Title: Glimpses into Loop Quantum Gravity

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Collection: Puzzles in the Quantum Gravity Landscape: viewpoints from different approaches

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Abstract: The organizers requested an overview of Loop Quantum Gravity (LQG). However, this is an impossible task, especially for a 30 minute talk, given that the subject is over three decades old with tens of thousands of papers. Instead, I will outline some of the key distinguishing features of LQG and discuss an illustrative application. Hopefully these topics will complement those that will be covered by Bianca Dittrich and Lee Smolin, without too much of an overlap.

Glimpses into Loop Quantum Gravity

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For further details on topics covered in the talk, see, e.g.,

https://www.youtube.com/watch?v=bm9KonBOkZw

(A longer talk at the QG@RRI conference last month), and

AA & E. Bianchi: A Short Review of Loop Quantum Gravity,
(Key issue Review, Rep. Prog. Phys. **84**, 042001 (2021).)

Puzzles in the QG Landscape, PI, October 23-27, 2023

1. LQG: Key Distinguishing Features

1.A. Emergent space-time geometry

At a fundamental level: A background independent theory of connections.
Phase space: Complex, $SU(2)$-valued pairs (A_a^i, E_i^a) on a 3-manifold M. No background metric $\Rightarrow$ dynamics determined by constraints: Simplest functions:

$$\mathcal{G}_i := \mathcal{D}_a E_i^a = 0, \qquad \mathcal{V}_a := E_i^b F_{ab}{}^i = 0, \qquad \mathcal{S} := \tfrac{1}{2}\mathring{\epsilon}^{ij}{}_k\, E_i^a E_j^b\, F_{ab}{}^k = 0.$$

They automatically constitute a first class system system!

- No background metric $\Rightarrow$ Hamiltonian is a linear combination of constraints: $H_{\Lambda,\vec{N},N}(A,E) = \int_M (\Lambda^i \mathcal{G}_i + N^a \mathcal{V}_a + N\mathcal{S})\mathrm{d}^3x$. As in any gauge theory, the first term generates an internal SU(2) gauge rotations. The second term generates gauge covariant (GC) Lie derivatives: Lifts of Diffeos on M to the $SU(2)$ bundle.

- Surprising recent result: the third term generates a generalized gauge covariant (GGC) Lie-derivative! $\dot{A}_a^i = \mathring{\epsilon}^{ij}{}_k\, \mathbb{L}_{\vec{N}_j} A_a^k$ and $\dot{E}_i^a \approx \tfrac{1}{2}\mathring{\epsilon}_{ij}{}^k\, \mathbb{L}_{\vec{N}_j} E_k^a$, where $N_j^a := N E_j^a$. (Furthermore, in the final picture, this turns out to be precisely the 'time evolution' in GR, transmuted to 'space-evolution' along N_j^a!) (AA & Varadarjan)

- The GGC Lie derivative, $\mathbb{L}_{\vec{U}_i} V_j := U_i^b \mathcal{D}_b V_j - V_j^b \mathcal{D}_b V_i^a) \equiv [U_i, V_j]^a$, provides a graded Lie-algebra. Claim: The corresponding infinite dimensional "graded Lie-group" embodies the entire content of GR dynamics! Understanding its structure is a fascinating open problem in mathematics.

Emergence of GR

• So far, just a curious background independent theory of connections. What does it have do to with gravity? GR, (-,+,+,+ as well as +,+,+,+), emerges as two 'real' sections of the gauge theory phase space. There is a precise dictionary from (A_a^i, E_i^a) to metric (ADM) variables (q_{ab}, p^{ab}) that yields:

Constraints: $(\epsilon = 1 \leftrightarrow +,+,+,+$ and $\epsilon = -1 \leftrightarrow -,+,+,+)$

$$C^a := -2D_b\, p^{ab} = 0, \quad \text{and} \quad C := -\tfrac{1}{2}\left(q^{\frac{1}{2}}\, \mathcal{R} + \epsilon\, q^{-\frac{1}{2}}\left(q_{ac}q_{bd} - \tfrac{1}{2}q_{ab}q_{cd}\right) p^{ab}\, p^{cd}\right) = 0.$$

