Title: Balance Laws as Test of Gravity - VIRTUAL

Speakers: Lavinia Heisenberg

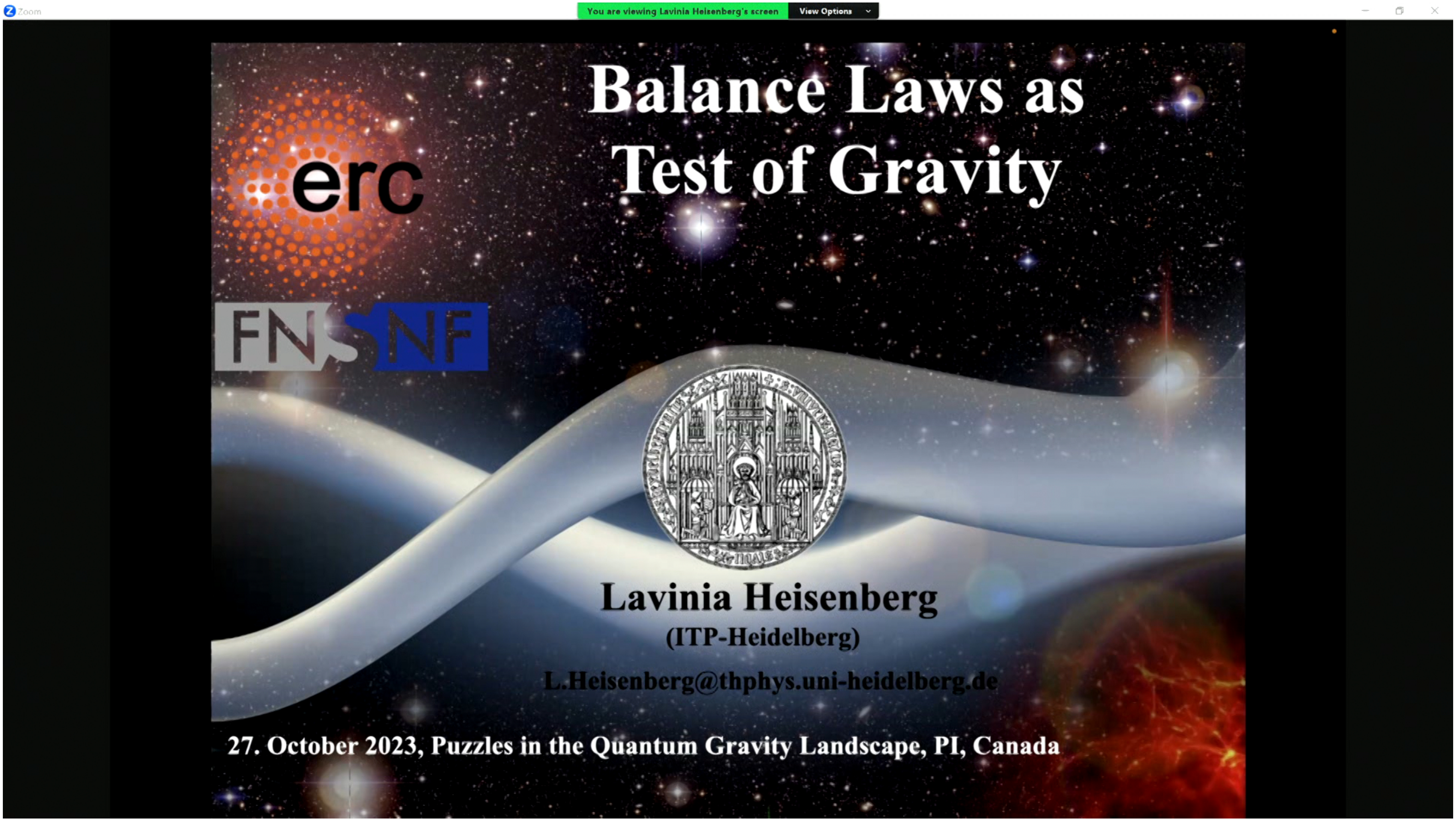
Collection: Puzzles in the Quantum Gravity Landscape: viewpoints from different approaches

Date: October 27, 2023 - 9:00 AM

URL: https://pirsa.org/23100017

Abstract: I will discuss how one can use balance laws in full non-linear general relativity in order to test waveform models (arXiv:2309.12505)

The presenter will be joining via Zoom for this talk.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Balance Laws as
Test of Gravity
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FNSNF
Lavinia Heisenberg
(ITP-Heidelberg)
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27. October 2023, Puzzles in the Quantum Gravity Landscape, PI, Canada

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Balance Laws as Test of Gravitational Waveforms

- A pedagogical introduction into this subject is given in arXiv:2309.12505

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Subject Areas:
Gravitational Wave Physics, Cosmology, Particle Physics

Balance Laws as Test of Gravitational Waveforms

Lavinia Heisenberg[1]

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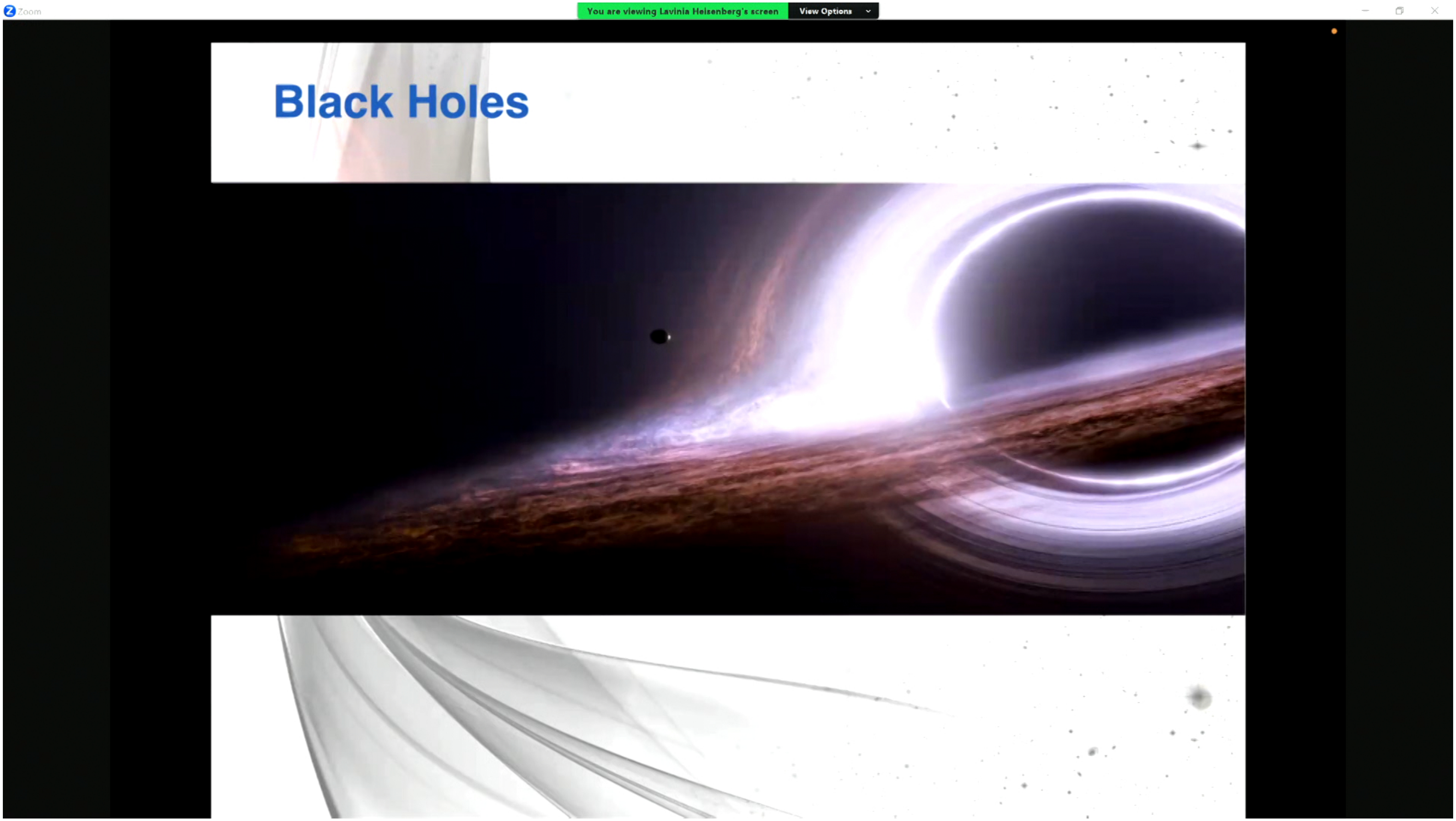
Gravitational waveforms play a crucial role in comparing observed signals to theoretical predictions. However, obtaining accurate analytical waveforms directly from general relativity remains challenging. Existing methods involve a complex blend of post-Newtonian theory, effective-one-body formalism, numerical relativity, and interpolation, introducing systematic errors. As gravitational wave astronomy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Black Holes

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Black Holes: The Schwarzschild Black Hole

The Schwarzschild black hole is a vacuum solution of Einstein's field equations which corresponds to

Black Holes: The Schwarzschild Black Hole

To derive the Schwarzschild solution, we impose

$$\begin{cases} \mathcal{L}_{\xi_T} g_{\mu\nu} &= 0 \\ \mathcal{L}_{\mathcal{R}_1} g_{\mu\nu} &= 0 \\ \mathcal{L}_{\mathcal{R}_2} g_{\mu\nu} &= 0 \\ \mathcal{L}_{\mathcal{R}_3} g_{\mu\nu} &= 0 \end{cases}$$

Stationarity (time–translation invariance)

Black Holes: The Schwarzschild Black Hole

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Insert this ansatz metric into the vacuum Einstein field equations, $G_{\mu\nu} = 0$

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Black Holes: The Schwarzschild Black Hole

Insert this ansatz metric into the vacuum Einstein field equations, $G_{\mu\nu} = 0$

- Boundary condition 1: Spacetime is asymptotically Minkowski,

$$\text{“}\lim_{r\to+\infty} g_{\mu\nu}(r) = \eta_{\mu\nu}\text{”}$$

Black Holes: The Schwarzschild Black Hole

Insert this ansatz metric into the vacuum Einstein field equations, $G_{\mu\nu} = 0$

- Boundary condition 1: Spacetime is asymptotically Minkowski,

$$\text{“} \lim_{r \to +\infty} g_{\mu\nu}(r) = \eta_{\mu\nu} \text{”}$$

- Boundary condition 2: Far away from the source, we should recover Newton's law

$$\ddot{x}^i = -\left\{ \begin{matrix} i \\ tt \end{matrix} \right\} = \frac{G\,M}{r^2}$$

Black Holes: The Schwarzschild Black Hole

The boundary conditions do not completely fix the solution. We still have gauge freedom!

Different gauge choices lead to different looking solutions. For instance:

Schwarzschild gauge:

$$\mathrm{d}s^2_{\mathsf{Sch}} = -\left(1-\frac{2M}{r}\right)\mathrm{d}t^2 + \left(1-\frac{2M}{r}\right)^{-1}\mathrm{d}r^2 + r^2\,\mathrm{d}\Omega^2$$

Painlevé–Gullstrand gauge:

$$\mathrm{d}s^2_{\mathsf{PG}} = -\left(1-\frac{2M}{r}\right)\mathrm{d}t^2 + 2\sqrt{\frac{2M}{r}}\mathrm{d}t\,\mathrm{d}r + \mathrm{d}r^2 + r^2\,\mathrm{d}\Omega^2$$

Black Holes: The Schwarzschild Black Hole

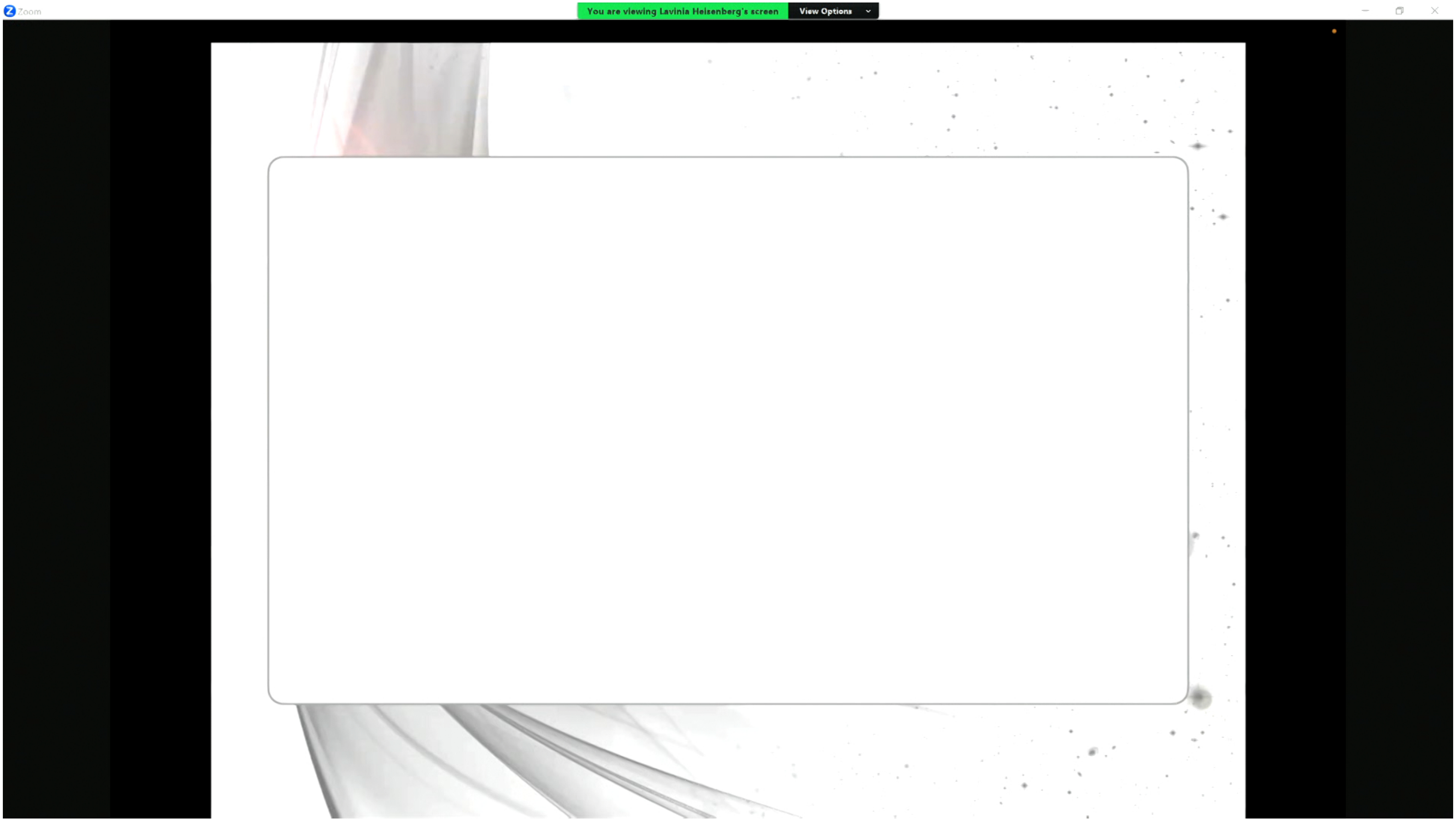
The solutions *look* different, but they are *physically equivalent*. In particular, they agree on all physical predictions such as

- Gravitational redshift & time dilation
- Bending of light
- Existence of event horizons
- etc.

