Title: Hamiltonian supermaps: Higher-order quantum transformations of unknown Hamiltonian dynamics

Speakers: Mio Murao

Series: Quantum Foundations

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Abstract: Supermaps are higher-order transformations taking maps as input. We consider quantum algorithms implementing supermaps for the input given by unknown Hamiltonian dynamics, which can be regarded as infinitely divisible unitary operations. We first show a quantum algorithm that approximately but universally transforms black-box Hamiltonian dynamics into controlled Hamiltonian dynamics utilizing a higher-order transformation called neutralization. Then, we present another universal algorithm that efficiently simulates linear transformations of any Hamiltonian consisting of a polynomial number of terms in system size, using only controlled-Pauli gates and time-correlated randomness. This algorithm for implementing Hamiltonian supermaps is an instance of quantum functional programming, where the desired function is specified as a concatenation of higher-order quantum transformations. As examples, we demonstrate the simulation of negative time-evolution and time-reversal, and perform a Hamiltonian learning task.

References:

Q. Dong, S. Nakayama, A. Soeda and M. Murao, arXiv:1911.01645v3
T. Odake, Hlér Kristjánsson, A. Soeda M. Murao, arXiv:2303.09788

Zoom Link: https://pitp.zoom.us/j/94278362588?pwd=MGszYk9uN1A3K1RTOVhYSGpkL1FQdz09

Hamiltonian supermaps: Higher-order quantum transformations of unknown Hamiltonian dynamics

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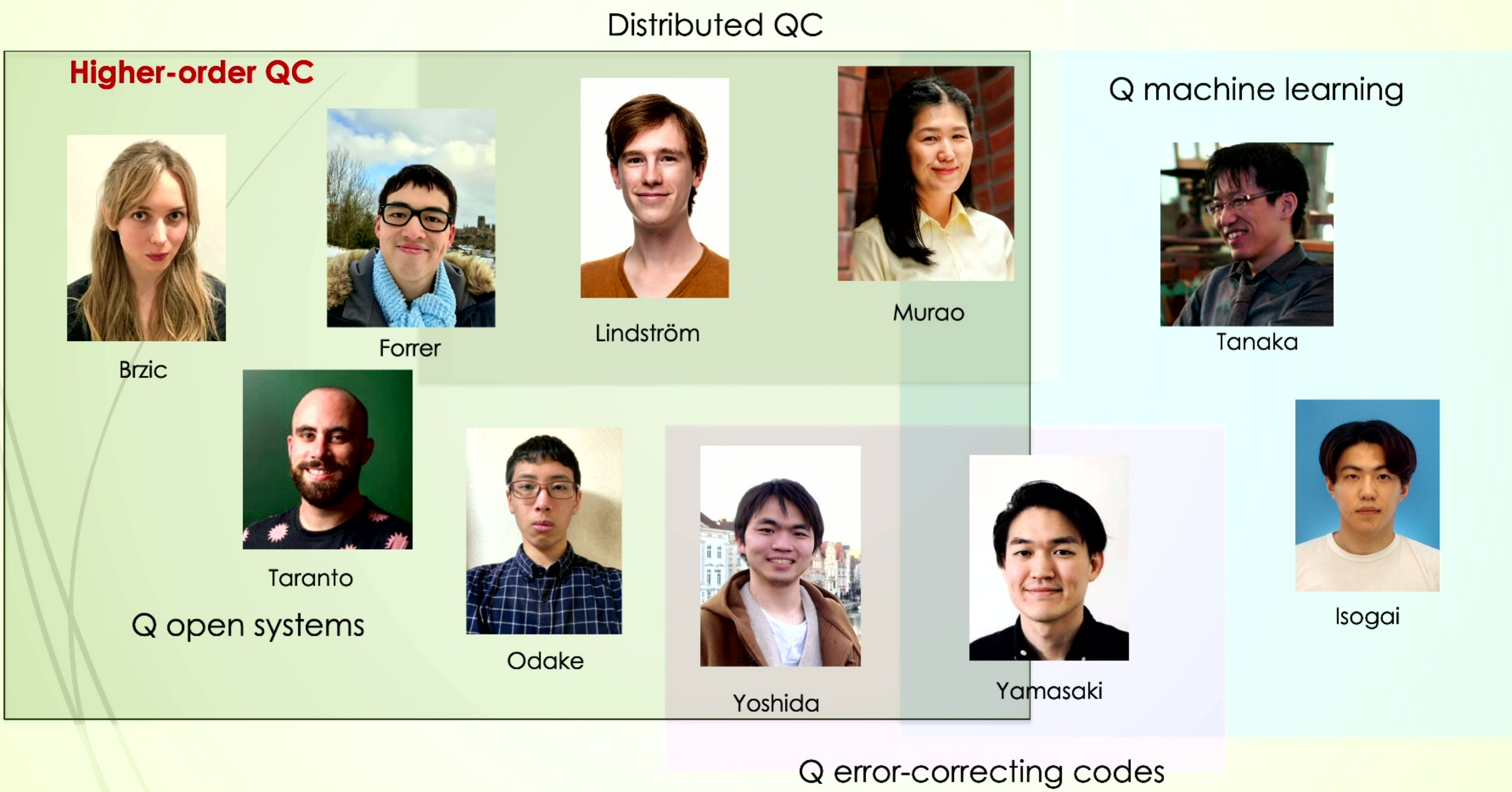
Our group members (April-September 2023)



Quantum computers (in my opinion)

Machines which can perform **any operations allowed in quantum mechanics**

- Not just a computer to compute classical computational problems.

➨ Understanding physics under **conditional operations and manipulations**

How laws of physics (= quantum mechanics) allow/restrict manipulations?
How they are different from the classical world?

Ultimately, how much we can “program” quantum systems?

(usual quantum computers)

Quantum computer for processing quantum **states**

variable

an arbitrary quantum **state**
(can be an **unknown** state)

ρ → physical system

→ $\rho' = f(\rho)$: output quantum state (can be unknown)

$f = ...$ command

A CPTP map, a quantum instrument, or a POVM

"quantum state processor" $\rho \to f(\rho)$

= more general quantum objects

Quantum computer for **processing quantum systems**

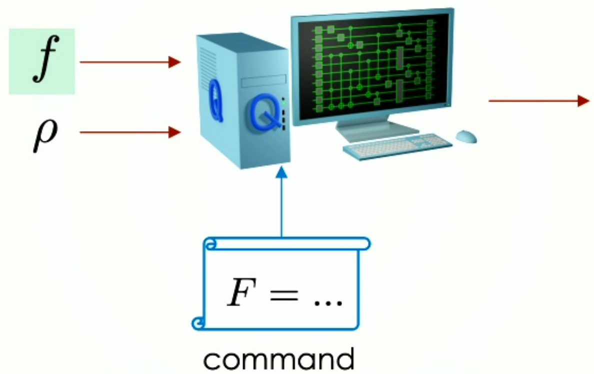
Processing quantum **maps** and **states** $\rho \to F(f)(\rho)$

where $f \to F(f)$ is a **supermap**, a map of a map

variables

An arbitrary quantum **map** (can be a **blackbox**)

An arbitrary quantum **state** (can be an **unknown state**)



$F(f)(\rho)$

not a super-operator in general, it can be a super-super operator

"Quantum **process** processor"

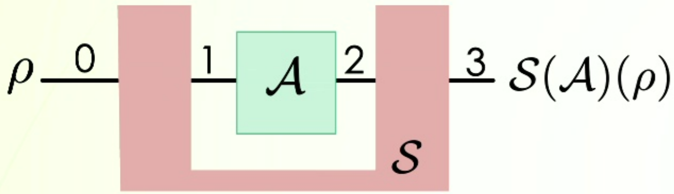
We call **"higher-order quantum operations/transformations"**

Also known as
- = **superchannels and superinstruments** (Calgary group, our group)
- = **quantum supermaps, quantum testers** (Paiva group)
- = **Processes**, or **process matrices** (Vienna group)

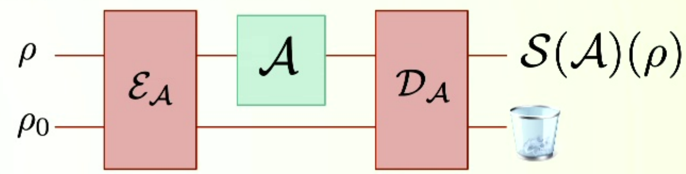
Formulation of higher-order quantum transformations (supermaps)

