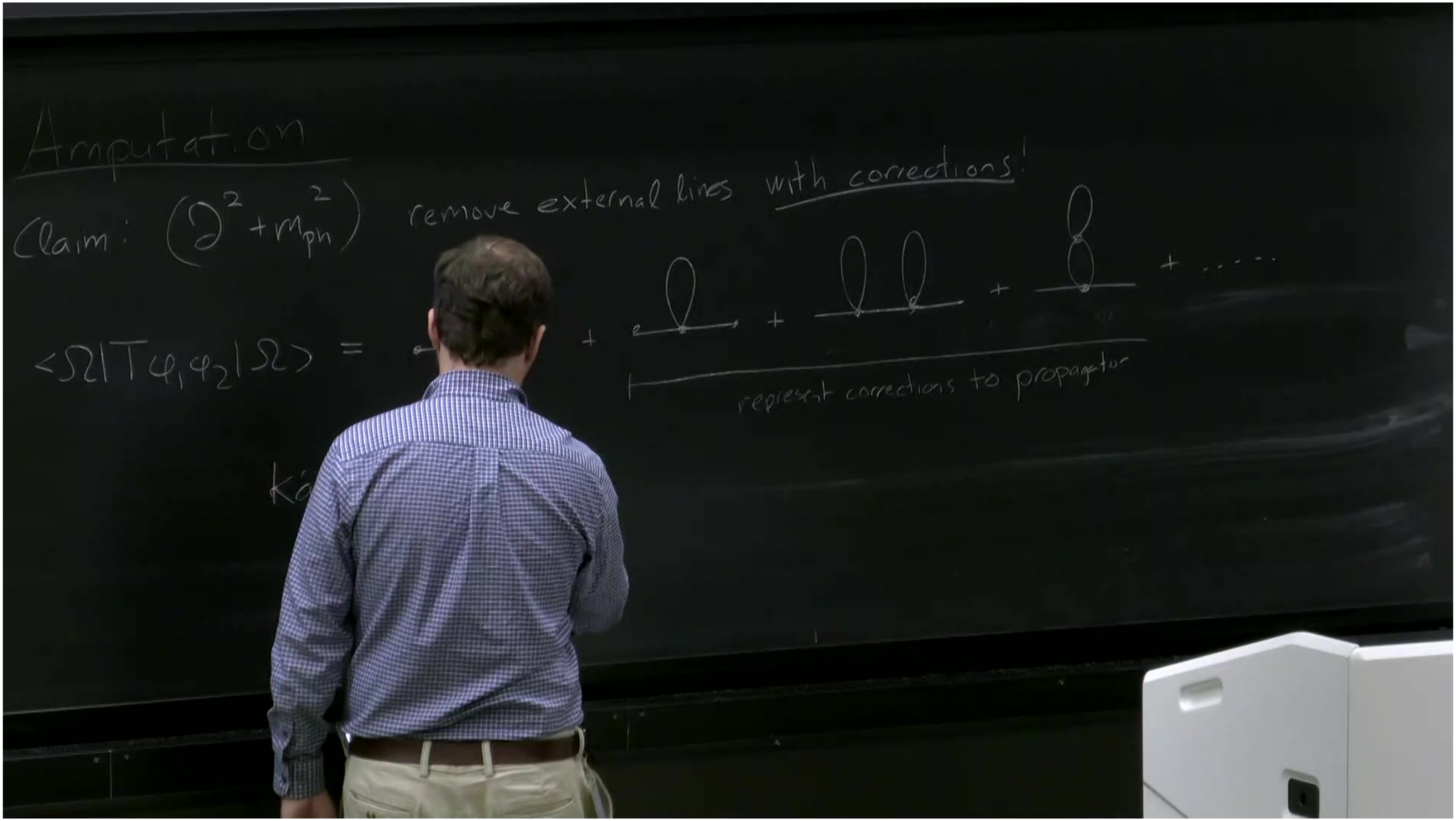
Amputation

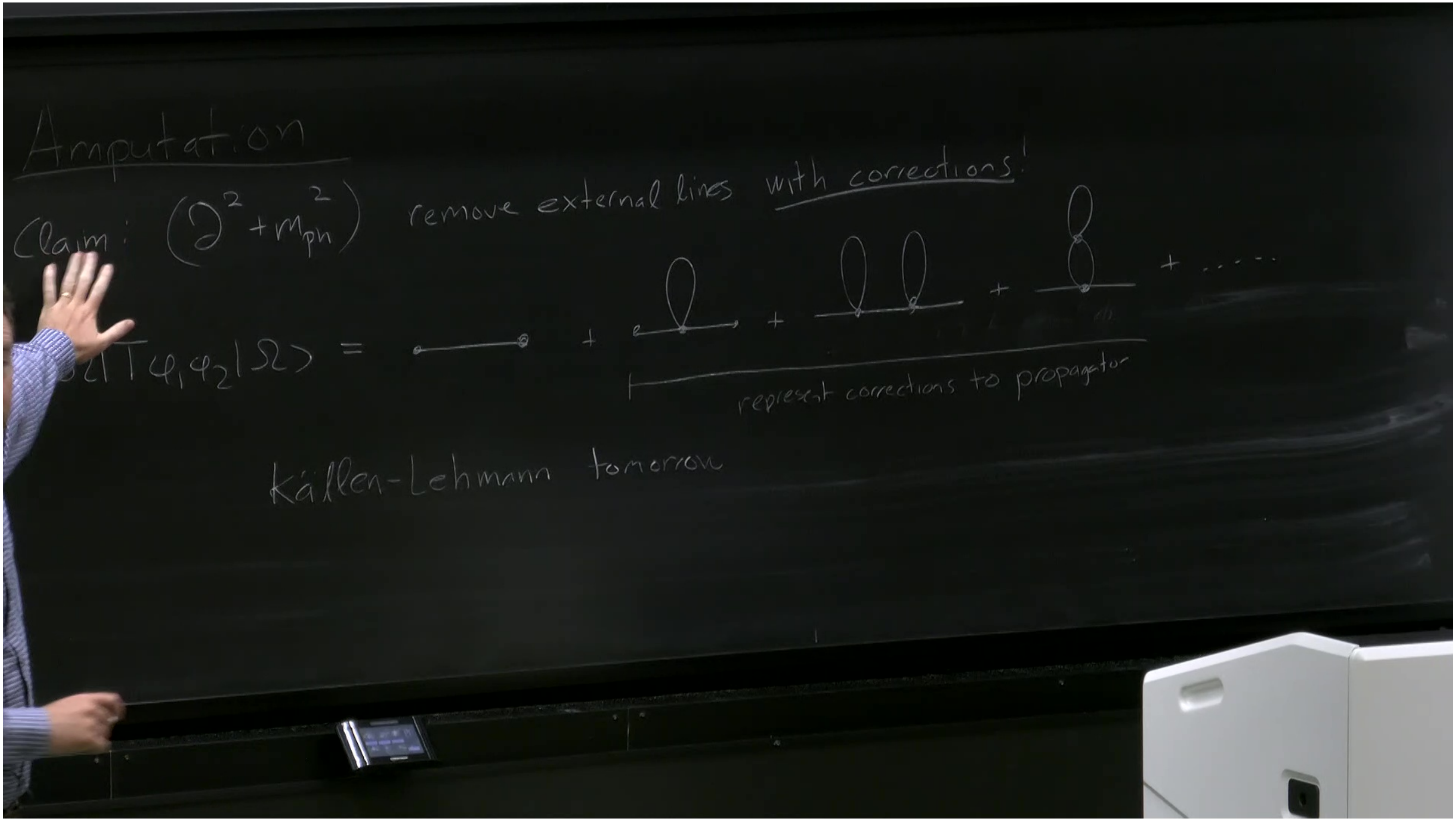
Claim: $(\partial^2 + m_{ph}^2)$ remove external lines with corrections!