Black Holes: The Schwarzschild Black Hole

Theorem (Birkhoff, 1922):
Any spherically symmetric solution of Einstein's vacuum field equations is necessarily the Schwarzschild solution. In particular, it is static.

Important consequence: spherically symmetric, pulsating stars do not emit gravitational waves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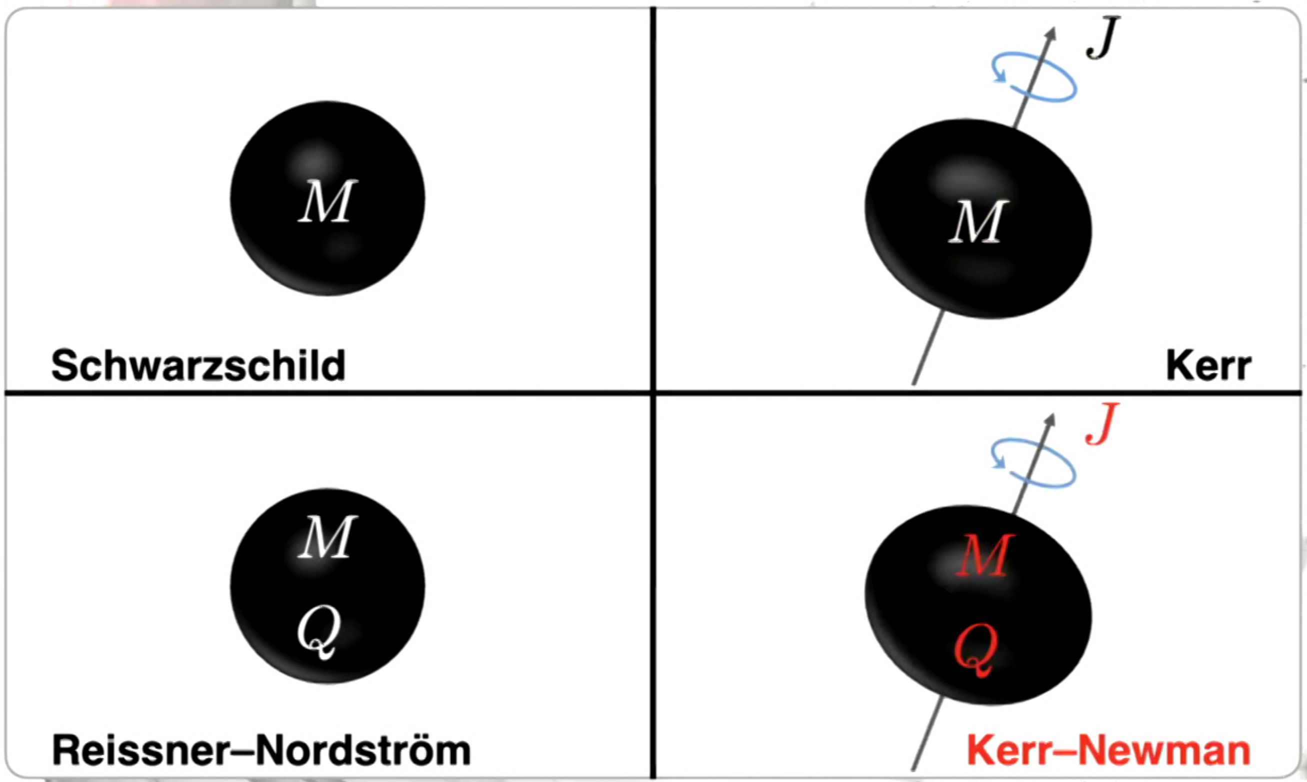
Stationary Black Hole Solutions



The No Hair Theorem

All stationary BH solutions of the Einstein field equations coupled to Standard Model matter are completely characterised by only three externally measurable parameters: Mass M, charge Q, angular momentum J.

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Black Holes Merger

- When two black holes merge the system is highly non–spherical symmetric and non–stationary.
- The no–hair theorem is broken.
- When two black holes merge they emit a GW signal

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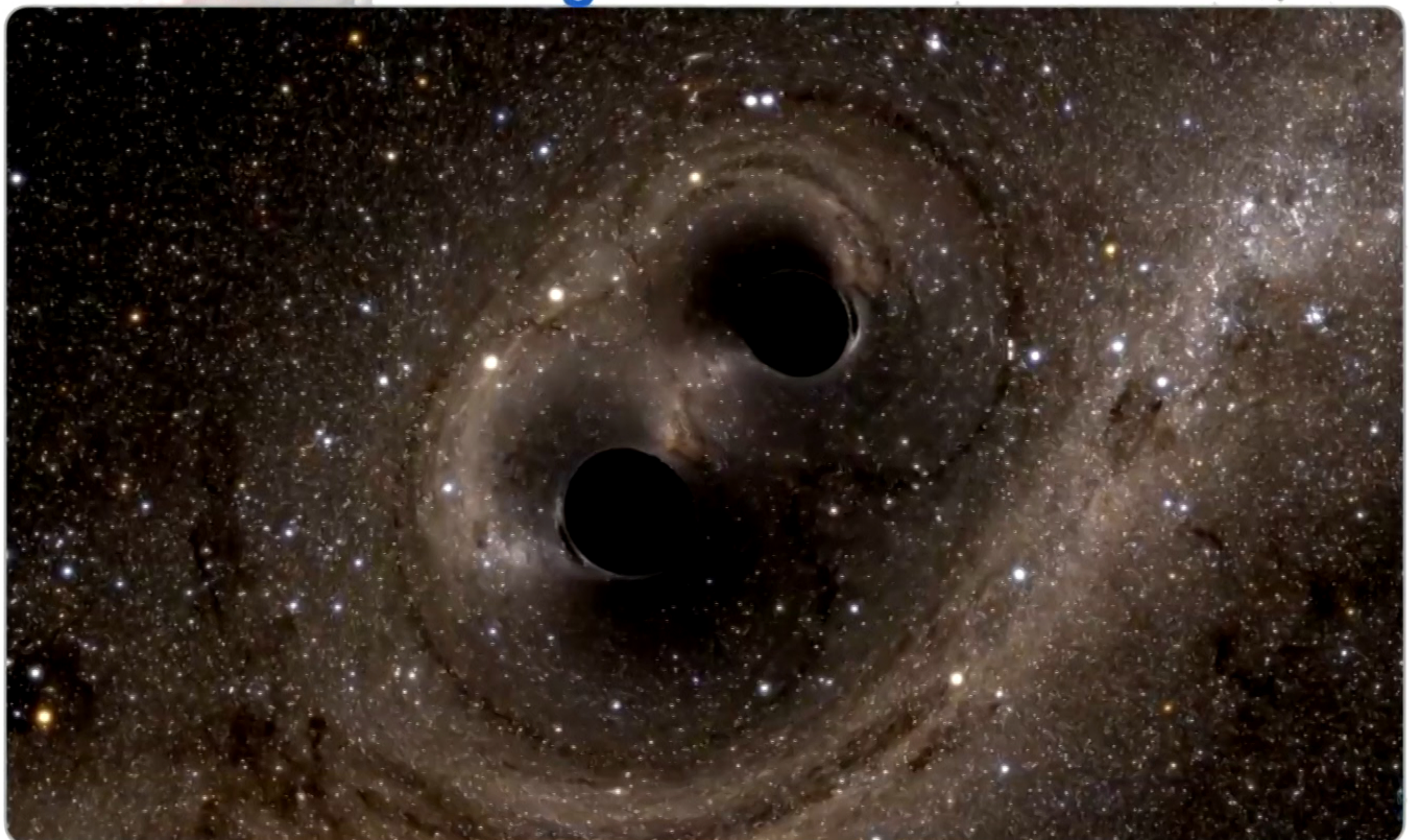
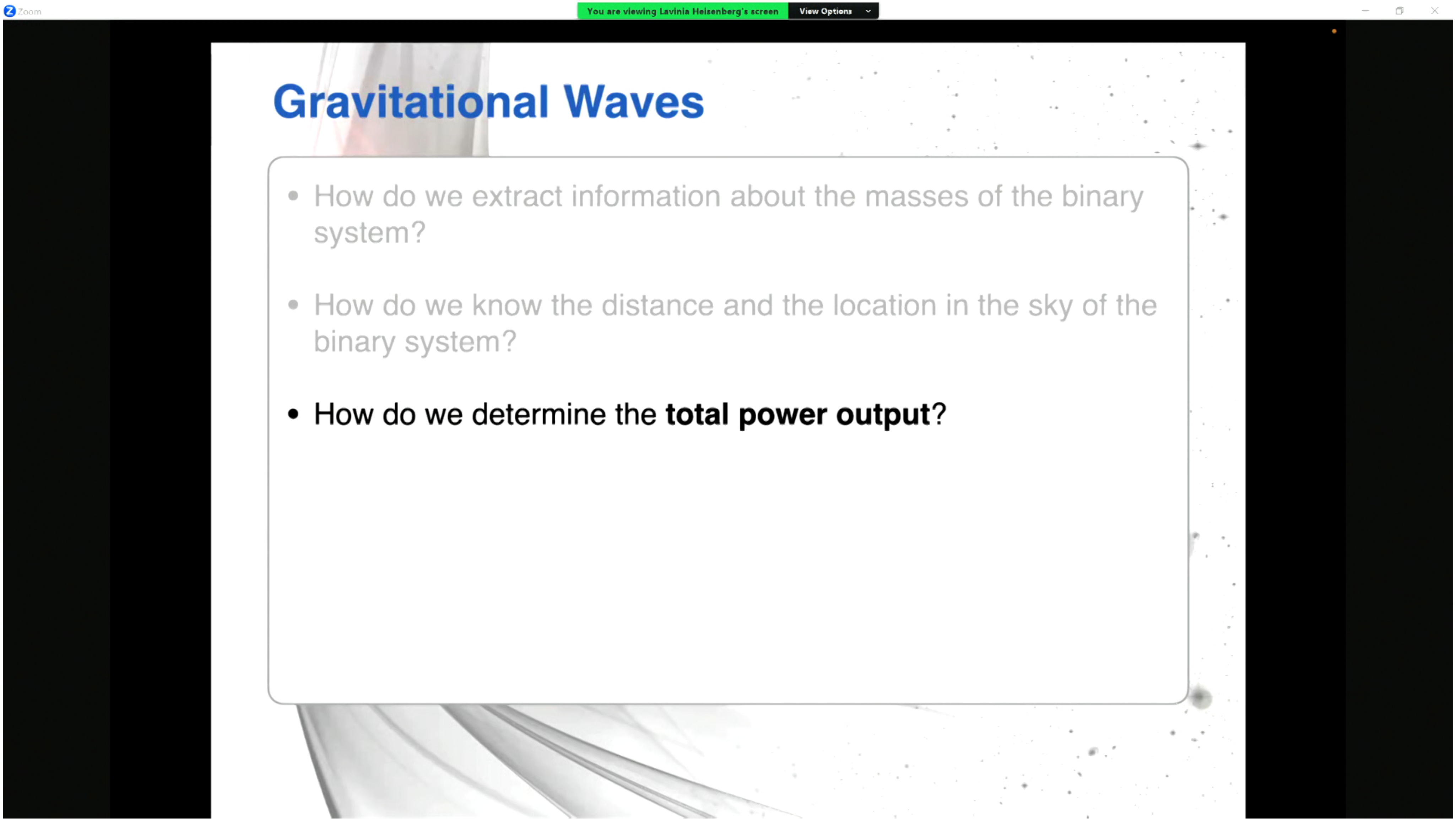
Black Holes Merger

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Gravitational Waves
How do we extract information about the masses of the binary system?
How do we know the distance and the location in the sky of the binary system?
How do we determine the total power output?

Gravitational Waves

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- How do we know the distance and the location in the sky of the binary system?
- How do we determine the total power output?
- How can we **test different theories of gravity**?

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Gravitational Waves

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Gravitational Waveforms!

