Title: Classical Physics Lecture - 100223

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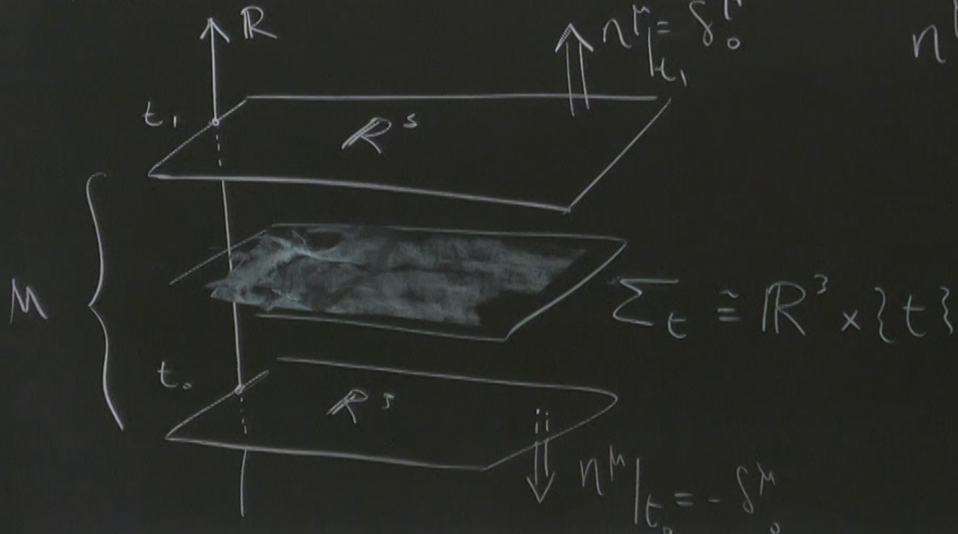
Lagrangian Field theory

- Spacetime setup:

$$M \cong [t_0, t_1] \times \mathbb{R}^3 \subset \mathbb{R}^{1,4}$$

Mikowski



n^μ = outgoin unit normal at ∂M

Lagrangian Field theory

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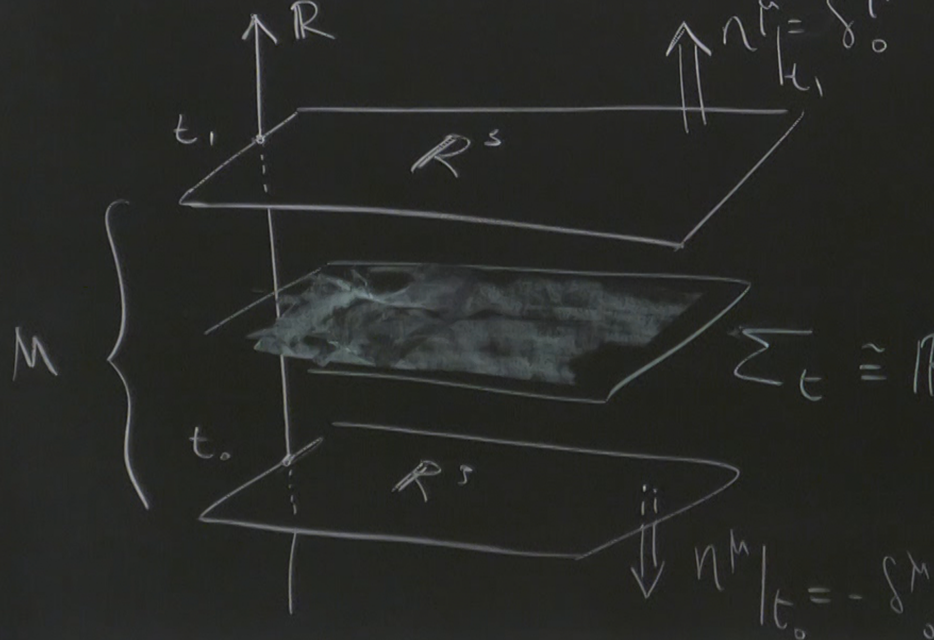

$\Sigma_t \cong \mathbb{R}^3 \times \{t\}$

Scalar Field

$\varphi : \mathbb{R}^{1,3} \longrightarrow \mathbb{R}$ real sc f.

$(\mathbb{C})$ complex sc f.