G. Chiribella, G. M. D'Ariano, and P. Perinotti, PRL (2008), PRA (2009)

The quantum comb formalism provides the condition to implement a supermap $\mathcal{S}$ by the conditions for the **Choi operator** of the quantum comb $\mathcal{J}_\mathcal{S}$ representing the supermap such that $\mathcal{S}(J_\mathcal{A}) = J_{\mathcal{S}(\mathcal{A})}$ $(\mathcal{J}_\mathcal{S} * J_\mathcal{A} = J_{\mathcal{S}(\mathcal{A})})$



$\mathcal{E}_\mathcal{A}, \mathcal{D}_\mathcal{A}$: unitaries



Single-slot quantum comb: $\mathcal{S}$ supermap

quantum circuit representation

Choi operator: $\mathcal{J}_\mathcal{S}$

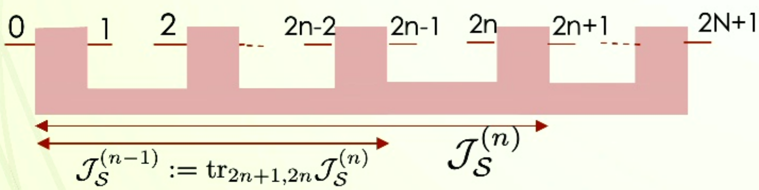
The Choi operator of a quantum comb:

$$\mathcal{J}_\mathcal{S} \in \mathcal{B}((\mathcal{H}_1 \otimes \mathcal{H}_2) \otimes (\mathcal{H}_0 \otimes \mathcal{H}_4))$$

(1) $\mathcal{J}_\mathcal{S} \geq 0 \Leftrightarrow \mathcal{S}$ is completely CP preserving

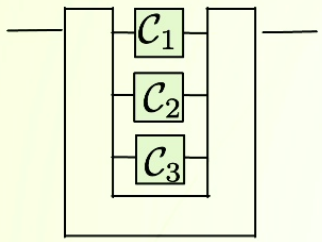
(2) $\mathrm{tr}_{123}\mathcal{J}_\mathcal{S} = I_0 \Leftrightarrow \mathcal{S}$ is trace preserving (deterministic)

(3) $\mathrm{tr}_3\mathcal{J}_\mathcal{S} = \mathrm{tr}_{23}\mathcal{J}_\mathcal{S} \otimes I_2$ ← the requirement of causal order

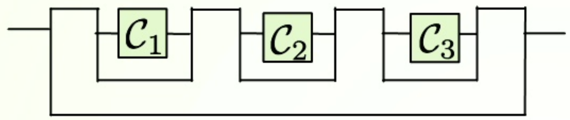


Multiple-slot case: $\mathrm{tr}_{2n+1}\mathcal{J}_\mathcal{S}^{(n)} = \mathrm{tr}_{2n,2n+1}\mathcal{J}_\mathcal{S}^{(n-1)} \otimes \frac{I_{2n}}{d_{2n}}$ for $n = 0, 1, \cdots, N$

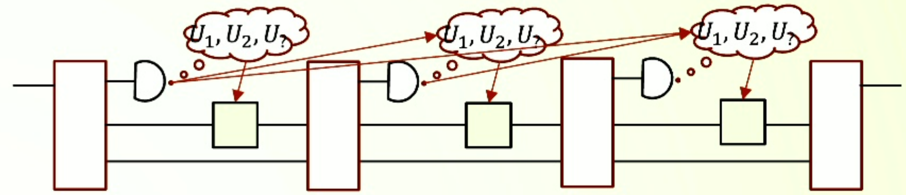
Causal order structures of higher-order quantum operations



parallel



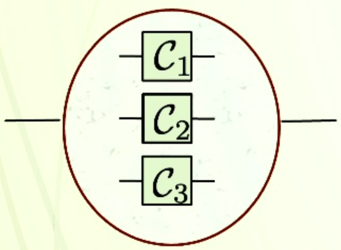
sequential



Dynamical ordering
(classical control of causal order)

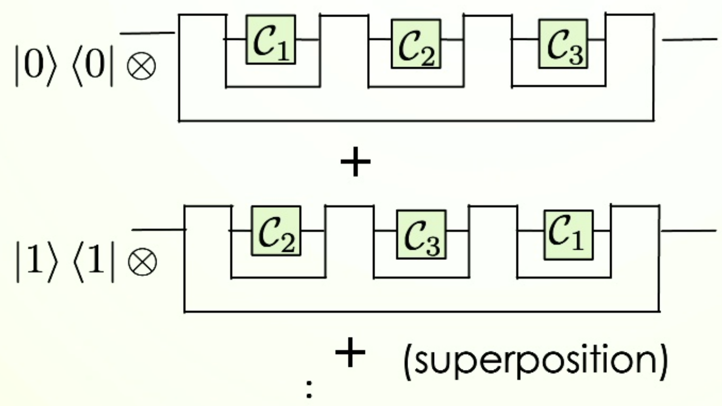
J. Wechs, H. Dourdent, A. A. Abbott, and C. Branciard, arXiv:2101.08796

Indefinite causal order:cannot be written by a q. circuit

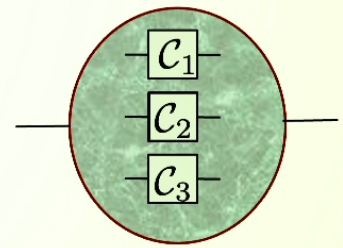


quantum switch-type

=



$\neq$



General Indefinite
causal order

O. Oreshkov, F. Costa, and C. Brukner, Nat Commun 3, 1092 (2012), G. Chilibella et al., Phys. Rev. A 88, 022318 (2013)

Why **higher-order** quantum operations? $f \rightarrow F(f)$

- Understanding the power of quantum computers as a **quantum system processor (manipulator)** including unknown quantum **states and maps (dynamics)**
- Higher-order also includes lower-order (states and normal maps as special cases)
- It may extend **usefulness of quantum computers**
- It may extend our **understanding of quantum mechanics**
- Their properties are **yet not fully understood** except the pioneering works in terms of quantum combs and process matrices

  G. Chiribella et al. PRL 2008, PRA 2009
  O. Oreshkov, F. Costa, and C. Brukner, Nature Comm, 2012
  O. Oreskov and C. Giarmatzi, NJP (2016)
  M. Araujo et al. NJP (2015), Quantum (2017)
- Function of functions $\rightarrow$ quantum **functional** programing?

  T. M. Rambo, J. B. Altepeter, P. Kumer and G. M. D'Ariano, Phys. Rev. A (2016)
  A. Bisio and P. Perinotti, Proc. Roy. Soc. A (2019)
- **Controllization of a unitary:** one of the higher-order quantum operations and key elements many quantum algorithms ➡ **A useful subroutine for Q algorithms**

A new frontier of quantum information!

In our group, we investigate
higher-order quantum transformations
for developing a new paradigm of functional quantum programming

- *Aiming to develop a new framework of* ***functional quantum programming***
  - *Analyze* ***implementation algorithms*** *of higher-order quantum transformations*
  - *Formulate* ***compositions*** *of higher-order quantum transformations for programming quantum algorithms in a functional programming manner*
- *Analyze* ***causal order structures*** *in higher-order quantum transformations*
- *Seeking applications for quantum simulation, sensors, and process controllers/processor*