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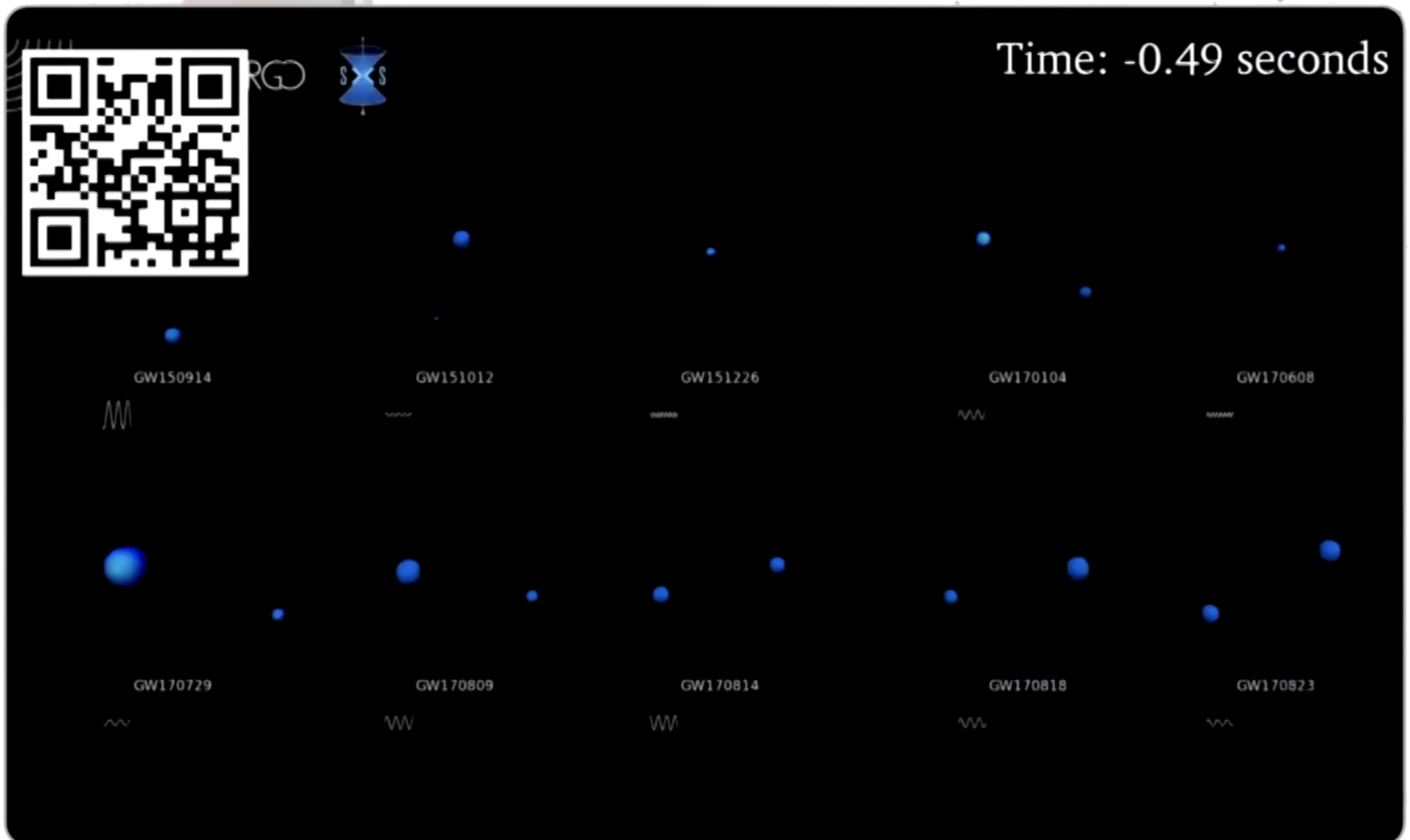
Gravitational Waveforms



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Gravitational Waveforms

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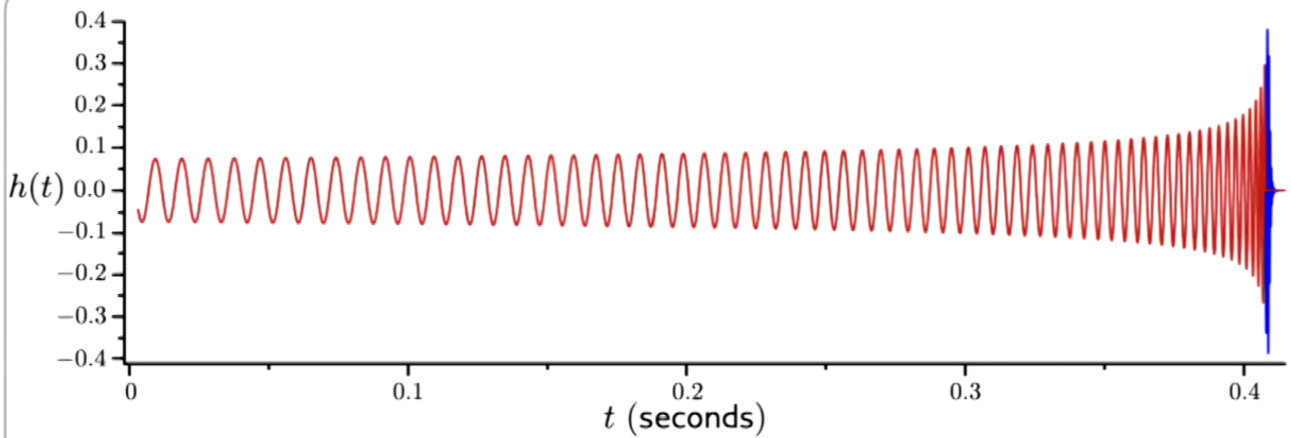
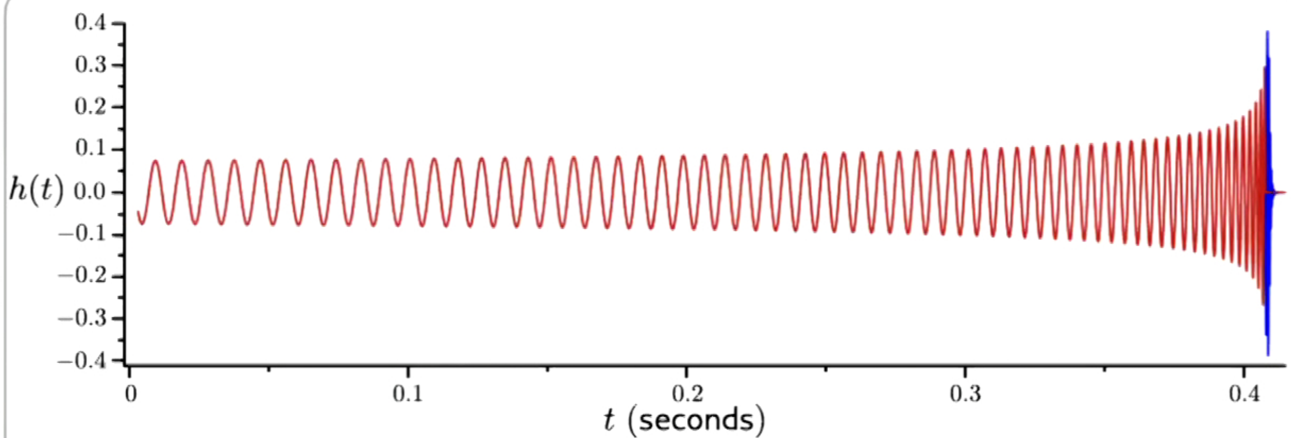
Gravitational Waveforms



- From frequency evolution, one infers the **masses**

Gravitational Waveforms



- From frequency evolution, one infers the masses
- From the amplitude and the masses, one infers the **distance**

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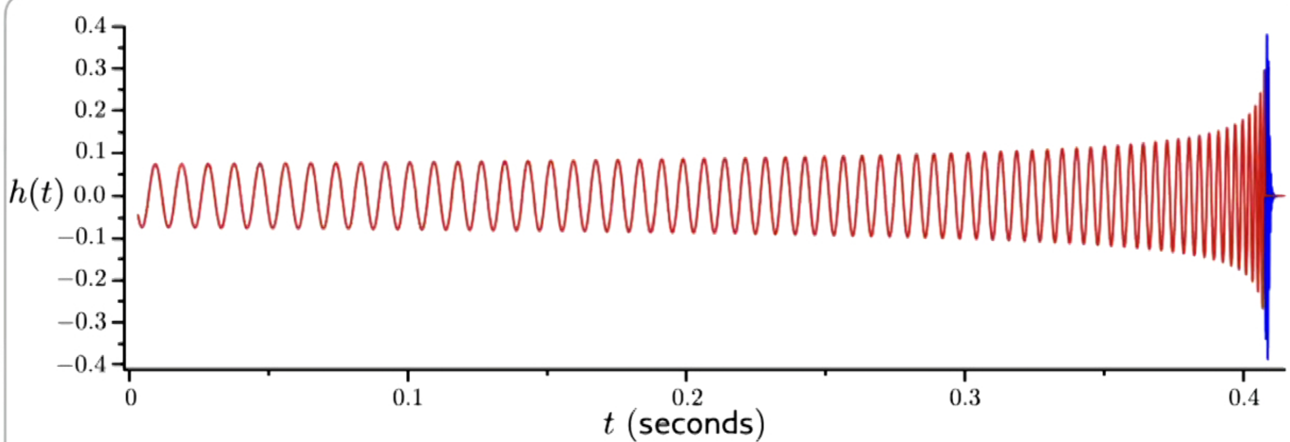
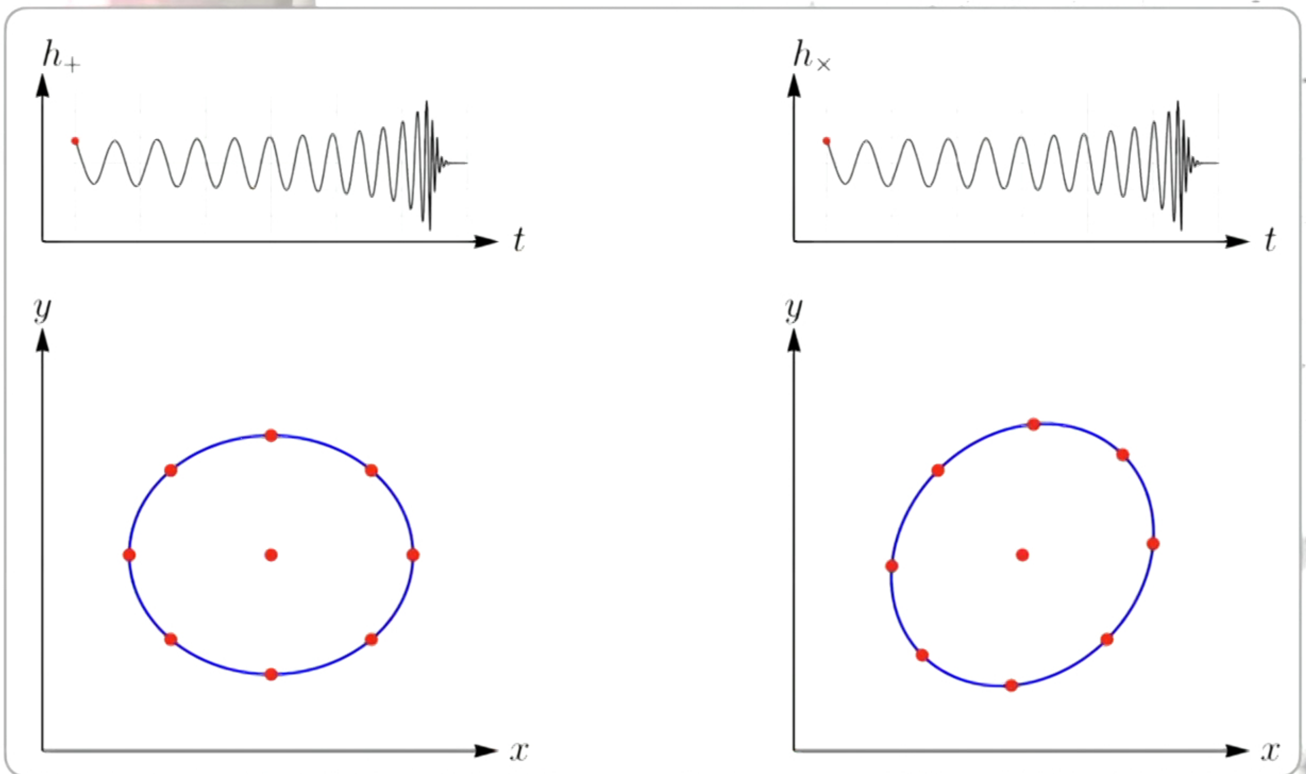
Gravitational Waveforms

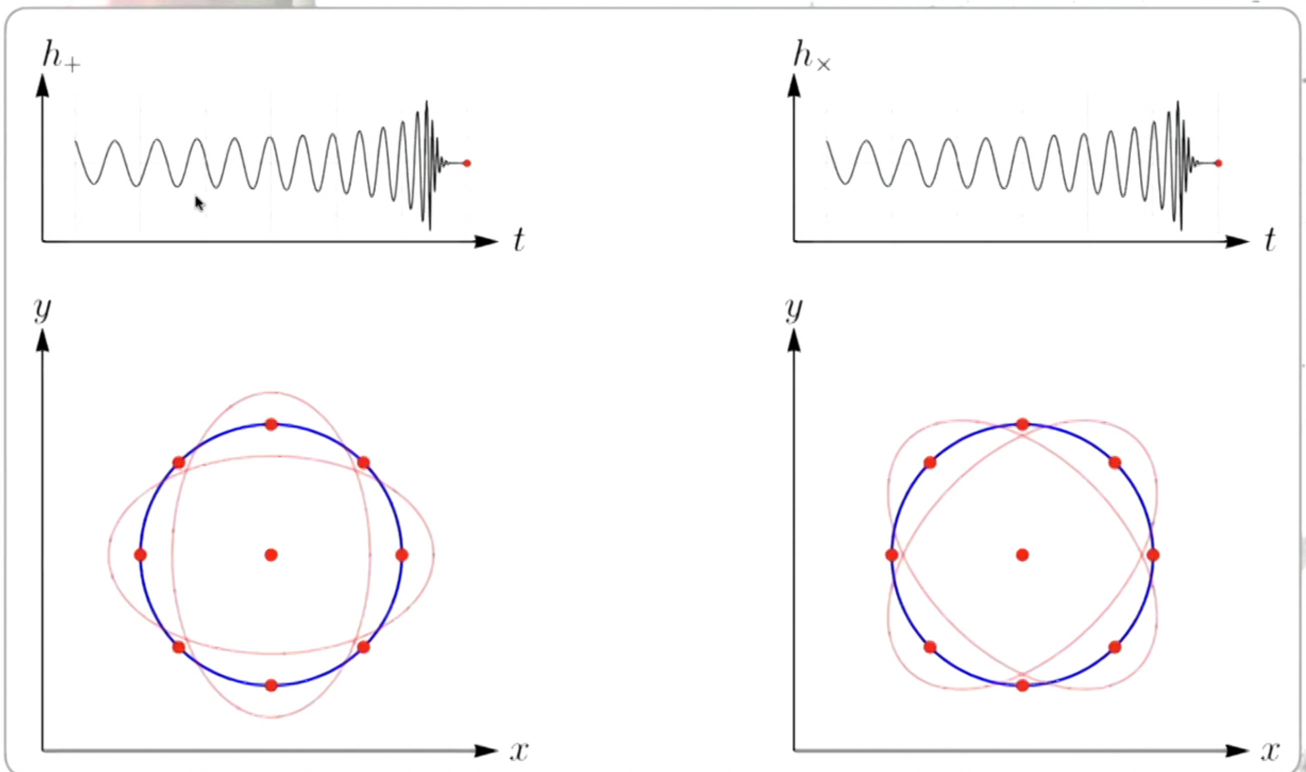


- From frequency evolution, one infers the masses
- From the amplitude and the masses, one infers the distance
- From time of arrival, amplitude, and phase at the detectors, one infers the **sky location**