Evolution Eqns:

$$\dot{q}_{ab} = 2N\, \left(q_{ac}q_{bd} - \tfrac{1}{2}q_{ab}q_{cd}\right) p^{cd}$$

$$\begin{aligned}\dot{p}^{ab} \;=\;& \epsilon\, q\left(q^{ac}q^{bd} - q^{ab}q^{cd}\right) D_c D_d N - \epsilon\, N\, q\left(q^{ac}q^{bd} - \tfrac{1}{2}q^{ab}q^{cd}\right)\mathcal{R}_{cd}\\ -\;& N\left(2\delta^a_d\delta^b_n q_{cm} - \delta^a_m\delta^b_n q_{cd} - \tfrac{1}{2}q^{ab}(q_{cm}q_{dn} - \tfrac{1}{2}q_{cd}q_{mn})\right)p^{cd}p^{mn}.\end{aligned}$$

• The RHS has complicated, non-polynomial dependence on (q_{ab}, p^{ab})! This is simply because these are 'composite fields' whose expressions in terms of the 'fundamental' ones $(A_a^i,\, E_i^a)$ are complicated. Analogy: Nuclear Physics $\leftrightarrow$ QCD.

• This gauge/gravity duality is unrelated to the AdS/CFT correspondence. Here, both sides are defined in the bulk; and the duality (i) does not need negative Λ nor SYSY nor extra dimensions, and (ii) the dictionary is complete, exact and explicit. The idea to arrive at QG starting from the gauge theory.

Power of Geometrization: Constraint Algebra

In metric variables the complete derivation of the Poisson bracket calculation takes over 10 pages (Thiemann's book, 2007)! Complete derivation in terms of connections:

$$
\begin{aligned}
\{C_M,\, C_N\} \;&\equiv\; \tfrac{1}{4}\int_\Sigma \mathrm{d}^3x\, M\, \mathring{\epsilon}^{ij}{}_k\, \big(\dot{E}^a_i\, E^b_j\, F_{ab}{}^k + E^a_i\, \dot{E}^b_j\, F_{ab}{}^k + E^a_i\, E^b_j\, \dot{F}_{ab}{}^k\big) \;-\; M \leftrightarrow N \\
&= \tfrac{1}{4}\int_\Sigma \mathrm{d}^3x\, M\, \mathring{\epsilon}^{ij}{}_k\, \Big[\mathring{\epsilon}_i{}^{mn}\, (\mathbb{L}_{\vec{N}_m} E^a_n)\, E^b_j\, F_{ab}{}^k \;-\; E^a_i\, E^b_j\, \mathring{\epsilon}^{km}{}_n\, \mathbb{L}_{\vec{N}m} F_{ab}{}^n\Big] - M \leftrightarrow N \\
&= \tfrac{1}{4}\int_\Sigma \mathrm{d}^3x\, M\, \Big[(\mathbb{L}_{\vec{N}_j} E^a_k - \mathbb{L}_{\vec{N}_k} E^a_j)\, E^{b\,j}\, F_{ab}{}^k \;+\; E^a_k E^b_j\, (\mathbb{L}_{\vec{N}j} F_{ab}{}^k - \mathbb{L}_{\vec{N}k} F_{ab}{}^j)\Big] - M \leftrightarrow N \\
&= \tfrac{1}{4}\int_\Sigma \mathrm{d}^3x\, \Big[2M\, \mathbb{L}_{\vec{N}_j} (E^a_k\, F_{ab}{}^k)\, E^{b\,j} \;+\; 2M\, E^{a\,(k} F_{ab}{}^{j)}\, (\mathbb{L}_{\vec{N}_k} E^b_j)\Big] - M \leftrightarrow N \\
&= \int_\Sigma \mathrm{d}^3x\, (\mathbb{L}_{\vec{N}j} M^a_j)\; E^b_k\, F^k_{ab} \quad \equiv\; C_{\vec{V}}\,.
\end{aligned}
\tag{1}
$$

Note: The right side equals the familiar ADM diffeomorphism constraint with the structure function $V^a = \epsilon\, q^{ab}(N\partial_b M - M\partial_b N)$ but 'geometrizes' it.

