Title: Branes & Strings, Freedom & Safety and a bit of Cosmology -- Some current problems in Quantum Gravity

Speakers: Kellogg Stelle

Collection: Puzzles in the Quantum Gravity Landscape: viewpoints from different approaches

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Abstract: What relations are there between the various ways of wrangling with quantum gravity? String theory is now much more than just a theory of strings -- branes and braneworlds abound. Some kind of effective theory for the familiar world needs to emerge. Are there ways that one could glimpse underlying structure from aspects of an effective theory? That happens for pions -- is there anything like that for gravity? Effective theories also involve higher derivatives, and those can summon up spirits (i.e. ghosts) from the vasty deep. Do asymptotic freedom or asymptotic safety give ways to exorcise them? And what might the effective theory tell us about the earliest times?

Branes, Safety and some Cosmology

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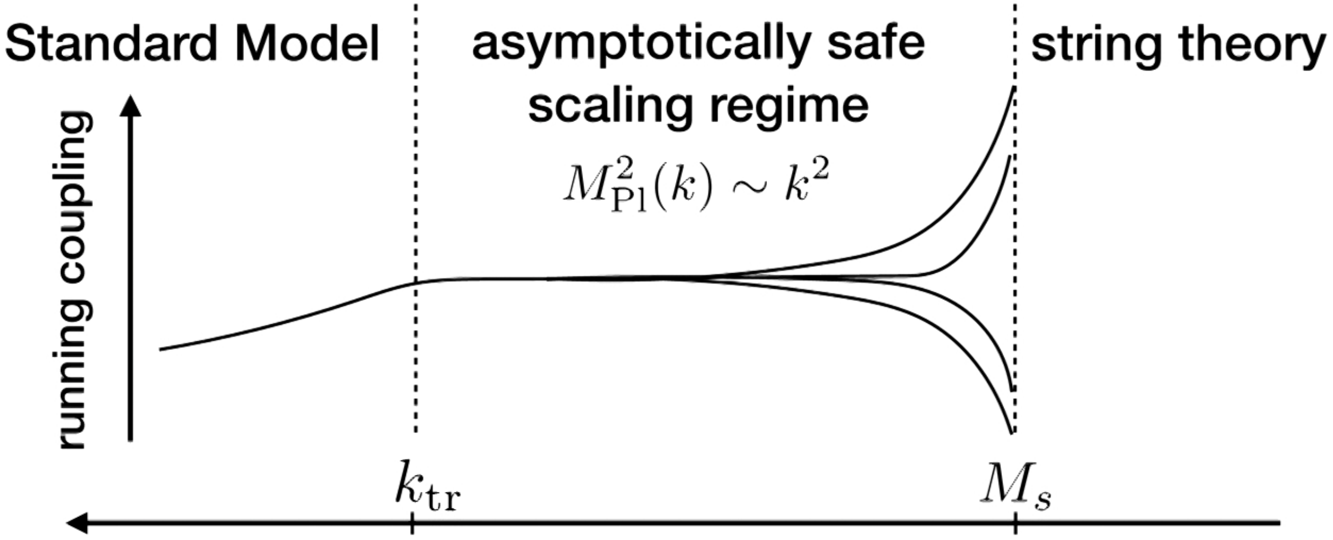
Puzzles in the Quantum Gravity Landscape Conference
Perimeter Institute
October 27, 2023

An overall perspective from Astrid



(de Alwis, Eichhorn, Held, Pawlowski, Schiffer & Versteegen, 1907.07894)

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Some Themes

1. Could we live on a brane?
2. Safe Gravity, Anisotropies and Inhomogeneity
3. No Boundary Proposal and Complex Metrics
4. Cosmology as a Filter on the Landscape

Could we live on a brane?

- String theory invites a perspective of gravity confined in a subspace of some higher D dimensional parent.
- A standard picture of how an effective 4d theory can emerge involves compactification of the (D-d) extra dimensions.
- But there could be other possibilities, suggested by the brane solutions of supergravity effective field theory.

Gravity Localisation

- The transverse-space structure of fluctuated brane solutions follows from a second-order differential equation. Consistent embeddings correspond to one type of boundary condition (essentially Dirichlet), but another type can be possible, with Robin boundary conditions.
- The consistent embeddings are basically smeared throughout the transverse dimensions, but in some cases one can obtain a different genuine concentration / localisation of gravity near the brane worldvolume.
- One example of this happens where the vacuum brane occurs in the D=10 embedding of the 1984 Salam-Sezgin theory.

