Title: UV complete 4-derivative scalar field theory

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Collection: Puzzles in the Quantum Gravity Landscape: viewpoints from different approaches

Date: October 24, 2023 - 10:10 AM

URL: https://pirsa.org/23100006

Abstract: A scalar field theory with 4-derivative kinetic terms and 4-derivative cubic and quartic couplings is presented as a proxy for quadratic gravity. Unitarity and positivity appear as the key issues in the scalar field theory, just as they do in quadratic gravity. We have extended some calculations to show how these issues are resolved in the high energy limit of the theory. The results also show how it is that differential cross sections can have good high energy behaviour.

UV complete 4-derivative scalar field theory

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Puzzles in the Quantum Gravity Landscape
Perimeter Institute
October 2023

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the QFT

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi(\Box + m^2)\partial^\mu\phi + \lambda_3(\partial_\mu\phi\partial^\mu\phi)\Box\phi + \lambda_4(\partial_\mu\phi\partial^\mu\phi)^2$$

- 4-derivatives, both in the interaction terms and the kinetic terms
- dimensionless real scalar field $\phi(x)$ and dimensionless couplings λ_3 and λ_4
- shift symmetry $\phi \to \phi + c$
- m^2 breaks the classical scale invariance

proxy for quadratic gravity

- Einstein action is supplemented with terms quadratic in curvature, and these terms bring in 4-derivatives
- 4-derivatives in kinetic and interaction terms
- both theories are renormalizable
- the shift symmetry is playing the role of coordinate invariance of the gravity theory
- the $m^2 \partial_\mu \phi \partial^\mu \phi$ is playing the role of the Einstein term
- at low energies this term dominates; left with a normal massless field with non-renormalizable interactions

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UV completeness

- both theories are UV complete, and so we can see what happens at energies much higher than m
- also refer to this as the $m \to 0$ limit
- ultra-Planckian energies in the case of gravity
- the story turns out to be very similar for the two theories, since it is really just about the physics of four derivatives
- scalar theory is easier to deal with, so will focus on that
- return to quadratic gravity at the end

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ghost

- propagator has massive pole with abnormal sign residue
- the negative norm state is said to be in conflict with unitarity
- what is actually meant by this is that the theory may have problems with positivity
- S-matrix unitarity can still be defined in the presence of negative norm states
- $S\mathbb{1}S^\dagger = \mathbb{1}$ where $\mathbb{1} = \sum_X \frac{|X\rangle\langle X|}{\langle X|X\rangle}$ reflects the negative norms
- S-matrix unitarity means that probability is conserved

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optical theorem

- also the optical theorem can be directly verified in perturbation theory by keeping track of minus signs
- the LHS is imag part of forward scattering amplitude, and its calculation is affected by any wrong-sign propagators
- the RHS is a scattering process into on-shell final states, and this is affected by any negative norms among these states
- it can thus be seen that the LHS and RHS of the optical theorem are both affected in such a way that it remains satisfied

positivity

- if the issue is positivity rather than unitarity then this at least leaves some room open for discussion
- there are certainly some abnormal minus signs floating around in calculations, but the question is whether physical quantities that should be positive, can end up being positive
- this needs some investigation
- our focus will be on the positivity constraint in the high energy limit, but we will return to lower energies