"scalar": transf. property under a $\underset{\Lambda^\mu{}_\nu}{\text{Lorentz}}$ & $\underset{a^\mu}{\text{transl.}}$ transf.:

$\varphi \xrightarrow{(\Lambda, a)} \varphi'$: $\varphi'(x) = \varphi(\Lambda^{-1}x - a) \iff \varphi'(x') = \varphi(x)$

Rmk

Vector field $A^\mu \xrightarrow{(\Lambda, a)} A'^\mu$: $A'^\mu(x) = (\Lambda^{-1})^\mu{}_\nu A^\nu(\Lambda^{-1}x - a)$

"Same" as: $\vec{E}(x) \xrightarrow{R} \vec{E}'(x) = (R\vec{E})(R^{-1}x)$

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$\Lambda^\mu{}_\nu$ a^μ

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Vector field $A^\mu \xrightarrow{(\Lambda, a)} A'^\mu$; $A'^\mu(x) = (\Lambda^{-1})^\mu{}_\nu A^\nu(\Lambda^{-1}x - a)$

("same" as: $\vec{E}(x) \xrightarrow{R} \vec{E}'(x) = (R\vec{E})(R^{-1}x)$)

	independent variable (source)	configuration space (target)	history
N-particle mech	$[t_0,t_1]\subset\mathbb{R}$ "time"	$Q=(\mathbb{R}^3)^N \ni q^i_\alpha$, $i \leftarrow 1,\dots,3$, $\alpha \leftarrow 1,\dots,N$	$\gamma:\mathbb{R}\longrightarrow Q=\mathbb{R}^{3N}$, $t\longmapsto q^i_\alpha=\gamma^i_\alpha(t)$
Field theory	$M=[t_0,t_1]\times\mathbb{R}^3\subset\mathbb{R}^{1,3}$ "spacetime"	Scalar f.: $Q=\mathbb{R}$ $(\mathbb{C})$ $\ni q$ vector f.: $Q=\mathbb{R}^4$ (vector pot. A^μ)	$\varphi:\mathbb{R}^{1,3}\longrightarrow Q=\mathbb{R}$, $x\longmapsto q=\varphi(x)$ $A:\mathbb{R}^{1,3}\longrightarrow Q=\mathbb{R}^4$, $x\longmapsto q^\mu=A^\mu(x)$

	1st derivative	Lagrangian
$Q = \mathbb{R}^{3N}$ $q^i_\alpha = \gamma^i_\alpha(t)$	velocity $v \in T_qQ$ $v^i_\alpha = \boxed{\partial_t}\boxed{\gamma^i_\alpha}(t)$	$L : (q, v, t) \mapsto L(q, v, t) \in \mathbb{R}$ ex: $L = \frac{1}{2} m v^2 - V(q)$
$\to Q = \mathbb{R}$ $\to q = \varphi(x)$ $\to Q = \mathbb{R}^4$ $\to q^\mu = A^\mu(x)$	1st jet $\cdot\ v_\mu = \boxed{\partial_\mu}\boxed{\varphi}(x)$ (scalar) $\cdot\ v_\mu^\nu = \boxed{\partial_\mu}\boxed{A^\nu}(x)$ (vector)	$\mathcal{L} : (q, v, x) \mapsto \mathcal{L}(q, v, t) \in \mathbb{R}$ ← "scalar" ex: (scalar) $\mathcal{L} = -\frac{1}{2} v^\mu v_\mu - V(q)$ ex: vector: see tutorial/tomorrow

Action functional

$$S[\gamma] = \int_{t_0}^{t_1} dt\, L(\gamma(t), \partial_t \gamma, t)$$

$\uparrow$ history

$$\delta S[\bar{\gamma}] = 0 \quad \text{with } \delta\gamma|_{t_0} = \delta\gamma|_{t_1} = 0$$

$$S[\varphi] = \int_M d^4x\, \mathcal{L}(\varphi, \partial_\mu \varphi, x)$$

$\uparrow$ history

$$\delta S[\bar{\varphi}] = 0 \quad \text{with } \delta\varphi|_{\partial M} = 0$$

$$\delta\varphi(t_0, \vec{x}) = \delta\varphi(t_1, \vec{x}) = 0$$

Scalar Field

$\varphi : \mathbb{R}^{1,3} \longrightarrow \mathbb{R}$ real sc f.

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"scalar": transf property under a Lorentz & transl transf: $\Lambda^\mu{}_\nu$, a^μ

$\varphi \xrightarrow{(\Lambda, a)} \varphi'$: $\varphi'(x) = \varphi(\Lambda^{-1}x - a) \Leftrightarrow \varphi'(x') = \varphi(x)$, $x' = \Lambda x + a$

Rmk

Vector field $A^\mu \xrightarrow{(\Lambda, a)} A'^\mu$: $A'^\mu(x) = (\Lambda)^\mu{}_\nu A^\nu(\Lambda^{-1}x - a) \Leftrightarrow A'^\mu(\Lambda x + a) = \Lambda^\mu{}_\nu A^\nu(x)$