Salam-Sezgin theory and its vacuum in D=10

D=6 SS

$$\mathcal{L}_{SS} = \frac{1}{2}R - \frac{1}{4g^2}e^{\bar\phi}F_{\mu\nu}F^{\mu\nu} - \frac{1}{6}e^{-2\bar\phi}G_{\mu\nu\rho}G^{\mu\nu\rho} - \frac{1}{2}\partial_\mu\bar\phi\partial^\mu\bar\phi - g^2e^{-\bar\phi}$$

$$G_{\mu\nu\rho} = 3\partial_{[\mu}B_{\nu\rho]} + 3F_{[\mu\nu}A_{\rho]}$$

Vacuum solution after embedding into D=10 Type I:

$$d\hat{s}^2_{10} = H_{SS}^{-\frac{1}{4}}\left(dx^\mu dx_\mu + dy^2 + \frac{1}{4g^2}\left[d\psi + \text{sech}\,2\rho\,(d\chi + \cos\theta\,d\varphi)\right]^2\right) + H_{SS}^{\frac{3}{4}}\,d\bar{s}_4^2$$

$$e^{\hat\phi} = H_{SS}^{\frac{1}{2}}\,, \quad \hat{A}_2 = \frac{1}{4g^2}\left[d\chi + \text{sech}\,2\rho\,d\psi\right]\wedge(d\chi + \cos\theta\,d\varphi)$$

Cvetič, Gibbons & Pope, Nucl. Phys. B677 (2004) 164; Crampton, Pope & KSS, 1408.7072

where $H_{SS} = \text{sech}\,2\rho$ and

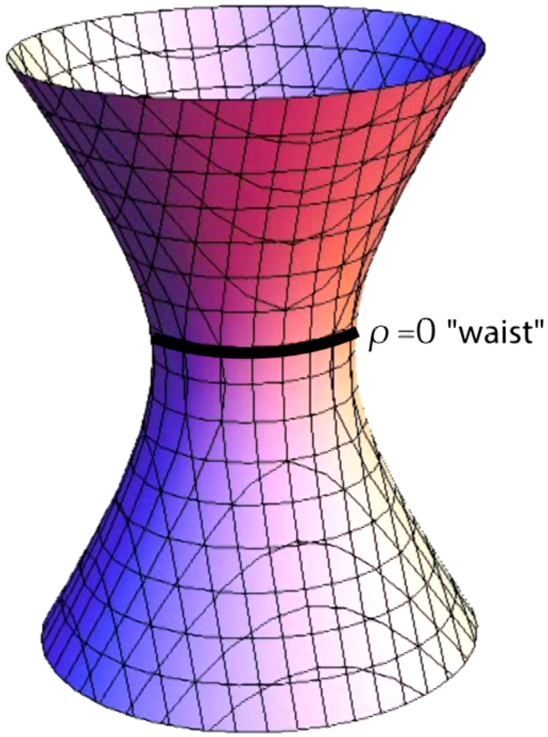
$$d\bar{s}_4^2 = \Big(\cosh 2\rho d\rho^2 + \frac{1}{4}\cosh 2\rho(d\theta^2 + \sin\theta d\varphi^2) + \frac{1}{4}\sinh 2\rho\tanh 2\rho(d\chi + \cos\theta d\varphi)^2\Big)$$

} 4d Eguchi-Hanson metric

The x^μ are the 4d worldvolume coordinates, which in the above "vacuum" state is flat; y is an S^1 circle coordinate.

Taking this solution as a brane vacuum, one can embed an *N=2, d=4* supergravity on the $\rho = 0$ world volume. But there is another possibility.

- The SS vacuum solution has a “hyperbolic” noncompact transverse space structure



- Instead of making a consistent embedding of *N=2, d=4* supergravity, which amounts to smearing the *d=4* fields up through the transverse space, one can use separation of variables $h_{\mu\nu}(x,\rho) = h_{\mu\nu}(x)\xi(\rho)$lial" ρ variable in the noncompact transverse space.

- The transverse Sturm-Liouville system turns out to have a Pöschl-Teller integrable structure which allows the transverse wavefunction spectrum to be solved explicitly. The spectrum contains a single zero mode followed by a gap and then a continuous spectrum above the gap edge.

- The zero mode $\xi(\rho) = \frac{2\sqrt{3}}{\pi}\log(\tanh\rho)$ is *normalisable.* The corresponding $h_{\mu\nu}(x)$ describe massless gravity on / near the $\rho = 0$ worldvolume. (Crampton, Pope & KSS, 1408.7072)

- One obtains a finite Newton constant

$$G_4 = \frac{\kappa_4^2}{32\pi} = \frac{3888\,\zeta(3)^2\,G_{10}\,g^5}{\pi^8 \ell_y}$$

- For the transverse Sturm-Liouville prob... is in this case is of Robin structure (mixed Dirichlet-Neumann):

$$\lim_{\rho\to 0^+}\Big(\sinh 2\rho\,\log\tanh\rho\,\xi'_{(\lambda)}(\rho) - 2\xi_{(\lambda)}(\rho)\Big) = 0$$