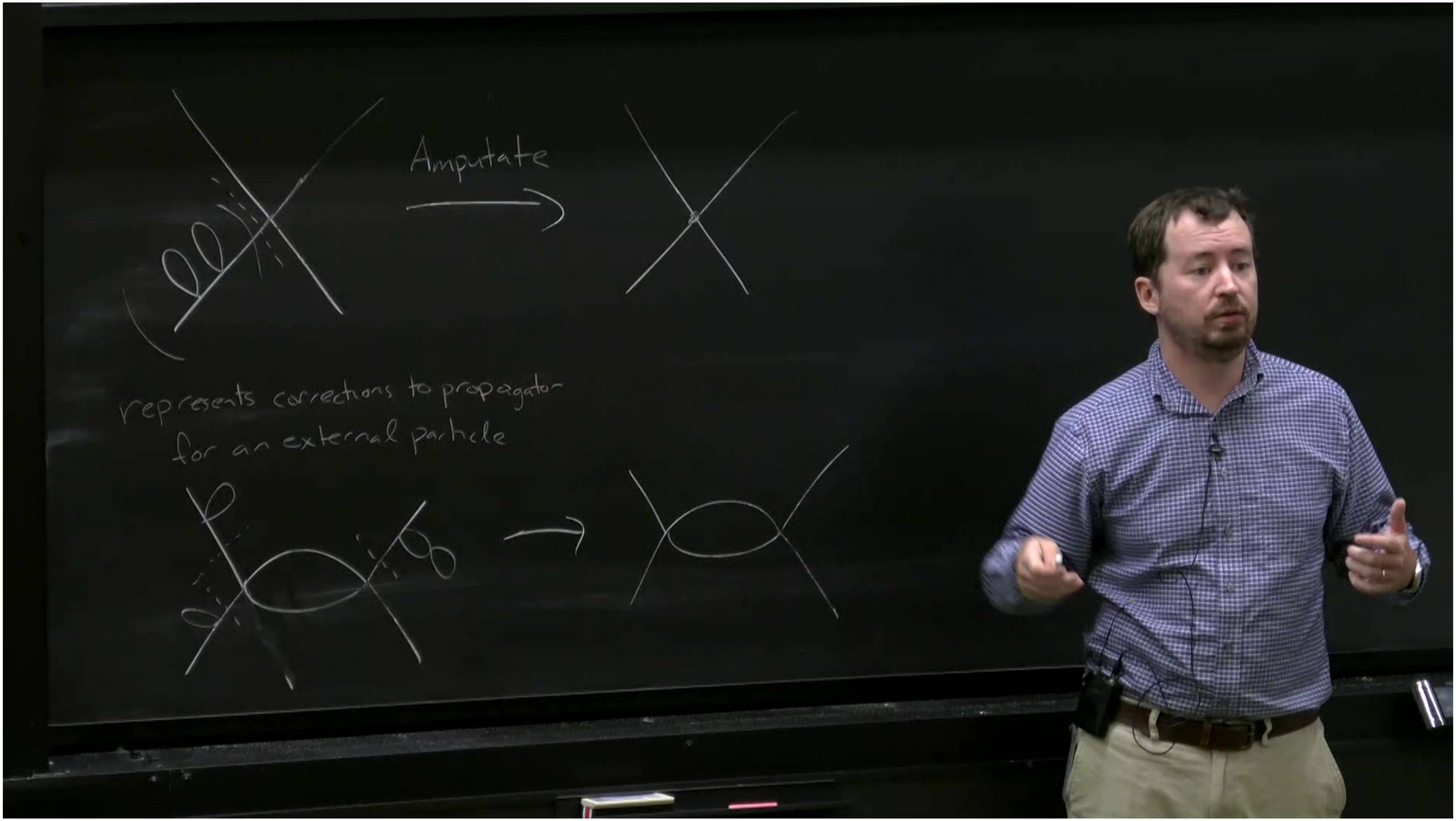
$$\langle \Omega | T \varphi_1 \varphi_2 | \Omega \rangle =$$

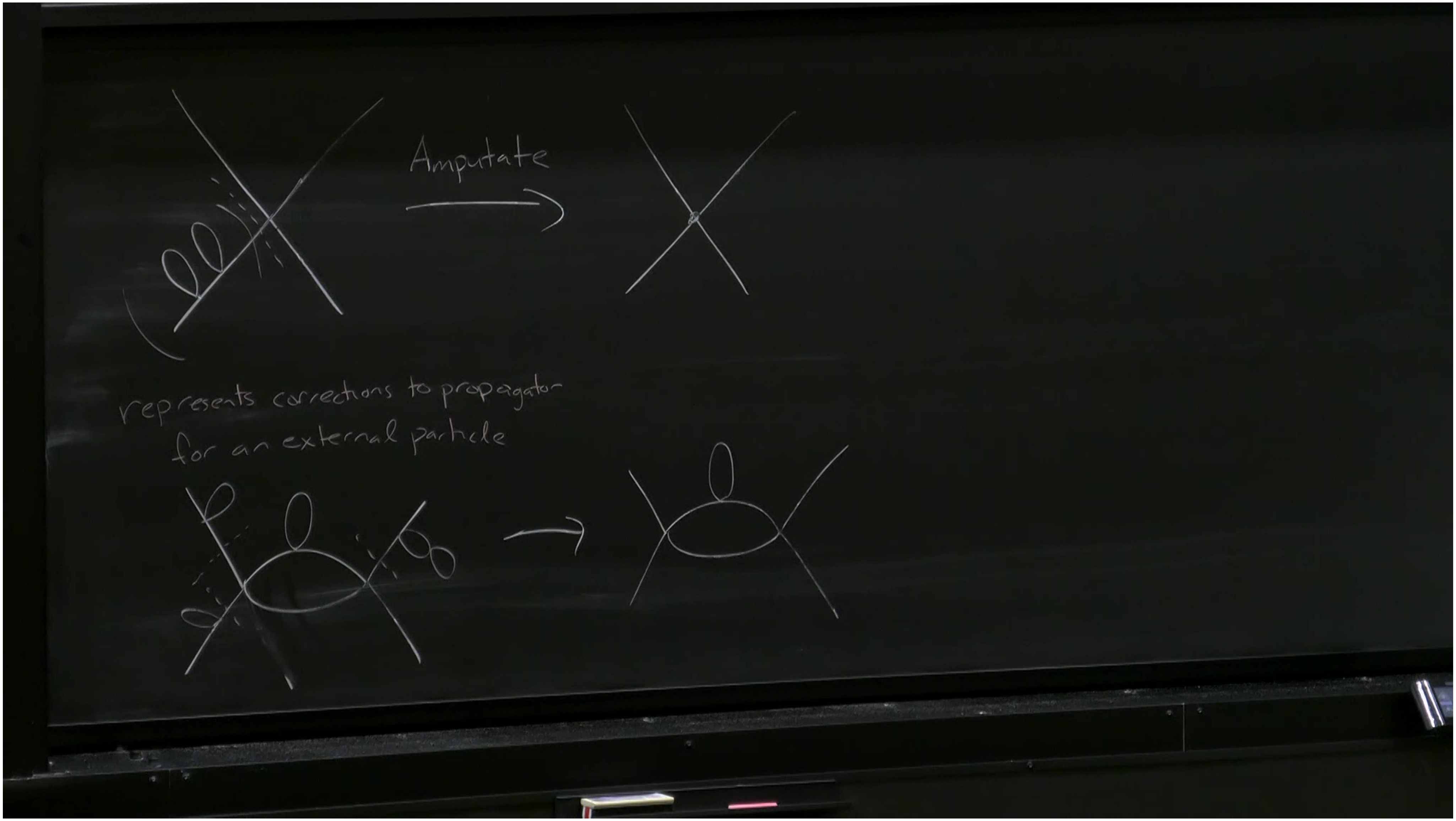


represent corrections to


Amputation
Claim: (∂² + m²ph) remove external lines with corrections!
<Ω|T φ1 φ2|Ω> =
+ ....
represent corrections to propagator


Amputation
Claim: $(\partial^2 + m_{ph}^2)$ remove external lines with corrections!
$\langle\Omega|T\varphi_1\varphi_2|\Omega\rangle =$
$+$
$+$
$+$
$+ \cdots$
represent corrections to propagator
Källen-Lehmann tomorrow