Varadarajan has promoted this geometric action of the Hamiltonian constraint to full LQG through careful and elaborate constructions.

2.B New Syntax

- Recall that Einstein discovered GR in two steps: (1) He first recognized that a curved space-time metric g_{ab} incorporates gravity, whence Riemannian geometry is the natural syntax for gravitational physics; and (2) subsequently, he found the correct dynamical equations that determines g_{ab}. LQG uses the same steps, but now for quantum gravity.

- The Heisenberg algebra (generated by (q_{ab}, p^{ab}) of geometrodynamics is replaced by the algebra of holonomies (or Wilson lines) electric fluxes (AA & Isham). Highly non-trivial result: $\mathfrak{A}$ admits a unique background independent representation (Lewandowski, Okolow, Sahlmann & Thiemann; Fleischhack). (Contrast with Minkowskian QFTs). The Hilbert space is $\mathcal{H} = L^2(\mathcal{A}, \mathrm{d}\mathring{\mu})$. Physically interesting operators are represented by well-defined SA (or unitary) operators on $\mathcal{H}$ (AA & Lewandowski, Baez, ...). Rigorous results; no hidden infinities.

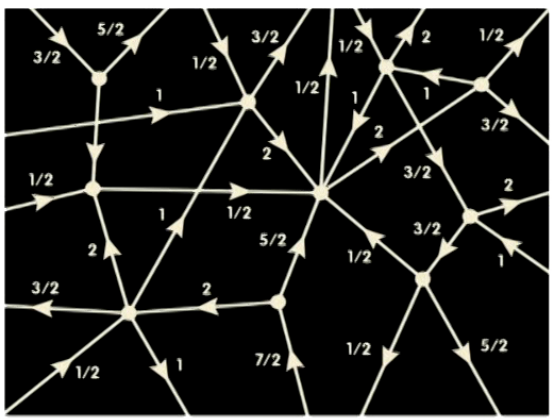
- On this Hilbert space, geometric operators have purely discrete eigenvalues. Thus Riemannian geometry is quantized exactly in the sense that energy and angular momentum are quantized for the hydrogen atom. This Quantum Riemannian Geometry is the LQG syntax to formulate and answer physical questions of QG (also for topics not covered in this talk: spin foams, investigation of black hole entropy & evaporation, ...).

2.C Quantum Riemannian Geometry

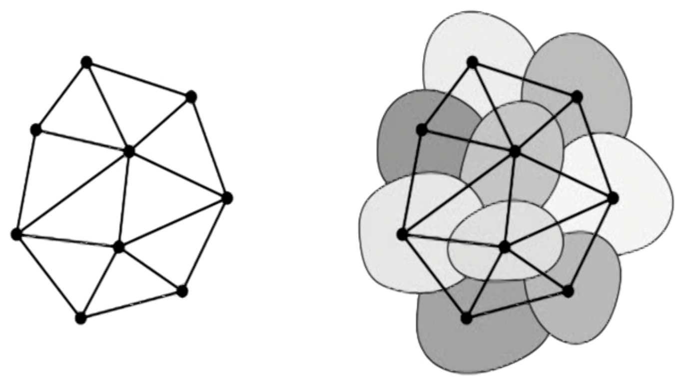
- Geometric observables, $\hat{A}_S$ and $\hat{V}_R$, are especially important to discuss physics (ALMMT, Rovelli & Smolin, Loll, AA & Lewandowski, ...). The smallest non-zero eigenvalue of $\hat{A}_S$ –called the area gap Δ– plays a key role in dynamics because curvature is defined in terms of holonomies around closed curves (the Wilson loops). BH entropy calculations yield $\Delta \approx 5.16\ell_{\rm Pl}^2$. $\hat{V}_R$ plays a key role in the definition of the Hamiltonian constraint operator (Thiemann).