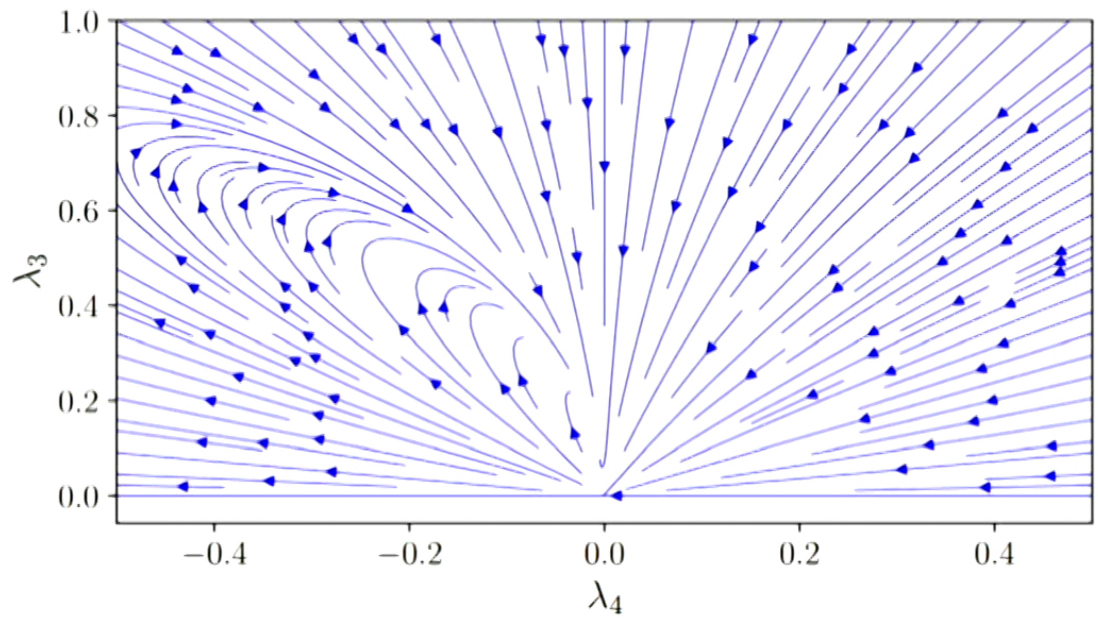
β-functions

- renormalization of $\partial_\mu \phi \Box \partial^\mu \phi$ term is treated as a standard wave function renormalization

$$\frac{d\lambda_3}{d \ln \mu} = -\frac{5}{4\pi^2}(\lambda_4 \lambda_3 + \frac{3}{4}\lambda_3^3)$$
$$\frac{d\lambda_4}{d \ln \mu} = -\frac{5}{4\pi^2}(\lambda_4^2 + \lambda_4 \lambda_3^2)$$

BH, Phys Lett B 138023 (2023)

renormalization group flow



- arrows point to the UV
- asymptotic freedom in UV
- some flows also show asymptotic freedom in IR

a new mass scale

- flow towards IR stops when the energy scale drops below m; this is the transition to the low energy theory
- the crossover to this low energy theory may occur at weak couplings, in which case the theory remains perturbative at all scales
- but for sufficiently small m, the flow towards the IR can result in large couplings
- creates a new mass scale through dimensional transmutation
- it is this that can be the origin of Planck mass in gravity

outline

- describe a simplified method for calculating in the high energy limit
- old method involves decomposing ϕ into two degrees of freedom
- in new method there appears to be only one degree of freedom at high energies
- calculate the optical theorem and a differential cross section as functions of λ_3 and λ_4
- this gives expressions that we can test for positivity
- positivity picks out the allowed region on the RG flow plane

a related issue

- four derivative interaction terms apparently produce diverging amplitudes at large momenta
- nonstandard cancellations take place at the cross section level (also in quadratic gravity)
- the reduction to effectively one degree of freedom at high energies clarifies what is happening
- makes more clear the origin of good high energy behaviour

mass derivative

- four derivative propagator $G^{(4)}(p^2, m^2)$ can be written in terms of the Feynman propagator

$$G^{(2)}(p^2, m^2) = \frac{1}{p^2 - m^2 + i\varepsilon}$$

as

$$G^{(4)}(p^2, m^2) = -\frac{G^{(2)}(p^2, m^2) - G^{(2)}(p^2, 0)}{m^2}$$

- thus in the $m \to 0$ limit (high energy limit)

$$\lim_{m \to 0} G^{(4)}(p^2, m^2) = \lim_{m \to 0} \left(-\frac{d}{dm^2}\right) G^{(2)}(p^2, m^2)$$

mass derivative in optical theorem

- the imaginary part of a forward scattering amplitude $\mathcal{A}_{i\to i}$ is extracted by cutting propagators and using

$$\mathrm{Im}(G^{(2)}(p^2,m^2)) = -i\pi\delta(p^2-m^2).$$

- the analog for $G^{(4)}$ in the $m\to 0$ limit is

$$\lim_{m\to 0}\mathrm{Im}(G^{(4)}(p^2,m^2)) = -i\pi \lim_{m\to 0}(-\frac{d}{dm^2})\delta(p^2-m^2)$$

- the additional operation, $-\lim_{m\to 0}\frac{d}{dm^2}$, also works on the RHS of the optical theorem