Amputate
represents corrections to propagator
for an external particle


Amputate
represents corrections to propagator
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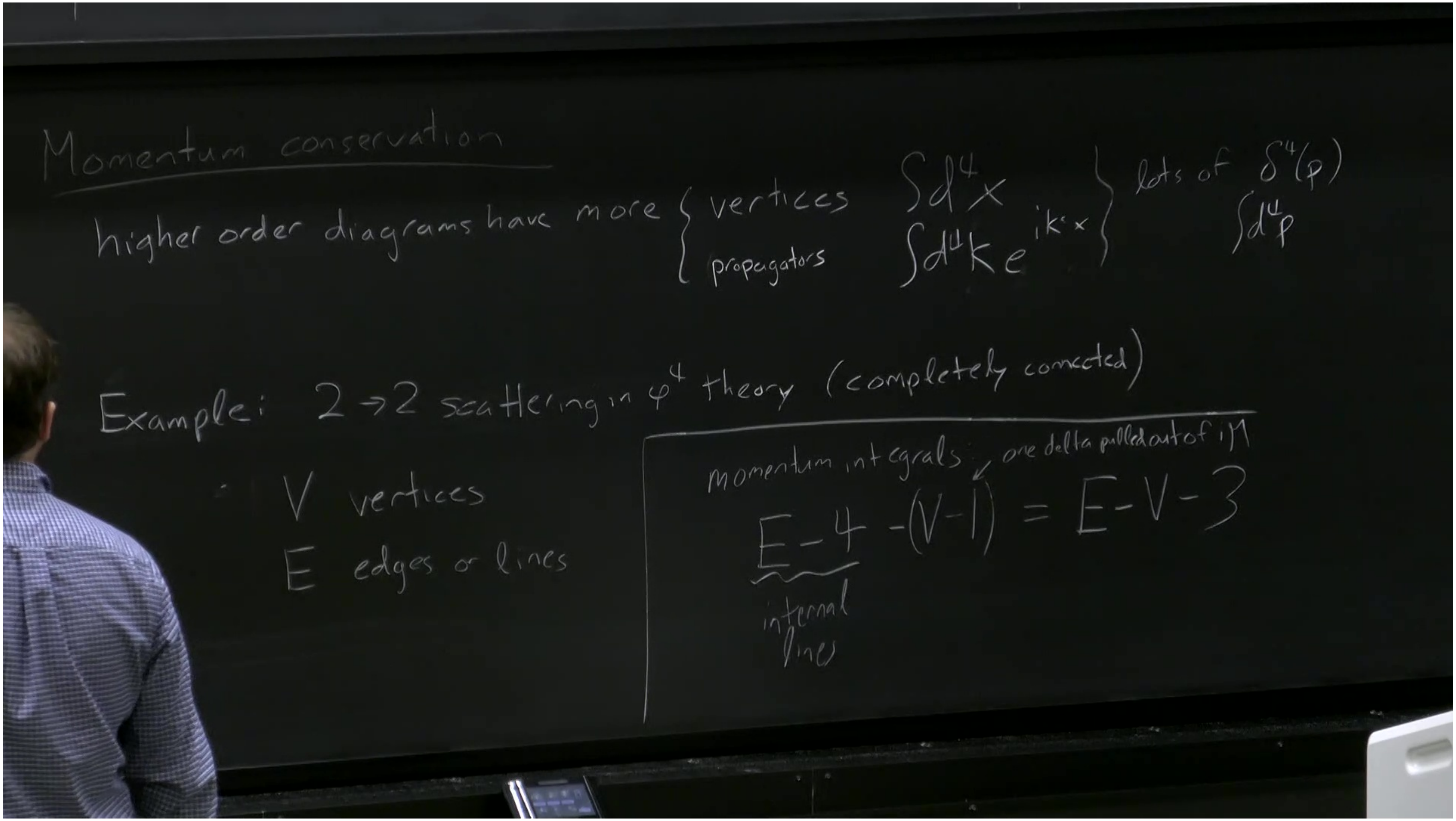
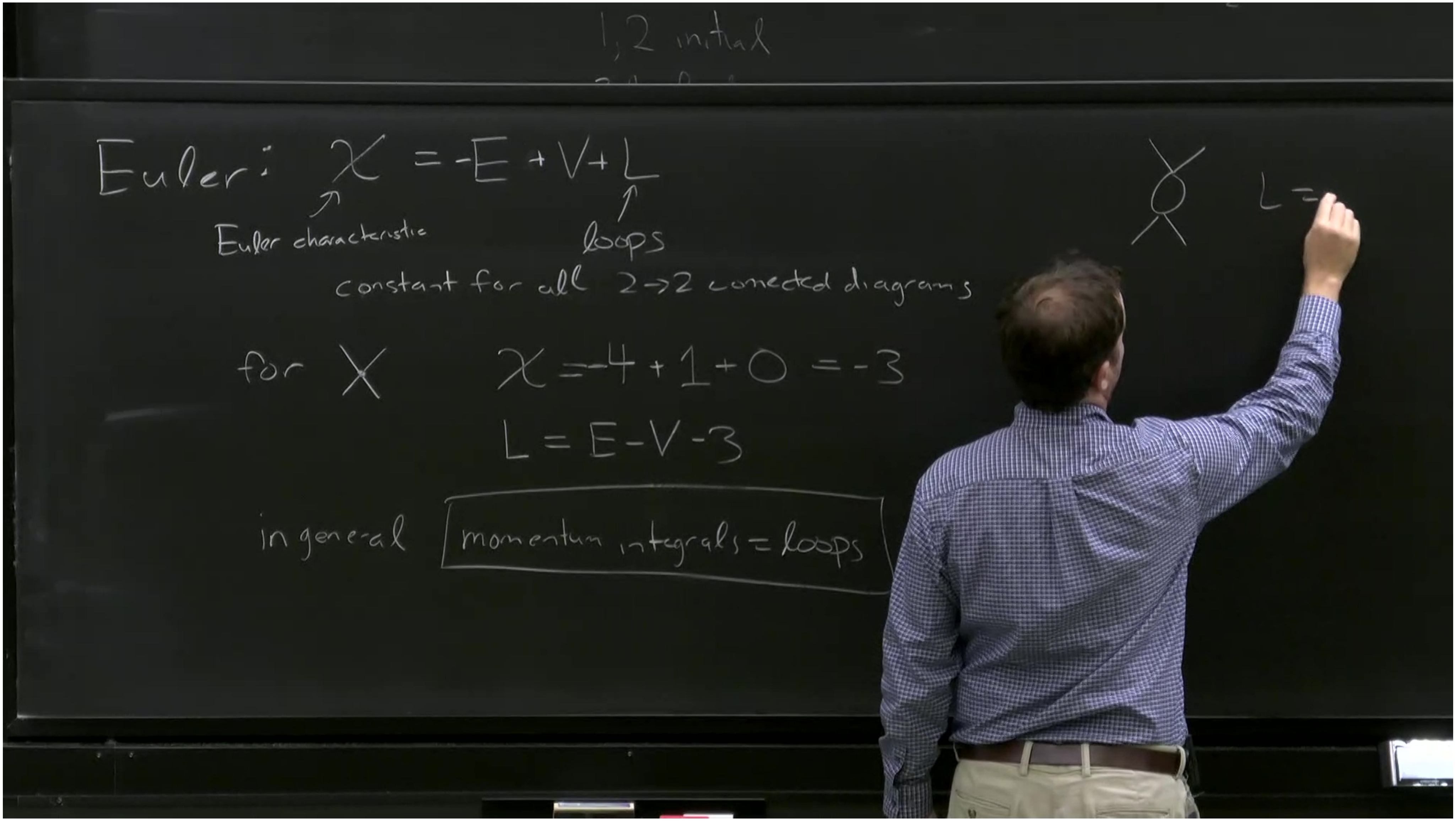
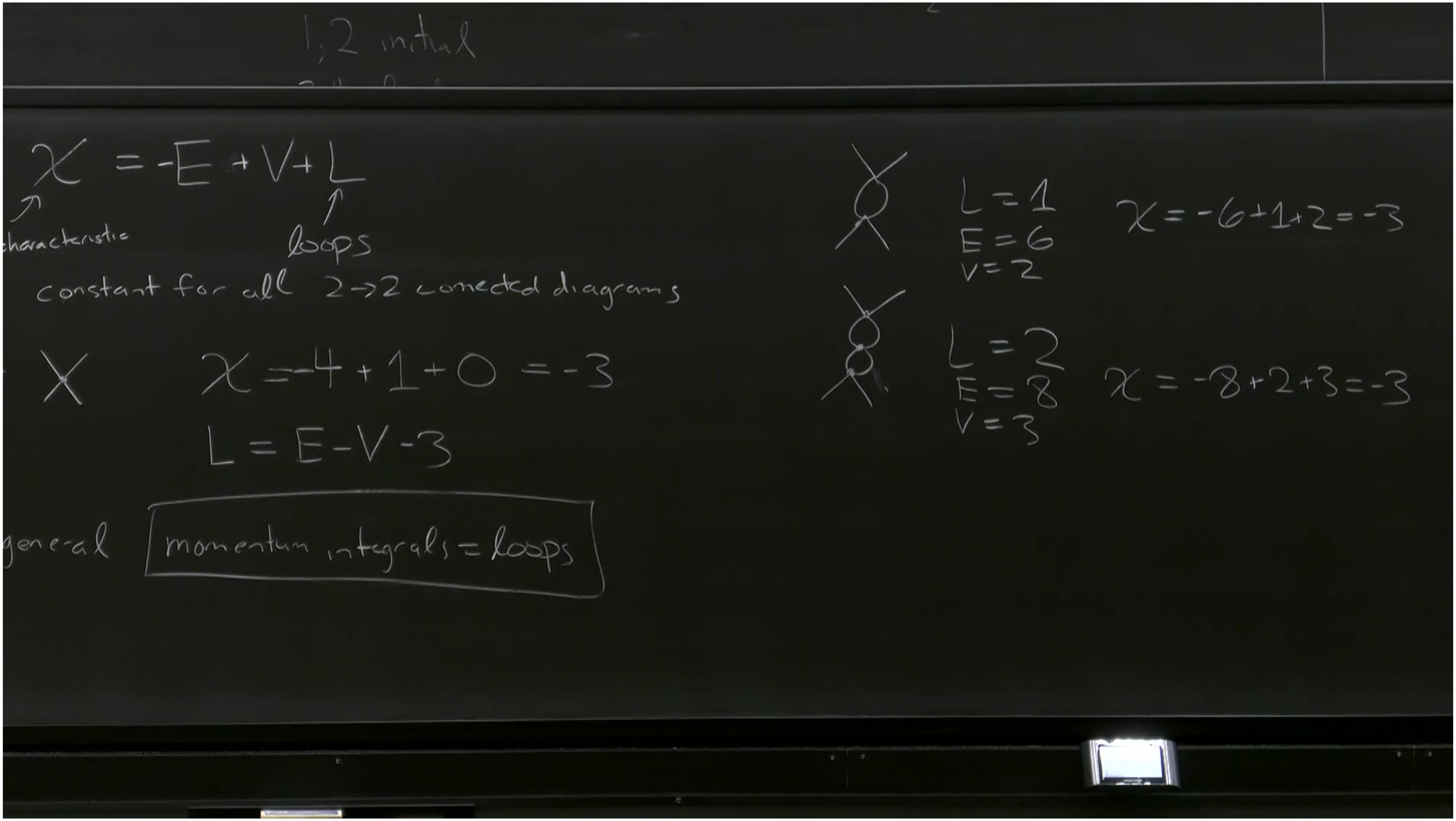