- The basis that digonalizes these SA operators on $\mathcal{H}$ is given by spin network states $\mathcal{N}_{\gamma,\vec{j},\vec{I}}(A)$ associated with decorated graphs (Rovelli & Smolin).

They provide truly powerful tools in calculations and facilitate visualization of Quantum Riemannian geometry.



Nodes + Lines + Arrows + Labels = Spin network



Vertex ↔ chunk of space; Link ↔ 2-surface

2.D Quantum Dynamics: Hamiltonian Approach

• In the Hamiltonian approach a la Dirac, quantum dynamics is incorporated by solving the quantum constraint equations: $\hat{C}_i \,|\Psi\rangle = 0$. First, we have to give precise meaning to the operators $\hat{C}_i$. The LQG kineamtics/syntax provides necessary tools (Thiemann, ...). Issue is still open in the WDW theory.

• In classical GR, the structure of the constraint algebra ensures the 4-d Diff covariance. In geometrodynamics, the action of the vector constraint has a has direct geometric meaning as the generator of 3-d diffeomorphisms which translates to their Poisson algebra. But for the Hamiltonian constraints, there is no such geometric interpretation.

• The key question for quantum dynamics has been: Is the Poisson algebra of constraints faithfully lifted to the quantum theory? Until recently, in LQG there were consistent liftings (Thiemann, ...) but not faithful: $[\hat{C}_M, \hat{C}_N]$ as well as $\hat{C}_{\vec{V}}$ vanished on an appropriate Habitat (Gambini, Lewandowski, Marolf, Pullin).

• Major recent advance: Faithful (and hence also anomaly free) lifting of the constraint algebra to the quantum level in full Euclidean LQG (Varadarajan). Crucially uses representation of time evolution as GCC-Lie-derivatives. For the Lorentzian LQG, there is generalized Wick transform (Thiemann, AA, Varadarajan) that provides a natural avenue.

Organization

1. Key Distinguishing Features of LQG ✓
2. Illustrative example of applications:

   A bridge between theory and observations of the early universe
3. Summary and Outlook.

I will summarize the work of many, many researchers, especially:
Agullo, Baez, Barbero, Bojowald, Dittrich, Engle, Freidel, Gambini, Giesel, Han, Lewandowski, Livine, Mena, Pawlowski, Pullin, Rovelli, Sahlmann, Singh, Smolin, Speziale, Thiemann, Varadarajan, Wilson-Ewing & their groups.

This is *not* meant to be a broad overview. Over 35 years, there have been tens of thousands of papers by hundreds researchers in LQG! I only aim to provide glimpses to convey a feel for some of the foundational ideas and the current status. There are many topics I cannot cover; especially black holes and Spinfoams. My apologies in advance.

2. Illustrative Application: LQC

• Quantum Cosmology: Study of the cosmological sector of QG using symmetry reduction. In effect one focuses on few observables of interest to dynamics of the universe, e.g., $(\rho, a_i, R, ..., \phi, ...)$, and traces out other degrees of freedom.

• In LQC: No unphysical matter or new boundary conditions. Rather, quantum geometry effects change Einstein's Eqs. Distinguishing feature: Use methods of full LQG to cosmological models. Again a uniqueness theorem as in full LQG (AA & Campiglia; Engle, Hanusch, Thiemann). It provides us with a new syntax, leading to a new Hamiltonian constraint (in place of the WDW equation): Now, the Area Gap plays a key role! And dynamics is relational ($\phi \sim$ time).

