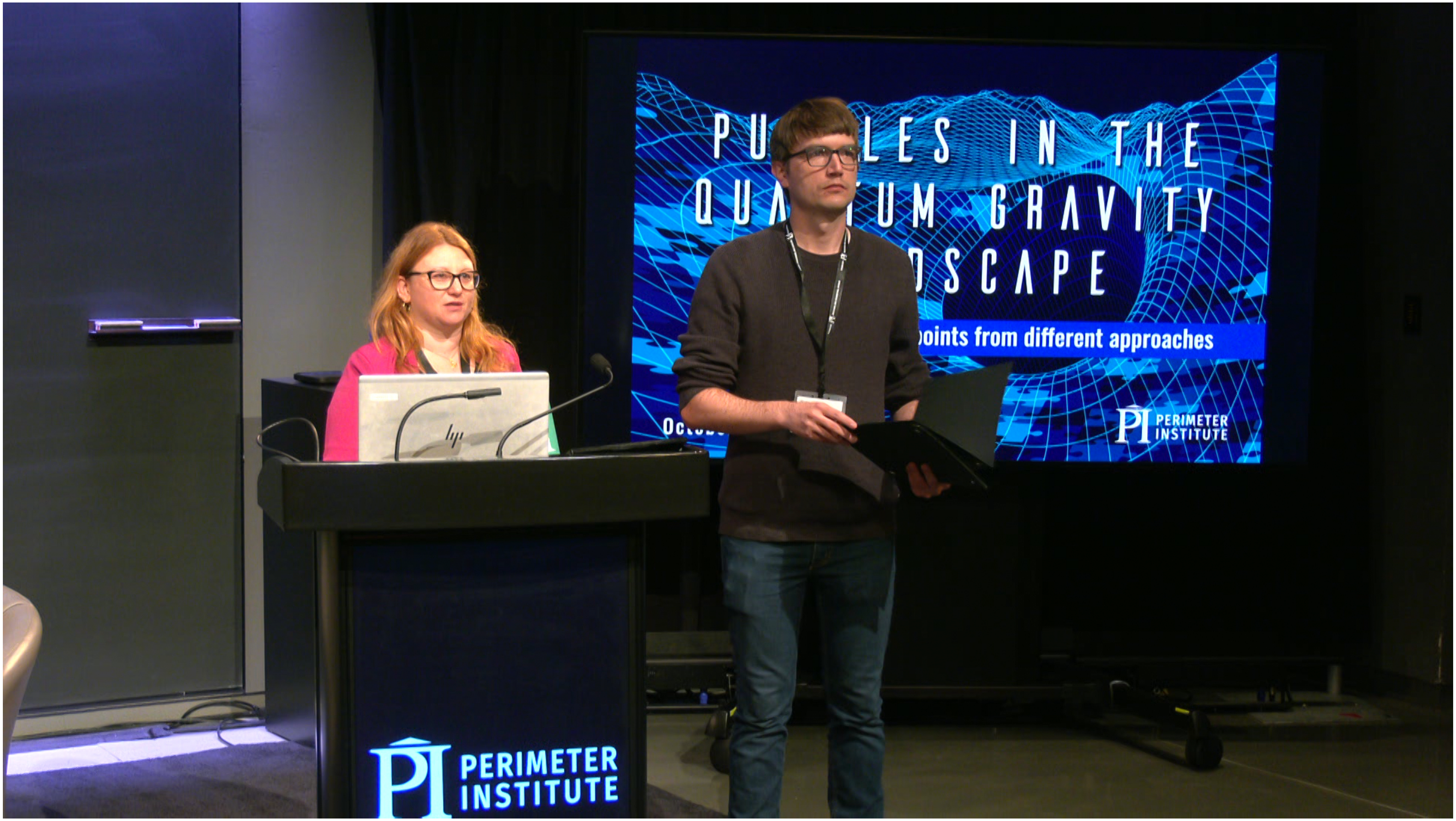
Title: Poster Prize Talk

Speakers:

Collection: Puzzles in the Quantum Gravity Landscape: viewpoints from different approaches

Date: October 26, 2023 - 1:10 PM

URL: https://pirsa.org/23100019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# Parameterizations of black-hole spacetimes beyond circularity

Class.Quant.Grav. 39 (2022) 13, 134002, arXiv:2203.00105v2

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October 26, 2023

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# Disclaimers

- None of the following black-hole (BH) spacetime metrics which follow arise from an action principle
  $\Longrightarrow$ Parameterized approach

- None of the following BH spacetime metrics are intrinsically quantum, just Quantum Gravity (QG) inspired
  $\Longrightarrow$ "beyond GR"

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  - How do they look like in terms of their spacetime symmetries?
- Focus on axisymmetric, stationary and asymptotically flat BH spacetimes
- Different approaches to answer those questions, among which the parameterized approach