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- For the transverse Sturm-Liouville problem, the boundary condition is in this case is of Robin structure (mixed Dirichlet-Neumann):
$$\lim_{\rho\to 0^+}\Big(\sinh 2\rho\,\log\tanh\rho\,\xi'_{(\lambda)}(\rho) - 2\xi_{(\lambda)}(\rho)\Big) = 0$$

- The resulting dynamics does not correspond to a consistent embedding smeared through the transverse space, but instead concentrates gravity and the other fields in the region near the worldvolume.
- Analogous d=4 gravity-concentrating systems exist in some other cases (using analysis of singular Sturm-Liouville systems originating with H. Weyl, then A. Zettl). The zero modes are normalisable, but there is no mass gap in these other cases.
  - Randall-Sundrum II (cf also Lisa's talk)
  - D3-branes on a resolved conifold over $S^5/\mathbb{Z}_3$
  - D3-branes on resolved conifolds over $Y^{p,q}$
  - D3-branes on a resolved cone over $T^{1,1}/\mathbb{Z}_2$

  Leung & KSS
- Note: the Salam-Sezgin geometry can also be related to the SLED program (Supersymmetry in Large Extra Dimensions)

(Aghababaie,Burgess,Parameswaran&Quevedo, hep-th.0304256)

Safe Gravity and Weyl curvature

(J-L. Lehners & K.S.S., 1909.01169)

- Niedermaier's approach to asymptotic safety was based on renormalizable gravity M. Niedermaier, PRL 103, 101303 (2009) & Nucl. Phys. B 833 (2010) 226

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{\kappa^2} R - \Lambda - \frac{1}{2\sigma} C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \frac{\omega}{3\sigma} R^2 \right]$$

Energy scale

$$\frac{1}{\kappa^2} = \frac{\mu^2}{g_N} \qquad \Lambda = \lambda \mu^4$$

dimensionless

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About that spin-2 ghost

- Thanks to Bob Holdom and John Donoghue for great discussions on why one shouldn't devote too much time to it!

Asymptotic Safety

- Couplings run:

$$\mu\frac{d}{d\mu}g_N = f_g(g_N, \lambda, \sigma, \omega)\,,$$

$$\mu\frac{d}{d\mu}\lambda = f_\lambda(g_N, \lambda, \sigma, \omega)\,,$$

$$\mu\frac{d}{d\mu}\sigma = -\frac{133}{160\pi^2}\sigma^2\,,$$

$$\mu\frac{d}{d\mu}\omega = -\frac{25+1098\omega+200\omega^2}{960\pi^2}\sigma$$

Note that John Donoghue has recent questions about the appropriateness of these standard $\mu d/d\mu$ beta functions

- Quadratic curvature terms are asymptotically free $\sigma \sim 1/\ln\mu \to 0$

Fradkin & Tseytlin 1981, 1982;
Avramidy & Barvinsky 1985

Asymptotic Safety

- Couplings run:

$$\mu\frac{d}{d\mu}g_N = f_g(g_N,\lambda,\sigma,\omega)\,,$$
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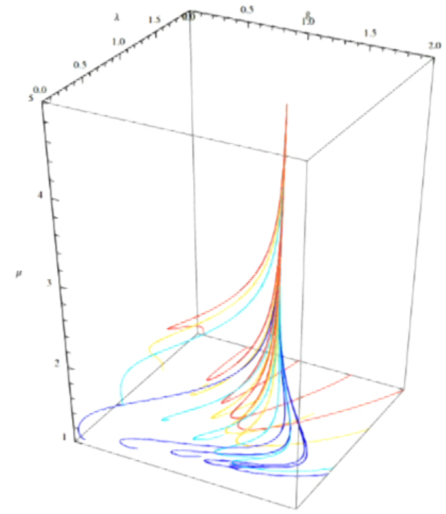


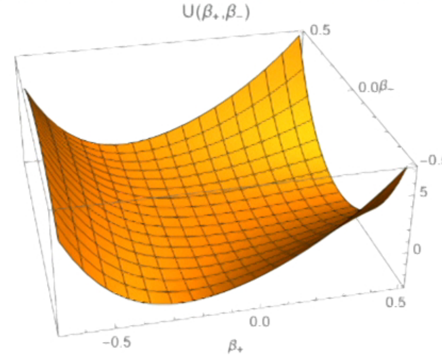
- Quadratic terms are asymptotically free
- Evidence for a stable non-trivial fixed point

$$\sigma^\star = 0, \omega^\star \approx -0.0228$$

Codello & Percacci hep-th/0607128; Codello, Percacci & Rahmede 0805.2909; D. Litim, to appear

Anisotropies: Bianchi IX metric

- Useful metric to analyse:



$$ds^2_{IX} = -dt^2 + \sum_m \left(\frac{l_m}{2}\right)^2 \sigma_m^2$$

$$l_1 = a\, e^{\frac{1}{2}\left(\beta_+ + \sqrt{3}\beta_-\right)}, \quad l_2 = a\, e^{\frac{1}{2}\left(\beta_+ - \sqrt{3}\beta_-\right)}, \quad l_3 = a\, e^{-\beta_+}$$