RHS of optical theorem

- each on-shell particle in a final state f should be assigned its own dummy mass m_j
- $|\mathcal{A}_{i\to f}|^2$ will depend on the values of these m_j's via the on-shell conditions
- take the mass derivatives, then the $m_j \to 0$ limits, then the phase space integral
- term on the RHS of the optical theorem corresponding to the final state f takes the form

$$\lim_{m_j \to 0} \left(\prod_{j=1}^{n} \left(-\frac{d}{dm_j^2}\right) |\mathcal{A}_{i\to f}(m_1..m_n)|^2 \right)$$

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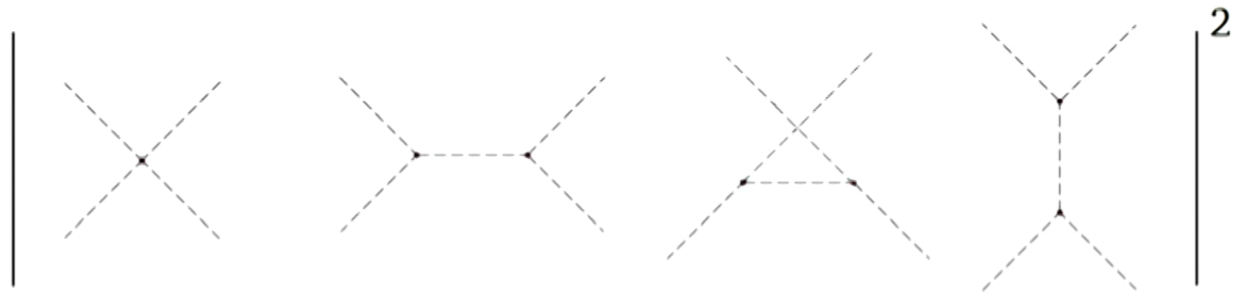
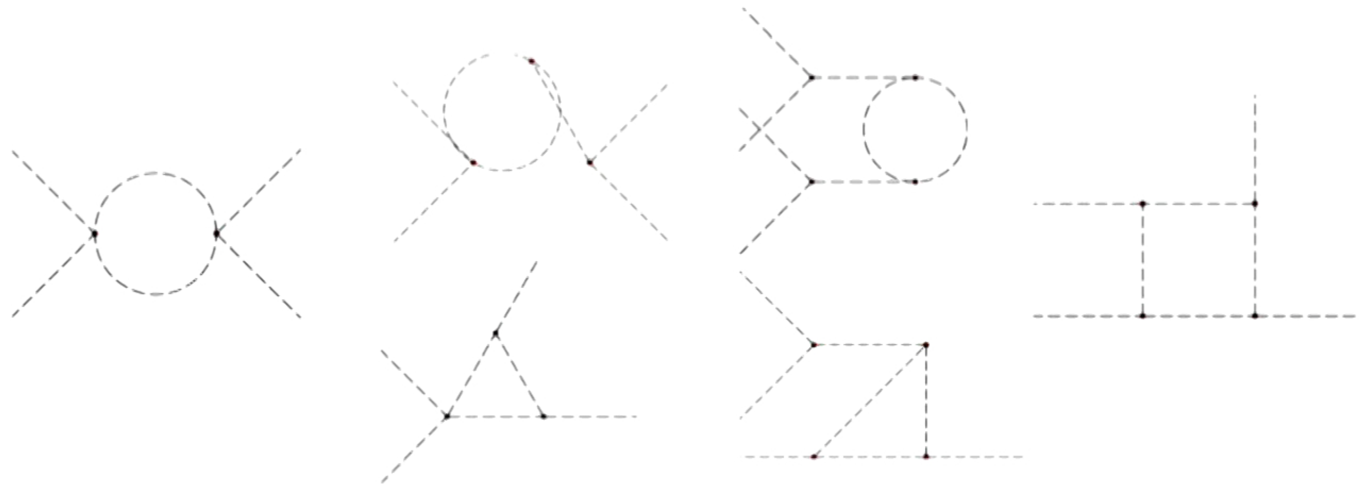
$|\mathcal{A}_{i\to f}(m_1..m_n)|^2$ for $\phi\phi \to \phi\phi$



- new method reproduces the usual sum over the $\phi\phi$ final states
  - in that sum, for each ϕ, choose one mass, then the other, insert a minus sign for the negative norm, and divide by m^2, due to the way field is normalized

now the LHS of optical theorem

- imaginary part of the forward scattering amplitude
- various diagrams are of order λ_4^2, $\lambda_4\lambda_3^2$ or λ_3^4