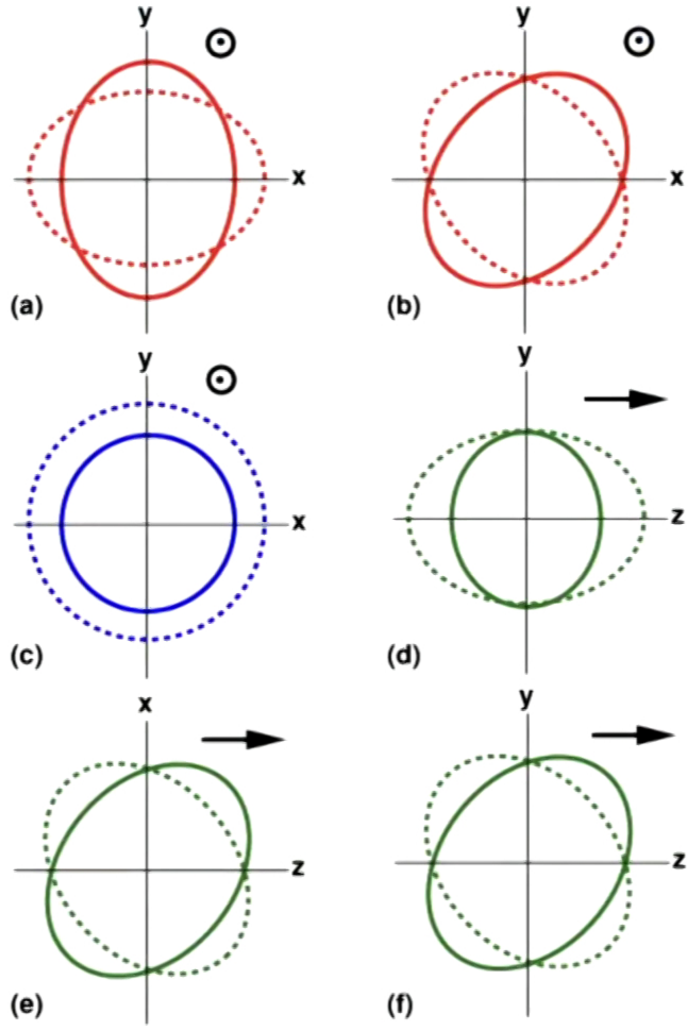
Gravitational Waves: Polarizations



Gravitational Waves: Polarizations



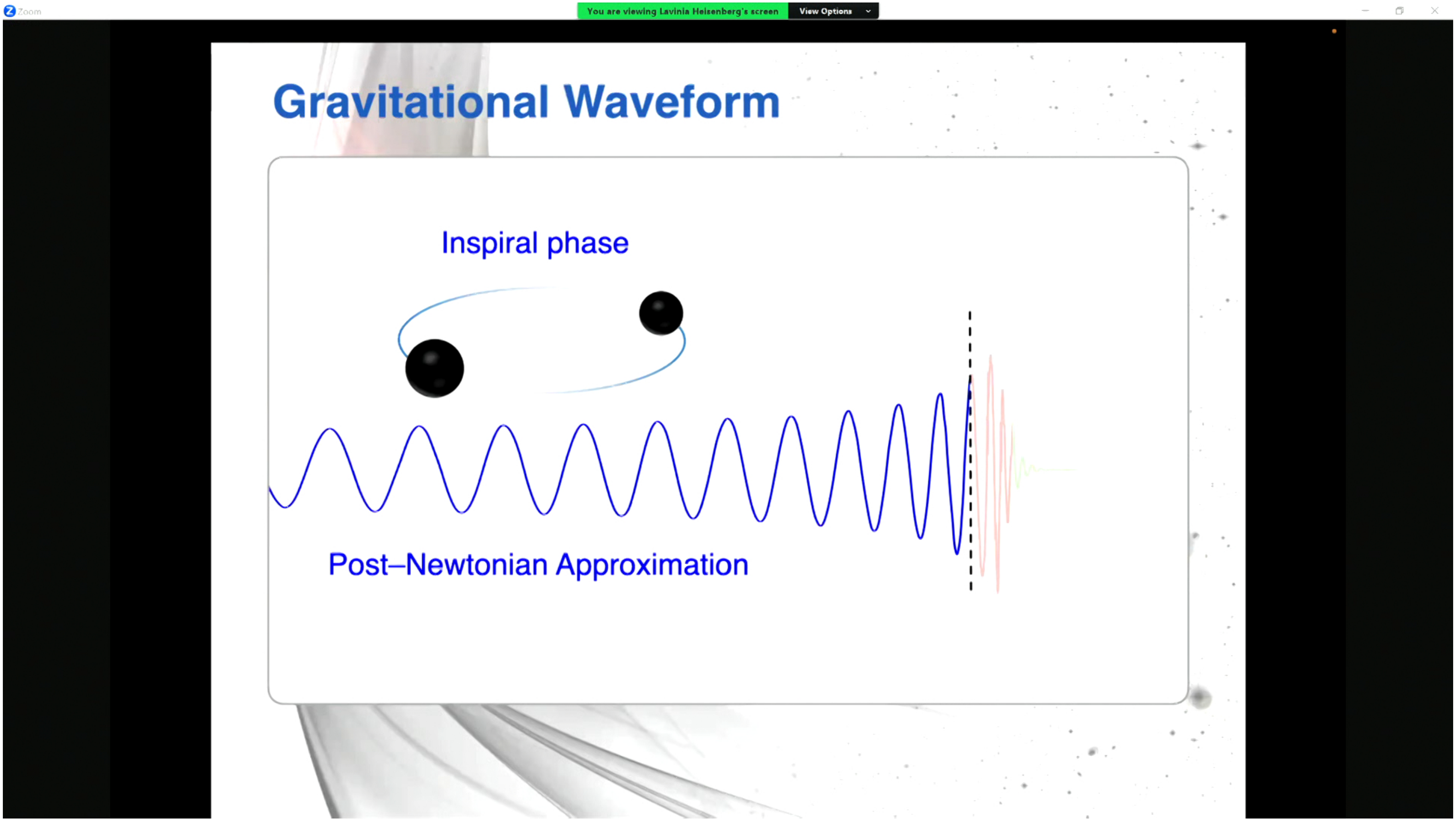
Gravitational–Wave Polarization



Gravitational Waveform

We have to solve Einstein's field equations in order to generate the GW waveform

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$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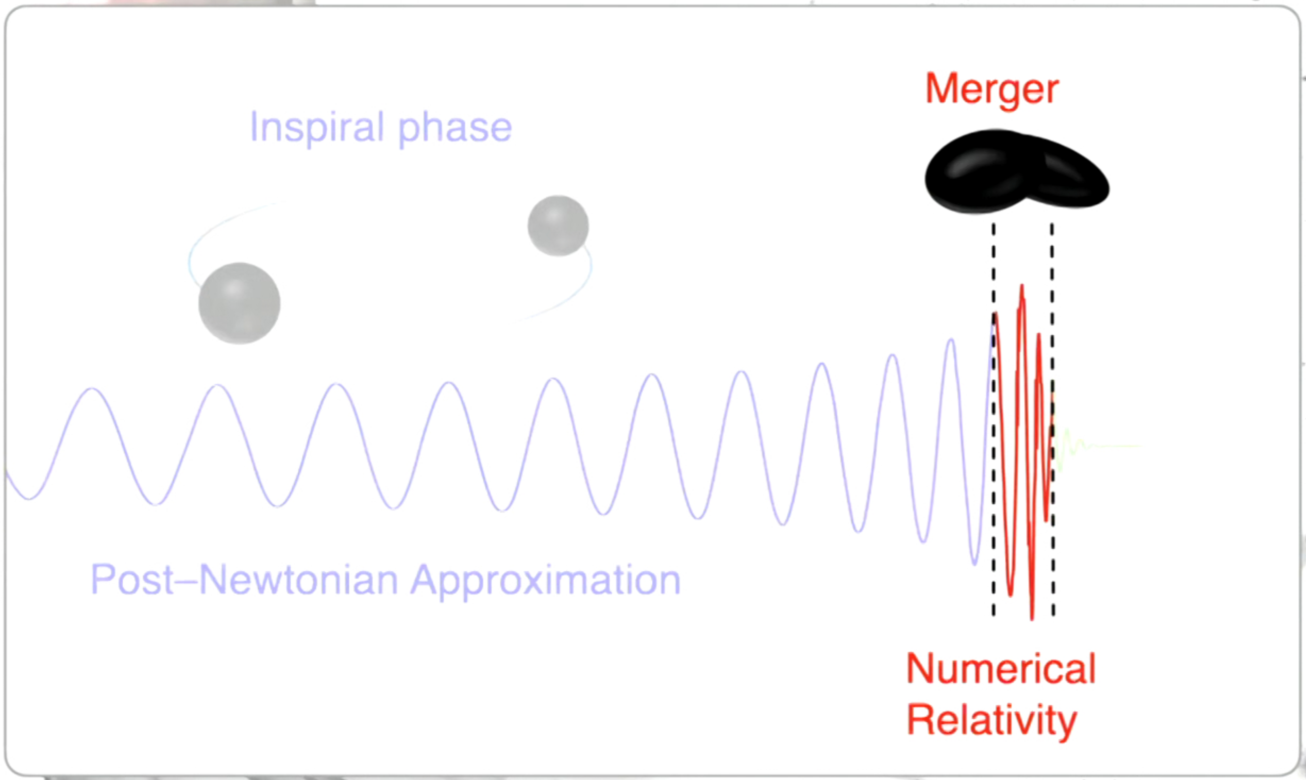
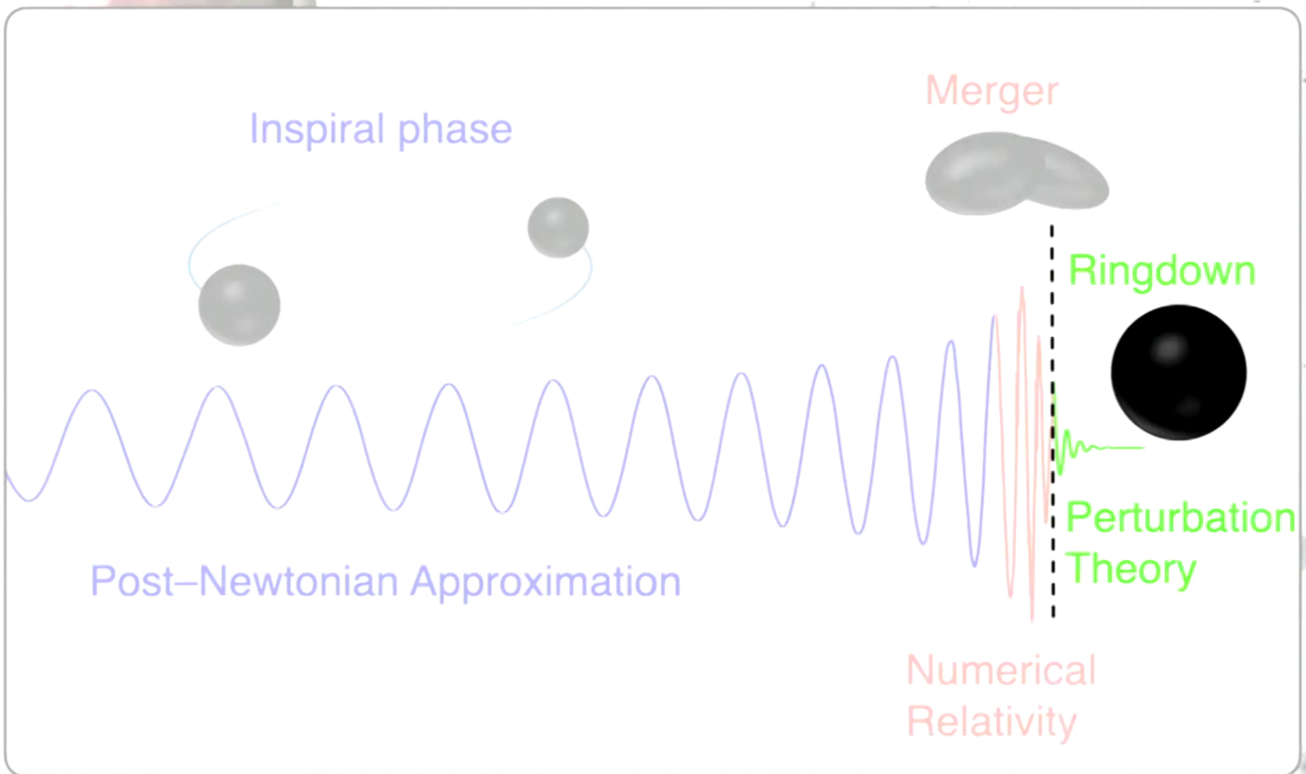
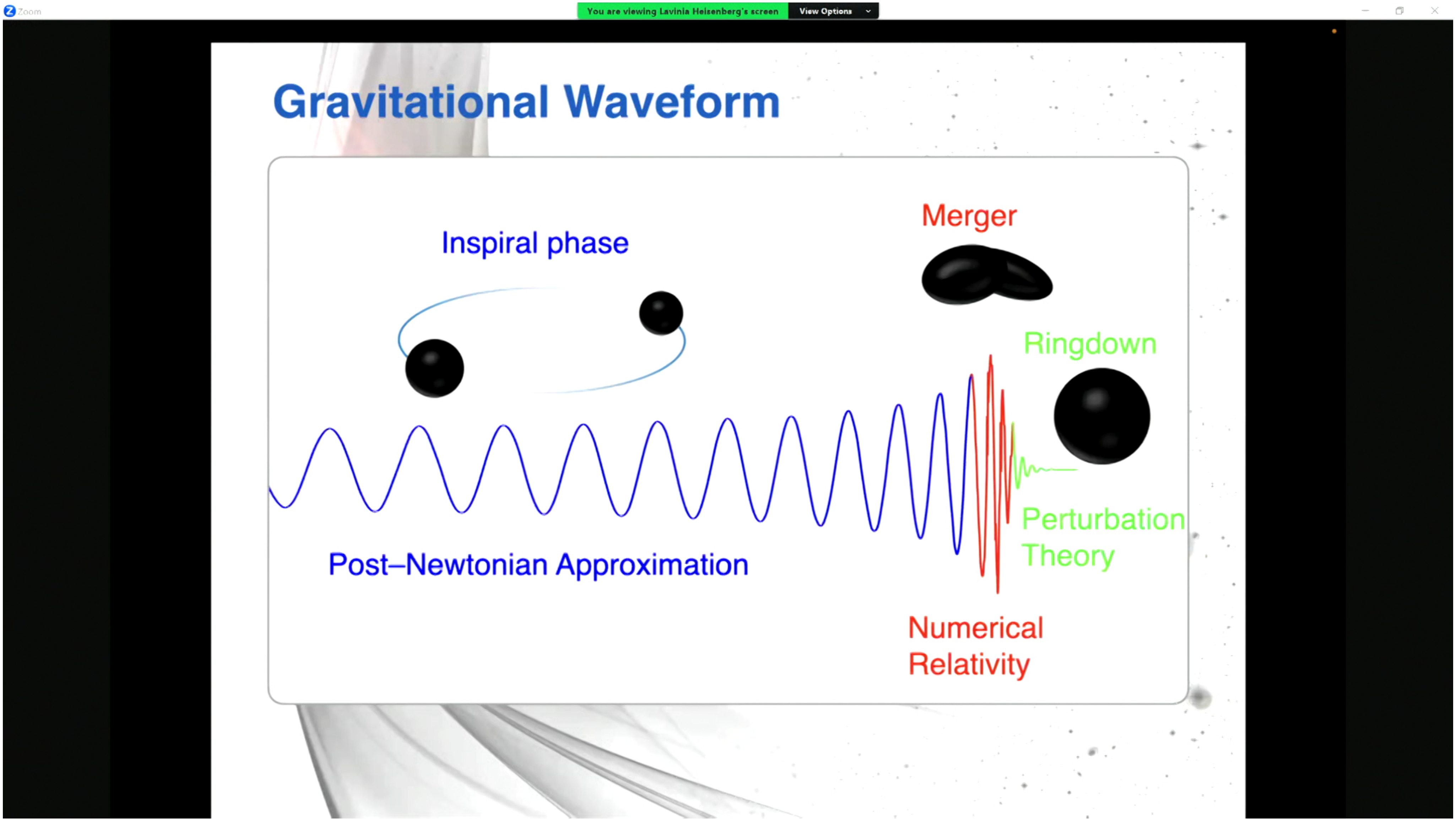
Gravitational Waveform
Inspiral phase
Post–Newtonian Approximation