- Leads to the classical action

$$S_{EH} = \int d^4x \sqrt{-g}\frac{R}{2} = 2\pi^2 \int dt a \left(-3\dot{a}^2 + \frac{3}{4}a^2(\dot{\beta}_+^2 + \dot{\beta}_-^2) - U(\beta_+, \beta_-)\right)$$

$$U(\beta_+, \beta_-) = -2\left(e^{2\beta_+} + e^{-\beta_+ - \sqrt{3}\beta_-} + e^{-\beta_+ + \sqrt{3}\beta_-}\right) + \left(e^{-4\beta_+} + e^{2\beta_+ - 2\sqrt{3}\beta_-} + e^{2\beta_+ + 2\sqrt{3}\beta_-}\right)$$

Anisotropy potential

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Bianchi IX: quadratic curvature terms

- Quadratic curvature terms

$$\int d^4x\sqrt{-g}R^2 = 2\pi^2\int dta^3\left[6\frac{\ddot{a}}{a}+6\frac{\dot{a}^2}{a^2}+\frac{3}{2}a^2(\dot{\beta}_+^2+\dot{\beta}_-^2)-U(\beta_+,\beta_-)\right]^2$$

$$-\int d^4x\sqrt{-g}\,C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}$$

$$=2\pi^2\int dt\Big\{3a^3\left[\left(\frac{\ddot{a}}{a}+H^2\right)(\dot{\beta}_-^2+\dot{\beta}_+^2)-\ddot{\beta}_-^2-\ddot{\beta}_+^2-(\dot{\beta}_-^2+\dot{\beta}_+^2)^2\right]$$

$$+4a\left[-(\ddot{\beta}_-+3H\dot{\beta}_-)U_{,\beta_-}-(\ddot{\beta}_++3H\dot{\beta}_+)U_{,\beta_+}-\left(2\frac{\ddot{a}}{a}+\dot{\beta}_-^2+\dot{\beta}_+^2\right)U\right]$$

$$+\frac{64}{3a}\Big(-e^{-8\beta_+}+e^{-5\beta_+-\sqrt{3}\beta_-}+e^{-5\beta_++\sqrt{3}\beta_-}-e^{-2\beta_+}+e^{\beta_+-3\sqrt{3}\beta_-}+e^{\beta_++3\sqrt{3}\beta_-}$$

$$-e^{\beta_+-\sqrt{3}\beta_-}-e^{\beta_++\sqrt{3}\beta_-}-e^{4\beta_+-4\sqrt{3}\beta_-}-e^{4\beta_++4\sqrt{3}\beta_-}+e^{4\beta_+-2\sqrt{3}\beta_-}+e^{4\beta_++2\sqrt{3}\beta_-}\Big)\Big\}$$

- Quadratic curvature terms, however, lead to the following scalings near t=0:

$$3s-3\,,\quad s-1-p-m\,,\quad s-1-p+m\,,\quad s-1+2p+2m\,,\quad s-1+2p-2m$$
$$s-1+2p\,,\quad s-1-4p\,,\quad 1-s-8p\,,\quad 1-s-5p-m\,,\quad 1-s-5p+m\,,\quad 1-s-2p$$
$$1-s+p+3m\,,\quad 1-s+p-3m\,,\quad 1-s+p+m\,,\quad 1-s+p-m\,,$$
$$1-s+4p+4m\,,\quad 1-s+4p-4m\,,\quad 1-s+4p+2m\,,\quad 1-s+4p-2m$$

- These lead to *mutually exclusive conditions*

$$s>1\,,\quad -\frac{1}{2}(s-1)<p<\frac{1}{4}(s-1)\,,\quad -\frac{1}{2}(1-s)<p<\frac{1}{8}(1-s)$$
$$p+m>s-1\,,\quad p+m<s-1\,,\quad p-m<s-1\,,\quad p-m>s-1$$

- Hence the quadratic gravity terms filter out universes that start out anisotropically.

Similarly for Inhomogeneities

- Lemaître-Tolman-Bondi metric:

$$ds^2 = -dt^2 + \frac{A'^2}{F^2}dr^2 + A^2(d\theta^2 + \sin^2\theta d\phi^2)$$

- The scale factor A now has dependence A(t,r) but F(r) only

$$\int d^4x\sqrt{-g}\,\frac{R}{2} = 2\pi\int dt\,dr\,\frac{A'}{F}\left(1 - F^2 + \dot{A}^2 - 2\frac{AFF'}{A'} + 2\frac{A\dot{A}\dot{A}'}{A'} + 2A\ddot{A} + \frac{A^2}{A'}\ddot{A}'\right)$$

