Title: Correlations, representations and transformers in physics and in AI

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Series: Quantum Foundations

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Abstract: I will begin by reviewing the general mathematical concept of representation. I will then show that representation theory is more generally applicable than one might expect, for example in quantum foundations, in quantum gravity and in machine learning. In those contexts, I will first talk about the notion of representation underlying phenomena of emergence in quantum gravity. I will then discuss how quantum reference frames might be viewed as representations. Finally, I will talk about how transformer models, such as GPT-4, construct representations of what they learn and, in that light, what it may take for machines to reach AGI or even consciousness.

Zoom link: https://pitp.zoom.us/j/99349397588?pwd=T056VjZXRWZWT28zY3VOZHFDcmQ3QT09

On the Role of Representations in Foundations, Quantum Gravity and AI

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General representations

A representation can be any structure-preserving map $\Phi : A \to B$.

Example: Abstract axioms of a vector space. Any concrete vector space is a representation.

Example: In QM, abstract states $|\psi\rangle$ and operators $\hat{f}$ possess position representation, momentum representation, etc, (in this case equivalent by Stone and von Neumann).

Example: every cohomology theory is a representation of the category of differentiable manifolds in the category of abelian groups.

Approximate representations

Examples:

- Classical phase space is approximate representation of underlying quantum Poisson algebra of QM
- Experimental quantum circuit is approximate representation of a quantum algorithm.
- If spacetime has some torsion, then the usual torsion-free spacetimes using the levi-civita connection are approximate representations.

Notice: Approximate representations can be good in some regimes and bad or impossible in other regimes.

Is QFT on curved spacetime a representation?

Is QFT on curved spacetime an approximate representation of a more general, underlying structure?

What if yes? The underlying structure could possess other representations:

- Quantum reference frames?
- Dualities such as AdS/CFT?
- Other regimes, such as quantum gravity at Planck scale?

Let us search for a more abstract, simpler structure of which QFT on curved spacetime is an approximate representation.

Is QFT on curved spacetime a representation?

Observation 1: Matter is describable through correlators

Namely in terms of quantum field theoretic n-point functions $G^{(n)}(x^{(1)}, x^{(2)}, \ldots, x^{(n)})$.

Remark: We could now study these information theoretically:

- QFT interactions constitute classical & quantum channels. (See recent papers by my group).
- Question, e.g.: Are the Feynman rules describable entirely through their classical and quantum channel capacities?

But for now, we move on to spacetime curvature:

Is QFT on curved spacetime a representation?

Observation 2: Spacetime is describable through correlators

The metric is expressible through $G^{(2)}(x,y)$:

$$g_{\mu\nu}(x) = -\frac{1}{2}\left(\frac{\Gamma(D/2-1)}{4\pi^{D/2}}\right)^{\frac{2}{D-2}} \lim_{x\to y} \frac{\partial}{\partial x^{\mu}}\frac{\partial}{\partial y^{\nu}}\; G^{(2)}(x,y)^{\frac{2}{2-D}}$$

$\Rightarrow$

Spacetimes are expressible as (M,g) and also as $(M,G^{(2)})$.