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**Fundamental (top-down)**

- Assumes specific form of new-physics (e.g. new fields, higher curvature terms etc.)
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**Parameterized (bottom-up)**

- Parameterized deviations from GR BHs - could be constrained observationally
- Theory agnostic
- Example: Non-circular parameterizations of BHs

# The parameterized approach

- 3 different approaches to study BH spacetimes beyond GR:

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### Principled-parameterized

[Eichhorn, Held '21a,b]

- Parameterized deviations from GR motivated by new-physics guiding principles
- Can lead to regular BH spacetimes
- Example: Kerr metric with specific upgrade $M \to M(r, \chi)$

### Parameterized (bottom-up)

- Parameterized deviations from GR BHs - could be constrained observationally
- Theory agnostic
- Example: Non-circular parameterizations of BHs

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[Papapetrou '66], [Weyl 1917], [Lewis '32], [Konoplya, Rezzolla, Zhidenko '16]

$$\xi_1^{\mu} R_{\mu}^{\ [\nu} \xi_2^{\kappa} \xi_1^{\lambda]} = 0 \text{ everywhere},$$
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$\Longrightarrow$ Implies a spacetime isometry.
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- Constant of motion from rank-2 Killing tensor: $C = K_{\mu\nu} u^\mu u^\nu$

  [Benenti, Francaviglia '79], [Johannsen '13], [Vigeland, Yunes, Stein '11]

- In vacuum GR $R_{\mu\nu} = 0$, so circularity holds.
  But some non-vacuum GR & beyond GR spacetimes break circularity

  [Ioka, Sasaki '03, '04], [Birkl, Stergioulas, Muller '11], [Minamitsuji '20], [Anson, Babichev, Charmousis, Hassaine '20]

# Parameterizations and symmetries

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|---|---|---|---|
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| add: | circularity | 5 | 4 (LP form) |
| | | | |

- Most general circular metric (BL preferred): [Johannsen '13]

$$ds^2_{RZ} = -g_{tt}dt^2 - 2g_{t\phi_{\rm BL}}dtd\phi_{\rm BL} + g_{\phi_{\rm BL}\phi_{\rm BL}}d\phi^2_{\rm BL} + g_{rr}dr^2 + g_{\theta\theta}d\theta^2$$

  - 5 non-zero metric components & 5 free functions ✓

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| BHs: | axisymmetry + stationarity | $10 \to 6$ | at least 6 |
| add: | circularity | 5 | 4 (LP form) |
| | + hidden constant | 5 | 10 (of 1 coord.) |

- Most general circular metric with new Carter-like hidden constant of motion: [Benenti, Francaviglia '79]

$$g^{\mu\nu}\partial_\mu\partial_\nu = \frac{1}{S_{x_1} + S_{x_2}}\left[\left(G^{ij}_{x_1} + G^{ij}_{x_2}\right)\partial x_i \partial x_j + \Delta_{x_1}\partial x_1^2 + \Delta_{x_2}\partial x_2^2\right]$$

with $i, j$ indices related to Killing coordinates, and $1, 2$ to explicit coordinates

## $2^{\mathrm{nd}}$ key result: more general non-circular parameterization

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- Proposed non-circular parameterization of deviations from Kerr
- Written in preferred Horizon-Penetrating (HP) coordinates:

$$
\begin{aligned}
ds^2_{\rm HP} \quad = \quad & -\left(\frac{r^2-2Mr+a^2\chi^2}{r^2+a^2\chi^2}\right)(1+\Delta_{\rm HP,\,1}(r,\chi))\,du^2+2\,(1+\Delta_{\rm HP,\,2}(r,\chi))\,dudr \\
& -4\frac{Mar}{r^2+a^2\chi^2}(1-\chi^2)(1+\Delta_{\rm HP,\,3}(r,\chi))dud\phi-2a(1-\chi^2)(1+\Delta_{\rm HP,\,4}(r,\chi))drd\phi \\
& +\frac{r^2+a^2\chi^2}{1-\chi^2}(1+\Delta_{\rm HP,\,5}(r,\chi))d\chi^2 \\
& +\frac{1-\chi^2}{r^2+a^2\chi^2}\left((a^2+r^2)^2-a^2\left(r^2-2Mr+a^2\right)\left(1-\chi^2\right)\right)(1+\Delta_{\rm HP,\,6}(r,\chi))d\phi^2
\end{aligned}
$$

satisfying:

- Asymptotic flatness & correct Newtonian limit: $\Delta_{\rm HP,\,i}(r,\chi)\sim\mathcal{O}\left(\frac{1}{r^2}\right)$
- Flat Minkowski limit as $M\to 0$: only reached for $\Delta_{\rm HP,\,i}(r,\chi)\sim M$ & no extra quantum hair

# Relating non-circular and circular parameterizations

- Challenge: directly relating non-circular HP parameterization to general circular BL parameterization