("same" as: $\vec{E}(x) \xrightarrow{R} \vec{E}(Rx) = (R\vec{E})(x)$)

history | 1st derivative | Lagrangian

Action functional

$$S[\gamma] = \int_{t_0}^{t_1} dt \; L(\gamma(t), \partial_t \gamma, t)$$

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$$S[\varphi] = \int_M d^4x \; \mathcal{L}(\varphi, \partial_\mu \varphi, x)$$

↑ history

$$\delta S[\bar{\varphi}] = 0 \quad \text{with } \delta\varphi\big|_{\partial M} = 0$$

$$\delta\varphi(t_0, \vec{x}) = \delta\varphi(t_1, \vec{x}) = 0$$

- boundary term vanishes because of b. cond. $\delta\varphi|_{\partial M} = 0$

- bulk term:

$$\delta S = \int d^4x \underbrace{\left(\frac{\partial \mathcal{L}}{\partial \varphi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \right)}_{=0} \overset{\text{arbitrary}}{\delta\varphi} \hat{=} 0$$

(in short-hand notation)

$= 0$ (Euler Lagrange)

$$\Big|_{(\varphi, \partial\varphi, x)} \partial_\mu \delta\varphi \Bigg) + \mathcal{O}(\epsilon^2)$$

$$\Bigg) \delta\varphi + \int_{\partial M} d^3\vec{x}\; n_\mu \frac{\partial \mathcal{L}}{\partial v_\mu}\Big|_{(\dots)} \delta\varphi + \mathcal{O}(\epsilon^2)$$

Euler-Lagrange

$\delta S[\bar\varphi]=0$ with $\delta\varphi|_{\partial M}=0$

$\varphi(x) \longmapsto \varphi(x)+\epsilon\,\delta\varphi(x)$

$$S[\varphi+\epsilon\delta\varphi] = \int_M d^4x\ \mathcal{L}(\varphi+\epsilon\delta\varphi, \partial_\mu\varphi+\epsilon\partial_\mu\delta\varphi, x)$$

$$= S[\varphi]+\epsilon\int_M d^4x\left(\frac{\partial\mathcal{L}}{\partial q}\bigg|_{(\varphi,\partial\varphi,x)}\delta\varphi+\frac{\partial\mathcal{L}}{\partial v_\mu}\bigg|_{(\varphi,\partial\varphi,x)}\partial_\mu\delta\varphi\right)+\mathcal{O}(\epsilon^2)$$

$$= S[\varphi]+\epsilon\int_M d^4x\left(\frac{\partial\mathcal{L}}{\partial q}\bigg|_{(\dots)}-\frac{\partial}{\partial x^\mu}\frac{\partial\mathcal{L}}{\partial v_\mu}\bigg|_{(\dots)}\right)\delta\varphi+\int_{\partial M}d^3\vec{x}\ n_\mu\frac{\partial\mathcal{L}}{\partial v_\mu}$$

$q^\mu = A^\mu(x)$

- boundary term vanishes because of b. cond. $\delta\varphi|_{\partial M} = 0$

- bulk term:

$$\delta S = \int d^4x \underbrace{\left(\frac{\partial \mathcal{L}}{\partial \varphi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)}\right)}_{=0 \quad \text{(Euler Lagrange)}} \delta\varphi \hat{=} 0$$

arbitrary

(in short-hand notation)

$\delta\varphi + \mathcal{O}(\epsilon^2)$

$|_{(\dots)}$

Ex: $\mathcal{L}(q, v_\mu, x) = -\frac{1}{2}\eta^{\mu\nu} v_\mu v_\nu - V(q)$

config $q \in \mathbb{R}$

1st jet $v_\mu \in \mathbb{R}^4$

$$S[\varphi] = \int d^4x \; -\frac{1}{2}\eta^{\mu\nu}(\partial_\mu \varphi(x))(\partial_\nu \varphi(x)) - V(\varphi(x))$$