M. Saravani, S. Aslanbeigi, A. Kempf, Physical Review D93, 045026 (2016)

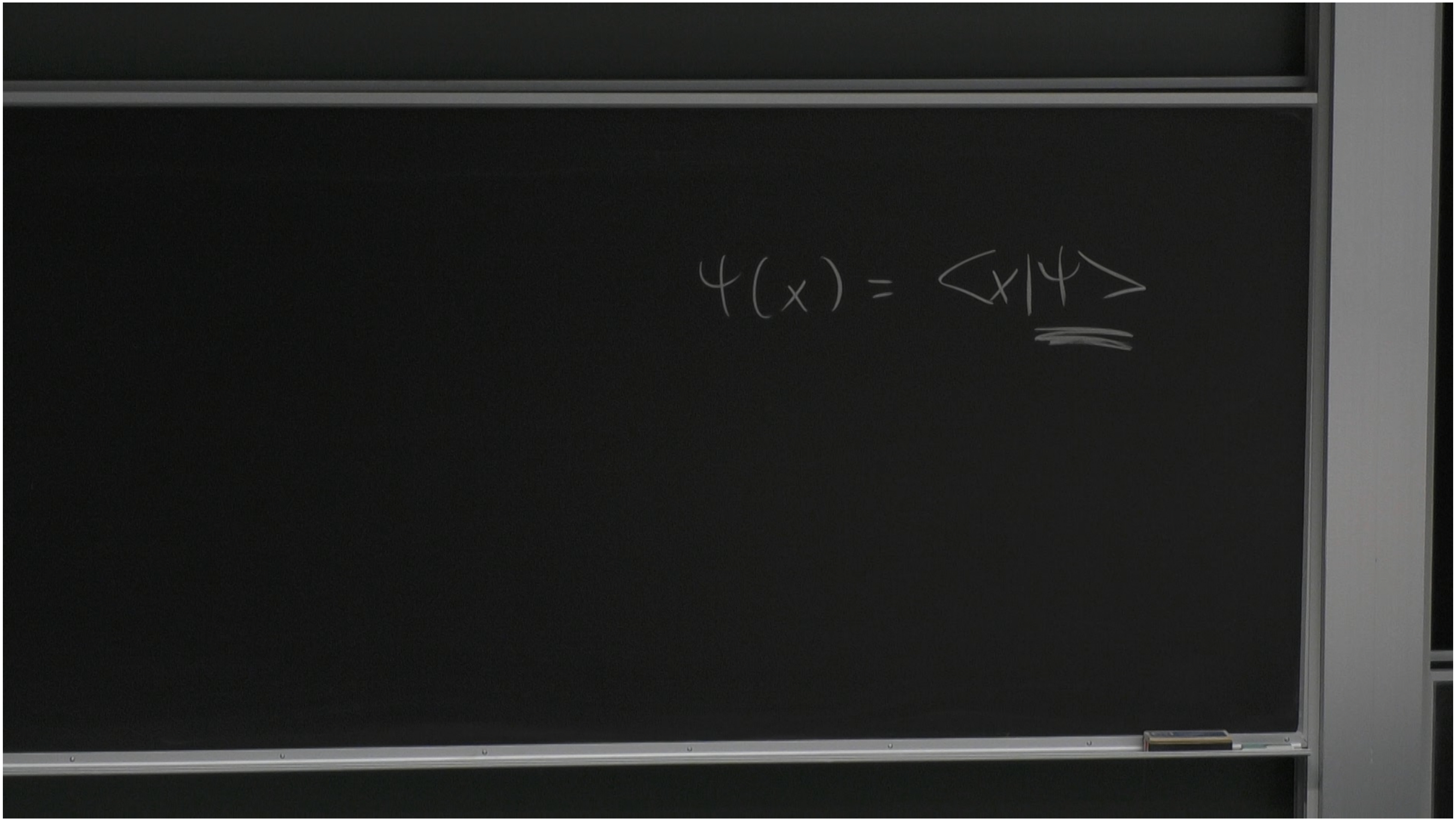
Summary so far

Both matter, and spacetime, are expressible through the QFT n-point functions:

$$G^{(n)}(x^{(1)}, x^{(2)}, \ldots, x^{(n)})$$

However:

- The correlators $G^{(n)}(x^{(1)}, x^{(2)}, \ldots, x^{(n)})$ depend on spacetime coordinates, which assumes that there is a differentiable spacetime manifold.
- **Can we find a more abstract coordinate-free formulation?**


$\psi(x) = \langle x|\psi\rangle$

QFT on curved spacetime, coordinate free

Recall: In QM, $G(x, x')$ can be written in any basis, they represent abstract Hilbert space operators.

Here too: The correlators $G^{(n)}(x_1, ..., x_n)$ represent abstract n-argument Hilbert space operators, $G^{(n)}$.

Is the set of $G^{(n)}$ operators a coordinate-free description of QFT on curved spacetime?

The $G^{(n)}$: equivalent description of spacetime and matter?

Given the operators $G^{(n)}$, how to represent them as field correlators $G^{(n)}(x_1, ..., x_n)$ on a spacetime manifold?

Via $G^{(2)}(x_1, ..., x_n)$, this would then yield also the metric.

Strategy:

In QM, we would use position operators to obtain position bases ...

... here in QFT, use "local interactions" $G^{(n)}$, $n > 2$ to obtain position bases.

The $G^{(n)}$: equivalent description of spacetime and matter?