Selected works on this topic

- Quantum algorithm for **projective measurement of energy** of **unknown** Hamiltonian systems, Nakayama, Soeda and Murao, Phys. Rev. Lett. (2015)
- Quantum algorithm for **inverting, transposing and conjugating unknown** unitary gates, Quintino, Dong, Shimbo, Soeda and Murao, Phys. Rev. Lett (2019)
- New **"Success-or-draw" strategy** for quantum algorithm for transforming **unknown** unitary gates, Dong, Quintiono, Soeda and Murao, Phys. Rev. Lett. (2021)
- **Deterministic exact** qubit-unitary inversion, Yoshida, Soeda and Murao, Phys. Rev. Lett (2023)

black box

Higher-order quantum operations for unitaries

- Examples useful for quantum programming: $\mathcal{S} : U \to f(U) = V_U$
  - **Replication** $V_U = U \otimes U$ G. Chiribella, G. M. D'Ariano, and P. Perinotti, PRL (2008)
  - **Inversion** $V_U = U^{-1} = U^{\dagger}$ "undo" M.T. Quintino, Q. Dong, A. Shimbo, A. Soeda and M. Murao, PRL (2019); S. Yoshida, A. Soeda and M. Murao, PRL (2023)
  - **Complex conjugation** in terms of a fixed basis $V_U = U^*$ J. Miyazaki, A. Soeda and M. Murao, PRR (2019); D. Ebler et al., IEEE Tran Info Theory (2023)
  - **Transposition** in terms of a fixed basis $V_U = U^T$ M.T. Quintino, Q. Dong, A. Shimbo, A. Soeda and M. Murao, PRA (2019)
  - **Controllization** up to phase $V_U = |0\rangle\langle 0| \otimes I + e^{i\theta_U}|1\rangle\langle 1| \otimes U$ Q. Dong, S. Nakayama, A. Soeda and M. Murao, arXiv:1911.01645v3
  - **Quantum switch** $V_{U_1,U_2} = |0\rangle\langle 0| \otimes U_1U_2 + |1\rangle\langle 1| \otimes U_2U_1$ G. Chiribella, G. M. D'Ariano, P. Perinotti, and B. Valiron, PRA (2013)
  - **Neutralization** $V_U = I$ ← also known as refocusing, resetting
  - **Homomorphic and anti-homomorphic higher-order functions** M.T. Quintino and D. Ebler, Quantum (2022)

Most of these cannot be universally implemented with a **single call** of U

Need to utilize **multiple calls** or **divisible calls** of the black box

In this talk,

We consider higher-order quantum transformations of **divisible unitaries,** in particular, Hamiltonian dynamics (Hamiltonian supermaps)

1. Universal **neutralization** and **controllization** algorithm of divisible unitaries and Hamiltonian dynamics

   Q. Dong, S. Nakayama, A. Soeda and M. Murao, arXiv:1911.01645v3 (some updates in 2021)

2. **Higher-order transformations of Hamiltonian dynamics**

   T. Odake, HléKristjánsson Akihito Soeda Mio Murao, arXiv:2303.09788

X: Unitary and isometry inversion with multiple calls of unitaries

Satoshi Yoshida's talk in the next week!

S. Yoshida, A. Soeda and M. Murao, PRL 131, 120602 (2023)

Universal neutralization and controllization algorithms of divisible unitaries (up to arbitrary phases)

Q. Dong, S. Nakayama, A. Soeda and M. Murao, 1911.01645v3

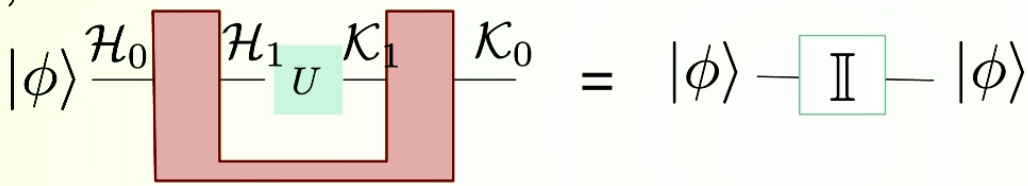
Update in 2021



Neutralization of unitary black boxes with d-dimensional U

Task: Find a higher-order quantum transformation implementing

$\forall |\phi\rangle$,



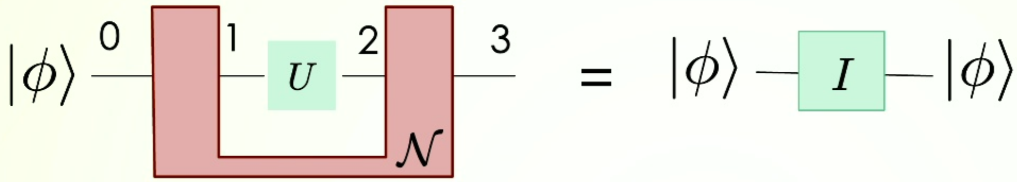
Irrespective of the input unitary, transforming it into an identity by the same higher-order quantum transformation

$$\forall U, U \rightarrow \mathbb{I}$$

Removing a coherent error (unknown unitary)

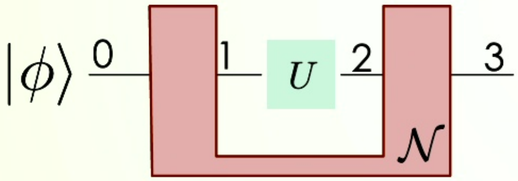
Neutralization comb for unitary operations

- higher-order quantum operation to neutralize a unitary U

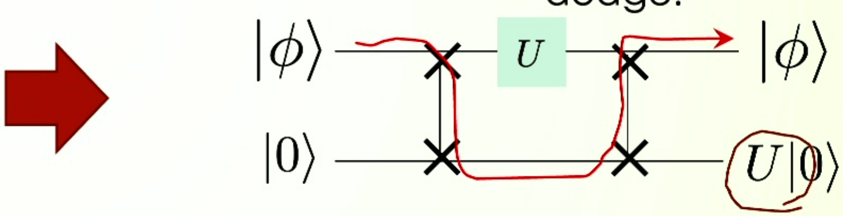


$\mathcal{N} : U \to I$

- The simplest implementation of a neutralization comb



$\mathcal{J}_{\mathcal{N}} = |I\rangle\rangle\langle\langle I|_{03} \otimes |0\rangle\langle 0|_1 \otimes I_2$



*This works for any $\mathcal{A}$

However, this way of dumping the action of the unitary into environment does not really "**erase**" the action from the whole universe....

We use properties of **invariance** to eliminate the action from our universe with d-dimensional U

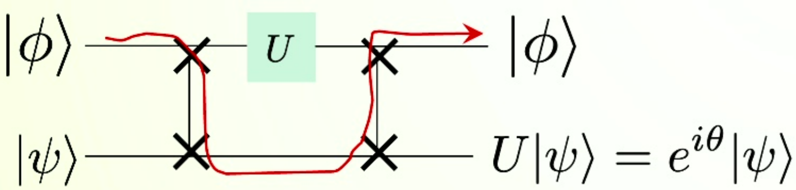
Divisible unitary operations and universal neutralization

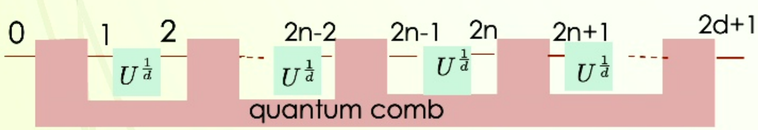
- If we can divide U into d parts of $U^{\frac{1}{d}}$, namely, $U^{\frac{1}{d}} \times d$

(or allowing **fractional queries**)

*Getting d-th root of unitary is not possible in general. But possible for Hamiltonian dynamics

Idea: To erase the unitary action from the whole universe, use an **invariant state** for the auxiliary (or environmental) state





There is no state to be invariant for all unitaries, but the totally antisymmetric state $|\Psi^-\rangle$ is invariant under

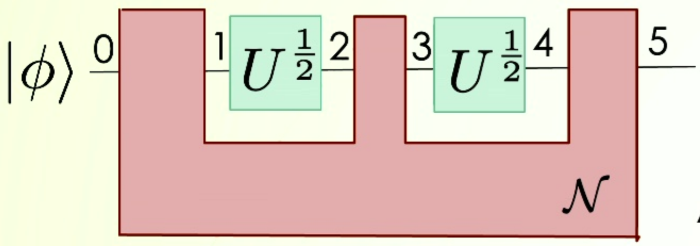
$$\underbrace{U^{\frac{1}{d}} \otimes U^{\frac{1}{d}} \otimes \cdots \otimes U^{\frac{1}{d}}}_{d \text{ tensor product}}$$

$$U^{\frac{1}{d}} \otimes U^{\frac{1}{d}} \otimes \cdots \otimes U^{\frac{1}{d}} |\Psi^-\rangle = \det[U^{\frac{1}{d}}]|\Psi^-\rangle$$

Use this property to construct a neutralization comb!