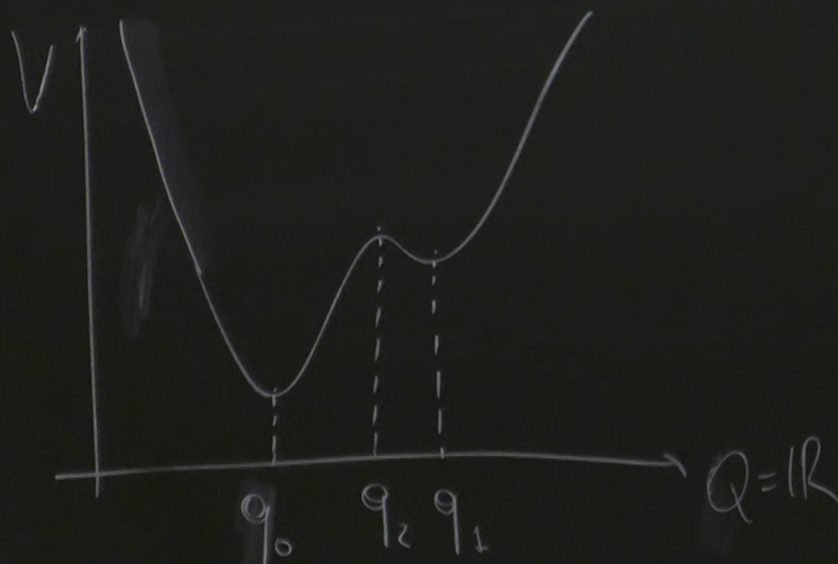
Euler-Lagrange

$$\frac{\partial \mathcal{L}}{\partial q}\bigg|_{(\varphi, \partial\varphi, x)} = -V'(q)\bigg|_{(\varphi, \partial\varphi, x)} = -V'(\varphi(x))$$

$$\frac{\partial \mathcal{L}}{\partial v_\mu}\bigg|_{(\varphi, \partial\varphi, x)} = -\eta^{\mu\nu} v_\nu\bigg|_{(\varphi, \partial\varphi, x)} = -\eta^{\mu\nu}\partial_\mu \varphi(x)$$

$$\to \quad 0 \overset{!}{=} -V'(\varphi(x)) + \partial_\mu\left(\eta^{\mu\nu}\partial_\nu \varphi(x)\right)$$

$$\Leftrightarrow \quad \Box\varphi(x) - V'(\varphi(x)) = 0$$



$$V'(q_0) = V'(q_1) = V'(q_2)$$

$$\Rightarrow \quad \varphi(x) = \begin{cases} q_0 \\ q_1 \\ q_2 \end{cases} \quad \text{(const!}$$

- q_2 unstable $\to$ (not inte
- q_0, q_1 are minima

$\hookrightarrow$ I call these

$\delta\varphi(t_0,\vec{x}) = \delta\varphi(t_1,\vec{x}) = 0$

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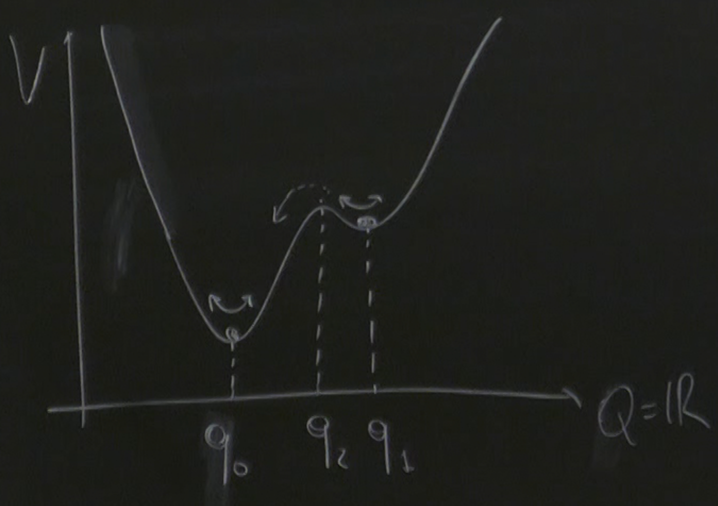

$V'(q_0) = V'(q_1) = V'(q_2) = 0$

$\Rightarrow \varphi(x) = \begin{cases} q_0 \\ q_1 \\ q_2 \end{cases}$ (const!) is a solution

- q_2 unstable → (not interesting)
- q_0, q_1 are minima → stable under "small" perturbations

→ I call these configurations $\varphi(x)$

[vacuum tunnelling : Ruth Gregory's GP]

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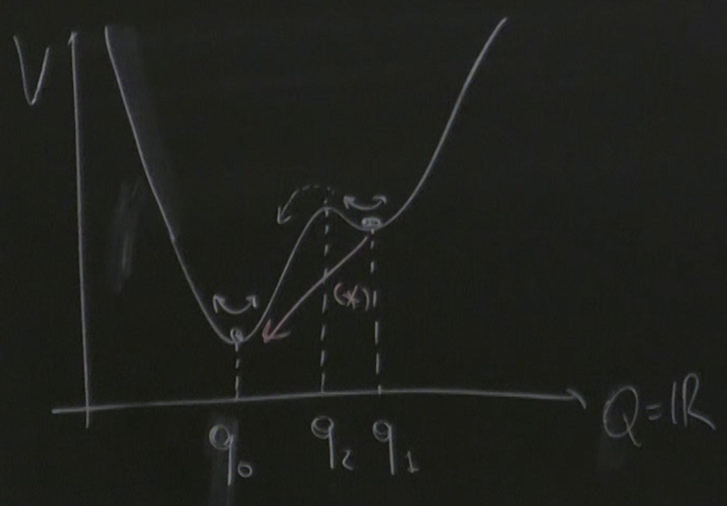