$$\sim t^{s+1} \qquad\qquad \sim t^{3s-1}$$

$$\int d^4x\sqrt{-g}\,R^2 = 8\pi\int dt\,dr\,\frac{A'}{A^2F}\left(1 - F^2 + \dot{A}^2 - 2\frac{AFF'}{A'} + 2\frac{A\dot{A}\dot{A}'}{A'} + 2A\ddot{A} + \frac{A^2}{A'}\ddot{A}'\right)^2$$

$$\sim t^{-s+1} \qquad\qquad \sim t^{3s-3}$$

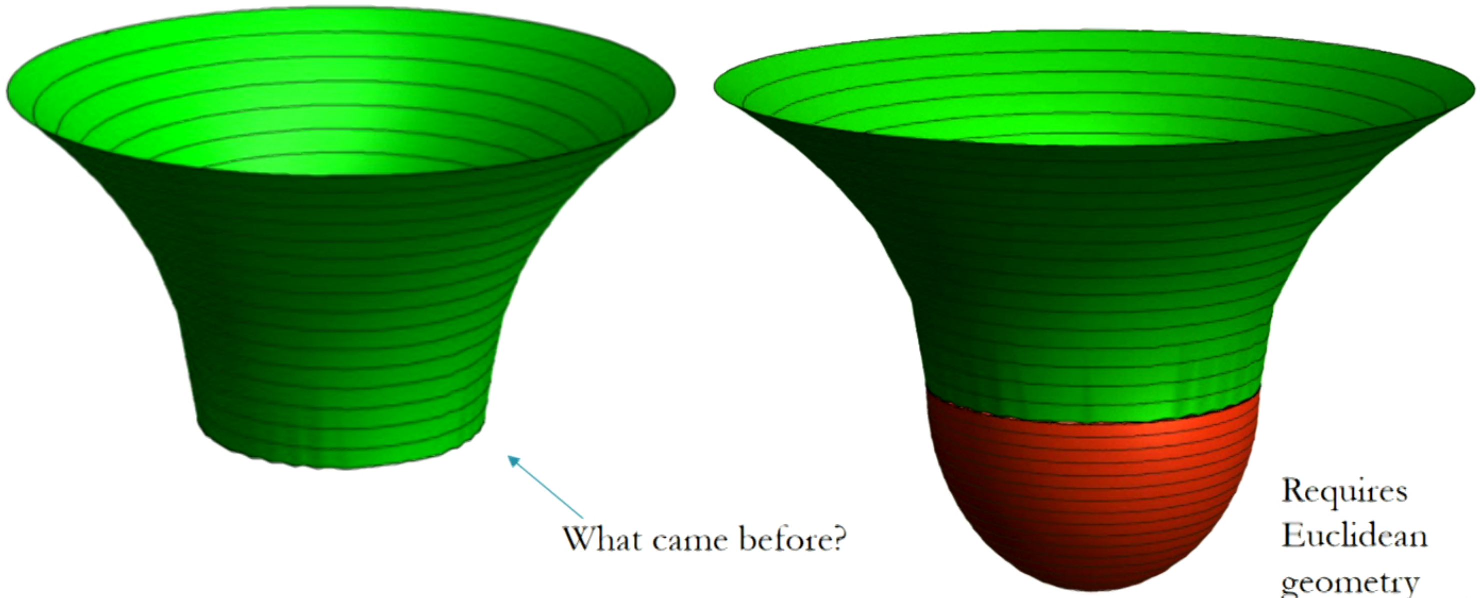
- Giving conflicting requirements: $s < 1$ and $s > 1$
- In the absence of inhomogeneities, however, one finds accelerated expansion $A \sim t^s$ with $s > 1$.

The No-Boundary Proposal and Complex Metrics

(cf also Bianca's talk)

Feldbrugge, Lehners & Turok, 1703.02076; Lehners, 2209.14669; Lehners & Quintin, 2309.03272

- If the geometry is smoothly rounded off, then
  - An infinite regression is avoided
  - The initial singularity is avoided
  - We may not need to specify any boundary conditions



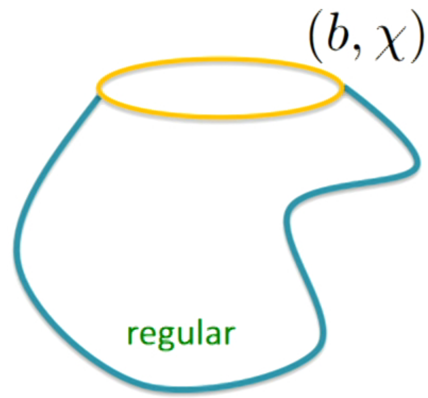
No Boundary Expectations

- The ideas were spelled out in detail by Hartle & Hawking (Phys.Rev.D28 (1983) 2960-2975)

- Initial proposal: the wave function of the universe should be calculated via a path integral defined by a sum over compact, regular, Euclidean metrics

$$\Psi(final) = \int_{no-boundary}^{final} e^{-S_E/\hbar}$$

$$\Psi(b,\chi) = \int_{\mathcal{C}} \mathcal{D}a\mathcal{D}\phi e^{-S_E(a,\phi)}$$
$$\approx e^{-S_{E,ext}(b,\chi)}$$