• One finds that quantum geometry creates a brand new repulsive force in the Planck regime, overwhelming classical attraction. The Big Bang is replaced by a Big Bounce: $\rho_{\rm sup} = \frac{18\pi G\hbar^2}{\Delta^3}$. Analyzed in detail using the Hamiltonian, path integral, and consistent histories frameworks. (AA, Bojowald, Corichi, Campiglia, Craig, Henderson, Lewandowski, Martin-Benito, Mena, Pawlowski, Singh, Sloan, Wilson-Ewing, ...) All strong curvature singularities are resolved in cosmological models (Singh).

Unforeseen Interplay between UV and IR

• Detailed calculations show that quantum geometry effects, that dominate at the Planck scale leading to singularity resolution, dissipate quickly after the bounce and become negligible already at the onset of inflation where curvature is $10^{-12}\,\ell_{\rm Pl}^{-2}$. Nonetheless they can leave observable signatures in the CMB.

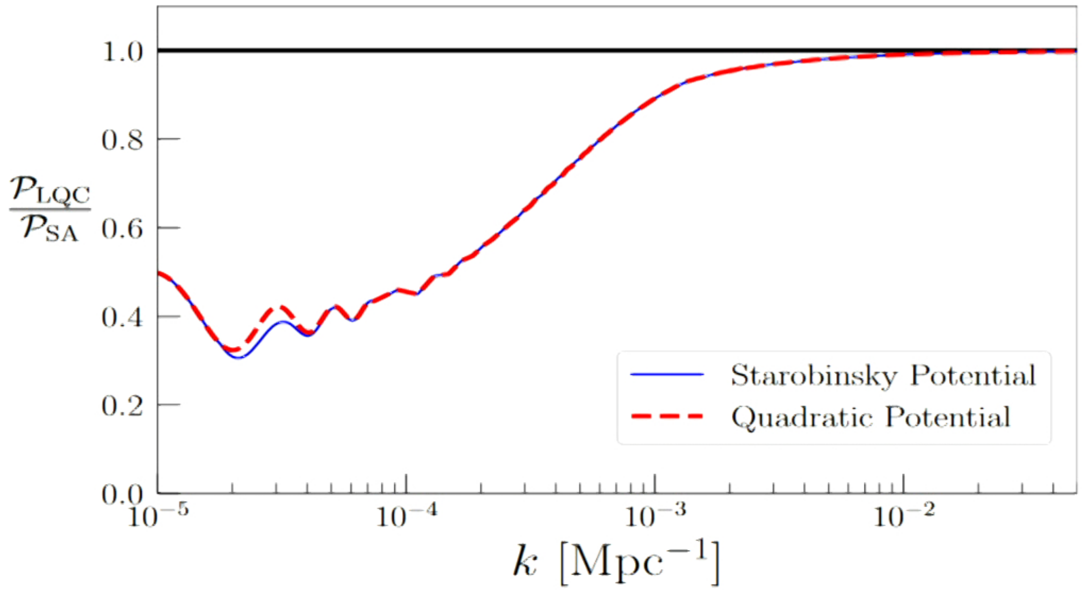
• In LQC the curvature has a universal upper bound, achieved at the bounce. Thus, curvature radius is non-zero and provides a length new scale $\ell_{\rm LQC}$. During their pre-inflationary evolution, perturbation modes with $\lambda_{\rm phy} \ll \ell_{\rm LQC}$ at the bounce are unaffected by curvature. So they arrive in the BD vacuum at the 'onset' of inflation. But those with $\lambda_{\rm phy} \gtrsim \ell_{\rm LQC}$ are not. Therefore, the LQC effects do leave signatures are in on CMB, but only in the infrared. This is the unforeseen interplay between UV and IR.

• Detailed calculations: kinematics and dynamics use the same principles as full LQG. But to make observational predictions additional inputs are needed to select states of the background FRLW quantum geometry and, as in the standard inflation, of perturbations. In the approach I will discuss, these are provided by certain principles that are motivated by LQG quantum geometry at the bounce, and a quantum extension of Penrose's Weyl curvature hypothesis.