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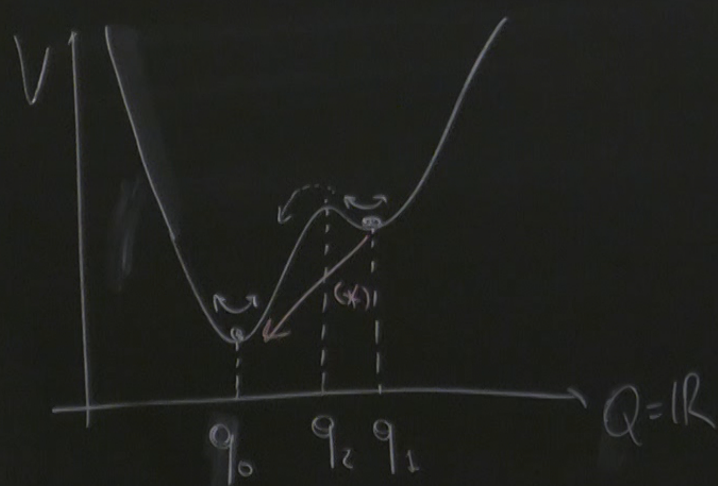

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$\rightarrow$ I call these configurations $\varphi(x) = \begin{cases} q_0 & \text{True vacuum (absolute min)} \\ q_1 & \text{False vacuum (local min)} \end{cases}$

[vacuum tunnelling : Ruth Gregory's GP]

$\phi(x) = \varphi(x) - q_0$ "small" perturbation.

$V(\phi(x)) = V$

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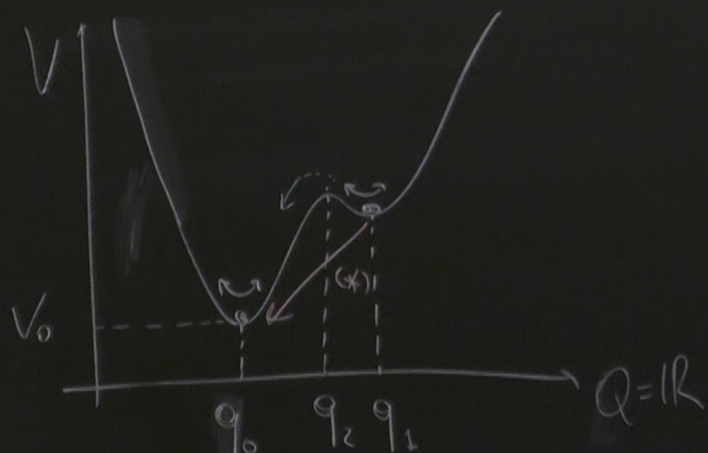