Clean* neutralization comb (d=2 example)

$$U^{\frac{1}{2}} \times 2 \rightarrow I$$



$$= \ |\phi\rangle - \boxed{I} - |\phi\rangle$$

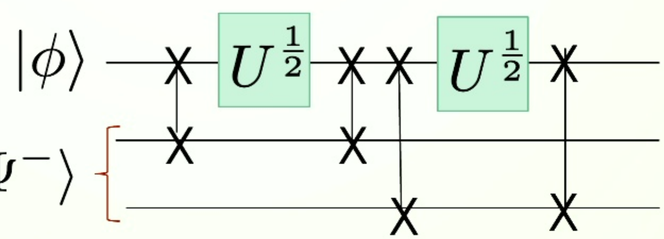
higher-order quantum operation to **neutralize** any unitary U

A Choi operator of the neutralization comb can be chosen as

$$\mathcal{J}_N = |I\rangle\rangle\langle\langle I|_{05} \otimes |\Psi^-\rangle\langle\Psi^-|_{13} \otimes I_{24}$$

invariant state

A circuit implementation of the clean neutralization comb



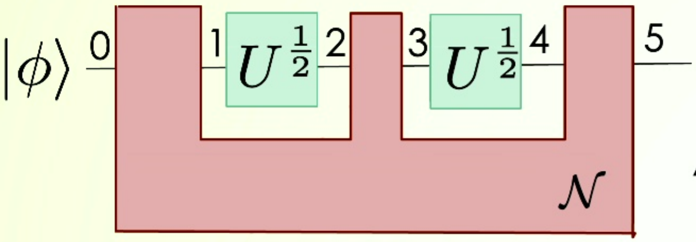
$$\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle) \ =: |\Psi^-\rangle$$

The totally anti-symmetric state of d=2

d-partite qudit state auxiliary system required

*"clean" implementation of a supermap is introduced by Z. Gavorová, M. Seidel, and Y. Touati, arXiv:2011.10031

Clean neutralization comb (d=2 example)



$= |\phi\rangle$ —[I]— $|\phi\rangle$

$U^{\frac{1}{2}} \times 2 \rightarrow I$

higher-order quantum operation to **neutralize** any unitary U

A Choi operator of the neutralization comb can be chosen as

$$\mathcal{J}_N = |I\rangle\rangle\langle\langle I|_{05} \otimes |\Psi^-\rangle\langle\Psi^-|_{13} \otimes I_{24}$$

invariant state

!! By coherently applying this neutralization comb or an identity-comb (just apply a sequence of the unitaries) depending on the control qubit, we can perform controlled-*U* by using the divisible *U*!

Controlled unitary: $C_U := |0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes U$

Neutralized *U*

Just applying *U* without comb

But this comb is not a pure comb (the Choi operator is not a pure state)!

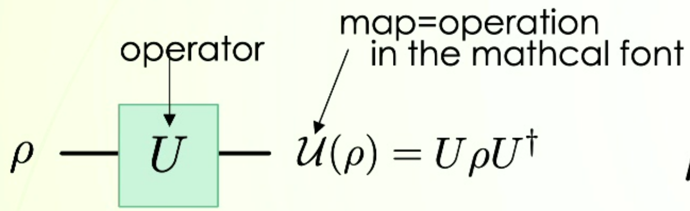
How to define controlled-general operations?

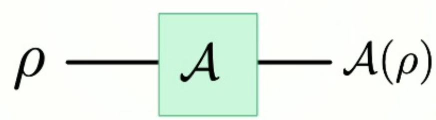
Controlled general quantum operations?

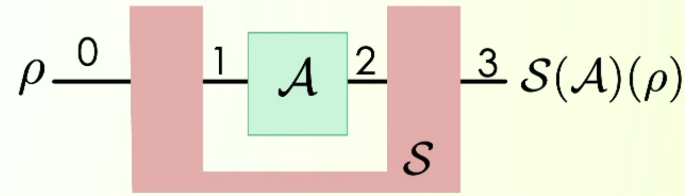
Quantum operations:

- Unitary operations
- Quantum operations: CPTP maps
- Higher-order quantum operations: quantum combs* (supermaps, transformations of maps)

*G. Chiribella, G. M. D'Ariano, and P. Perinotti, Phys. Rev. A (2009)
*G. Chiribella, G. M. D'Ariano, and P. Perinotti, Phys. Rev. Lett. (2008)







Unitary operation: U
operator

Quantum operation: $\mathcal{A}$
map,
superoperator

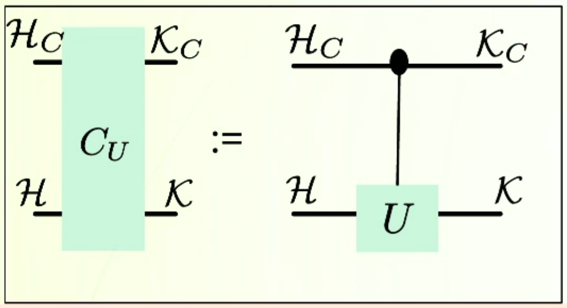
quantum comb: $\mathcal{S}$
supermap,
super-superoperator

- **How to define "controlled" quantum operations?**
- **Are controlled general quantum operations useful?**

We drop the phase factor from here

Quick review of controlled-unitary operations (2)

- The simplest example of a superposition of quantum programs



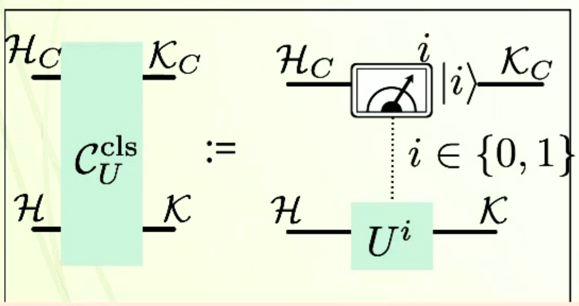
Coherently controlled of unitaries ="superposed"

- Choi operator of $\mathcal{C}_U$: $J_{\mathcal{C}_U}$ on $\mathcal{H}_C \otimes \mathcal{K}_C \otimes \mathcal{H} \otimes \mathcal{K}$

$$J_{\mathcal{C}_U} = |00\rangle\langle 00| \otimes J_I + |11\rangle\langle 11| \otimes J_U$$
$$+|00\rangle\langle 11| \otimes |I\rangle\rangle\langle\langle e^{i\theta_U} U| + |11\rangle\langle 00| \otimes |e^{i\theta_U} U\rangle\rangle\langle\langle I|$$

where $|I\rangle\rangle = \sum_{m=0}^{d-1} |m\rangle|m\rangle$, $|U\rangle\rangle = (I \otimes U)|I\rangle\rangle$, $J_U = |U\rangle\rangle\langle\langle U|$

off-diagonal terms



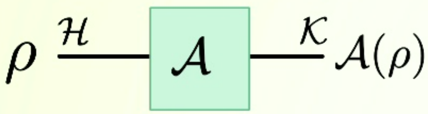
incoherently controlled unitaries

- Choi operator of $\mathcal{C}_U^{\mathrm{cls}}$ (not a unitary): $J_{\mathcal{C}_U^{\mathrm{cls}}}$

$$J_{\mathcal{C}_U^{\mathrm{cls}}} = |00\rangle\langle 00| \otimes J_I + |11\rangle\langle 11| \otimes J_U$$

"off-diagonal" terms are missing

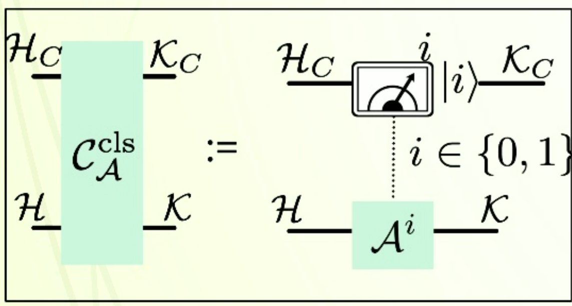
Quick review of general quantum operations (CPTP maps)



Quantum operation: $\mathcal{A}$

- Kraus representation: $\mathcal{A}(\rho) = \sum_i K_i \rho K_i^\dagger$ where $\sum_i K_i^\dagger K_i = I$
- Choi operator: $J_{\mathcal{A}} = (\mathrm{id} \otimes \mathcal{A})|I\rangle\rangle\langle\langle I| = \sum_i |K_i\rangle\rangle\langle\langle K_i|_{\mathcal{HK}}$

  where $|K_i\rangle\rangle = \sum_{mn} \langle m|K_i|n\rangle \cdot |n\rangle|m\rangle$



incoherently controlled $\mathcal{A}$

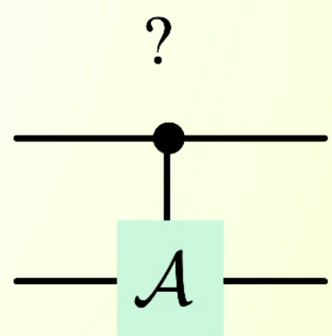
$\mathcal{A}^0 := \mathrm{id}, \ \mathcal{A}^1 := \mathcal{A}$