Gravitational Waveform



Gravitational Waveform

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Gravitational Waveform
Merger
Inspiral phase
Ringdown
Perturbation Theory
Post–Newtonian Approximation
Numerical Relativity

Gravitational Waveform Models

These three phases of a waveform require different approximation techniques:

- Post–Newtonian Approximation for the inspired phase
- Numerical Relativity for the highly non–linear merger phase
- Perturbation Theory for the ringdown phase

Gravitational Waveform Models

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There are many different waveform models. The non–exhaustive list includes

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Gravitational Waveform Models

To get the full waveform, it is necessary to combine these approximated phases together into a so–called **waveform model**

There are many different waveform models. The non–exhaustive list includes

- SXS waveform models
- Pure and Hybridised NR Surogate models
- Effective One Body models
- IMPRPhenomD, IMPRPhenomXPHm, IMPRPhenomTPHM and many more derived models!

Gravitational Waveform Models

Discussing these models in detail is outside the scope of this talk.

What is important to us, is what these models have in common and what distinguishes them:

- Waveform models are essential tools for the **detection** of GWs and for **parameter estimation** of coalescing binary systems

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Gravitational Waveform Models

- Each model depends on **ten intrinsic parameters** and **four extrinsic parameters**

- Intrinsic parameters: The 2 masses of the constituents
  The individual spins (6 components)
  Eccentricity of orbit (1 parameter)
  Orientation of orbit (1 parameter)

- Extrinsic parameters: Luminosity distance from source
  3 parameters determining the orientation

Gravitational Waveform Models

- Each model makes simplifying assumptions regarding physics

- Examples:
  a) Effective One Body models assume a small mass–ratio of the binary

  b) Many of the older model neglect the so–called GW memory effect

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Gravitational Waveform Models

- Each model makes simplifying assumptions regarding physics
- Examples:
  a) Effective One Body models assume a small mass–ratio of the binary

  b) Many of the older model neglect the so–called GW memory effect
- Each model needs input from numerical relativity (NR) for the merger phase. But NR simulations are expensive
  $\Longrightarrow$ Can only cover finitely many points in parameter space
  $\Longrightarrow$ Each model needs to interpolate between NR data points

Gravitational Waveform Models

- Each approximation technique introduces errors. These errors can propagate through the waveform

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Given that waveform models play such a crucial role in detecting GW events and extracting information from the signal, we have to ask some crucial questions:

Gravitational Waveform Models

Given that waveform models play such a crucial role in detecting GW events and extracting information from the signal, we have to ask some crucial questions:

- Which of the many models can we trust when analyzing observational data?
- Which models is closest to what full, non–linear GR would predict for the physics of compact binary coalescence?
- **Can we test which model makes the smallest error and provides the best approximation?**

Gravitational Waveform Models

Yes! We can use the so–called Balance Laws!

The Need for Testing Waveform Models

Understanding how GWs are described in the full, non–linear theory opens the possibility to develop exact mathematical tests for waveform models:

Balance Laws

This is a recent idea (2019) based on arXiv:1906.00913

Compact binary coalescences: Constraints on waveforms

Abhay Ashtekar,* Tommaso De Lorenzo,† and Neev Khera‡

Institute for Gravitation and the Cosmos & Physics Department,
Penn State, University Park, PA 16802, U.S.A.

Balance Laws: The Basic Idea

The idea behind the balance law approach is simple:

- Binary systems lose energy through emission of GWs
- The total energy loss has to balance the total energy carried away by GWs
- This balancing of energies allows us to test and compare different models

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Mechanical Balance Laws

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Balance Laws as Test of Gravitational Waveforms

Lavinia Heisenberg[1]

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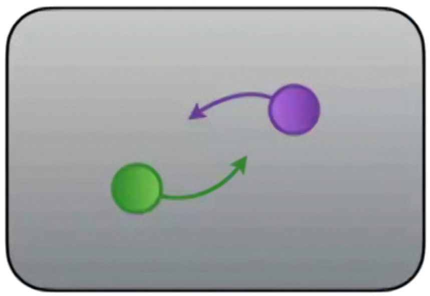
Mechanical Balance Laws

We illustrate the idea in more detail using a mechanical analogue

Consider a (relativistic or non–relativistic) **dissipative** mechanical system



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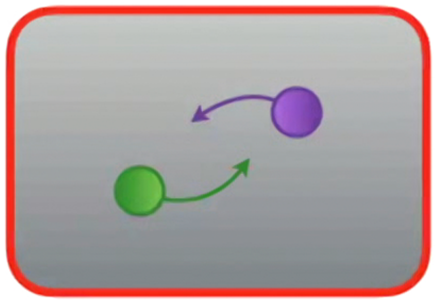

Boundary of system

Mechanical Balance Laws

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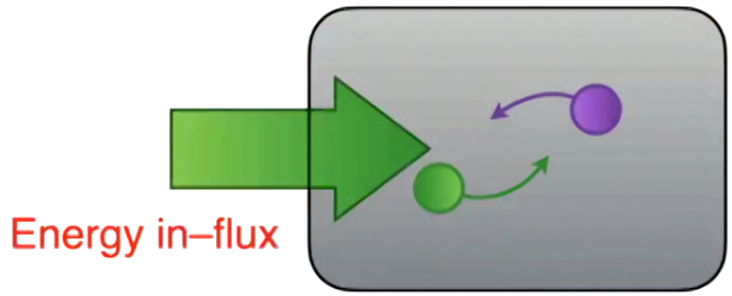

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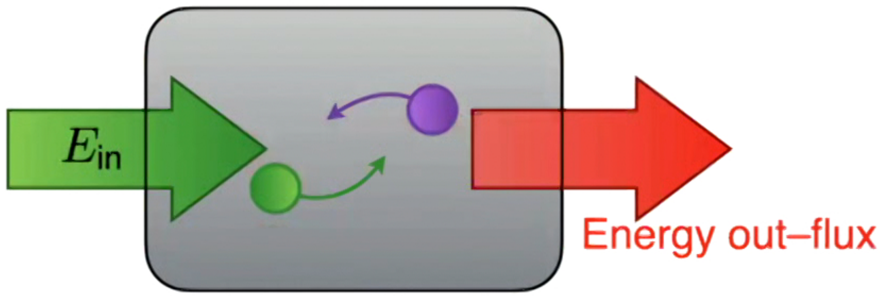
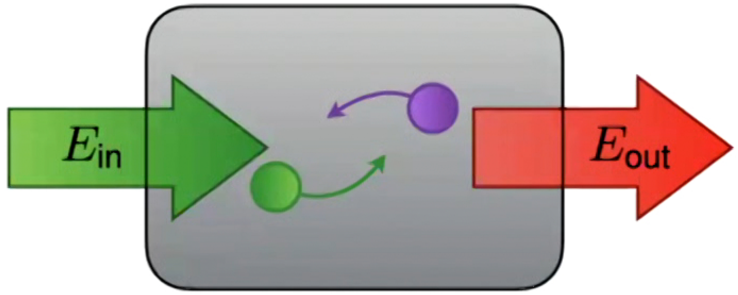


Mechanical Balance Laws



Energy in system not conserved!

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But total in/out flux of energy has to balance total gain/loss of energy in the system!

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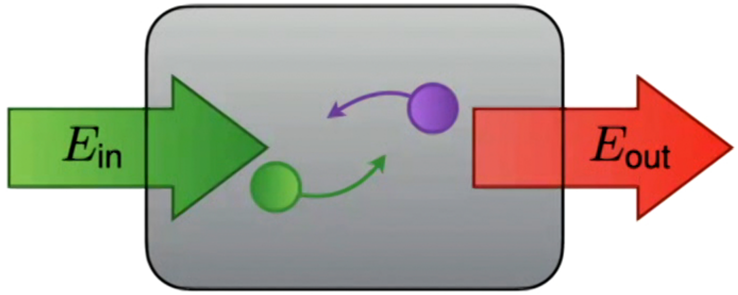
Mechanical Balance Laws



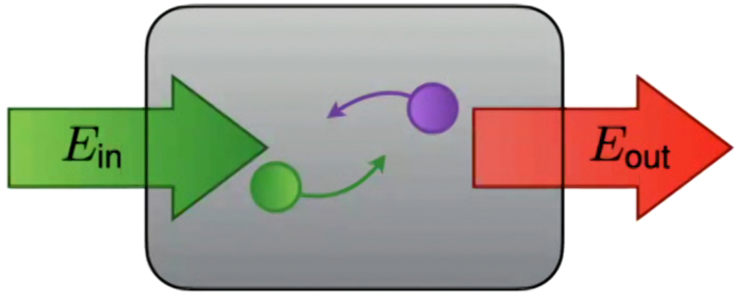
Imagine we measure the energy of the system at t_{initial} and t_{final}

Measurement yields E_{initial} and E_{final}

Then one can prove that

$$E_{\text{final}} - E_{\text{initial}} = -\int_{t_{\text{initial}}}^{t_{\text{final}}} \frac{\partial L}{\partial t}\, \mathrm{d}t$$

Mechanical Balance Laws



$$E_{\text{final}} - E_{\text{initial}} = -\int_{t_{\text{initial}}}^{t_{\text{final}}} \frac{\partial L}{\partial t}\,\mathrm{d}t = \boxed{E_{\text{in}} - E_{\text{out}}}$$

Total energy in/out flux through boundary of system

Mechanical Balance Laws

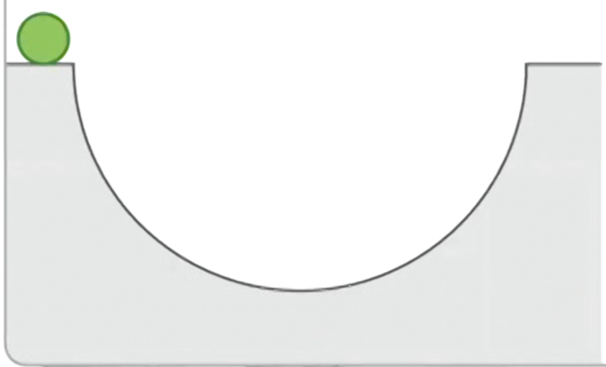
The strategy is as follows:

1. Derive equations of motion from Lagrangian L

2. Make simplifying assumptions and / or approximate equations and / or solve numerically if they are too difficult to solve analytically

Example: Rolling Ball with Friction

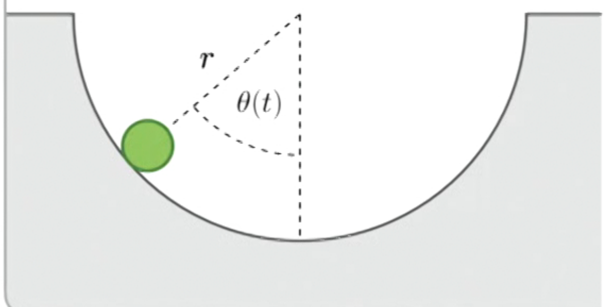
Consider a ball rolling down a half–pipe with **friction**



Example: Rolling Ball with Friction

Friction term

$$\begin{cases} \ddot{\theta}(t) + \boxed{g\,\mu\,\dot{\theta}(t)} + \frac{g}{r}\,\sin\theta(t) = 0 \\ \dot{\theta}(0) = \omega_0 \\ \theta(0) = \theta_0 \end{cases}$$



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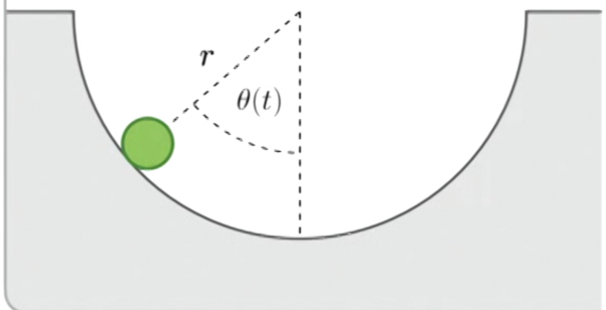

Non–linear term

$$\begin{cases} \ddot{\theta}(t) + g\,\mu\,\dot{\theta}(t) + \boxed{\frac{g}{r}\ \sin\theta(t)} = 0 \\ \dot{\theta}(0) = \omega_0 \\ \theta(0) = \theta_0 \end{cases}$$

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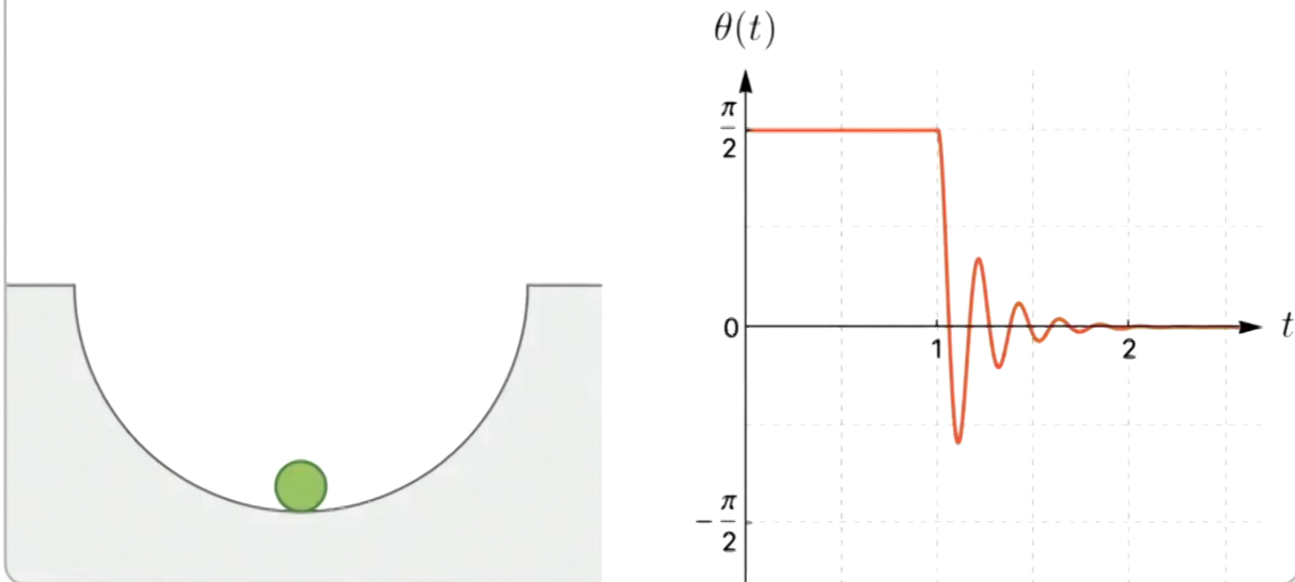
Example: Rolling Ball with Friction

$$\theta(t) = \mathrm{e}^{-\frac{1}{2} g\, t\, \mu} \left\{ \theta_0 \cos\left(t \sqrt{\tfrac{g}{r}} \sqrt{\left|1 - \tfrac{1}{4} g\, \mu^2\, r\right|}\right) + \left(\omega_0 + \tfrac{1}{2} g\, \mu\, \theta_0\right) \sqrt{\tfrac{r}{g}} \frac{1}{\sqrt{\left|1 - \frac{1}{4} g\, \mu^2\, r\right|}} \sin\left(t \sqrt{\tfrac{g}{r}} \sqrt{\left|1 - \tfrac{1}{4} g\, \mu^2\, r\right|}\right) \right\}$$

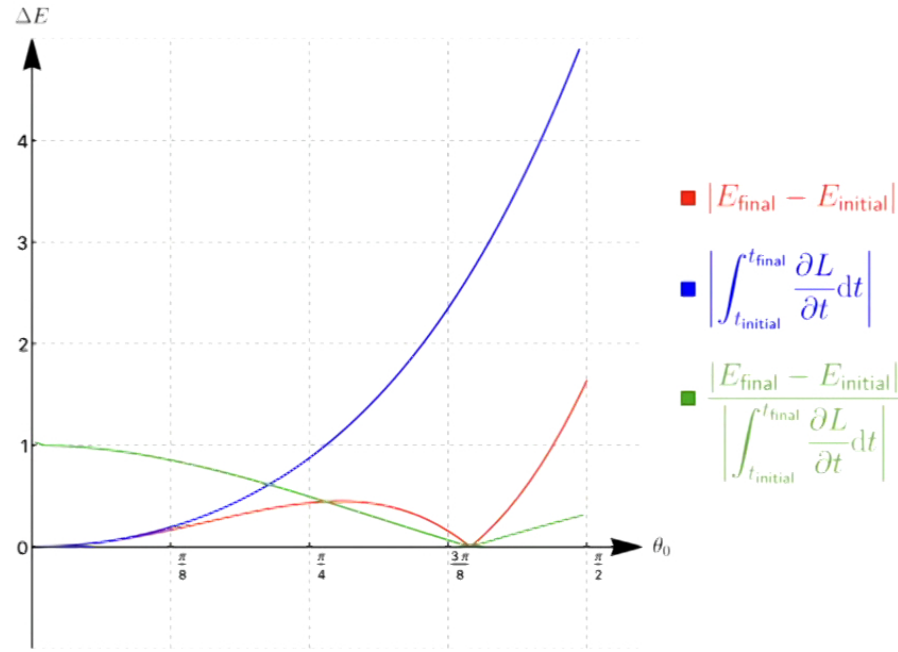


Example: Rolling Ball with Friction

Fixed values:

$m,\ \mu,\ g,\ r$

$\omega_0,\ t_{\text{initial}},\ t_{\text{final}}$



Variable:

$$\theta_0 \in \left[0, \frac{\pi}{2}\right]$$

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Example: Rolling Ball with Friction

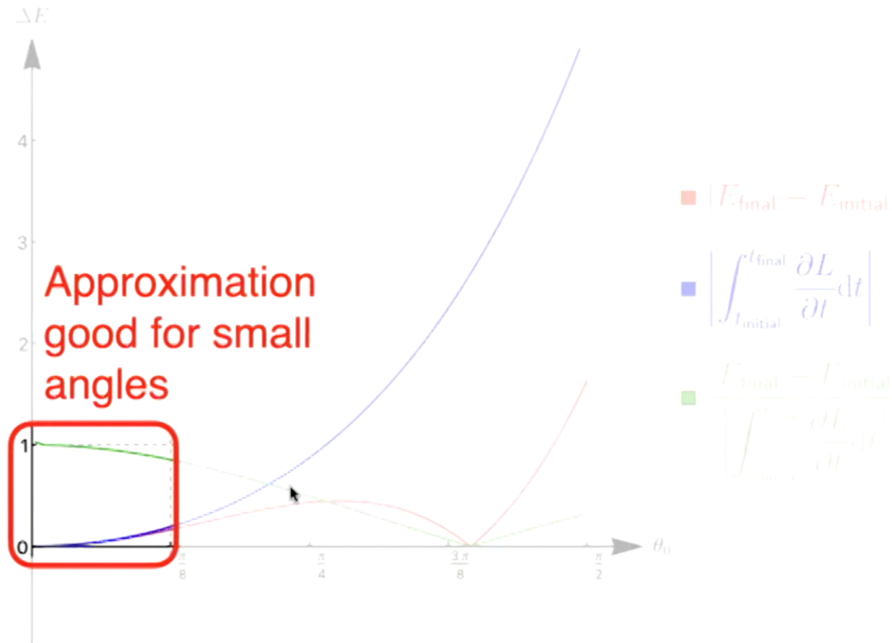
Fixed values:

$m, \ \mu, \ g, \ r$

$\omega_0, \ t_{\text{initial}}, \ t_{\text{final}}$

Variable:

$\theta_0 \in \left[0, \frac{\pi}{2}\right]$



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Example: Rolling Ball with Friction

Fixed values:

$m,\ \mu,\ g,\ r$

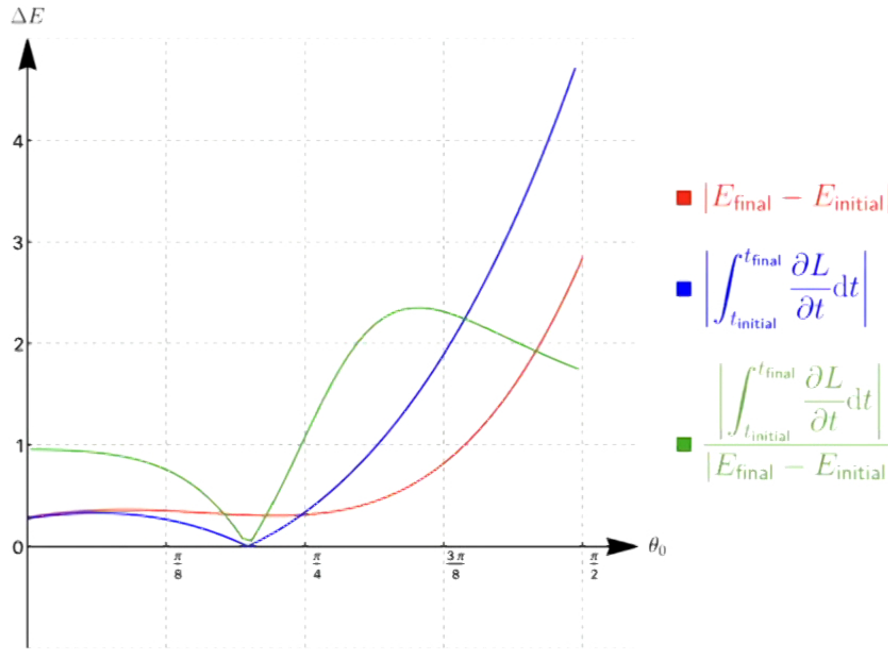
$t_{\text{initial}},\ t_{\text{final}}$

Changed value:

ω_0

Variable:

$\theta_0 \in \left[0, \frac{\pi}{2}\right]$



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From Mechanical to GR Balance Laws

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Subject Areas:
Gravitational Wave Physics, Cosmology, Particle Physics

Balance Laws as Test of Gravitational Waveforms

Lavinia Heisenberg[1]

[1]Institute for Theoretical Physics, Philosophenweg 16, 69120 Heidelberg, Germany

Gravitational waveforms play a crucial role in comparing observed signals to theoretical predictions. However, obtaining accurate analytical waveforms directly from general relativity remains challenging. Existing methods involve a complex blend of post-Newtonian theory, effective-one-body formalism, numerical relativity, and interpolation, introducing systematic errors. As gravitational wave astronomy

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From Mechanical to GR Balance Laws

- Recall that balance laws are **exact mathematical relations** of a theory (**no** approximations)

- To develop balance laws for coalescing binary systems, we need to understand GWs in the **full, non–linear theory**

From Mechanical to GR Balance Laws

- Recall that balance laws are **exact mathematical relations** of a theory (**no** approximations)
- To develop balance laws for coalescing binary systems, we need to understand GWs in the **full, non–linear theory**
- A detailed introduction into this subject is given in arXiv:2201.11634

Gravitational Waves in Full, Non-Linear General Relativity

Fabio D'Ambrosio*[1], Shaun D. B. Fell†[2], Lavinia Heisenberg‡[2,1], David Maibach§[2], Stefan Zentarra¶[1], and Jann Zosso‖[1]

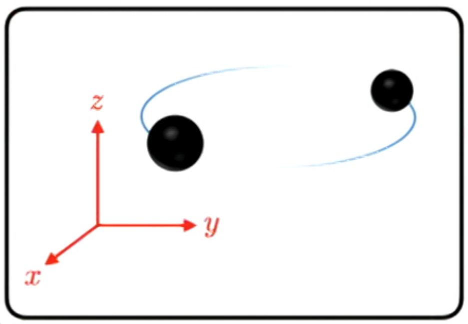
[1] *Institute for Theoretical Physics, ETH Zurich, Wolfgang-Pauli-Strasse 27, 8093 Zurich, Switzerland*
[2] *Institut für Theoretische Physik, Philosophenweg 16, 69120 Heidelberg, Germany*