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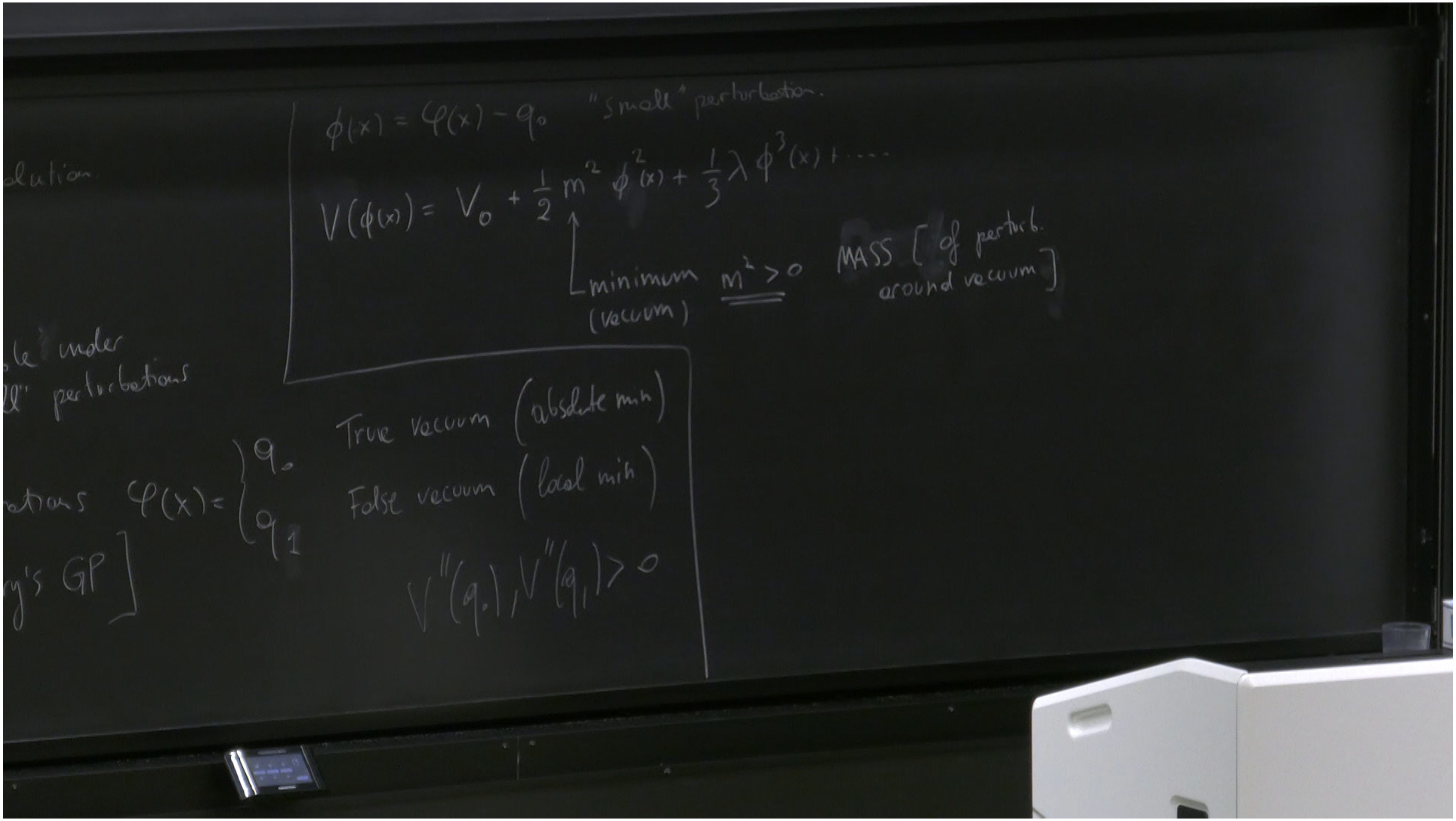
$\phi(x) = \varphi(x) - q_0$ "small" perturbation.
$V(\phi(x)) = V_0 + \frac{1}{2} m^2 \phi^2(x) + \frac{1}{3}\lambda \phi^3(x) + \ldots$
minimum (vacuum) $m^2 > 0$ MASS [of perturb. around vacuum]
under
perturbations
$\varphi(x) = \begin{cases} q_0 \\ q_1 \end{cases}$
True vacuum (absolute min)
False vacuum (local min)
$V''(q_0), V''(q_1) > 0$

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$$\text{EL}: \ \Box\phi - m^2\phi = \underbrace{\lambda\phi^2}_{\substack{\text{self interaction} \\ (\text{"small"})}} + \dots$$

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minimum (vacuum) $\underline{\underline{m^2 > 0}}$ MASS [of perturb. around vacuum]

True vacuum (absolute min)

False vacuum (local min)

$V''(q_0), V''(q_1) > 0$

EL : $\Box\phi - m^2\phi = \underbrace{\lambda\phi^2}_{\text{self interaction ("small")}} + \dots$

$\phi(x) = \int \frac{d^4p}{(2\pi)^4} e^{ip_\mu x^\mu} \hat{\phi}(p)$

$-p^2 - m^2 = 0$ i.e. $p^2 = -m^2$

Noether

$$\varphi(x),\ A_\mu(x),\ \dots \longrightarrow \varphi^I(x)$$

Variation: $\quad s : (q, v_\mu, x) \mapsto s(q, v_\mu, x)$

$$\tilde{\delta}_s \varphi^J(x) = s^J(\varphi^J(x), \partial_\mu \varphi^J(x), x)$$

Infinitesimal sym iff $\quad \tilde{\delta}_s \mathcal{L}(\varphi, \partial\varphi, x) = \frac{d}{dx^\mu} R^\mu_s(\varphi, \partial\varphi, \dots, x) \equiv \partial_\mu R^\mu + \partial_\mu \varphi \dots$

$\Leftrightarrow$ action is invariant up to a boundary term

Noether current: $\quad J^\mu_s = \sum_I \underbrace{\boxed{\frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi^I)}}}_{\equiv \pi^\mu_I} \tilde{\delta}_s \varphi^I - R^\mu_s(\varphi, \partial\varphi, \dots)$

$\varphi^I(x)$

$\ldots, x) \mapsto S(q, v, x)$

$\ldots(\varphi^J(x), \partial_\mu \varphi^J(x), x^\mu)$

$$\mathcal{L}(\varphi, \partial\varphi, x) = \frac{d}{dx^\mu} R^\mu_s(\varphi, \partial\varphi, \ldots, x) \equiv \partial_\mu R^\mu + \partial_\mu\varphi \frac{\partial R^\mu}{\partial\varphi} + \partial_\mu(\partial_\nu\varphi)\frac{\partial R^\mu}{\partial(\partial_\nu\varphi)} + \ldots$$

a boundary term

$$= \sum_I \underbrace{\boxed{\frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi^I)}}}_{\equiv \pi^\mu_I} \tilde{\delta}_s \varphi^I - R^\mu_s(\varphi, \partial\varphi, \ldots)$$