- Choi operator of incoherently controlled $\mathcal{A}$

$$J_{\mathcal{C}^{\mathrm{cls}}_{\mathcal{A}}} = |00\rangle\langle 00| \otimes J_I + |11\rangle\langle 11| \otimes J_{\mathcal{A}}$$

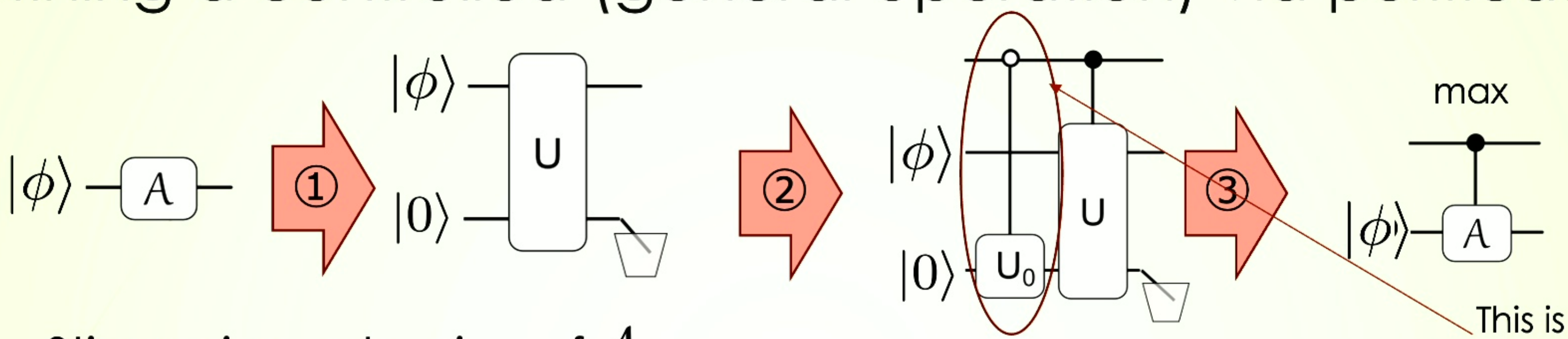
No coherence! Not a superposition!

How to coherently control (="superpose") id and $\mathcal{A}$ as much as possible?



Keeping the "off-diagonal terms" as much as possible?

Defining a controlled-(general operation) via purification



Stinespring extension of $\mathcal{A}$

This is the key idea!

② Introducing U_0 , satisfying

$$U_0|\phi\rangle \otimes |0\rangle = |\phi\rangle \otimes \sum_{i=1}^{n} \alpha_i |i\rangle \quad \sum_i |\alpha_i|^2 = 1$$

① Taking **mutually orthogonal** Kraus operators of $\mathcal{A}$: $\{K_i\}$
Define

$$U|\phi\rangle \otimes |0\rangle = \sum_{i=1}^{n} K_i |\phi\rangle |i\rangle$$

taking from i=1 ($K_0 = 0$) is a key tip!

③ Define the most coherently controlled general operation by maximizing the HS norm of $\mathrm{Tr}[K^\dagger K]$

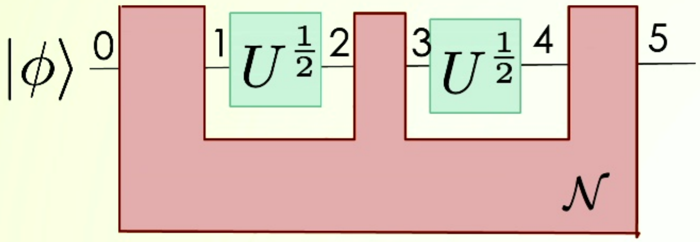
$$K := \sum_i \alpha_i^* K_i$$

choose the "largest" K to obtain max coherence

Related work: A. A. Abbott, J. Wechs, D. Horsman, M. Mhalla, and C. Branciard, arXiv:1810.09826
Quantum 4 (Sep, 2020)

Controlled-(neutralization comb) (d=2 example)

$U^{1/2} \times 2 \rightarrow \mathbb{I}$



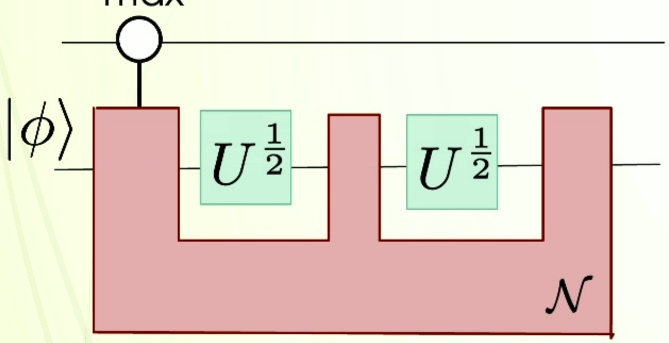
$= \quad |\phi\rangle - \boxed{\mathbb{I}} - |\phi\rangle$

A higher-order quantum transformation to **neutralize** a black box unitary U

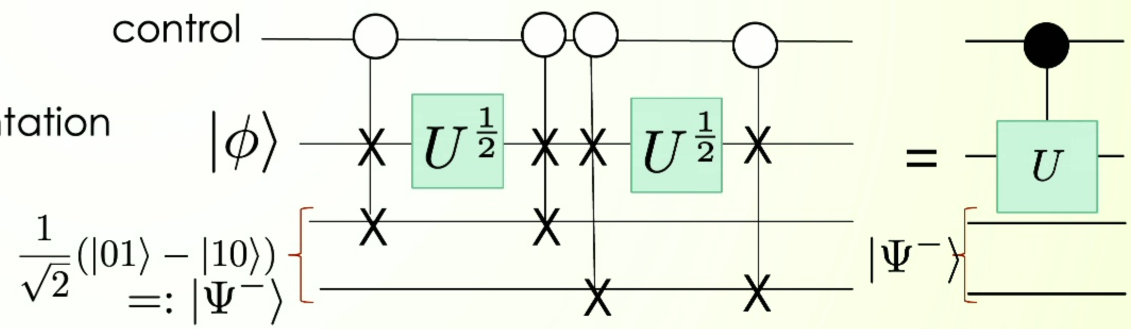
choose $|S_i\rangle\rangle \propto |I\rangle\rangle_{05} \otimes |\Psi^-\rangle_{13} \otimes (I \otimes \sigma_i)|\Psi^-\rangle_{24}$

Choi vector in terms of a Kraus operator

⇩ Max. coherent controllization of two different combs (the other is an identity comb)



implementation ⇨



Choi operator of the most coherently controlled controlled-comb:

$$\mathcal{J}_{\mathcal{C}_{\mathcal{N}}^{S_0}} \propto |11\rangle\langle 11| \otimes \mathcal{J}_{S_{\mathrm{id}}} + |00\rangle\langle 00| \otimes \mathcal{J}_{\mathcal{N}}$$

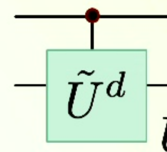
$$+|11\rangle\langle 00| \otimes |I\rangle\rangle\langle\langle S_0| + |11\rangle\langle 00| \otimes |S_0\rangle\rangle\langle\langle I|$$