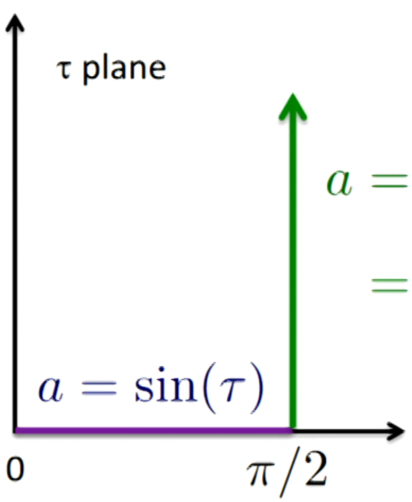


- Path integral is over all four-geometries that are regular in the past and that reach specified (real) values a=b, $\phi=\chi$
- Universe is *finite, non-singular and self-contained*
- Saddle point approximation when $S/\hbar \gg 1$: the dominant geometries are extrema of the action $\delta S = 0$
- With the required boundary conditions these are typically *complex* – these are called *"fuzzy" instantons*

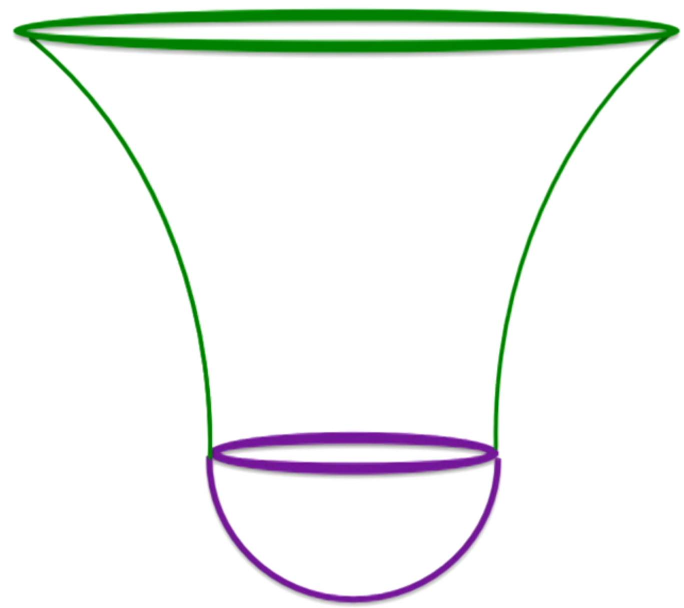
Hartle & Hawking, 0803.1663

Hawking's Prototype Instanton: complex de Sitter

- Here there is no scalar field, only a cosmological constant $\Lambda = 3\,H^2$
- Half of S^4 is glued to dS^4



$$a = \sin(\frac{\pi}{2} + it)$$
$$= \cosh t$$



Louko-Sorkin-Kontsevich-Segal-Witten criterion

- Proposal: a metric is *allowable* if a generic quantum field theory can be defined on it, in the sense that its path integral *converges* for all p-form actions:

  - Louko & Sorkin (gr-qc/9511023)
  - Kontsevich & Segal (2105.10161)
  - Witten (2111.06514)
  - JLL (2111.07816 & 2209.14669)
  - Jonas, Lehners & Quintin (2205.15332)

- Proposal: a metric is *allowable* if a generic quantum field theory can be defined on it, in the sense that its path integral converges for all p-form actions:

$$|e^{\frac{i}{\hbar}S}| < 1 \rightarrow Re\left[\sqrt{g}g^{i_1 j_1} \cdots g^{i_{p+1} j_{p+1}} F_{i_1 \cdots i_{p+1}} F_{j_1 \cdots j_{p+1}}\right] > 0$$

- Justification: only spin ≤ 1 fields (scalars and gauge fields) have local covariant stress-energy tensor [Weinberg-Witten theorem]
- Require real fields because we want to define a Hilbert space (want this to be defined locally, not via analytic continuation)
- Kontsevich&Segal provide arguments that QFT is well defined under these asssumptions and that they might be able to replace the standard axioms of QFT

Simple form for the metric

- Locally write the D-dim metric in diagonal form: $g_{ij} = \delta_{ij}\lambda_i$
- Then for example

$$\mathrm{Re}(\sqrt{g}) > 0 \rightarrow -\frac{\pi}{2} < \frac{1}{2}\Big(\Sigma_i Arg(\lambda_i)\Big) < \frac{\pi}{2}$$