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  $\Longrightarrow$ Use a counting argument as proof-of-principle:

$$
\text{14 free functions:} \quad \begin{array}{l} 6 \text{ from } \Delta_{\mathrm{HP,i}}, 1 \le i \le 6 \\ + \, 8 \text{ from coord. transfo. preserving } du = dt + \dots \\ \quad \& \; d\phi = d\phi_{\mathrm{BL}} + \dots \end{array}
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- \quad \text{9 constraints:} & \begin{array}{l} 5 \text{ from vanishing } g_{\mu\nu} \text{ in } ds^2_{RZ} \\ + \; 4 \text{ diff. constraints on coord. transfo. functions} \end{array} \\
\hline
\text{(at least)} & 5 \text{ free functions of } r, \chi \text{ remain}
\end{array}
$$

  $\Longrightarrow$ Matches most general circular parameterization in BL coordinates ✓
  1 function too many for LP form [Papapetrou '66], [Kundt, Trumper '66], [Wald '84]

# $3^{\text{rd}}$ key result: non-circular spacetimes seem to have specific image features

- Example included in non-circular HP parameterization: regular BH - obtained via principled-parameterized approach - with upgrade $M \to M(r, \chi)$

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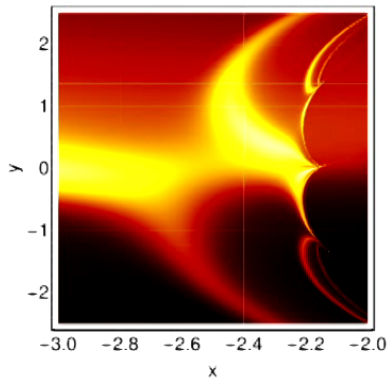
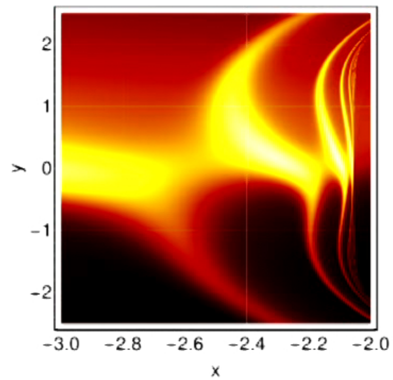
- Example included in non-circular HP parameterization: regular BH - obtained via principled-parameterized approach - with upgrade $M \to M(r,\chi)$

- Obtained by taking in non-circular parameterization $ds^2_{\mathrm{HP}}$
$\Delta_{\mathrm{HP},2/4/5} = 0$, $\Delta_{\mathrm{HP},1} = \frac{2r(M - M(r,\chi))}{r^2 + a^2\chi^2 - 2Mr}$, $\Delta_{\mathrm{HP},3} = \frac{M(r,\chi) - M}{M}$,
$\Delta_{\mathrm{HP},6} = -\frac{2a^2(M(r,\chi) - M)r(\chi^2 - 1)}{r^4 + a^4\chi^2 + a^2 r\left(2M + r(r - 2M)\chi^2\right)}$ [HD, Eichhorn, Held '22]




Non-circular $M(r,\chi)$

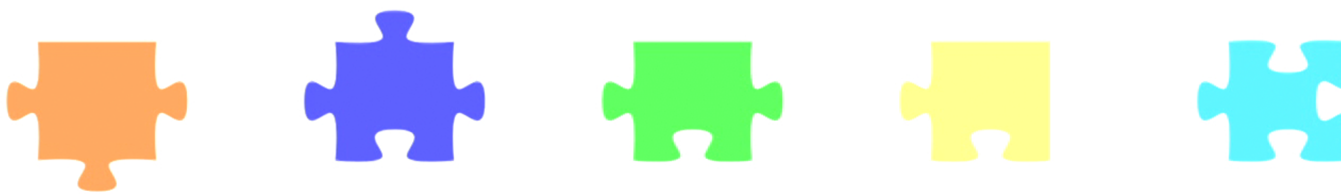



Circular $M(r)$

- Non-circular: 3 specific image features
  - $\Longrightarrow$ Cusps (in shadow boundary & photon rings)
  - $\Longrightarrow$ Dent at $y = 0$ axis
  - $\Longrightarrow$ Broken reflection symmetry about $y = 0$ axis

# Puzzling open questions

1. How to relate the proposed non-circular parameterization (in HP coord.) to the most general circular parameterization (in BL coord.)?



# Puzzling open questions

1. How to relate the proposed non-circular parameterization (in HP coord.) to the most general circular parameterization (in BL coord.)?
2. What is the minimal, general, non-circular parameterization of axisymmetric, stationary and asymptotically flat BH?
3. Can we establish a one-to-one map between non-circularity and the 3 specific image features seen in BH shadow images?
4. What is the impact of the non-constant angular velocity - of a non-circular spacetime - at the event horizon on BH thermodynamics?