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calculate diagrams for

- can again use mass derivatives and $m_j \to 0$
- limit is smooth since the imaginary part has no infrared divergences
- can thus use the massless $G^{(4)}$ propagator directly

$$G^{(4)}(p^2, 0) = -\frac{1}{(p^2 + i\varepsilon)^2}$$

- square of a Feynman propagator can be handled by standard methods for calculating one-loop diagrams
- 4-derivative vertices still complicate the calculation

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optical theorem for $\phi\phi \to \phi\phi$

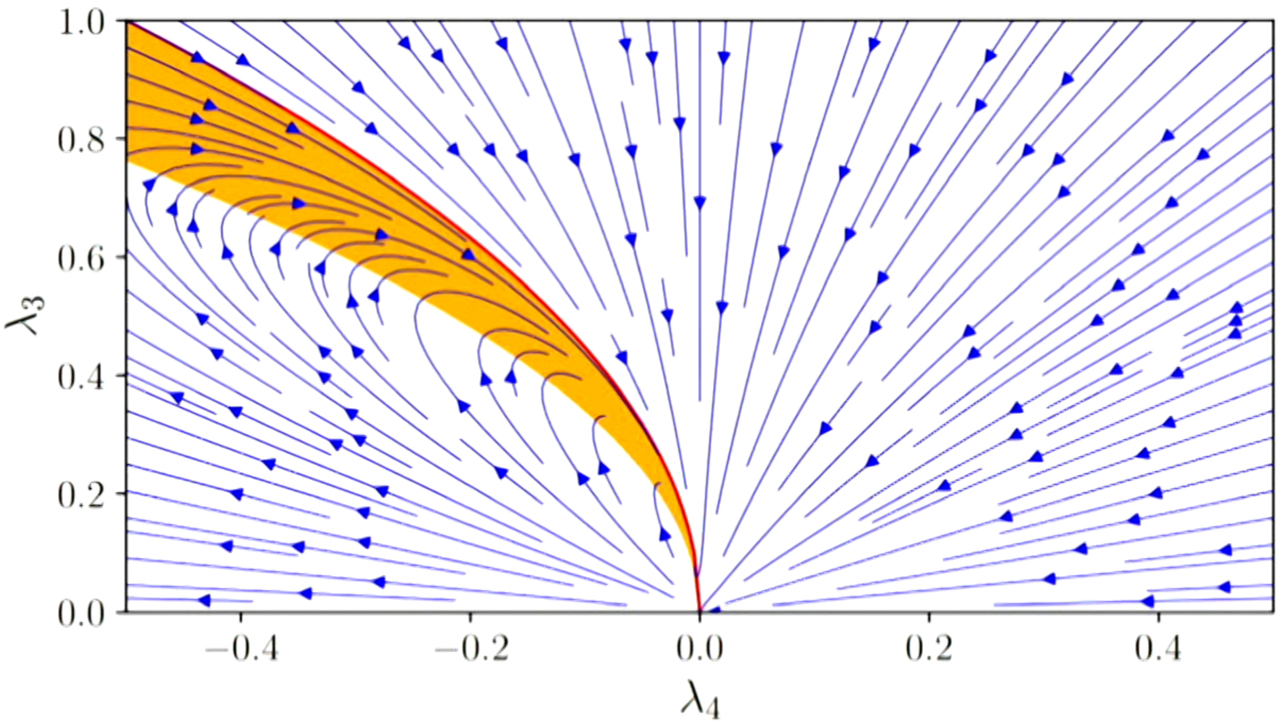
- calculate the two sides of the optical theorem independently

$$\text{LHS} = \text{RHS} = \frac{s^2}{6\pi}(6\lambda_3^4 + 19\lambda_3^2\lambda_4 + 14\lambda_4^2)$$

- the equality demonstrates S-matrix unitarity
- both sides calculated without decomposing ϕ field into positive and negative norm parts
- RHS naively goes like s^4, being the square of amplitudes that go like s^2
- is reduced to s^2 behaviour because of $-\frac{d}{dm^2}$ applied twice

the positivity constraint

- LHS = RHS is negative for $-\frac{6}{7} < \lambda_4/\lambda_3^2 < -\frac{1}{2}$
- this region is shaded orange, and the red line is $\lambda_4 = -\frac{1}{2}\lambda_3^2$



meaning of red line

- the red line marks the boundary between two sets of flows that are qualitatively different
- the flows below this line will eventually enter the orange region in the UV
- thus all such flows are forbidden
- the allowed flows are on or to the right of the red line
- these couplings are asymptotically free in the UV and can become strong in the IR

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two degrees of freedom.