- More generally, one requires that for any subset S of the set (1,2,…,D) one must have $\mathrm{Re}(\sqrt{g}\Pi_{i\in S}\lambda_i^{-1}) > 0$
- Putting all combinations for all p-forms together requires

$$\Sigma \equiv \Sigma_i |Arg(\lambda_i)| < \pi$$

Boundary of an allowable domain

- Real Lorentzian metrics have

$$Arg(\lambda_0) = \pm\pi,\ Arg(\lambda_i) = 0$$

- Hence they are on the boundary of the allowable domain
- One can regulate them, e.g. for FLRW:

$$ds^2 = -(1 \mp i\epsilon)dt^2 + a(t)^2 dx^2$$

- ε>0 corresponds to the standard iε prescription, propagation forwards in time by $e^{-iHt-\epsilon H}$
- Vice versa for ε<0
- But one cannot cross the ε=0 line

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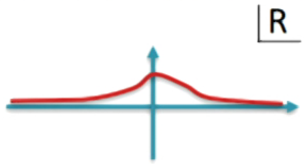
Eliminating pathological metrics

- LSKSW criterion eliminates zero-action wormholes

$$ds^2 = R'(u)^2 du^2 + R(u)^2 d\Omega_3^2$$



$$\Sigma = |Arg(R'(u)^2)| + 3|Arg(R(u)^2)|$$

when R crosses the imaginary axis $\Sigma \geq (D-1)\pi$

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But scalars can originate from the metric

- Dimensional reduction, require ϕ to be normalised

$$g_{MN}dx^M dx^N = e^{2a\phi} g_{\mu\nu} dx^\mu dx^\nu + e^{2b\phi} g_{ij} dx^i dx^j$$

$$a = -\frac{D-d}{d-2} b \qquad b = \sqrt{\frac{d-2}{(D-d)(D-2)}}$$

- If the 4-dimensional metric is Euclidean, then obtain

$$|\mathrm{Im}\,(\phi)| < \sqrt{\frac{D-2}{D-4}} \frac{\pi}{6\sqrt{2}} \approx \frac{\pi}{10}$$

- Note: imaginary part of ϕ (not argument) is bounded

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- A similar effect occurs for conformal transformations to Einstein frame
- No-boundary solutions require complex scalar fields in general:

$$\mathrm{Im}(\phi_{SP}) = -\frac{V_{,\phi}}{V}\frac{\pi}{2}$$

- This is allowed:
  - if the scalars originate from the metric
  - if the potential is flat enough

$$\frac{|V_{,\phi}|}{V} \lessapprox \frac{1}{5}$$

- This constraint arises already from looking only at South Pole values - *stronger conditions are expected* from analysing the full instanton

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D=8 Starobinsky model

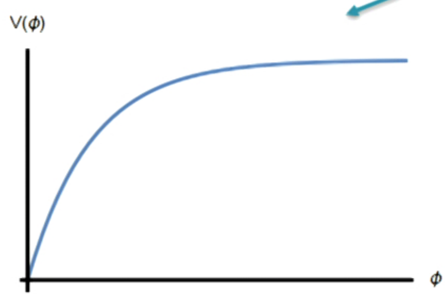
Lehners, Leung & KSS, 2209.08960

- R^4 corrections and 4-form flux are present in 10-dimensional supergravity
- Consider a toy model in 8 dimensions, compactified on S^4

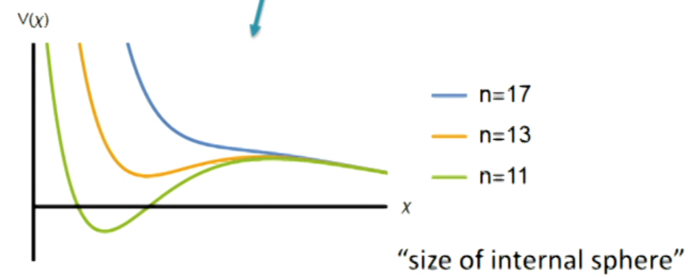
$$S = \frac{1}{2}\int d^8x\sqrt{-\hat{g}}\left(\hat{R} + \alpha\hat{R}^4 - \frac{1}{2\cdot 4!}q^2F_{(4)}^2\right)$$

Ketov; Otero et al.

$$F_{(4)} = 2n_4\mathrm{vol}(S^4)$$



Inflationary potential for the scalaron ϕ



Small flux: negative minimum
Medium flux: positive, stable minimum
Large flux: positive but decompactifies