$$S = \sum_i \alpha_i^* S_i \xrightarrow{\text{optimize}} S = S_0$$

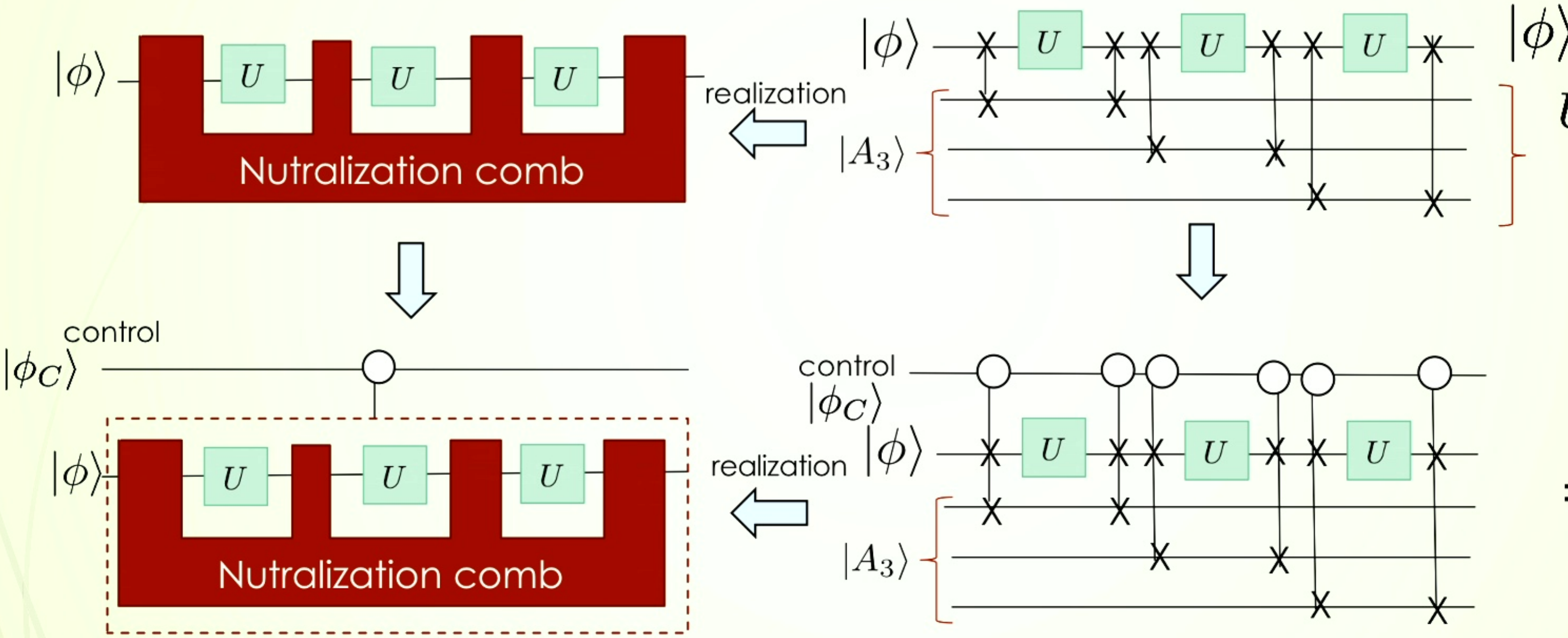
(Controllization up to the global phase)

Controllization as a "controlled-comb"

- Neutralization (for d=3)

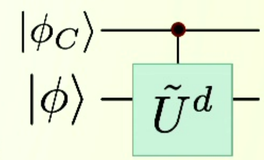


$$\tilde{U}^d = U^d / \det[U]$$



$$U \otimes U \otimes U|A_3\rangle = \det[U]|A_3\rangle$$

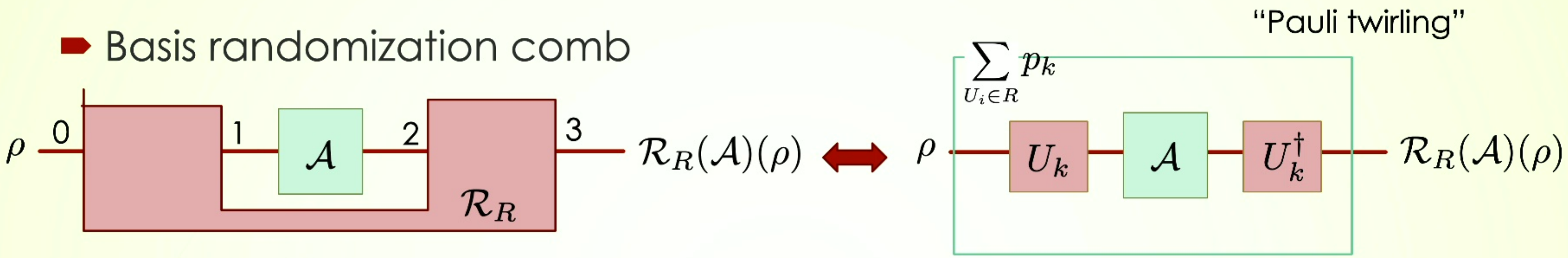
$$= \quad |\phi_C\rangle, |\phi\rangle \; \tilde{U}^d \otimes \det[U]|A_3\rangle$$



$$|output(\phi)\rangle = \alpha|0\rangle \otimes |\phi\rangle \otimes U^{\otimes d}|A_d\rangle + \beta|1\rangle \otimes U^d|\phi\rangle \otimes |A_d\rangle$$
$$= \{\alpha|0\rangle \otimes |\phi\rangle + \beta|1\rangle \otimes U^d/\mathrm{Det}(U)|\phi\rangle\} \otimes \mathrm{Det}(U)|A_d\rangle$$

Approximate neutralization without an auxiliary system

- Basis randomization comb



choose U_k randomly from a set $R = \{U_k\}$ Examples of $\{U_k\}$: Pauli operations $P = \{I, X, Y, Z\}$
Clifford operations Cl , etc.

1-design 3-design

The basis randomization comb with Pauli or Clifford operations transforms

$$J_{\mathcal{A}} = \sum_{i,j} c_{i,j} |\sigma_i\rangle\rangle\langle\langle\sigma_j| \Rightarrow J_{\mathcal{R}_{\mathcal{R}}(\mathcal{A})} = \sum_i c_{i,i} |\sigma_i\rangle\rangle\langle\langle\sigma_i|$$

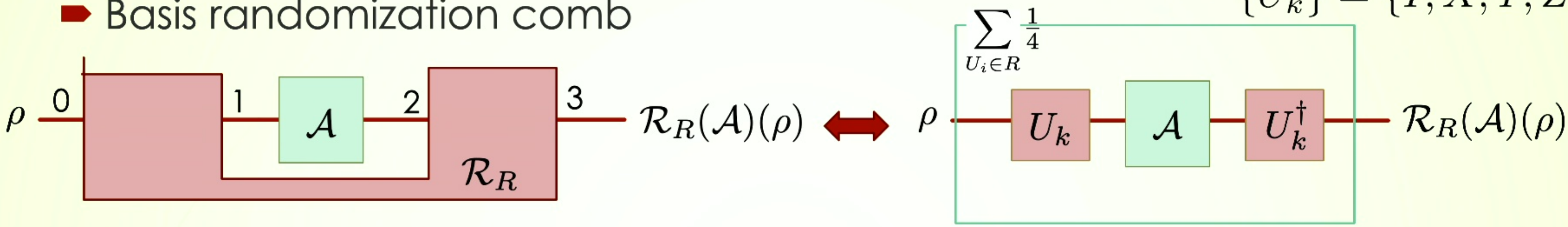
"diagonalizes" the Choi operator in terms of Pauli basis $\{|\sigma_i\rangle\rangle\}$

The Choi operator of the basis randomization comb: $\mathcal{J}_{\mathcal{R}_R} = \sum_{U_i \in R} |U_i\rangle\rangle\langle\langle U_i|_{01} \otimes |U_i^\dagger\rangle\rangle\langle\langle U_i^\dagger|_{23}$

Application to neutralize divided Hamiltonian dynamics

- Basis randomization comb

d=2 $\{U_k\} = \{I, X, Y, Z\}$



Hamiltonian dynamics $U(t) = e^{-iHt}$

Infinitesimal Hamiltonian dynamics $\delta U = e^{-iH\delta t} = I - iH\delta t + O(\delta t^2)$ δU

The Choi operator of the output map for **infinitesimal Hamiltonian dynamics** for the Pauli set

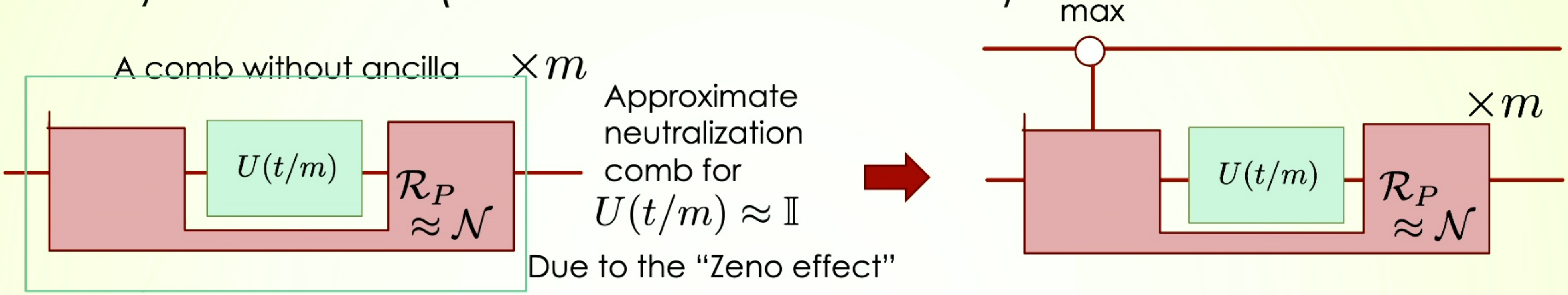
$$J_{\langle \delta U \rangle_{R_P}} = |I\rangle\rangle\langle\langle I| + \sum_k (-iU_k^\dagger H U_k \delta t)|I\rangle\rangle\langle\langle I| + \sum_k |I\rangle\rangle\langle\langle I|(-iU_k^\dagger H U_k \delta t)^\dagger + O(\delta t^2)$$