- consider the two fields constructed from ϕ,

$$\psi_1 = \frac{1}{m^2}(\Box + m^2)\phi$$
$$\psi_2 = \frac{1}{m^2}\Box\phi$$

- when expressed in terms of ψ_1 and ψ_2 the kinetic term of the Lagrangian becomes

$$-\frac{m^2}{2}\psi_1\Box\psi_1 + \frac{m^2}{2}\psi_2(\Box + m^2)\psi_2$$

- ψ_1 and ψ_2 are the two fields of definite mass (0 and m) and definite norm (+ and −)
- but we also see that $\phi = \psi_1 - \psi_2$

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one degree of freedom

- $\phi = \psi_1 - \psi_2$ is the only combination that appears in all the interaction terms

- we have introduced the operation $-\lim_{m\to 0} \frac{d}{dm^2}$ for every external ϕ line, and this also treats ψ_1 and ψ_2 on equal footing with a relative minus sign

- the external state matches the interacting state

- the apparent two degrees of freedom have been reduced to one

differential cross section for $\phi\phi \to \phi\phi$

- treat it as a single process and take four mass derivatives
- need the dependence on the set of four masses m_j
- before taking mass derivatives, would diverge as $\sim (s^2)^2/s$ for large s
- a term $\sim m_1^2 m_2^2 m_3^2 m_4^2/s$ is needed to survive four m_j^2-derivatives and $m_j \to 0$
- in the end we have a differential cross section that behaves like $1/s$ at large s times a function of the scattering angle
- reason for good high energy behaviour is now clear

result

- the differential cross section for $\phi\phi \to \phi\phi$ scattering at high energies

$$\frac{d\sigma}{d\Omega} = \Big(\left(\lambda_3^4 - 4\lambda_4^2\right)\sin(\theta)^6 + 24\lambda_4^2\sin(\theta)^4 + \left(-48\lambda_3^4 - 96\lambda_3^2\lambda_4\right)\sin(\theta)^2 + 64\lambda_3^4 + 128\lambda_3^2\lambda_4\Big) / \left(16\pi^2\sin(\theta)^4 s\right)$$

- this result is positive definite for any θ as long as $\lambda_4 \geq -\frac{1}{2}\lambda_3^2$
- this is the same constraint as before!
- interesting special case on the red line:

$$\frac{d\sigma}{d\Omega} = \frac{3\lambda_4^2}{2\pi^2 s} \quad \text{when } \lambda_4 = -\frac{1}{2}\lambda_3^2$$

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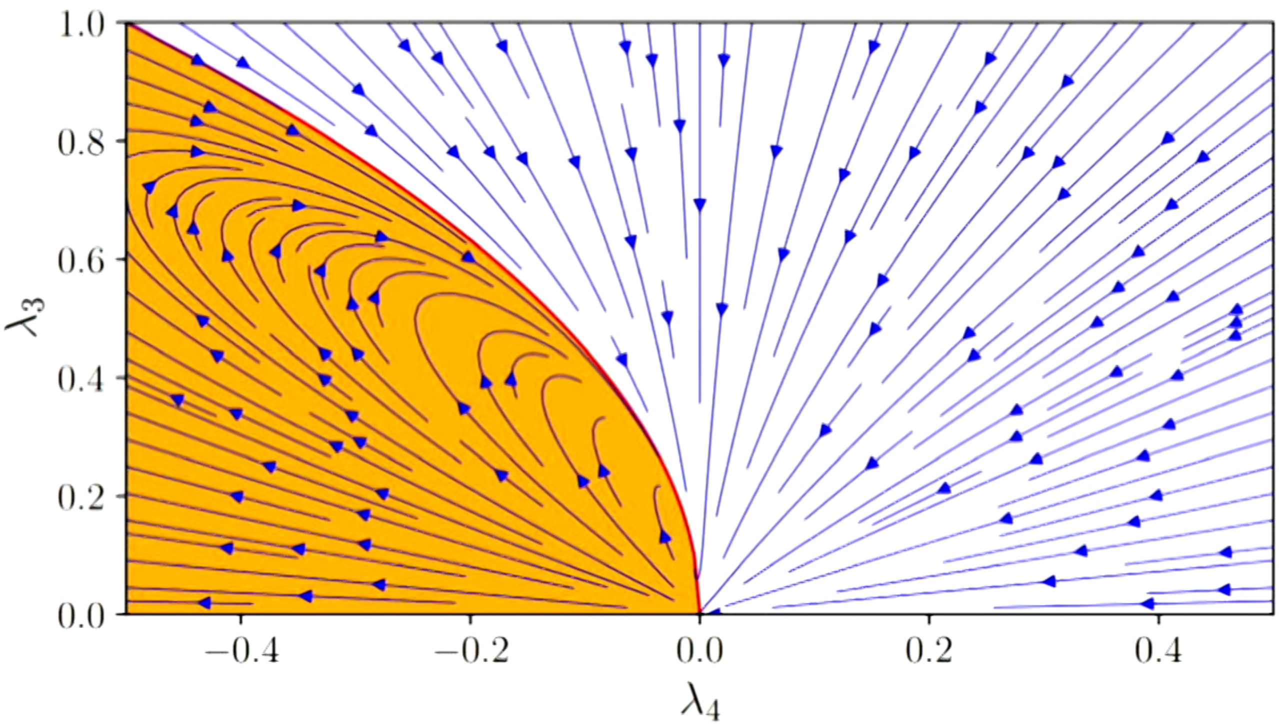
the $\lambda_4 \geq -\frac{1}{2}\lambda_3^2$ constraint