Change in the primordial Spectrum

(AA, Gupt, Jeong, Sreenath)

Scalar modes: Standard Inflation predicts a nearly scale invariant primordial power spectrum: Standard Ansatz (SA): $\mathcal{P}_{\mathcal{R}}(k) = A_s(\frac{k}{k_\star})^{n_s-1}$. In LQC, while the primordial spectrum *is* nearly scale invariant on small angular scales (large k), there is power suppression on large angular scales: $\mathcal{P}_{\mathcal{R}}(k) = f(k)\ A_s(\frac{k}{k_\star})^{n_s-1}$. The suppression factor $f(k) = 1$ for large k and $f(k) < 1$ for small k.



Predictions for CMB

• A window of opportunity: Overall, standard inflation is in excellent agreement with observations. But there are anomalies. Statistical significance of any one anomaly is small. But two or more of them imply that we live in an exceptional universe. Can one alleviate this tension?

Planck 2018 Results. I. Overview and the cosmological legacy of Planck ...if any anomalies have primordial origin, then their large scale nature would suggest an explanation rooted in fundamental physics. Thus it is worth exploring any models that might explain an anomaly (even better, multiple anomalies) naturally, or with very few parameters.

• LQC Predictions: Two anomalies are naturally alleviated, preserving all the successes of standard inflation: (i) Power Suppression for $\ell < 30$ (This scale naturally descends from LQC dynamics and the choice of states); and, (ii) the lensing amplitude anomaly (which, e.g., was called "a crisis" by Di Valentino, Melchiorri & Silk).

• Direct Observational Check on the area gap Δ: Make the area gap a variable in the LQC analysis of CMB, and find the posterior probability of its value using the Planck data: The value $\Delta = 5.17\ell_{\rm Pl}^2$ from the BH entropy calculations lies within 1 σ. An increase of area gap by a factor of 10, for example, is observationally ruled out at 95% confidence level (AA, Gupt, Sreenath). Unforeseen synergy and a bridge from observations to LQG quantum geometry.

ΛCDM+SA versus ΛCDM+LQC

Parameter	SA	LQC
$\Omega_b h^2$	0.02238 ± 0.00014	0.02239 ± 0.00015
$\Omega_c h^2$	0.1200 ± 0.0012	0.1200 ± 0.0012
$100\theta_{MC}$	1.04091 ± 0.00031	1.04093 ± 0.00031
τ	0.0542 ± 0.0074	0.0595 ± 0.0079
$\ln(10^{10} A_s)$	3.044 ± 0.014	3.054 ± 0.015
n_s	0.9651 ± 0.0041	0.9643 ± 0.0042

Comparison between the Standard Ansatz (SA) and LQC.
The mean values and marginalized probability distributions for the six cosmological parameters calculated using C_ℓ^{TT}. Currently, the relative error in the measurement of optical depth τ is 13.65% while that in other 5 parameters is less than 0.4%. In LQC, the value of τ increased by 9.8% relative to the SA! Independent missions will measure τ more accurately.

3. Summary and Outlook

- LQG provides a specific syntax for quantum gravity that descends from a background independent gauge theory. This paves way to a non-perturbative, background independent approach. In this talk I sketched the Hamiltonian approach and its application to the early universe. It illustrates that the approach has sufficiently matured to make contact with observations.

- There have been significant advances also on 3 other fronts: (i) Spinfoams The path integral approach to dynamics. (*Many* analytic results, & numerical computations are getting mature); (ii) Quantum aspects of black holes (Horizon entropy, singularity resolution, black hole evaporation beyond Hawking effect); (iii) Interface with Quantum Information (Intertwiner as well as boundary state entanglement in spin-nets, quantum nature of the Coulombic interaction, testing the "dressed metric" framework using photononics).

- LQG is very much an ongoing program. Many interesting problems remain in all sub-areas. Different groups are working on various open issues: Conceptual, Mathematical, Numerical aspects of LQG, and applications to astrophysical and cosmological observations/phenomenology.