Momentum conservation
higher order diagrams have more { vertices $\int d^4x$; propagators $\int d^4k\, e^{ik\cdot x}$ } lots of $\delta^4(p)$ $\int d^4p$
Example: $2 \to 2$ scattering in φ^4 theory (completely connected)
V vertices
E edges or lines
momentum integrals: one delta pulled out of $i\mathcal{M}$
$\underbrace{E-4}_{\text{internal lines}} - (V-1) = E - V - 3$


1,2 initial
Euler: χ = -E + V + L
Euler characteristic
loops
constant for all 2→2 conected diagrams
for X
χ = -4 + 1 + 0 = -3
L = E - V - 3
in general
momentum integrals = loops
L =


1;2 initial
χ = -E + V + L
characteristic
loops
constant for all 2→2 connected diagrams
χ = -4 + 1 + 0 = -3
L = E - V - 3
general
momentum integrals = loops
L = 1
E = 6
V = 2
χ = -6 + 1 + 2 = -3
L = 2
E = 8
V = 3
χ = -8 + 2 + 3 = -3

3, 4 final

nman rules for $i\mathcal{M}$

$\mathcal{M}$ = sum of all completely connected, amputated diagrams

(interesting part)

internal line $\xrightarrow[\vec{p}]{}$ $= \dfrac{i}{p^2 - m^2 + i\varepsilon}$

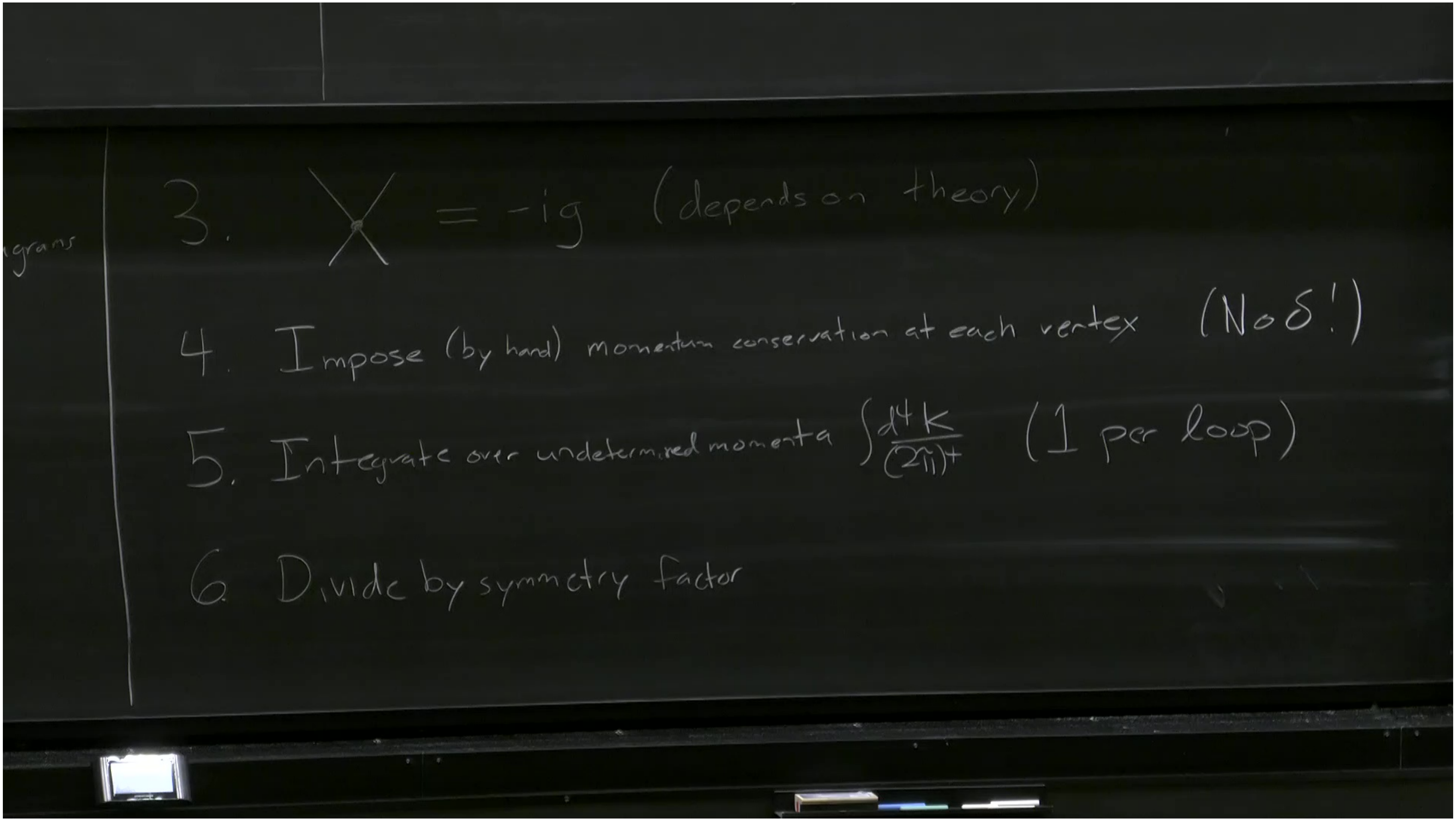
arrow next to line

external lines $\bullet\!\!\xrightarrow[\vec{p}]{}$ $= 1$

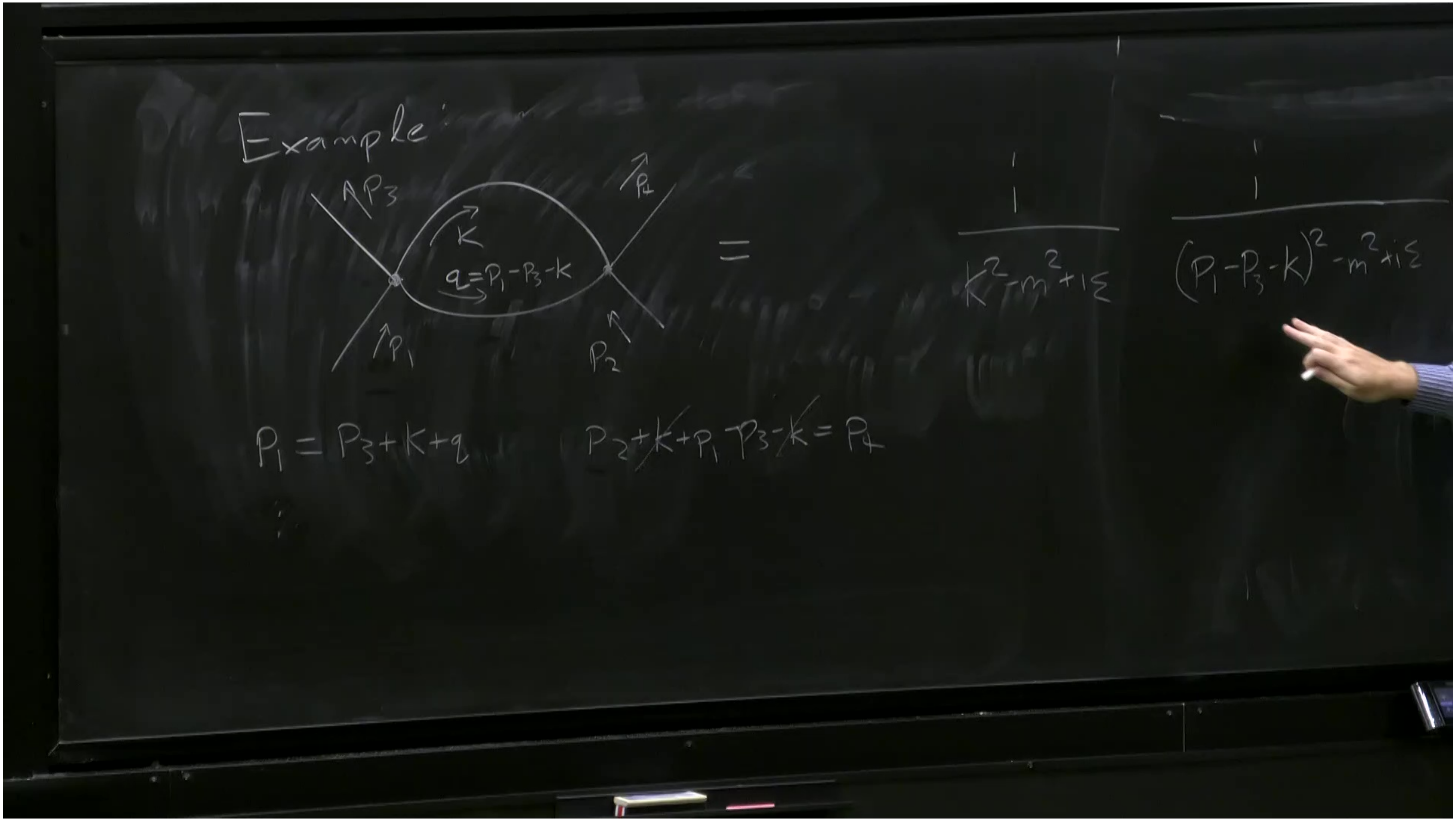
grams

3.  $= -ig$ (depends on theory)

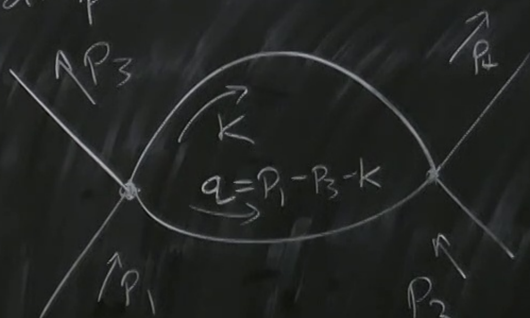
4. Impose (by hand) momentum conservation at each vertex



3. = $-ig$ (depends on theory)
4. Impose (by hand) momentum conservation at each vertex (No δ!)
5. Integrate over undetermined momenta $\int \frac{d^4k}{(2\pi)^4}$ (1 per loop)
6. Divide by symmetry factor


Example:
p_3
p_4
k
$q = p_1 - p_3 - k$
p_1
p_2
=
$k^2 - m^2 + i\varepsilon$
$(p_1 - p_3 - k)^2 - m^2 + i\varepsilon$
$p_1 = p_3 + k + q$
$p_2 + \cancel{k} + p_1 - p_3 - \cancel{k} = p_4$

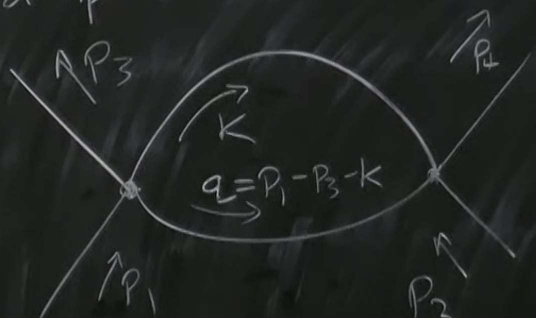
Example:



$$= (-ig)^2 \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m^2 + i\varepsilon} \, \frac{i}{(p_1 - p_3 - k)^2 - m^2 + i\varepsilon}$$

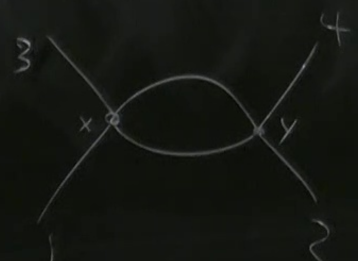
$$p_1 = p_3 + k + q \qquad p_2 + \cancel{k} + p_1 - p_3 - \cancel{k} = p_4$$