Gravitational Balance Laws

We consider a binary system of compact astrophysical objects

We choose our coordinate system such that in the distant past, the observer is at rest relative to the centre of mass of the system

Due to the emission of GWs, there is an out–flux of energy



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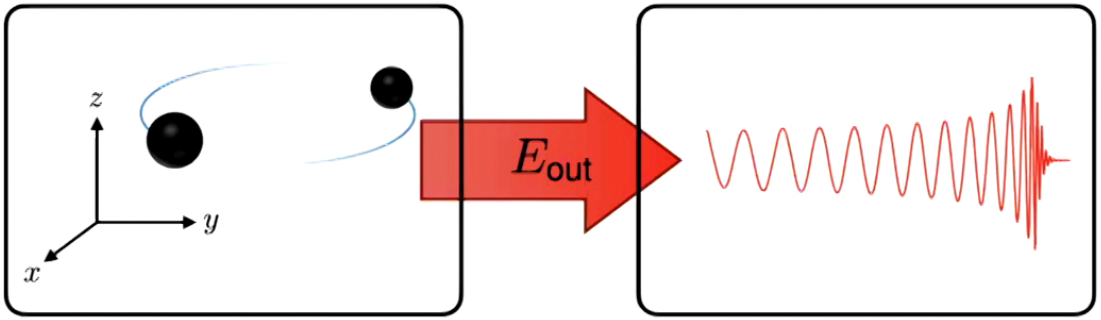
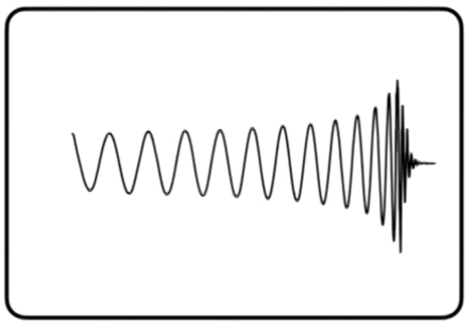
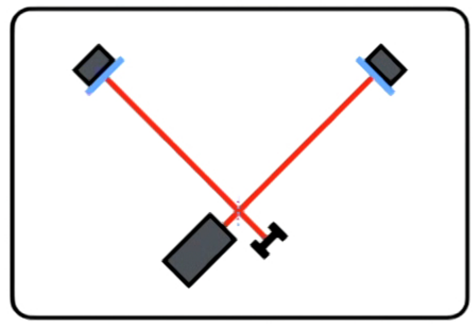


Gravitational Balance Laws

The signal is measured by interferometers which are “infinitely” far away



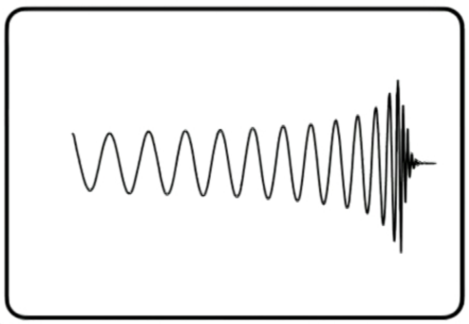


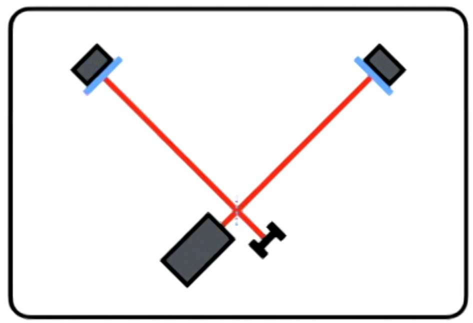
Gravitational Balance Laws

The signal is measured by interferometers which are "infinitely" far away

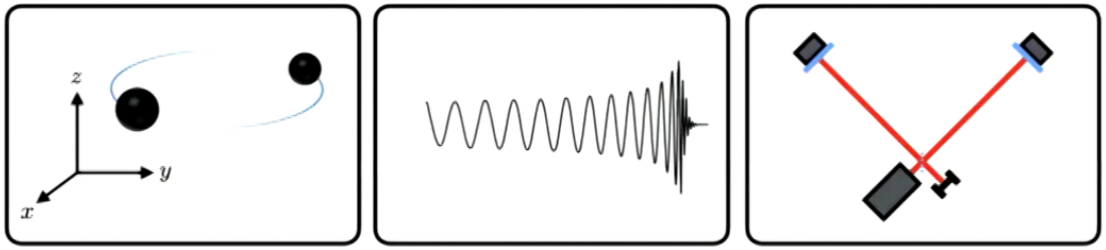
The signal allows us to determine

- The kick velocity; $\vec{v}_{\text{kick}}$
- The total mass of the binary system; $M_{\text{binary}} := M_{i^0}$
- The mass of the remnant; $M_{\text{remnant}} := M_{i^+}$





Gravitational Balance Laws



$$c^2\left(\frac{M_{\text{remnant}}}{\gamma(v_{\text{kick}})^3\left(1-\frac{\vec{v}_{\text{kick}}}{c}\cdot\hat{x}\right)^3}-M_{\text{binary}}\right) = -\frac{1}{4}\frac{D_{\text{L}}^2\,c^3}{G}\int_{-\infty}^{\infty}\left(\dot{h}_+^2+\dot{h}_\times^2\right)\mathrm{d}t$$

$$+\frac{1}{2}\frac{D_{\text{L}}\,c^4}{G}\,\mathrm{Re}\left[\eth^2\left(h_+-i\,h_\times\right)\right]\Bigg|_{t=-\infty}^{t=+\infty}$$

Gravitational Balance Laws

Observe the close resemblance with the mechanical balance laws

$$E_{\text{final}} - E_{\text{initial}} = -\int_{t_{\text{initla}}}^{t_{\text{final}}} \frac{\partial L}{\partial t}\, \mathrm{d}t$$

$$c^2 \left(\frac{M_{\text{remnant}}}{\gamma(v_{\text{kick}})^3 \left(1 - \frac{\vec{v}_{\text{kick}}}{c} \cdot \hat{x}\right)^3} - M_{\text{binary}} \right) = -\frac{1}{4}\frac{D_{\text{L}}^2 c^3}{G} \int_{-\infty}^{\infty} \left(\dot{h}_+^2 + \dot{h}_\times^2\right) \mathrm{d}t + \frac{1}{2}\frac{D_{\text{L}}\, c^4}{G} \text{Re}\left[\eth^2 \left(h_+ - i\, h_\times\right)\right]\Big|_{t=-\infty}^{t=+\infty}$$

Energy difference between initial and final state of system

Gravitational Balance Laws

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$$+ \frac{1}{2} \frac{D_{\text{L}}\, c^4}{G} \text{Re}\left[\eth^2 \left(h_+ - i\, h_\times\right)\right] \Big|_{t=-\infty}^{t=+\infty}$$

Total energy lost by system

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The GW Memory Effect

$$\frac{1}{2}\frac{D_{\mathrm{L}}\,c^4}{G}\mathrm{Re}\left[\eth^2\left(h_+ - i\,h_\times\right)\right]\Big|_{t=-\infty}^{t=+\infty}$$

Observe that the memory term is essentially the difference of two values:

$h_{+,\times}$ in the distant future minus $h_{+,\times}$ in the distant past

The GW Memory Effect

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$$h_{+,\times}\big|_{t=-\infty} = 0$$

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Observe that the memory term is essentially the difference of two values:

$h_{+,\times}$ in the distant future minus $h_{+,\times}$ in the distant past

In the distant past, before any GW has been emitted, one expects

$$h_{+,\times}|_{t=-\infty} = 0$$

Similarly, in the distant future after the GW has long passed, one expects

$$h_{+,\times}|_{t=+\infty} = 0$$

The GW Memory Effect

This difference has a clear interpretation in terms of a ring of test masses:

- A GW sets a ring of test masses into oscillatory motion
- The proper distance between the test masses is a function of $h_{+,\times}$
- After the wave has passed, one expects $h_{+,\times} = 0$ and thus the **original ring configuration is restored**
- **However:** GR predicts that in general $h_{+,\times} \neq 0$ after the passage of the wave

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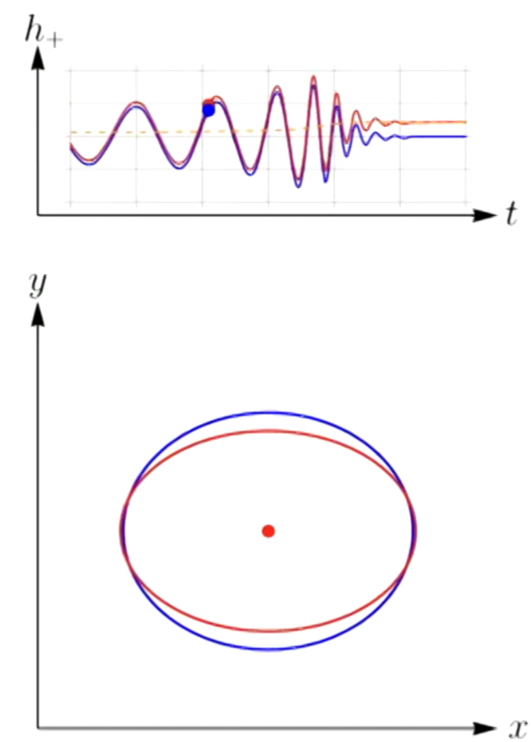
The GW Memory Effect

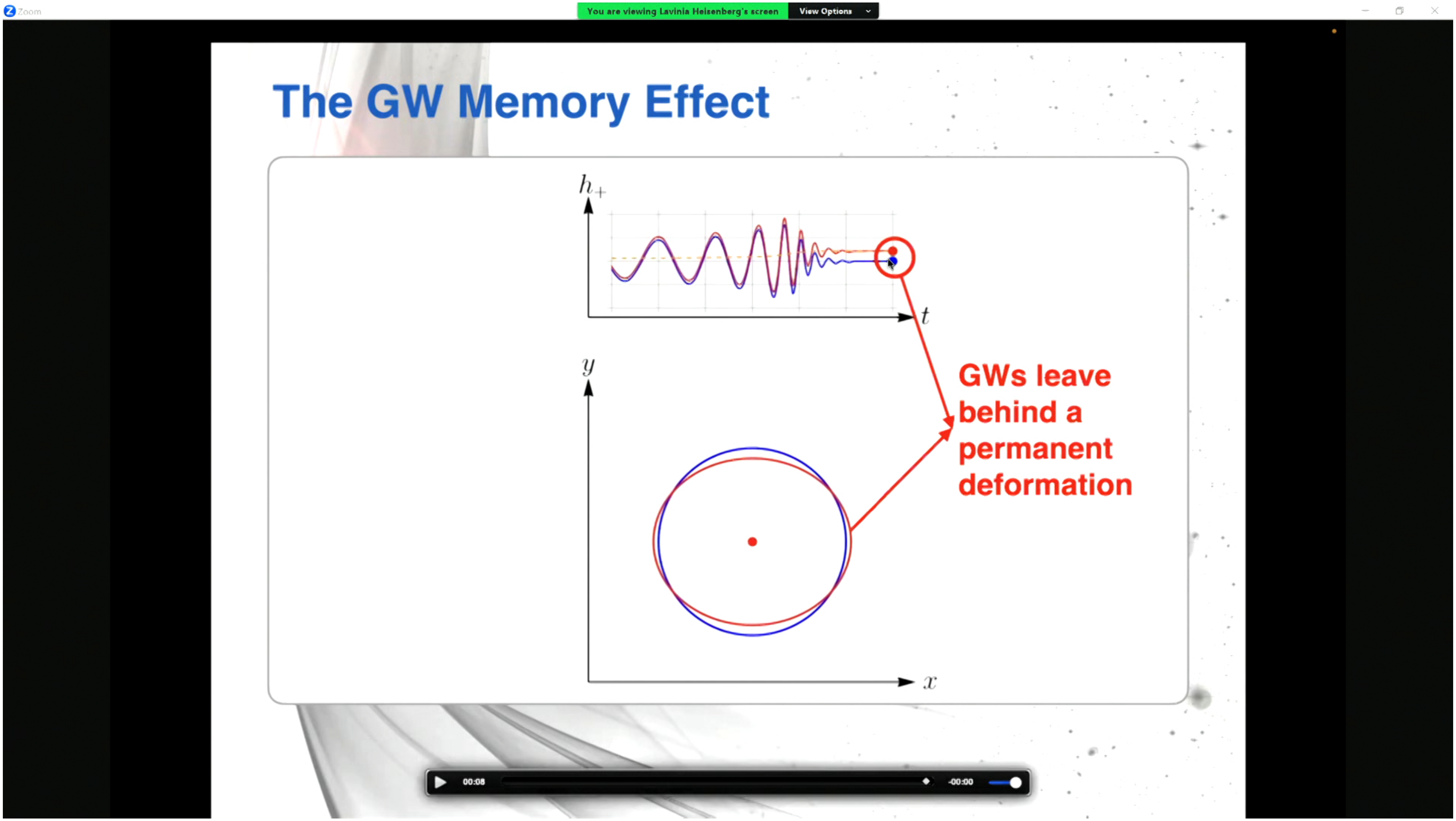
This difference has a clear interpretation in terms of a ring of test masses:

- This means that the proper distance between the test masses is **permanently changed**!
- In turn, this means that the shape is **permanently changed**!