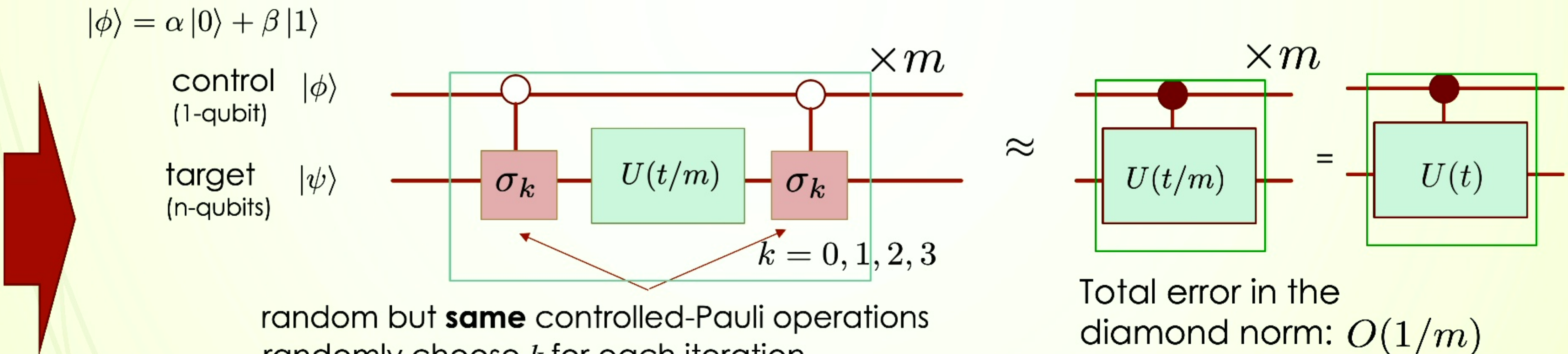
Instead of taking infinitesimal, divide the Hamiltonian dynamics into *m* parts and neutralize each

$U(\frac{t}{m}) = U^{-iHt/m}$

We can show that for $(U^{-iHt/m})^m$, the output map becomes $\mathrm{id} + O(1/m)$ due to the "Zeno effect"

Approximate controllization algorithm for Hamiltonian dynamics $U(t) = e^{-iHt}$ by a controlled-(basis randomization-comb)





random but **same** controlled-Pauli operations
randomly choose k for each iteration

Total error in the diamond norm: $O(1/m)$

An approximate controllization algorithm **without extra memory space**

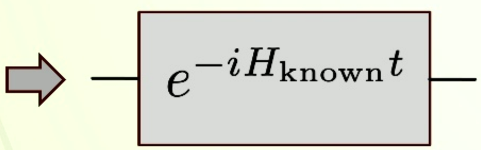
Higher-order transformations of Hamiltonian dynamics

Applying quantum combs to the **sliced Hamiltonian dynamics**, the dynamics of <u>what kind of transformation f</u> of Hamiltonian can be simulated?

Standard
Hamiltonian simulation:

Simulating dynamics of known Hamiltonian

$$H_{\text{known}} = \begin{pmatrix} * & \cdots & * \\ \vdots & \ddots & \vdots \\ * & \cdots & * \end{pmatrix}$$

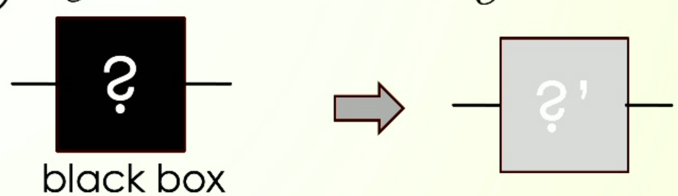


E.g. qDRIFT, QSVT-based

Higher-order
Hamiltonian simulation:

Simulating dynamics of a function of unknown Hamiltonian using its dynamics

① e^{-iHt}



② Function f
$H \mapsto f(H)$

[1] E. Campbell, PRL **123**, 070503 (2019). [2] Low, Guang Hao, and Isaac L. Chuang, Quantum **3**, 163 (2019).

Hermitian-preserving

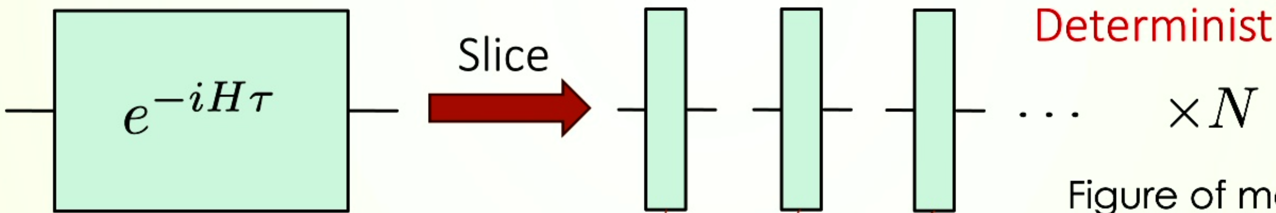
We found an algorithm to implement any linear map f

such that $f(I) \propto I$

Higher-order transformation $e^{-iHt} \mapsto e^{-if(H)t}$

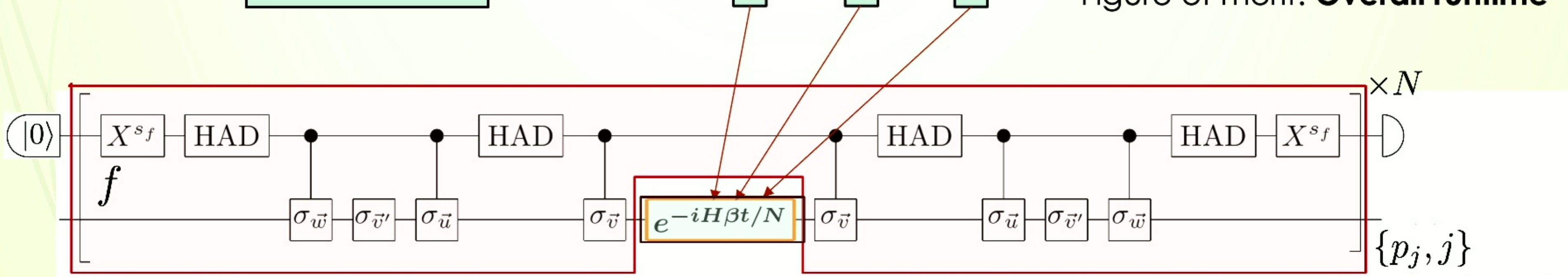
by a linear map f of unknown Hamiltonian using its sliced dynamics

E.g. negative time-evolution (simulating inverse unitary) e^{-iHt} ➡ e^{+iHt}



Deterministic & Approximate

Figure of merit: **Overall runtime**



Pauli transfer matrix (PTM) $\gamma_{\vec{w},\vec{u}}$ to represent the map f

n-qubit Hamiltonian system

Hermitian preserving linear map f

$$\text{s.t. } f(\sigma_{\vec{u}}) = \sum_{\vec{w}} \gamma_{\vec{w},\vec{u}} \sigma_{\vec{w}} \Leftrightarrow$$

$$H = \sum_{\vec{u}} c_{\vec{u}} \sigma_{\vec{u}} \Leftrightarrow \begin{pmatrix} 0 \\ c_{(0,\ldots,1)} \\ \vdots \\ c_{(3,\ldots,3)} \end{pmatrix} \in \mathbb{R}^{4^n} \Rightarrow \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \gamma_{(0,\ldots,1),(0,\ldots,1)} & \cdots & \gamma_{(0,\ldots,1),(3,\ldots,3)} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \gamma_{(3,\ldots,3),(0,\ldots,1)} & \cdots & \gamma_{(3,\ldots,3),(3,\ldots,3)} \end{pmatrix}$$