- positivity has picked out the running couplings that flow to strong coupling in the infrared

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QCD analogy

- we have been able to work in the asymptotically free regime where the perturbative degrees of freedom are appropriate
- this is similar to using perturbative QCD to study scattering of quarks and gluons at very high energies, even though quarks and gluons are not the asymptotic states
- they are not because of strong interactions at intermediate energies
- so what are the asymptotic states in the 4-derivative theory?

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asymptotic states

- with strong interactions, the poles of the bare propagators need not correspond to true asymptotic states of the theory
- another example, the sigma meson — has a width of order its mass ~ 0.5 GeV — and is in no way an asymptotic state
- for both the scalar theory and the gravity theory, we need to suppose that the ghost is not an asymptotic state
- as long as asymptotic states have positive norm, all probabilities are positive
- virtual effects of the ghost are okay

shift symmetry

- the massless field $\psi_1 = \frac{1}{m^2}(\Box + m^2)\phi$ transforms under the shift symmetry $\psi_1 \to \psi_1 + c$ in the same way as ϕ does
- we can suppose that this shift symmetry is not broken by the strong interactions
- then ψ_1 survives as a true asymptotic state
- we end up with one degree of freedom, at whatever energy scale is used to probe the theory
- this is either ψ_1, or $\psi_1 - \psi_2$ in the perturbative high energy description

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quadratic gravity

$$S = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}\left(R + \frac{R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2}{m_G^2} - \frac{R^2}{6m_S^2}\right)$$

- introduces massive spin-2 ghost and massive scalar
- Gm_G^2 is a dimensionless and asymptotically free coupling
- spin-2 sector has graviton and ghost — completely analogous to ψ_1 and ψ_2
- our comments about ψ_1 and ψ_2 also apply to the graviton and the ghost — except that the shift symmetry is replaced by coordinate invariance

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ghost summary

- in high energy limit the ghost is entering on an equal footing as a perturbative degree of freedom
- it plays an intrinsic role in achieving positivity and well-behaved cross sections
- this high energy picture is independent of whether or not the ghost is an asymptotic state
- but in the full theory, not being an asymptotic state seems to be required

quadratic gravity gives change of perspective

- on very small — standard QFT picture at ultra-Planckian energies
- on very large — arbitrarily large, horizonless, classical solutions, called 2-2-holes

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what is a 2-2-hole?

- gravitationally bound ball of relativistic gas
- compactness essentially the same as a black hole
- integrate the entropy density of the gas to get the total entropy S_{22}

$$T_\infty S_{22} = T_{\rm BH} S_{\rm BH} = \frac{M}{2}$$

$$\frac{S_{22}}{S_{\rm BH}} = 0.7548 N^{\frac{1}{4}} \left(\frac{m_G}{m_{\rm Pl}}\right)^{\frac{1}{2}} \gtrsim 1$$

- N is number of species, $S_{22} \propto N^{\frac{1}{4}}$ is a feature, not a bug