$\delta\varphi(t_0,\vec{x}) = \delta\varphi(t_1,\vec{x}) = 0$

Noether

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Noether thm

hell)

π^{Γ}

Γ

$\Big) \tilde{\delta}\varphi + \frac{d}{dx^{\mu}}\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \tilde{\delta}\varphi\right)$

EL eqs $\hat{=} 0$

$$\frac{d}{dt} Q_s(t) = \int_{\Sigma_t} d^3\vec{x}\, \frac{\partial}{\partial x^0} J_s^0(x^0, \vec{x})$$

$$\hat{=} -\int_{\Sigma_t} d^3\vec{x}\, \frac{\partial}{\partial x^i} J_s^i = \oint_{\text{infinity}} s_i J_s^i \equiv 0$$

the fields vanish rapidly enough at ∞.

Noether thm

If $\tilde{\delta}_s \varphi$ is an infinitesimal sym, then

$$\boxed{\frac{d}{dx^\mu} J^\mu_s \hat{=} 0}$$

and

$$Q_s(t) = \int_{\Sigma_t} d^3\vec{x} \; J^0_s(\varphi, \partial\varphi, \dots)$$

is conserved (does not depend on t)

Pf

- $\tilde{\delta}_s \mathcal{L} = \frac{d}{dx^\mu} R^\mu_s$ (off shell)
- $\tilde{\delta}_s \mathcal{L} = \left(\frac{\partial \mathcal{L}}{\partial \varphi} \tilde{\delta}\varphi + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \frac{d}{dx^\mu} \tilde{\delta}\varphi \right)$

$$= \left(\frac{\partial \mathcal{L}}{\partial \varphi} - \frac{d}{dx^\mu} \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \right) \tilde{\delta}\varphi + \frac{d}{dx^\mu} \Big(\frac{\partial \mathcal{L}}{\partial(\partial_\mu\varphi)} \dots$$

$$\Rightarrow \frac{d}{dx^\mu} \left(\pi^\mu \tilde{\delta}_s \varphi - R_s \right) = EL \text{ eqs} \dots$$

then

Pf

- $\tilde{\delta}_s \mathcal{L} = \frac{d}{dx^\mu} R_s^\mu$ (off shell)
- $\tilde{\delta}_s \mathcal{L} = \left(\frac{\partial \mathcal{L}}{\partial \varphi} \tilde{\delta} \varphi + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \frac{d}{dx^\mu} \tilde{\delta} \varphi \right)$

$$= \left(\frac{\partial \mathcal{L}}{\partial \varphi} - \frac{d}{dx^\mu} \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \right) \tilde{\delta} \varphi + \frac{d}{dx^\mu} \Big(\underbrace{\frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)}}_{\pi^\mu} \tilde{\delta} \varphi \Big)$$

$$\Rightarrow \frac{d}{dx^\mu} \left(\pi^\mu \tilde{\delta}_s \varphi - R_s \right) = EL\,eqs \hat{=} 0$$

$$\frac{d}{dt} Q_s(t) = \int_{\Sigma_t} d^3\vec{x}\, \frac{\partial}{\partial x^0} J_s^0 (x^0, \vec{x})$$