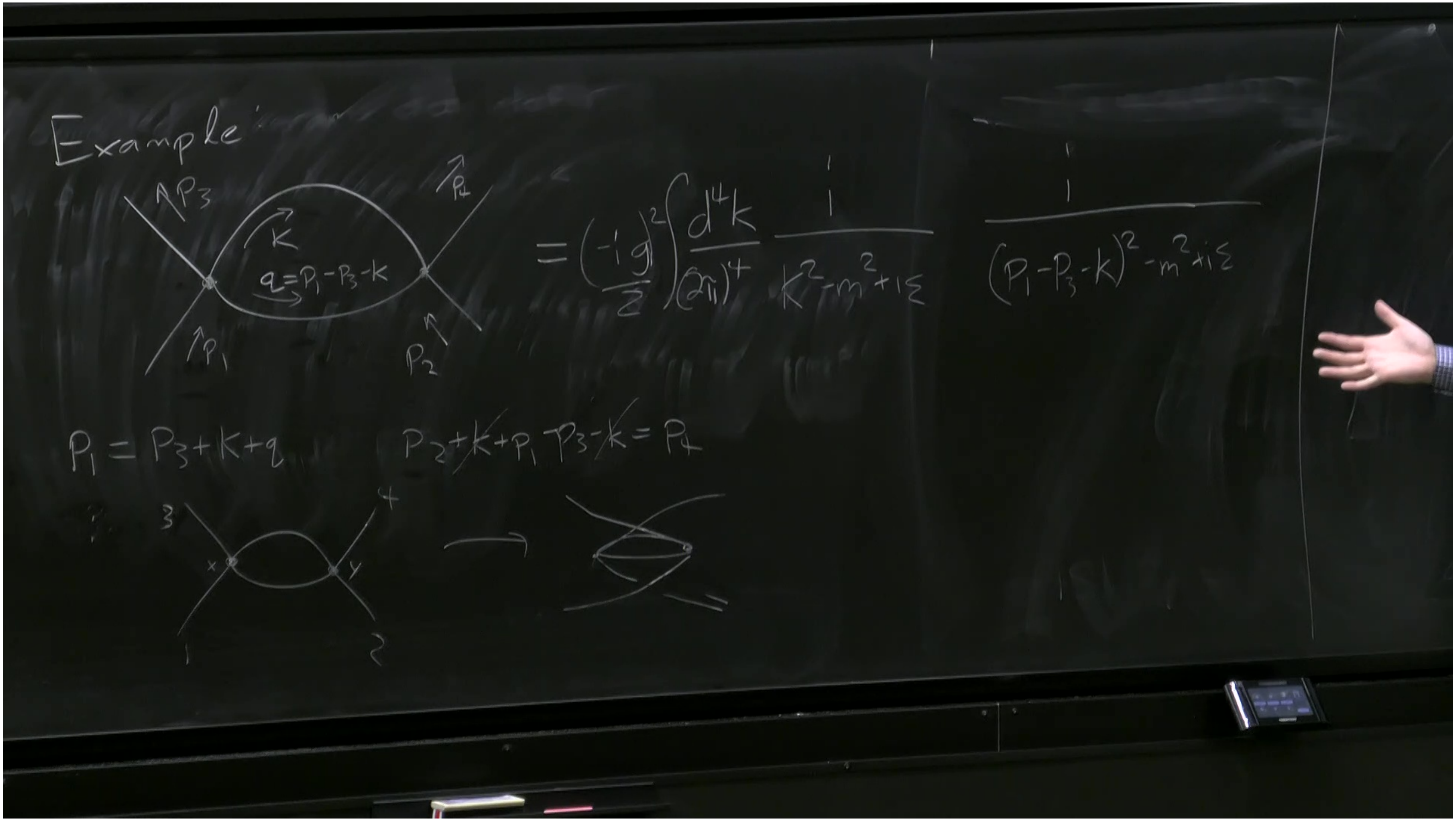
Example:

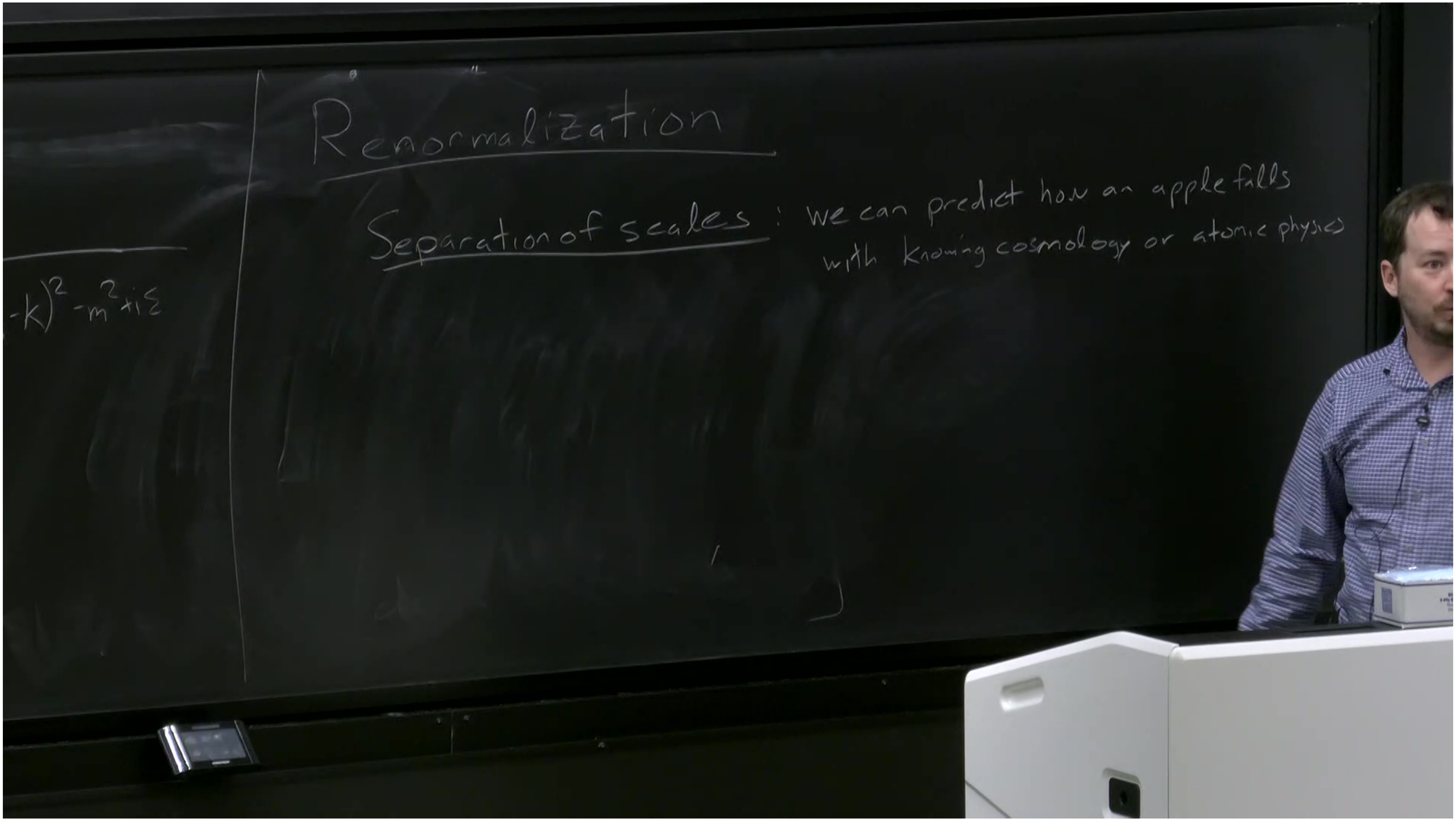


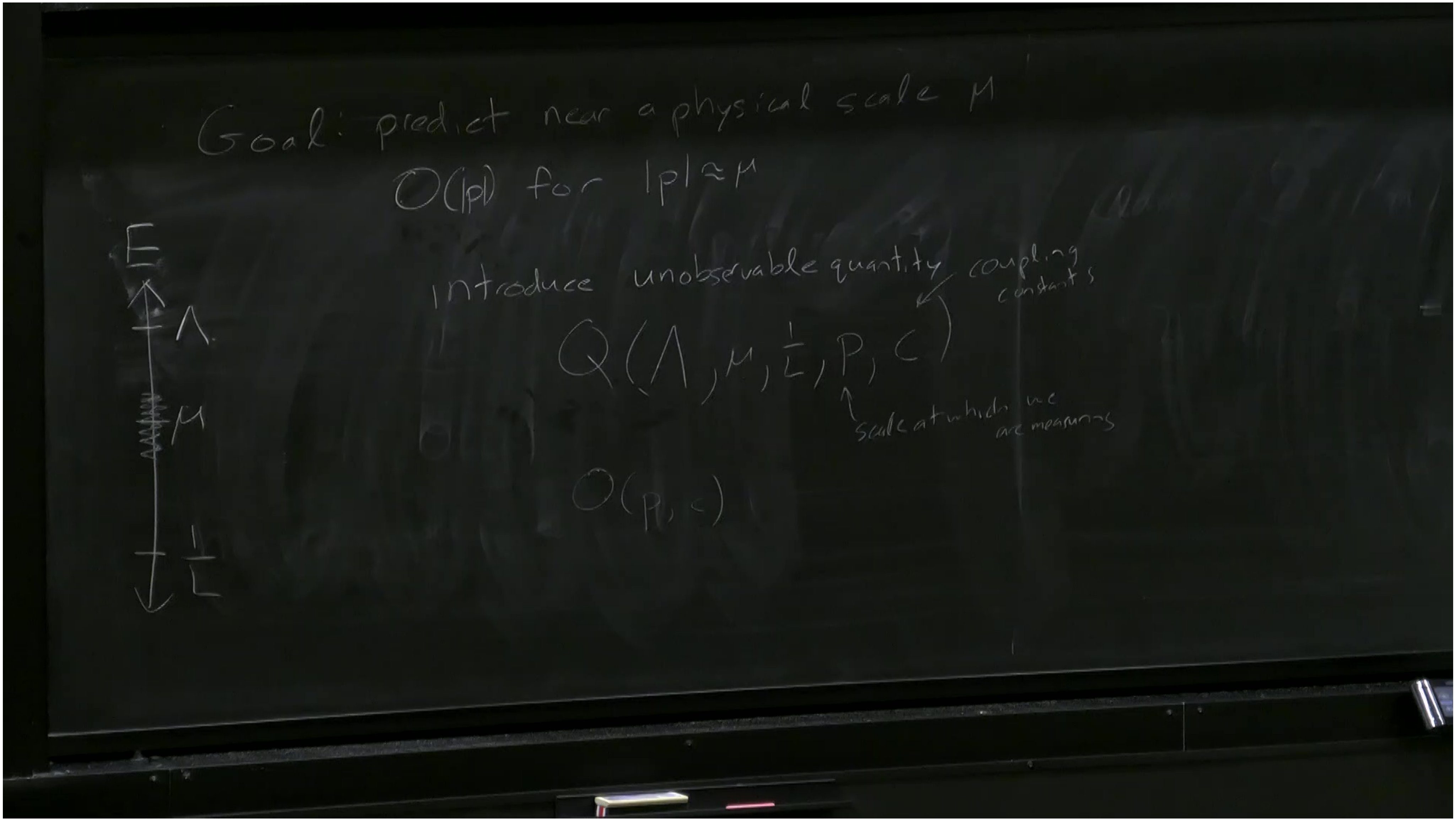
$$= \frac{(-ig)^2}{2}\int \frac{d^4k}{(2\pi)^4}\,\frac{i}{k^2-m^2+i\epsilon}\,\frac{i}{(p_1-p_3-k)^2-m^2+i\epsilon}$$

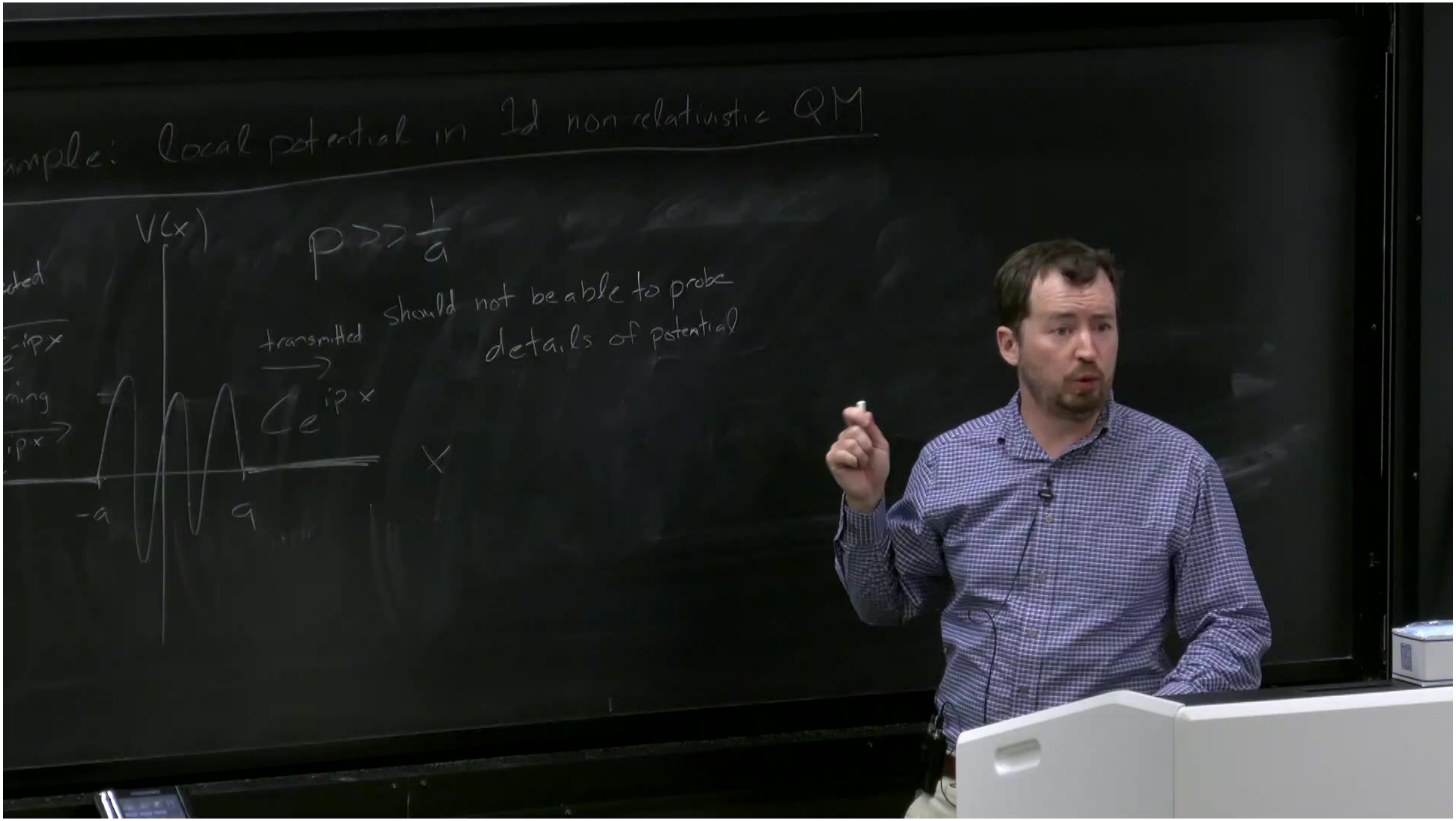
$p_1 = p_3+k+q$ $\qquad p_2+\cancel{k}+p_1-p_3-\cancel{k}=p_4$