Rescaling and dimensional reduction

- 2 steps:
  - Rewrite as gravity + scalar in Einstein frame

$$\hat{g}_{\mu\nu} \equiv e^{\frac{2}{\sqrt{42}}\phi} g_{\mu\nu}$$

scalaron

$$e^{\sqrt{\frac{6}{7}}\phi} = 1 + 4\alpha \hat{R}^3$$

  - Dimensional reduction on 4-sphere

$$ds_8^2 = e^{-\frac{2}{\sqrt{3}}\chi} ds_4^2 + e^{\frac{1}{\sqrt{3}}\chi} d\Omega_4^2$$

size of 4-sphere

$$F_{(4)} = 2n_4 \mathrm{vol}(S^4)$$

quantised

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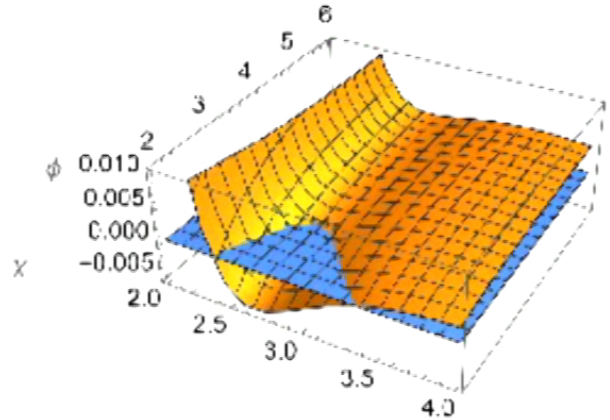
- This results in a theory of gravity coupled to 2 canonically normalised scalar fields with a potential:

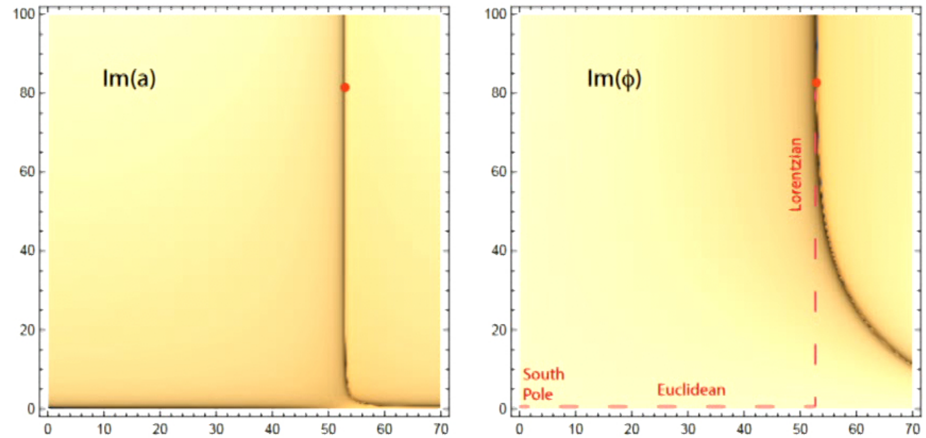


$$V(\phi,\chi) = \tilde{\alpha}\left(1 - e^{-\sqrt{\frac{6}{7}}\phi}\right)^{\frac{4}{3}} e^{-\frac{2}{\sqrt{3}}\chi} + n_4^2 e^{-2\sqrt{3}\chi} - 6e^{-\sqrt{3}\chi}$$

- The shape of the potential depends crucially on the amount of flux n_4

(cf Fernando's talk on the importance of fluxes for moduli stabilization)

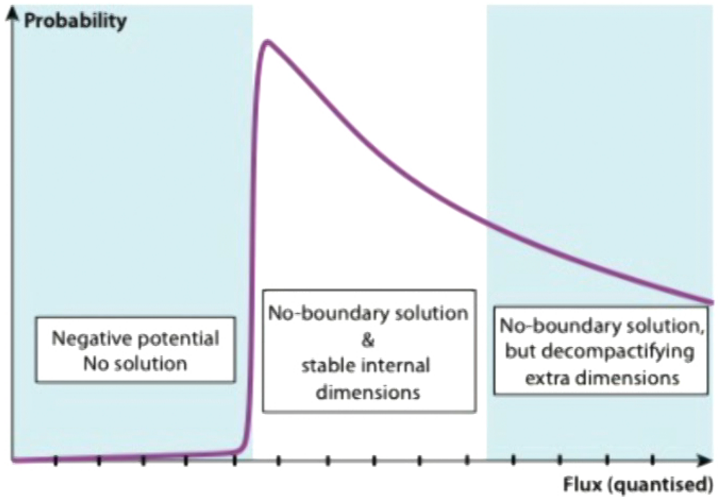
Evolution to real time and fields



χ constant on valley floor

Density plots of the imaginary values of the scale factor and the scalar field, in the complex time plane. The South Pole resides at the origin; the horizontal axis corresponds to Euclidean time, the vertical axis to Lorentzian time. Darker colors indicate smaller imaginary values, so the black lines show the locus of real field values. The dashed line in the right panel indicates the ``Hartle-Hawking'' contour. One can see that at late times, overlapping dark lines emerge in both plots, indicating that one approaches a real, classical solution of the equations of motion. This solution reaches the final values a_1=200, ϕ_1=6 at time τ = 53.185+83.538i, as marked by the red dot.

No-boundary as a filter



- No-boundary wave function selects a range of fluxes, and provides a probability distribution on solutions
- Cosmology can act as a filter on the landscape, significantly reducing the number of viable solutions

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