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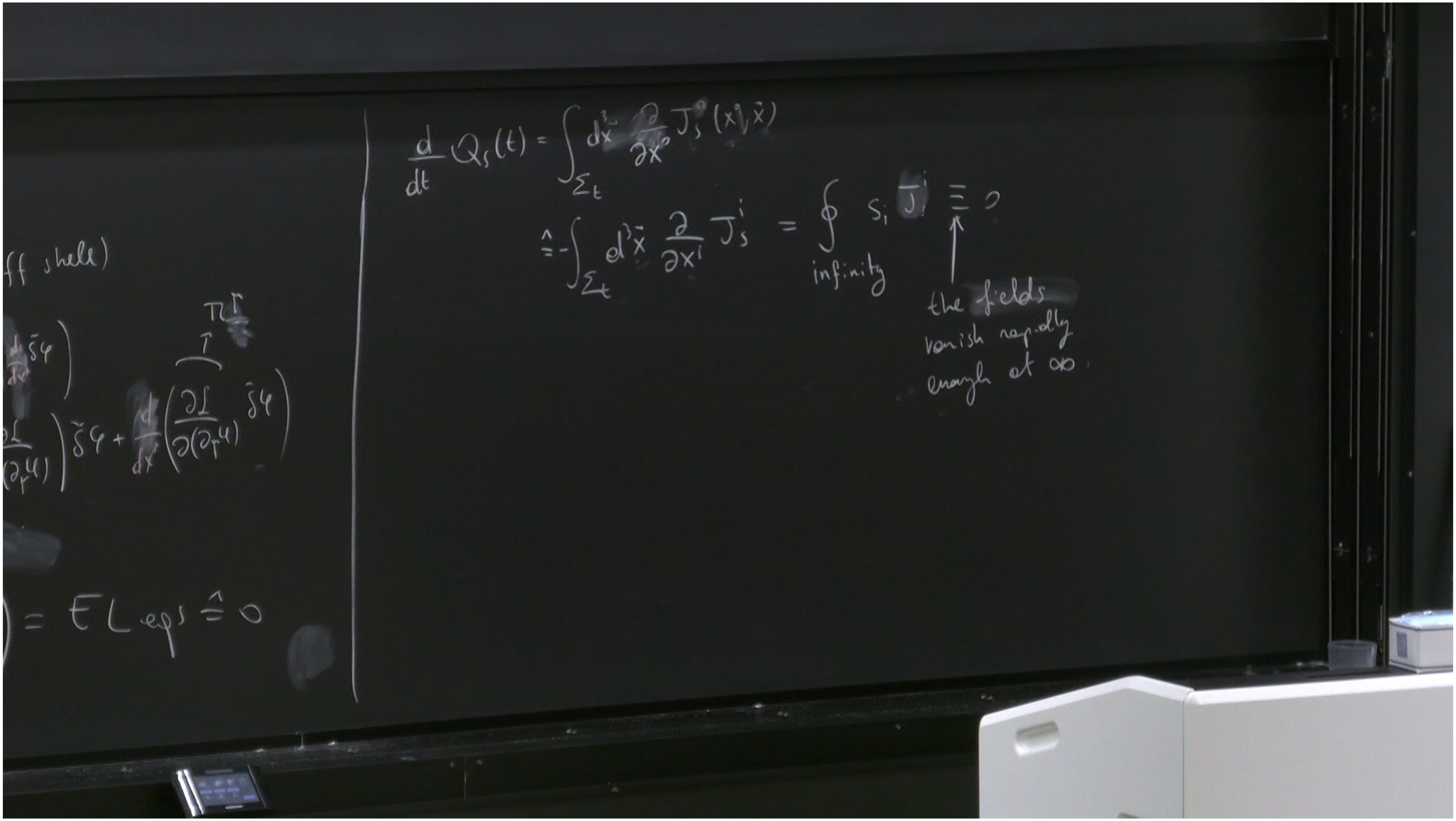
the fi... vanish ... enough



ff shell)
$\frac{d}{dt} Q_s(t) = \int_{\Sigma_t} d^3\vec{x} \frac{\partial}{\partial x^0} J_s^0 (x^0, \vec{x})$
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the fields vanish rapidly enough at ∞
$= EL\ eqs \hat{=} 0$

Ex

a complex scalar field $\varphi : M \to \mathbb{C}$

$$\mathcal{L} = -\partial_\mu \varphi^* \partial^\mu \varphi - m^2 \varphi^* \varphi - V(\varphi^* \varphi)$$

Two ways of thinking about this as:

$$\varphi = \begin{pmatrix} \varphi^{Re} \\ \varphi^{Im} \end{pmatrix} \qquad \boxed{\begin{aligned} \varphi &= \varphi^{Re} + i \varphi^{Im} \\ \varphi^* &= \varphi^{Re} - i \varphi^{Im} \end{aligned}}$$

E.o.m. for φ^*: $\Box \varphi - m^2 \varphi - V'(\varphi^* \varphi)\varphi = 0$

— // — φ: compl. conj. eq. of motion.

Sym $\varphi \to e^{i\alpha} \varphi$

$$\left[\begin{aligned} \tilde{\delta}_s \varphi &= i\varphi \\ \tilde{\delta}_s \varphi^* &= -i\varphi^* \end{aligned} \right.$$

$\tilde{\delta}_s \mathcal{L} = 0 \to R_s^\mu =$

$$J^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \tilde{\delta}_s \varphi +$$

Ex: $\nabla_\mu J^\mu \hat{=} 0$.

$$x \longmapsto \varphi^\mu = A^\mu(x)$$

Sym $\varphi \to e^{i\alpha}\varphi$

$$\begin{cases} \tilde{\delta}_s \varphi = i\varphi \\ \tilde{\delta}_s \varphi^* = -i\varphi^* \end{cases}$$

$$\tilde{\delta}_s \mathcal{L} = 0 \longrightarrow R^\mu_s = 0$$

$$J^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \tilde{\delta}_s \varphi + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi^*)} \tilde{\delta}_s \varphi^* = \mathrm{Im}\left(\varphi \nabla^\mu \varphi^*\right)$$

Ex: $\nabla_\mu J^\mu \hat{=} 0$.