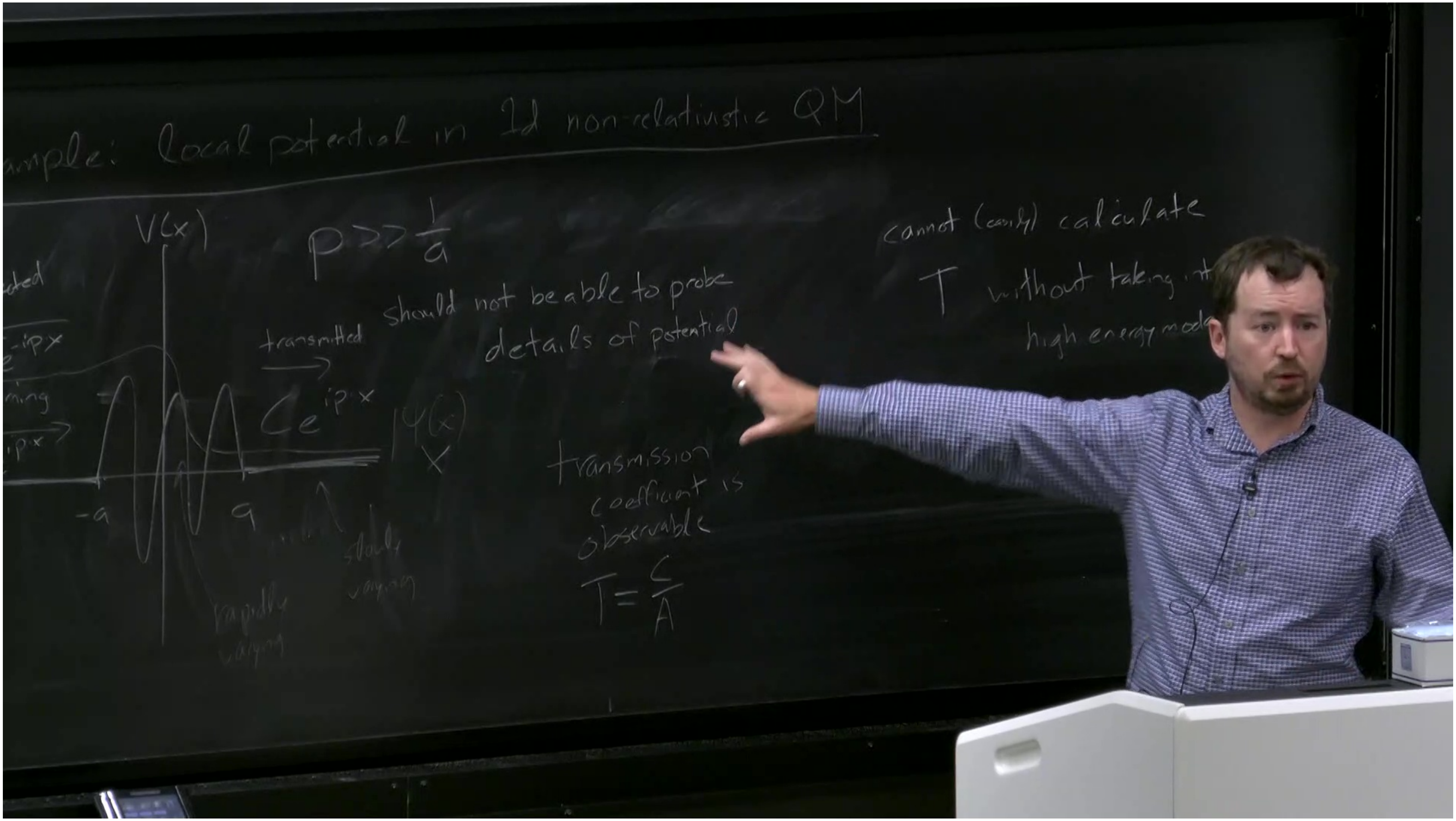


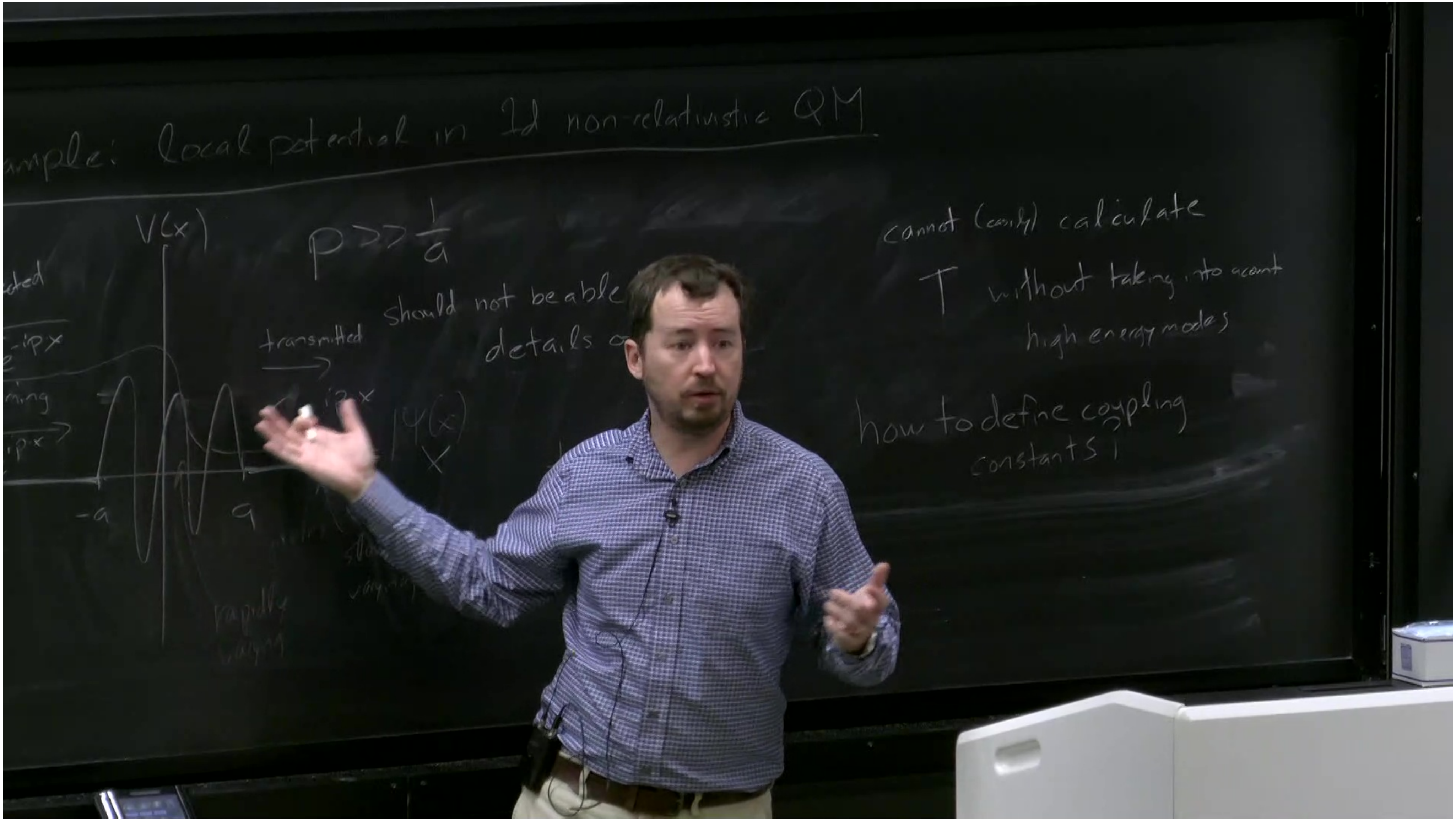

Example:
$q = p_1 - p_3 - k$
$= \left(\frac{-ig}{2}\right)^2 \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m^2 + i\varepsilon} \frac{i}{(p_1 - p_3 - k)^2 - m^2 + i\varepsilon}$
$p_1 = p_3 + k + q$
$p_2 + \cancel{k} + p_1 - p_3 - \cancel{k} = p_4$


Renormalization
Separation of scales : we can predict how an apple falls
with knowing cosmology or atomic physics
$-k)^2 - m^2 + i\varepsilon$


Goal: predict near a physical scale μ
O(|p|) for |p| ≈ μ
E
Λ
μ
1/L
introduce unobservable quantity
coupling constants
Q(Λ, μ, 1/L, p, c)
scale at which we are measuring
O(p, c)


ample: local potential in 1d non-relativistic QM
V(x)
p >> 1/a
should not be able to probe
details of potential
transmitted
Ce^{ipx}
-a
a
x


ample: local potential in 1d non-relativistic QM
V(x)
p >> 1/a
should not be able to probe
details of potential
transmitted
Ce^{ipx}
ψ(x)
-a
a
rapidly varying
slowly varying
transmission coefficient is observable
T = C/A
cannot (easily) calculate
T without taking int
high energy mode


ample: local potential in 1d non-relativistic QM
V(x)
$p >> \frac{1}{a}$
should not be able
details
transmitted
$|\psi(x)|$
-a
a
rapidly varying
cannot (easily) calculate
T without taking into acount
high energy modes
how to define coupling
constants?