If the theory's interactions are local, then:

- The vertices, i.e., $G^{(n)}$ for $n > 2$ are diagonalizable.
- A diagonalizing basis is a coordinate system. We obtain:

$$G^{(n)}(x^{(1)}, x^{(2)}, \ldots, x^{(n)}) \qquad \text{for all} \quad n$$

- Now that we have $G^{(2)}(x, y)$, we also obtain the metric:

$$g_{\mu\nu}(x) = -\frac{1}{2}\left(\frac{\Gamma(D/2-1)}{4\pi^{D/2}}\right)^{\frac{2}{D-2}} \lim_{x\to y} \frac{\partial}{\partial x^\mu}\frac{\partial}{\partial y^\nu}\; G^{(2)}(x,y)^{\frac{2}{2-D}}$$

(This proves that you can hear the shape of a nonlinear drum!)

Do all $G^{(n)}$ describe a spacetime and matter?

No, because generic $G^{(n)}$, for $n > 2$, are not exactly diagonalizable.

- In "low energy" regimes, the $G^{(n)}$ for $n > 2$ may be approximately diagonalizable.
- In "high energy" regimes, the $G^{(n)}$ generally possess no representation as QFT correlators on a spacetime.

A. Kempf, Front. Phys., Vol.9, 655857 (2021), https://arxiv.org/abs/2110.08278

Summary so far

- Spacetime and matter are describable by a collection of abstract n-point correlators $G^{(n)}$
- A priori, the set of space representations could include quantum reference frames and dualities such as AdS/CFT.

However:

- Generic $G^{(n)}$ are at best approximately representable as QFT correlators on a spacetime manifold.
- In general, the abstract correlators $G^{(n)}$ do not possess spacetime representations.

In this way, QFT in spacetime could be an (approximate) representation of more fundamental abstract n-point correlators of a more general pre-geometric theory.

A theory of the $G^{(n)}$?

Notice: Via a functional Fourier transform the $G^{(n)}$ are equivalent to an action:

$$Z[J] = N \int e^{iS[\phi]+i\int d^4x\ J(x)\phi(x)}\ D[\phi]$$

But instead of studying actions as usual, maybe we should study the information theory of the $G^{(n)}$ directly:

Information theory's building blocks are n-point correlators:

- n = 1: von Neumann and Rényi entropies etc,
- n=2: mutual information, coherent information, classical & quantum channel capacities, negativity, etc,
- n=3,4,..: higher-order web of informational relationships.

Remark: Why should information theory be so fundamental?

Why expect information theory could play such an important role?

The notion of information is applicable even in the most counter-intuitive regimes, e.g., even when physical notions such as distance, curvature, mass etc fail.

If information is the basic currency of physical processes, should study the $G^{(n)}$?

Concrete example: Abstract free boson and fermion theory

This is work with Marcus Reitz and Barbara Ŝoda:
arXiv:2303.01519

Strategy:

Take N_b free boson and N_f free fermion species on curved Euclidean spacetime and generalize to the abstract theory.

What regimes does the abstract theory have?

Spoiler: E.g., spacetime dimension depends on energy scale.

The Path Integral's Bosonic Action

Re-write UV cutoff action of N_b boson species basis independently:

$$S_b = \frac{1}{2}\sum_{i=1}^{N_b} \mathrm{Tr}\left((\Delta + m^2)|\phi\rangle_i\langle\phi|_i\right)$$

Notice: Existence of position bases no longer assumed.

Convenient: use the eigenbasis of Δ:

$$S_b = \sum_{i=1}^{N_b}\sum_{n=1}^{N} \lambda_n (\phi_n^i)^2.$$

The λ_n are the eigenvalues of wave operator $\Delta + m^2$.

The Path Integral's Gravity Action

Hawking & Gilkey showed that:

$$N = \frac{1}{16\pi^2}\int d^4x\sqrt{g}\left(\frac{\bar{\Lambda}^2}{2} + \frac{\bar{\Lambda}}{6}R + O(R^2)\right)$$

with N = Number of eigenvalues of Δ below UV cutoff Λ.

Thus, with $\mu = \frac{6\pi}{\bar{\Lambda}}$ **the gravity action is, with corrections:**

$$\begin{aligned} S_g &= \mu N && \text{in eigenbasis of wave operator} \\ &= \mu \mathrm{Tr}(\mathbf{1}) && \text{basis independently} \end{aligned}$$

The Gravity & Matter Path Integral

The full path integral now reads:

$$Z = \sum_{N=1}^{\infty} \int_{m^2}^{\Lambda} \mathcal{D}\lambda \int \mathcal{D}\phi \int \mathcal{D}\theta \mathcal{D}\bar{\theta} \; e^{-\beta S} \; \frac{\Lambda^{N(\frac{N_f}{2}-1)}}{(N-1)!} .$$

with

$$S = S_g + S_b + S_f$$

The Λ can be omitted in natural units in which $\Lambda = 1$.