PTM of “physically realizable linear map”

$$\sigma_{\vec{v}} := \sigma_{v_1} \otimes \cdots \otimes \sigma_{v_n}$$

$$\beta := 2 \sum_{\vec{w},\vec{u}} |\gamma_{\vec{w},\vec{u}}|$$

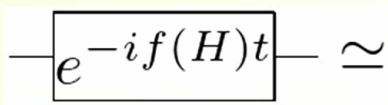
$$p_j := \frac{2|\gamma_{\vec{w},\vec{u}}|}{16^n \beta} \propto |\gamma_{\vec{w},\vec{u}}|$$

$$j = (\vec{u}, \vec{u}', \vec{v}, \vec{w})$$

$f(I) \propto I$

The algorithm

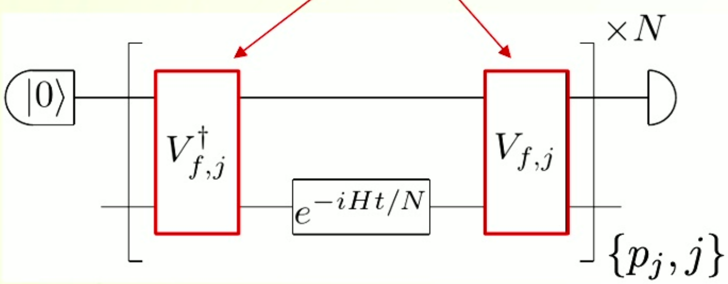
Pauli transfer matrix $\gamma_{\vec{w},\vec{u}}$ encode info of the map f



$\simeq$

$$\begin{pmatrix} f(H) & 0 \\ 0 & -f(H) \end{pmatrix} = \sum_j p_j V_{f,j} \begin{pmatrix} H & 0 \\ 0 & H \end{pmatrix} V_{f,j}^\dagger$$

Correlated randomness



$$e^{-iUHU^\dagger t} = U e^{-iHt} U^\dagger$$

$$p_j := \frac{2|\gamma_{\vec{w},\vec{u}}|}{16^n \beta} \propto |\gamma_{\vec{w},\vec{u}}|$$

$$j = (\vec{u}, \vec{u}', \vec{v}, \vec{w})$$

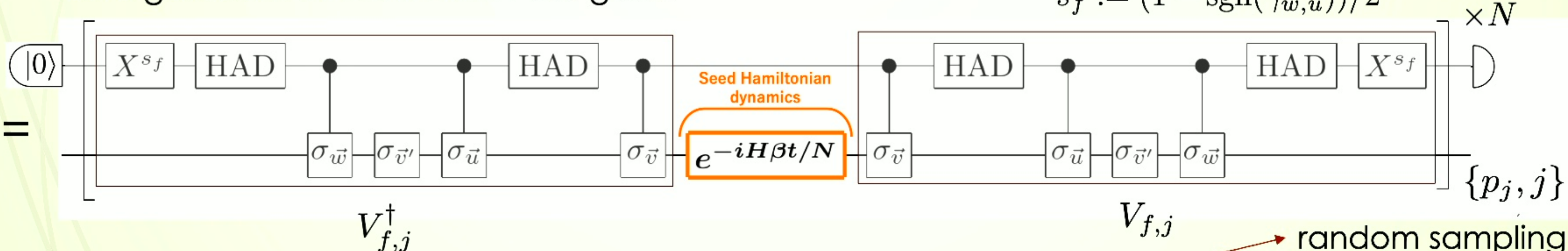
Runtime: $O(\beta^2 t^2 n/\epsilon)$

$\beta := 2\sum_{\vec{w},\vec{u}} |\gamma_{\vec{w},\vec{u}}|$ $\quad \epsilon$: allowed error variance 4ϵ

Extra gates are all controlled-Pauli gates

$$s_f := (1 - \mathrm{sgn}(\gamma_{\vec{w},\vec{u}}))/2$$

=

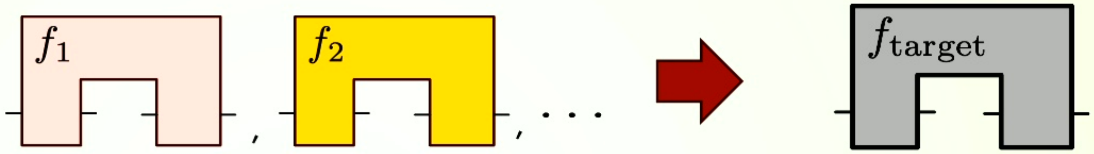


random sampling

similarly to Q-Drift quantum simulation
E. Campbell, PRL 123, 070503 (2019).

Construction by Quantum functional programming

Quantum functional programming:



E.g. composition of higher-order transformations

Our algorithm: construct a subroutine by a composition of seven functions

$$(\text{Subroutine function}) = g^{(7)}_{f,\vec{w},\vec{u}} \circ g^{(6)} \circ g^{(5)}_{\vec{w}} \circ g^{(4)} \circ g^{(3)}_{\vec{u}} \circ g^{(2)} \circ g^{(1)}$$

$V_{f,j}$ is naturally derived from this construction

Application 1: Negative time-evolution

Linear map: $f(H) = -H$

$$e^{-iHt} \mapsto e^{+iHt}$$

$$\gamma^{\text{neg}}_{\vec{w},\vec{u}} := \begin{cases} -\delta_{\vec{w},\vec{u}} & (\vec{u} \in J) \\ 0 & (\text{otherwise}) \end{cases}$$

support of H

- Applicable to Block encoding of unknown Hamiltonian

  *S. Lloyd et al. Hamiltonian singular value transformation and inverse block encoding, arXiv: 2104.01410 (2021)

Application 2: time-reversal evolution

Linear map: $f(H) = H^T$

$$e^{-iHt} \mapsto e^{-iH^T t} b = e^{-iH^* t}$$

$$\gamma^{\text{rev}}_{\vec{w},\vec{u}} := \begin{cases} (-1)^{s_{\vec{w}}} \delta_{\vec{w},\vec{u}} & (\vec{u} \in J) \\ 0 & (\text{otherwise}) \end{cases}$$

Both algorithms:

- Runtime is $O(|J|^2 t^2 n/\epsilon)$, which is exponential in n in general, but when H has a sparse support, it can be polynomial, t^2 slowing down

Application 3: Hamiltonian single parameter learning

filter out all irrelevant parameters

Linear map: $f_{\vec{v}}(H) = c_{\vec{v}} Y \otimes I \otimes \cdots \otimes I$ $\quad e^{-iHt} \mapsto e^{-ic_{\vec{v}}Yt} \otimes I \otimes \cdots \otimes I$

$H =: \sum c_{\vec{u}} \sigma_{\vec{u}}$

Rotation axis (arbitrary)

Measure by QPE

ϵ: precision, δ: failure probability

- Total evolution time: $O(\log(\delta)/\epsilon)$ (Heisenberg limit)
- Runtime: $O(\|H\|_{\mathrm{op}}^2 n/s^2)$ ($\mathrm{poly}(n)$ for sparse H) $\quad s$: Allowed RMS of error

c.f. Heisenberg limit for restricted situation [4], Runtime of full tomography of e^{-iHt} [3]:
$O(\mathrm{poly}(\exp(n), 1/s))$

[3] S. Kimmel et al. PRA **92**, 062315 (2015), [4] H. Y. Huang, et al., Phys. Rev. Lett. 130, 200403 (2020).

Summary

Higher-order quantum transformations: a new frontier of quantum information

- Implementation algorithms of higher-order quantum operations of **divisible unitaries** (especially **unknown Hamiltonian dynamics**) are presented

1. Universal **neutralization** and **controllization** algorithm of divisible unitaries and Hamiltonian dynamics
   Q. Dong, S. Nakayama, A. Soeda and M. Murao, arXiv:1911.01645v3 (some updates in 2021)

2. **Hermitian preserving Linear transformations** of Hamiltonian dynamics
   T. Odake, HléKristjánsson Akihito Soeda Mio Murao, arXiv:2303.09788

- New ideas are developed:

1. Make use of multiply correlated random unitary operations
2. Quantum algorithm design by the functional programming manner

Thank you for your attention!