The GW memory effect is the prediction that GWs lead to a permanent displacement of test masses

The GW Memory Effect

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The GW Memory Effect
h_+
t
y
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GWs leave behind a permanent deformation
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Simple Waveform Model

- arXiv:1810.06160

A Complete Analytic Gravitational Wave Model for Undergraduates

Dillon Buskirk [a] and Maria C. Babiuc Hamilton [b]

Department of Physics, Marshall University, Huntington, WV 25755, USA

Abstract

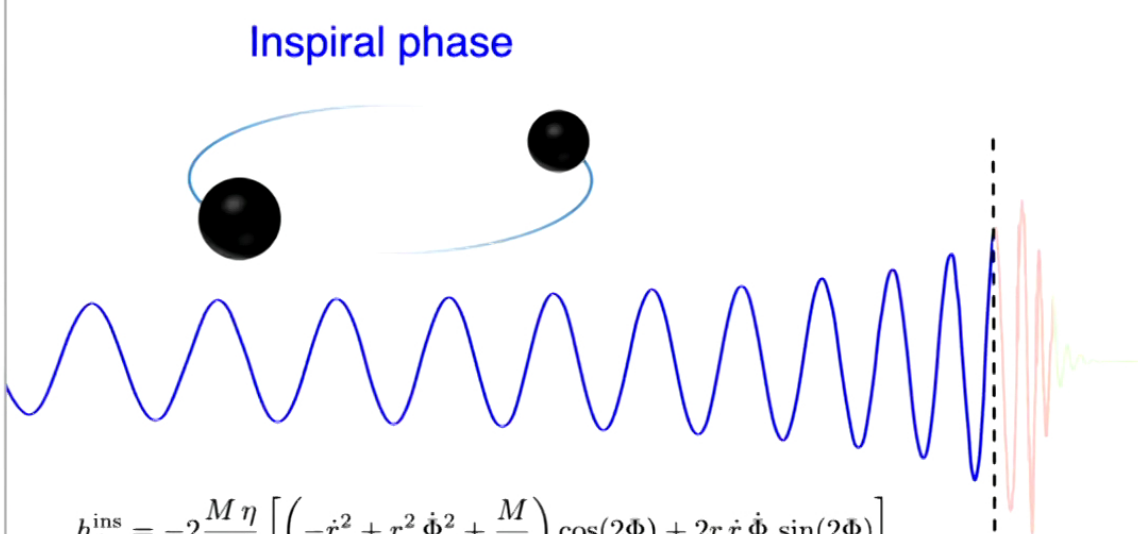
Gravitational waves are produced by orbiting massive binary objects, such as black holes and neutron stars, and propagate as ripples in the very fabric of spacetime. As the waves carry off orbital energy, the two bodies spiral into each other and eventually merge. They are described by Einstein's equations of General Relativity. For the early phase of the orbit, called the inspiral, Einstein equations can be linearized and solved through analytical approximations, while for the late phase,

Simple Waveform Model

Let us consider a simple model assuming:

- two objects with masses m1 and m2
- zero initial spin
- orbiting each other with a time-dependent separation r(t)
- time dependent orbital frequency w(t)

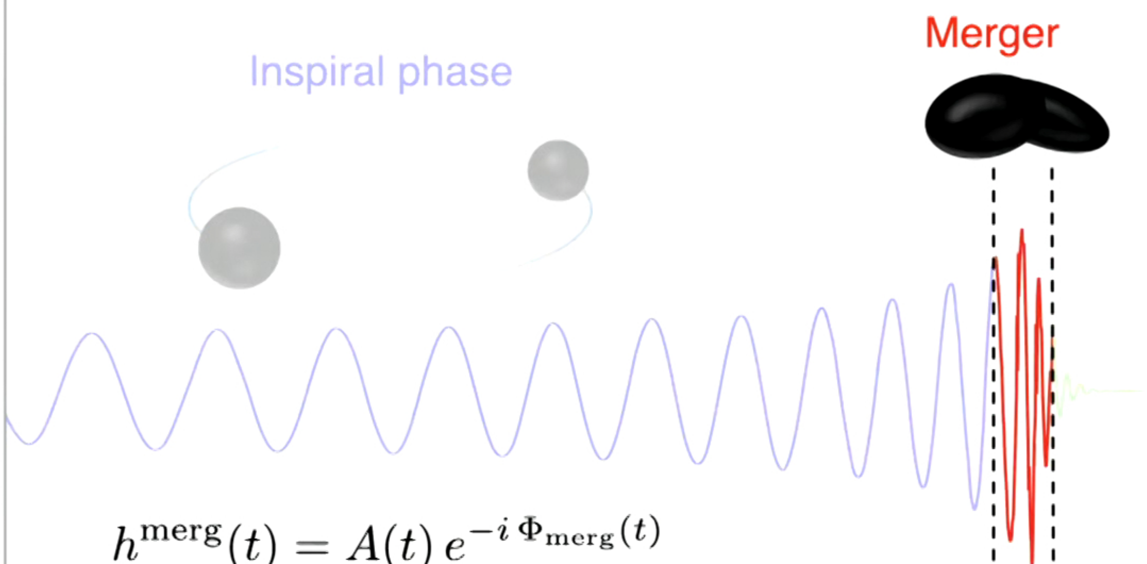
Gravitational Waveform



$$h_{+}^{\rm ins} = -2\frac{M\,\eta}{D_{\rm L}}\left[\left(-\dot{r}^2 + r^2\,\dot{\Phi}^2 + \frac{M}{r}\right)\cos(2\Phi) + 2r\,\dot{r}\,\dot{\Phi}\,\sin(2\Phi)\right]$$

$$h_{\times}^{\rm ins} = -2\frac{M\,\eta}{D_{\rm L}}\left[\left(-\dot{r}^2 + r^2\,\dot{\Phi}^2 + \frac{M}{r}\right)\sin(2\Phi) - 2r\,\dot{r}\,\dot{\Phi}\,\cos(2\Phi)\right],$$

Gravitational Waveform

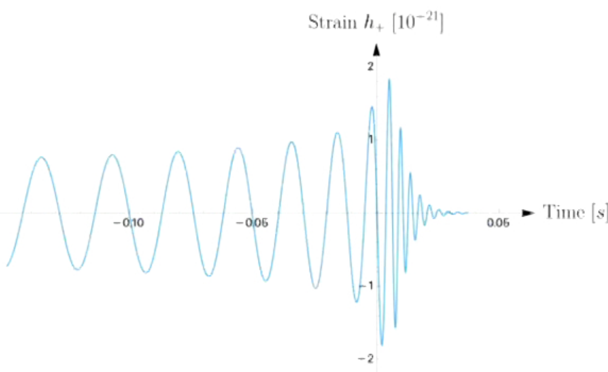


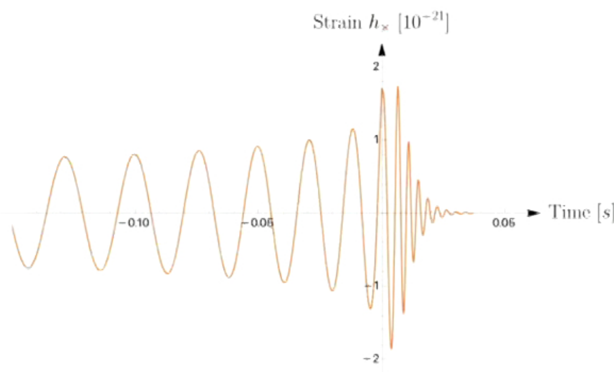
$$h^{\rm merg}(t) = A(t)\, e^{-i\,\Phi_{\rm merg}(t)}$$

A numerical fit to NR simulations.

Gravitational Waveform

The inspiral and merger-bringdown waveforms are matched and merged into a single, analytical waveform model.





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Gravitational Balance Laws

$$c^2\left(\frac{M_{\mathsf{remnant}}}{\gamma(v_{\mathsf{kick}})^3\left(1-\frac{\vec{v}_{\mathsf{kick}}}{c}\cdot\hat{x}\right)^3}-M_{\mathsf{binary}}\right)=-\frac{1}{4}\frac{D_{\mathsf{L}}^2\,c^3}{G}\int_{-\infty}^{\infty}\left(\dot{h}_+^2+\dot{h}_\times^2\right)\mathrm{d}t$$

$$+\frac{1}{2}\frac{D_{\mathsf{L}}\,c^4}{G}\mathsf{Re}\left[\eth^2\left(h_+-i\,h_\times\right)\right]\Big|_{t=-\infty}^{t=+\infty}$$

$$c^2\left(M_{\mathrm{remnant}}-M\right)$$

Gravitational Balance Laws

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$$c^2\left(M_{\text{remnant}}-M\right)$$

No memory!

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Gravitational Balance Laws

$$c^2\left(\frac{M_{\mathsf{remnant}}}{\gamma(v_{\mathsf{kick}})^3\left(1-\frac{\vec{v}_{\mathsf{kick}}}{c}\cdot\hat{x}\right)^3}-M_{\mathsf{binary}}\right) = -\frac{1}{4}\frac{D_{\mathsf{L}}^2\,c^3}{G}\int_{-\infty}^{\infty}\left(\dot{h}_+^2+\dot{h}_\times^2\right)\mathrm{d}t + \frac{1}{2}\frac{D_{\mathsf{L}}\,c^4}{G}\,\mathrm{Re}\left[\eth^2\left(h_+ - i\,h_\times\right)\right]\Big|_{t=-\infty}^{t=+\infty}$$

$$c^2\left(M_{\mathrm{remnant}}-M\right) = -\frac{D_{\mathrm{L}}^2\,c^3}{4G}\int_{-\infty}^{+\infty}\left(\dot{h}_+^2+\dot{h}_\times^2\right)dt$$

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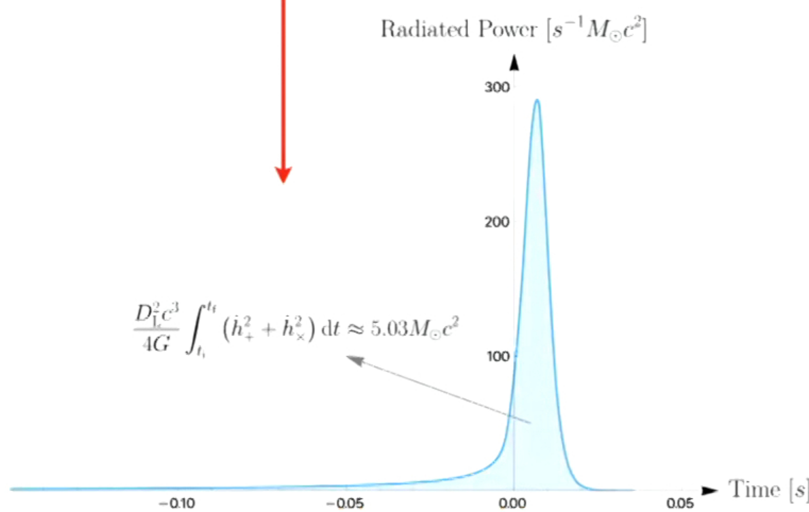
Gravitational Balance Laws

$$c^2\left(M_{\text{remnant}} - M\right) = -\frac{D_L^2\, c^3}{4G}\int_{-\infty}^{+\infty}\left(\dot{h}_+^2 + \dot{h}_\times^2\right) dt$$



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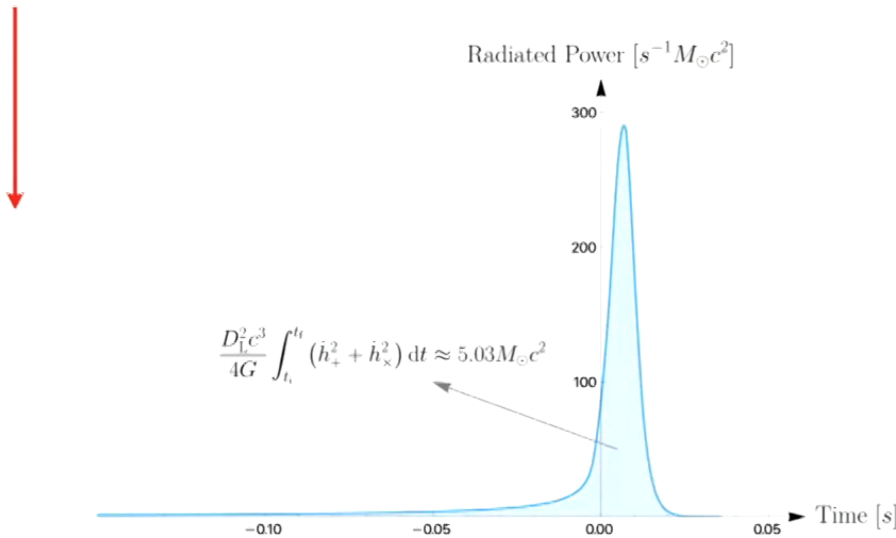

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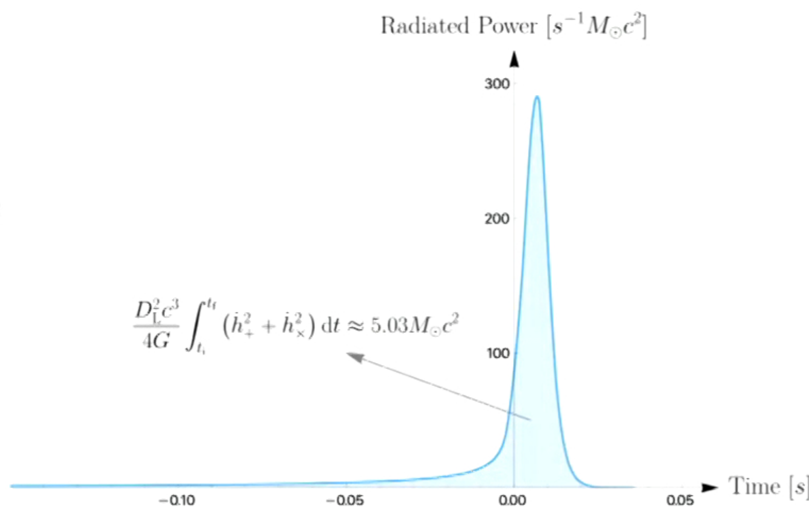
Gravitational Balance Laws

$$c^2\left(M_{\mathrm{remnant}} - M\right) = -\frac{D_{\mathrm{L}}^2\, c^3}{4G}\int_{-\infty}^{+\infty}\left(\dot{h}_+^2 + \dot{h}_\times^2\right) dt$$

$$c^2\left(M - M_{\mathrm{remnant}}\right) = 3M_{\odot}c^2$$



Summary

Using Balance Laws we can

- Compare and improve waveform models
- Determine which model performs better in which region of parameter space