The Path Integral can be evaluated

The path integral evaluates to:

$$Z = C\ m^{d-2} \exp\left[2\ C\ \frac{\Lambda^{d/2} - m^d}{d}\right] \tag{1}$$

Here:

$$C := (2\pi)^{\frac{N_b}{2}} e^{-\beta\mu} \beta^{\mu\beta_{max}} \Lambda^{1-\frac{N_f}{2}}$$

with

$$\beta_{max} := \frac{2N_f - N_b}{2\mu}$$

$$d := 2 - N_b + N_f$$

We can now calculate spacetime properties.

Dimension of Spacetime, when existing

To this end, calculate scaling of eigenvalue density:

$$\rho(\lambda) = \frac{1}{Z}\sum_N \int \mathcal{D}'\lambda \int \mathcal{D}\phi \int \mathcal{D}\theta \int \mathcal{D}\bar{\theta} e^{-\beta S}$$
$$\propto \lambda^{N_f/2 - N_b/2}$$

Compare with Weyl scaling:

Weyl found that whenever the eigenfunctions of Δ are dominated by the spacetime dimension, not by curvature, then:

$$\rho(|\lambda) \propto \lambda^{d/2-1}. \qquad (2)$$

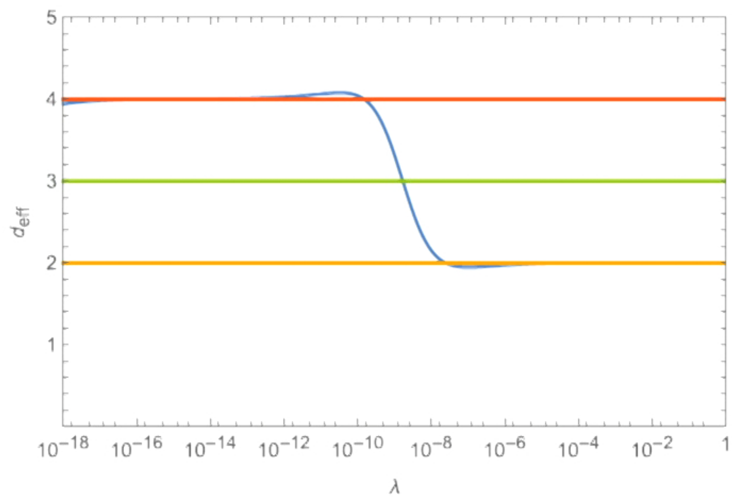
Therefore:

$$d_{eff} = 2 + N_f - N_b$$

Energy Scale Dependence of d_{eff}

Similarly, for nontrivial boson and fermion mass spectra:

$$d_{eff}(\lambda) = -2\lambda \frac{\partial \log(p(\lambda))}{\partial \lambda} + 2 \tag{3}$$



Example: $d_{eff}(\lambda)$ for 32 fermion and 30 boson species at low mass, and 1 fermion and 3 boson species at medium mass.

Summary

Results:

- Obtained dimensions and volumes of emergent spacetimes

  ... depending on gravity & boson pull vs. fermion pressure

- That balance

  ... depends on energy scale through mass spectrum, yields

  ... energy-dependent spacetime dimensions & emergence

Conclusions

- QFT on curved spacetime can be viewed as an approximate representation of an abstract structure of n-point correlators $G^{(n)}$.
- This yields a theory that also contains pre-geometric regimes.
- The representation theory may also contain quantum reference frames and dualities.

The abstract structure of n-point correlators $G^{(n)}$ is information theoretic. It might be said to be 'just structure'.

The physics of the emergence of spacetime and matter would be the mathematics of the emergence of representability as fields on a manifold.

Representations in AI and in biology

Transformers, such as GPT-4 have been called statistical parrots, that learn nothing but correlators.

But it is has also been shown that in the learning phase, to be efficient, they build an internal representation of what they learn.

E.g., they learn board game representations from only tables of moves.

Representations that contain the representer

Animals too build an internal representation of the world as they encounter it, for prediction purposes.

Some species build a representation that contains themselves, and recognize themselves in mirrors.

Representations that contain themselves

Some species even represent the representation inside themselves.

At this point, the representation becomes recursive - reflective - may this be the beginning of consciousness?

If so, how to build a conscious machine? Make it represent its own representing.

Zoomed out view of Representations

Maybe Carl Sagan already anticipated it all:

The cosmos is within us. We are made of star-stuff. We are a way for the universe to know itself.